Useful Forecasting: Belief Elicitation for Decision Making

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Abstract

Having information about an uncertain event is crucial for informed decision making. This paper introduces a simple framework in which 1) a principal uses the reported beliefs of multiple agents to make a decision and 2) the agents reporting a belief are affected by the decision. Naturally, the question arises how the principal can incentivize the agents to report their belief truthfully. I show that in this setting a direct reporting mechanism using a scoring rule to incentivize belief reports leads to truthful reporting by all agents as the unique Nash equilibrium under precisely two conditions, preference diversity and no pivotality. Moreover, if the principal can only consult a single agent the only mechanism that can guarantee truth-telling requires perfect knowledge of the agent's preferences.

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1. Introduction

Having information about an uncertain event is crucial for informed decision making. Sometimes a principal in charge of making a decision is poorly informed themselves but is able to consult a group of experts for advice. In this case the principal may be interested to obtain predictions about the event from the experts. However, if the interests of the uninformed principal and the experts are misaligned, the experts may have an incentive to misreport their information in order to influence the decision of the principal. Such situations can occur, for example, in organizations where a manager is faced with a decision between two projects. Some workers from sales or technical departments may be better informed than the manager about the potential success of different projects. However, these workers may not report their beliefs about the project's success truthfully as they have additional interests such as job security. Further examples that require a decision by a principal based on a forecast from (a group of) expert(s) can also be found in many other settings. Such settings include the forecasting of economic variables and a resulting decision, e.g. predictions of the EU member states and a decision by the ECB or forecasting of factors for national security as further explained by one example below.

As the leading example for this paper, consider the decision problem for Obama during the hunt for Bin Laden in 2011. Obama is faced with a decision to attempt capture or wait. If Bin Laden is at his suspected home it is best to attack, otherwise it would be better to wait. Obama himself does not know if Bin Laden is at home, however, his team of experts each have some belief about the probability that Bin Laden is at the suspected location. As documented by Friedman and Zeckhauser (2014) Obama asked all agents to provide him with a precise probability estimate. It is documented that some advisers actively misreported their belief in order to influence Obama's decision. One of the main reasons for misreporting may be that some experts have different preferences over the actions than Obama. Moreover, Obama did not provide explicit monetary incentives for accurate judgements. Naturally, the question arises if the principal can incentivize the experts to report their beliefs truthfully. Can monetary incentives in the form of scoring rule payments lead to truthful reporting by the expert(s)?

I consider a setting in which the state of the world is revealed after the principal chooses an action and (state-contingent) monetary transfers (e.g. rewards) between the principal and the agent(s) are possible. In this case, scoring rule mechanisms are widely regarded as the main tool to elicit private beliefs. Scoring rules are designed to incentivize a person to state their belief truthfully, even if the belief is impossible to verify. However, scoring rule mechanisms typically assume that the monetary reward is the only factor a person considers when reporting a belief. A frequently omitted aspect of belief elicitation is how the resultant belief is used. It is likely that an agent may not only care about the monetary reward from the scoring rule but also care about how his belief is used in a subsequent decision problem. This paper introduces a simple framework in which agents have private information about the state of the world and preferences over the two actions available to a principal. The principal has no knowledge about the state of the world and also does not know the action preferences of the experts. The principal's objective is to choose the best possible action.

I show that in this setting a scoring rule mechanism and a natural decision rule lead to truthful reporting by all agents as the unique Nash equilibrium under precisely two conditions, diversity and no pivotality. The condition on preference diversity requires that at least one agent must prefer each action. No pivotality requires that the true mean aggregate belief must not be close to the decision threshold of the principal. Or in other words, no agent must unilaterally be able to change the action being chosen by the principal if all other agents report their beliefs truthfully. Note that this condition is more likely to be satisfied the more experts the principal consults. Moreover, the only time it does not hold is when the principal is (nearly) indifferent between the two actions. In a setting where the principal can only consult a single agent the problem is more difficult as the condition is never satisfied. The only scoring rule mechanism that makes truth-telling a (strictly) dominant strategy requires perfect knowledge of the expert's preferences.

Taken together, the results imply clear recommendations for a principal faced with a decision under uncertainty and expert advisers that may be impacted by the principal's decision. The principal should provide monetary incentives for accurate forecasts to the experts. Such incentives (even if arbitrarily small in theory) guarantee truthful forecasts by the experts under two simple conditions which the principal can influence. The principal should ensure that she consults experts with differing preferences. Even in large and homogeneous groups, consulting a single expert with different action preferences can lead to truth-telling by the whole group. Moreover, the expert should consult as many experts as possible to ensure no pivotality of an individual expert.

The rest of the paper is organized as follows. Section 2 briefly discusses related work on belief elicitation and decentralized decision making. Section 3 introduces the model and describes the background on scoring rule mechanisms. Section 4 analyzes a simplified setting with only one expert. Section 5 considers the case with finitely many experts. Finally, section 6 provides a discussion of some alternative mechanisms.

2. Literature

This paper draws on the large literature around scoring rules (e.g. Savage, 1971; Winkler et al., 1996, ...) and connects it to the problem of decentralized decision making (as introduced by Holmström, 1977 and 1984) and strategic information transmission (Crawford and Sobel, 1982).

The vast majority of papers on belief elicitation and scoring rules do not consider subsequent decision making problems. Gneiting and Raftery (2007) provide an overview of existing work and develop a clean unifying framework on scoring rules to elicit beliefs from a single agent. Two main types of mechanisms exist that are used to elicit beliefs from a group of agents: prediction markets and prediction polls. Conitzer (2009) studies different mechanisms based on prediction markets. He provides a clear framework, linking predictions markets and mechanism design, as well as characterizing mechanisms that are incentive compatible. Contrary to prediction markets a prediction poll is a more direct method of belief elicitation. Atanasov et al. (2017) show that prediction polls can be a good alternative to prediction markets in terms of accuracy.

A few papers (mainly in computer science) have considered settings where beliefs are elicited using a scoring rule and the resultant belief(s) are used by a principal to make a decision (Berg and Rietz (2003), Oesterheld and Conitzer (2019), Othman and Sandholm (2010), Chen and Kash (2011), Chen et al. (2011) and Dimitrov and Sami (2010)). The main difference to the model in this paper is that all of the above papers assume agents to be decision-agnostic. As far as I am aware, Boutilier (2012) is the only one who analyzes a setting where an agent has preferences over the decision made by the principal. The focus lies on a setting with one agent. He suggests the use of a 'compensation function' which assumes that the principal has (im)precise knowledge of the action preferences of the agent. Combining the knowledge of the action preferences with a proper scoring rule allows the creation of a mechanism that induces truth-telling. Gimpel and Teschner (2014) and Choo et al. (2019) investigate a similar question experimentally. Both papers consider a framework with multiple agents that receive utility depending on the decision made by a principal. A prediction markets is used to elicit beliefs. Both papers find that agents misreport their belief strategically.

This paper also relates to the general class of delegation problems as introduced by Holmström (1977, 1984). He poses the problem of an uninformed principal that has to make a decision under uncertainty. The principal may consult a group of informed but biased agents. He suggests that the optimal mechanism for the principal is to delegate the decision to the agent, letting him choose from some (constrained) set of alternatives. Alonso and Matouschek (2008) extend the framework and provide a general characterization of the solution to the delegation problem. Contrary to the model in this paper, they assume that monetary transfers between the principal and the agent are not possible. Krishna and Morgan (2004) study a very similar problem and allow for monetary transfers. They also analyze the role of commitment from the principal and provide a characterization of optimal contracts. A crucial assumption in their model is that the bias of the agent is common knowledge. Other differences to this paper are that the contracts cannot depend on the realized state and the utility of the principal and agent are based on quadratic loss.

The model of this paper is also related to the broader mechanism design problem with correlated information (Cremer and McLean, 1985, 1988; McAfee and Reny, 1992). McAfee and Reny (1992) study a setting where a principal makes a decision that matters to everyone and agents have correlated private information. They find that while having private information typically allows for large rents this is not necessarily the case if the information is correlated. In related work, Riordan and Sappington (1988) find that ex-post information can be used to eliminate rents ex-ante for a privately informed party. While none of these papers discuss the use of scoring rules to elicit beliefs, the general ideas are closely related to the findings of this paper.

3. Model

3.1. General setup

Consider a (female) principal who faces a decision problem $A = \{a_1, a_2\}$. There are two states of the world $\Omega = \{\omega_1, \omega_2\}$. Nature draws the state ω_2 with probability $\bar{\mu} \in [0,1]$ and vice versa ω_1 is drawn with probability $1-\bar{\mu}$. The principal has expected utility preferences that depend on the action choice and her belief about the state of the world,

$$EU^P = \begin{cases} 0 & \text{if } a_1 \text{ is chosen} \\ \mu^P - \alpha & \text{if } a_2 \text{ is chosen} \end{cases}$$

where $\mu^P \in [0,1]$ denotes the principal's (posterior) belief about the state being ω_2 and $\alpha \in [0,1]$ indicates a certainty threshold for the choice of a_2 over a_1. The principal is assumed to have no information about the state of the world ex-ante. She adopts the aggregate belief of all agents as her posterior belief.[1] Her preferences imply that for a (posterior) belief $\mu^P < \alpha$ the principal prefers to choose a_1 and for a (posterior) belief $\mu^P > \alpha$ she prefers to choose a_2. In case of indifference, $\mu^P = \alpha$, I assume the principal chooses a_2.

There are n different Bayes-rational and risk-neutral (male) agents. Each agent i receives an independent signal $\mu_i \in \{\mu^1, ...\mu^K\}$, with $0 < \mu^1 < ... < \mu^K$ and $\mu^k - \mu^{k-1} = \epsilon$, which is correlated with $\bar{\mu}$.[2] This signal denotes the agents belief.[3] Each individual belief may be highly inaccurate but in aggregate, thanks to the law of large numbers, errors tend to cancel, such that with $n \to \infty$, $\frac{1}{n}\sum_{i \in N} \mu_i = \bar{\mu}$, where N denotes the set of all agents.[4] For a finite number of agents, $\tilde{\mu}$ denotes the average true belief of all agents. Similar to the principal, each agent i has preferences over the potential actions being selected by the principal,

$$U_i = \begin{cases} 0 & \text{if } a_1 \text{ is chosen} \\ u_i & \text{if } a_2 \text{ is chosen} \end{cases}$$

[1] I leave this deliberately abstract to avoid modeling a formal Bayesian updating problem of the principal. A further extension could be considered in the future.

[2] The beliefs of all agents are assumed to be discrete with steps of ϵ such that the optimal strategy for each agent is well-defined. In practice it is also plausible that agents cannot report all real numbers to the principal.

[3] An alternative interpretation might be that all agents receive the same signal but interpret it differently.

[4] This is idea is similar to the standard statistical model of collective wisdom. For a further micro-foundation see Hong and Page (2012)

with $u_i \in \mathbb{R}$. This implies that, in the absence of other incentives, for $u_i < 0$ agent i prefers the choice of a_1 and vice versa. Agent i's belief and action preference can be summarized as the type, $\theta_i = (\mu_i, u_i)$. The set of all possible types is denoted by $\Theta = [0, 1] \times \mathbb{R}$. Agents know that the principal does not have any information about the state of the world and that she will adopt the agents' aggregate belief as her posterior. In other words, the agents know that the principal makes a decision solely based on the information provided by the agents.

The principal's objective is to choose the best action, maximizing her own expected utility. To do so she relies on information from the agents about the state of the world. The principal adopts a posterior belief μ^P equal to the average of all reported beliefs from the agents, $\bar{r} = \frac{1}{n}\sum_{i \in N} r_i$, where r_i is the reported belief of agent i.[5] Ideally, she would choose a_1 if $\bar{\mu} < \alpha$ and a_2 if $\bar{\mu} \geq \alpha$. Knowing the agents' action preferences is only indirectly relevant to the principal. The principal implements a specific direct mechanism $\mathcal{M}(M, S, d)$ in which the message space M for agents is restricted to belief reports $r \in \{\mu^1, ..., \mu^K\}$, transfers from principal to each agent are based on a scoring rule $S(r, \omega) \to \mathbb{R}$, and the decision rule $d(\bar{r}) \to \{a_1, a_2\}$ is given by:

$$d(\bar{r}) = \begin{cases} a_2 & \text{if } \bar{r} \geq \alpha \\ a_1 & \text{if } \bar{r} < \alpha \end{cases}$$

The scoring rule function determines the payoffs for all agents based on their reported belief and the realization of the state. The following section explains further details about scoring rules. Note that the specific decision rule studied in this paper is intuitive in the sense that it is ex-post incentive compatible for the principal. Figure 1 provides a summary timeline of the events.

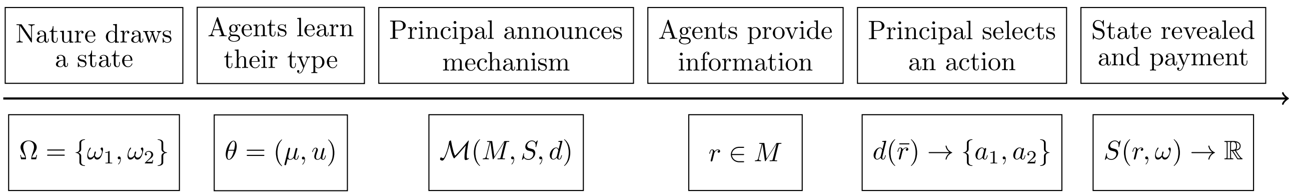

Figure 1: Summary timeline

[5] For simplicity I assume that aggregating beliefs by taking the simple average is optimal. Other methods of aggregation exist. This paper focuses mainly on the elicitation problem.

3.2. Scoring Rules

Stimulating agents to report their belief truthfully requires some incentive for accurate belief reports. The realized state is observable before transfers are implemented, hence allowing transfers to depend on the state. Scoring rules are a common method of incentivizing truthful belief reports based on state realizations.

A scoring rule is a function $S : [0,1] \times \Omega \to \mathbb{R}$ chosen by the principal which determines a monetary payoff $S(r, \omega)$ based on the reported belief r and the state of the world ω. Given some scoring rule S and a true belief μ, the agent's expected payoff from reporting r is defined as follows:

$$E_\mu S(r) := \mu S(r, \omega_2) + (1 - \mu) S(r, \omega_1).$$

In the absence of outside incentives a scoring rule that is considered proper leads agents to report their belief truthfully in order to maximize their expected payoff. Formally, a scoring rule S is considered proper, whenever

$$E_\mu S(\mu) \geq E_\mu S(r)$$

for every $r \neq \mu$ and every $\mu \in [0,1]$.

Following Gneiting and Raftery (2007), a scoring rule is characterized by a sub-differentiable function $G : [0,1] \to \mathbb{R}$ such that $E_\mu S(\mu) := G(\mu)$ for any $\mu \in [0,1]$. Consider the sub-tangent of $G(\mu)$ at r, $t_r(\mu) := a_r + b_r \mu$. Evaluating the tangent at zero and one shows the payoffs given a reported belief and the state of the world, i.e. $S(r, \omega_1) := t_r(0)$ and $S(r, \omega_2) := t_r(1)$. Figure 2 illustrates the payoffs following some example function G. For some reported belief r, the tangent of G at r shows the payoff $S(r, \omega_1)$ if the state is given by ω_1 and $S(r, \omega_2)$ if the state is given by ω_2. This implies that if an agent reports $r \neq \mu$ his expected score is given by:

$$E_\mu S(r) = G(r) + G'(r) \cdot (\mu - r).$$

Figure 2 also illustrates the expected cost of misreporting a belief. This cost is given by the Bregman divergence $d_G(r, \mu) := E_\mu S(\mu) - E_\mu S(r)$. As shown by Gneiting and Raftery (2007) the underlying function G determines whether a scoring rule is proper. A scoring rule is (strictly) proper if and only if G is (strictly) convex. Proper scoring rules have the desirable characteristic that in the absence of action preferences it is optimal for the agent

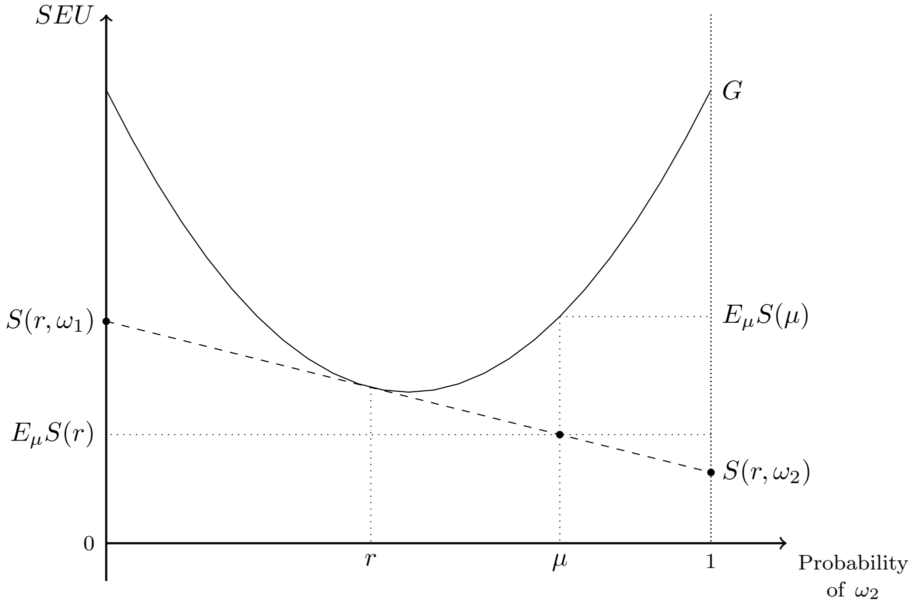

Figure 2: Some example function G leads to the scoring rule payoffs as shown by the tangent of G at r.

to report his belief truthfully. From here on I refer to (strictly) proper scoring rules as a scoring rule that makes truth-telling a (strictly) dominant strategy in the absence of action preferences.

4. Single Agent

4.1. Benchmark

Before analyzing the setting with n different agents it is useful to understand the behavior of a single agent. The principal announces a mechanism $\mathcal{M}$ that restricts messages to (discrete) belief reports $r \in \{\mu^1, ...\mu^K\}$[6] which are incentivized by some proper scoring rule S. The principal commits to a decision rule d such that:

$$d(r) = \begin{cases} a_2 & \text{if } r \geq \alpha \\ a_1 & \text{if } r < \alpha \end{cases}$$

Note that I assume in case of indifference, $r = \alpha$, the principal chooses a_2. The agent has some belief μ about the probability of the state being ω_2 and some action preferences as given by $u \in \mathbb{R}$. This leads to the following expected utility for the agent,

[6]For convenience, I assume that the decision threshold α is included in the set of feasible belief reports.

$$EU(r) = \begin{cases} E_\mu S(r) + u & \text{if } r \geq \alpha \\ E_\mu S(r) & \text{if } r < \alpha \end{cases}$$

Expected utility depends on two factors, accuracy of the reported belief and the action being chosen by the principal. The two factors are additively separable, hence resulting in a trade-off for the agent. On the one hand he would like to report his belief truthfully to maximize the expected payoff from the scoring rule. On the other hand misreporting his belief may influence the action chosen by the principal and hence the outside utility of the agent. This implies that for any proper scoring rule and any outside utility $u \geq 0$ only two reports may be optimal for the agent, reporting truthfully, $r = \mu$, or reporting a belief exactly at the threshold, $r = \alpha$. Conversely, if $u \leq 0$ the only two reports that may be optimal are reporting truthfully, $r = \mu$, or reporting a belief just below the threshold, $r = \alpha - \epsilon$. Taking the scoring rule and outside utility as given it is easy to show that there exists a precise threshold $c_- \in [0, 1]$ such that an agent with a belief $\mu \in (c_-, \alpha)$ finds it optimal to over-report and, conversely, an agent with a belief $\mu \in (\alpha, c_-)$ finds it optimal to under-report. The threshold c_- is such that the benefit from misreporting is exactly equal to the cost of misreporting, i.e. $u = E_{c_-} S(c_-) - E_{c_-} S(\alpha)$ if $u > 0$ and respectively $u = E_{c_-} S(c_-) - E_{c_-} S(\alpha - \epsilon)$ if $u < 0$. This leads to the optimal report, r^* as given below:

$$r^*(\mu, u, \alpha, G) = \begin{cases} \begin{cases} \mu & \text{if } \mu \notin [c_-, \alpha] \\ \alpha & \text{if } \mu \in [c_-, \alpha] \end{cases} & \text{if } u > 0 \\ \begin{cases} \mu & \text{if } \mu \notin [\alpha, c_-] \\ \alpha - \epsilon & \text{if } \mu \in [\alpha, c_-] \end{cases} & \text{if } u < 0 \end{cases}$$

For $\mu = c_-$ the agent is indifferent between reporting truthfully, $r = \mu$, and misreporting to influence the action choice, $r = \alpha$ or $r = \alpha - \epsilon$. For the remainder of the paper I assume the agent reports his belief truthfully in this case.

4.2. Optimal Scoring Rule

Attaching a decision directly to the report made by the agent gives rise to potential misreporting, as shown above. A proper scoring rule that normally makes truth-telling the unique optimal strategy for the agent does not necessarily have the same result if the agent has preferences over the actions. This section characterizes precisely which type of scoring rule mechanism preserves the property that truth-telling is optimal for any belief.

It is useful to define a new function $G^{net} : [0,1] \to \mathbb{R}$ which shows the utility from making a truthful report, including both the scoring rule payoff and the utility that follows from a certain action choice:

$$G^{net}(\mu) = \begin{cases} G(\mu) + u & \text{if } \mu \geq \alpha \\ G(\mu) & \text{if } \mu < \alpha \end{cases}$$

Similarly to before one can define the sub-tangent of the function G^{net} at $r \in \{0, \epsilon, 2\epsilon, ..., 1\}$ as $t_r^{net}(\mu)$. As G^{net} may be non-continuous the tangent $t_r^{net}(\mu)$ shows precisely when a truthful report leads to a lower expected payoff than some other report $r \neq \mu$.

Figure 3 illustrates the different incentives of the agent for some outside utility and some scoring rule incentives. For any belief μ the utility of reporting truthfully is illustrated. The constant labeled u shows the outside utility the agent receives if the principal chooses a_2. The two convex functions show the agents expected utility if he reports his belief truthfully plus the outside utility he receives. Note that the dashed parts of the convex functions are hypothetical for the agent as the principal only chooses a_2 with a report $r \geq \alpha$. The highlighted piecewise convex function therefore shows the expected utility of the agent if he reports his belief truthfully, G^{net}. As explained above the agent may have an incentive to misreport his belief. The intersection of the tangent, t^{net} for $r = \alpha$ with the function G is given by c_-. This implies that for any true belief $\mu \in (c_-, \alpha)$ the agent's expected utility from reporting $r = \alpha$, such that the principal chooses a_2, is strictly higher than reporting $r = \mu$, i.e. $EU_\mu(\alpha) > EU_\mu(\mu)$ $\forall \mu \in (c_-, \alpha)$. Therefore, for any true belief $\mu \in (c_-, \alpha]$, reporting $r = \alpha$ is optimal.

When trying to construct a scoring rule that makes truth-telling a dominant strategy for any belief μ, outside incentives of the agent must be taken into account. As illustrated in Figure 3, unless G^{net} is convex, the agent may have some belief that would make it optimal to misreport. The following Theorem shows that indeed only one type of scoring rule mechanism leads to a guaranteed truthful report.

Theorem 1. *For any belief, μ, and some fixed outside preferences, u, truth-telling is a*

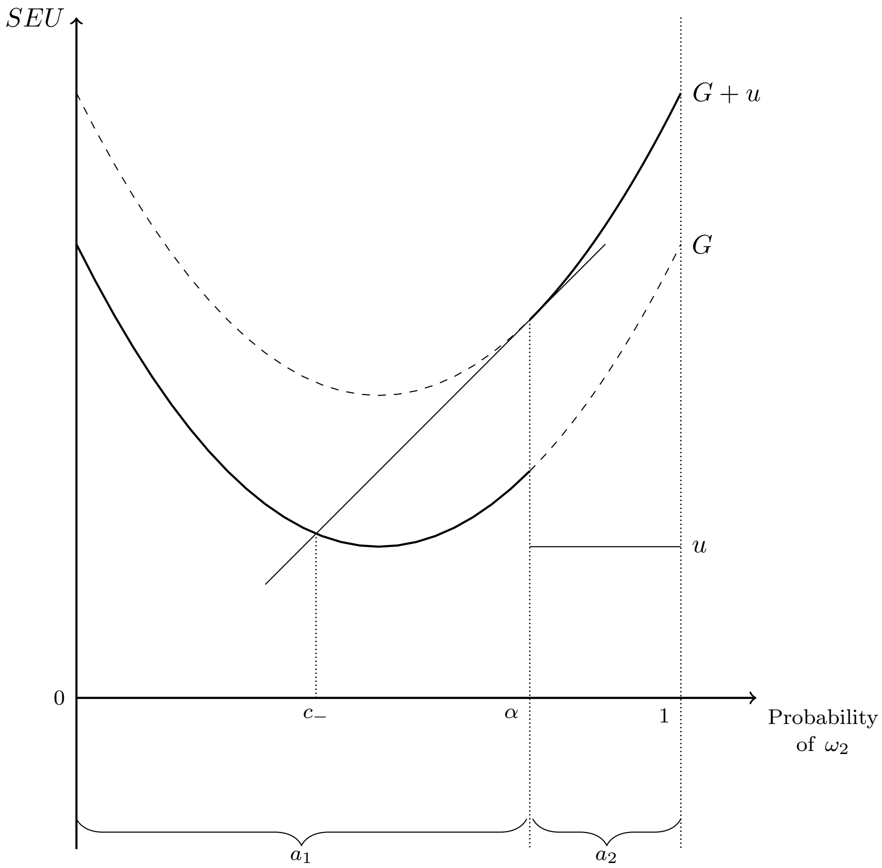

Figure 3: Incentive structure for some example scoring rule and some outside utility. For illustrative purposes a QSR is used. The highlighted convex function illustrates the expected utility of the agent for reporting his belief truthfully. For $\mu \in (c_-, \alpha]$ reporting $r = \alpha$ is optimal.

dominant strategy if and only if the scoring rule is given by S^* with

$$S^*(r,\omega) = \begin{cases} S(r,\omega) & \text{if } r \geq \alpha \\ S(r,\omega) + u & \text{if } r < \alpha \end{cases}$$

where $S(r,\omega)$ is any proper scoring rule.

Proof. See appendix. □

Theorem 1 shows the unique scoring rule mechanism that guarantees a truthful belief report, $r = \mu$, for any belief μ. However, this scoring rule requires the principle to have precise knowledge of the action preferences of the agent. Action preferences are by definition unknown to the principal. No mechanism exists that guarantees a truthful report for any possible belief if the agent has unobservable action preferences.

5. Multiple Agents

This section focuses on a direct mechanism where agents report their beliefs simultaneously and independently.[7] There are n different agents that all have some knowledge about the state of the world. The principal announces a mechanism $\mathcal{M}$ that restricts messages to belief reports, $r_i \in \{0, \epsilon, 2\epsilon, ..., 1\}$ which are again incentivized by a proper scoring rule, S. The principal commits to a decision rule d such that:

$$d(r_1, ..., r_n) = \begin{cases} a_2 & \text{if } \bar{r} \geq \alpha \\ a_1 & \text{if } \bar{r} < \alpha \end{cases}$$

where $\bar{r} = \frac{1}{n} \sum_{i \in N} r_i$

5.1. Agent Behavior

The suggested mechanism implies that the utility for each agent depends not only on his own report but also on the average report of all other agents, $\tilde{r}_{-i} := \frac{1}{n-1} \sum_{j \neq i} r_j$. Formally, the expected utility of agent i is given by:

$$EU_i(r_i) = \begin{cases} E_{\mu_i} S(r_i) + u_i & \text{if } \bar{r} \geq \alpha \\ E_{\mu_i} S(r_i) & \text{if } \bar{r} < \alpha \end{cases}$$

The main difference to the scenario with just a single agent is that individual agents may not be able to influence the principal's decision. If the principal only consults a single agent this agent knows with certainty that submitting a report $r_i \geq \alpha$ leads to action a_2 being chosen. This is not the case with multiple agents. Only agents that are pivotal can submit a report that may affect the action being chosen. Formally, pivotality is defined as follows.

Definition 1. (Pivotality) Agent i is considered to be pivotal if $\tilde{r}_{-i}$ is such that $\frac{n-1}{n}\tilde{r}_{-i} < \alpha \leq \frac{n-1}{n}\tilde{r}_{-i} + \frac{1}{n}$.

Given some $\tilde{r}_{-i}$, $\bar{r} = \frac{n-1}{n}\tilde{r}_{-i}$ is the lowest possible aggregate report that agent i can achieve by reporting $r_i = 0$. Conversely, $\bar{r} = \frac{n-1}{n}\tilde{r}_{-i} + \frac{1}{n}$ is the highest possible aggregate report that agent i can achieve by reporting $r_i = 1$. If the threshold α is such that agent i can influence the action choice of the principal, i.e. reporting $r_i = 0$ leads to $\bar{r} < \alpha$ and

[7] This mechanism is also referred to as a 'prediction poll'.

reporting $r_i = 1$ leads to $\bar{r} \geq \alpha$, this agent is considered pivotal. Pivotality plays a crucial role for the agent when deciding on a belief to report. If an agent is not pivotal he has no incentive to misreport his true belief. Besides characterizing when an agent is pivotal it is useful to define a pivotal report for each agent, $c_{i,+} \in \mathbb{R}$. If agent i reports $c_{i,+}$ it leads to an aggregate report exactly at the decision threshold $\bar{r} = \alpha$ for any reported beliefs by the other agents, $\tilde{r}_{-i}$. Formally, this pivotal report is given by:

$$c_{i,+} := \alpha + (n-1)(\alpha - \tilde{r}_{-i})$$

Note that an agent may not be pivotal, in which case $c_{i,+} \notin [0, 1]$. If the agent is pivotal then for any $r_i \geq c_{i,+}$ the principal chooses action a_2 and, respectively, for any $r_i < c_{i,+}$ the principal chooses action a_1. The optimal report for each agent is defined below.

Proposition 1. *The optimal report for agent i is given by:*

$$r_i^* = \begin{cases} \mu_i & \text{if } c_{i,+} \notin [0,1] \\ \begin{cases} \mu_i & \text{if } \mu \notin (c_{i,-}, c_{i,+}] \\ c_{i,+} & \text{if } \mu \in (c_{i,-}, c_{i,+}] \end{cases} & \text{if } c_{i,+} \in [0,1] \end{cases}$$

if $u_i \geq 0$, and,

$$r_i^* = \begin{cases} \mu_i & \text{if } c_{i,+} \notin [0,1] \\ \begin{cases} \mu_i & \text{if } \mu \notin (c_{i,+}, c_{i,-}] \\ c_{i,+} - \epsilon & \text{if } \mu \in (c_{i,+}, c_{i,-}] \end{cases} & \text{if } c_{i,+} \in [0,1] \end{cases}$$

if $u_i < 0$

As before, $c_{i,-}$ is the report such that agent i is indifferent between misreporting and reporting truthfully, i.e. $u_i = E_{c_{i,-}} S(c_{i,-}) - E_{c_{i,-}} S(c_{i,+})$. If $\mu_i = c_{i,-}$, I assume the agent reports his belief truthfully.

The optimal report for each agent is similar to the case with only a single agent. The main difference is that while an agent can always influence the decision if he is the only one that is consulted, this is not necessarily the case with multiple agents. As the number of agents increases it is less likely that an individual agent is pivotal to the decision. In this case it is best for the agent to report his belief truthfully. Consider the example as shown in

Figure 4. The agent prefers a_2 over a_1. For simplicity assume that the action preferences u_i are sufficiently large relative to the incentives of the scoring rule mechanism such that $c_{i,-}$ plays no role. The optimal report given any value of $\tilde{r}_{-i}$ is then indicated by the red solid line. Every report on or above the diagonal dashed line leads to $\bar{r} \geq \alpha$ and hence a_2 being chosen by the principal. As shown above, if the agent is not pivotal only a truthful report is optimal for him. However, in the interval $[\frac{n\alpha-1}{n-1}, \frac{n\alpha-\mu_i}{n-1})$ it is optimal to over-report.

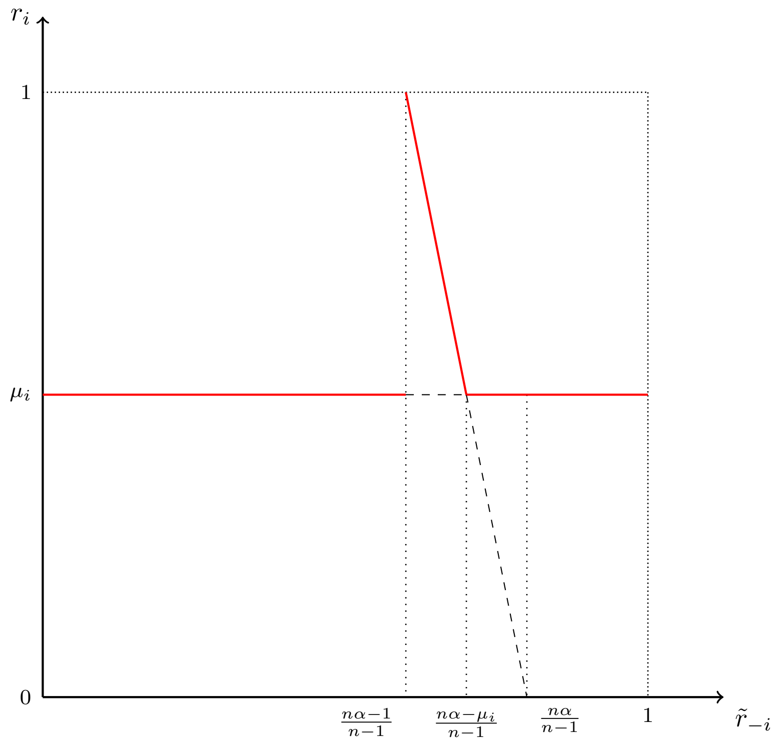

Figure 4: Best response function.

5.2. Nash Equilibria

For any type profile $(\theta_1, ..., \theta_n)$ only two kinds of (pure strategy) equilibria are feasible, the truth-telling equilibrium and a single misreporting equilibrium. The truth-telling equilibrium is defined as the equilibrium outcome in which every agent reports his belief truthfully, such that the final average is given by $\bar{r} = \tilde{\mu}$, where $\tilde{\mu}$ denotes the average true belief of all agents. An outcome is considered a misreporting equilibrium if at least one agent misreports his belief.

Ideally, the principal would like every agent to submit a truthful report. If the principal can only consult a single expert this cannot be guaranteed. With multiple experts truth-telling is the unique and strict Nash equilibrium under two conditions, diversity and no pivotality. The two conditions are formally defined below.

Theorem 2. *For any number of agents ($n \geq 2$), any strictly proper scoring rule S, all agents reporting their belief truthfully, $r_i = \mu_i \ \forall \ i$, is the unique and strict Nash equilibrium if,*

1) ***Diversity:*** *the profile of action preferences is such that for at least one agent i $u_i > 0$ and for at least one agent j $u_j < 0$, and*

2) ***No pivotality:*** $\tilde{\mu} \notin [\alpha - \frac{1}{n}, \alpha + \frac{1}{n})$.

Proof. The complete proof can be found in the appendix. The condition on $\tilde{\mu}$ guarantees existence of a truth-telling equilibrium and the condition on agents' preferences ensures uniqueness by eliminating any misreporting equilibrium. □

The 'diversity' condition states that at least one agent must prefer each action. This marks a main difference to the single expert scenario in which the expert can naturally only prefer one of the two actions. This condition on preferences eliminates any potential misreporting equilibria. It seems plausible that a principal has some degree of control over the group of experts in practice. The principal should thus ensure a minimal degree of diversity among the group of experts. While the principal does not know $\tilde{m}u$, ensuring that at least one agent prefers each action seems feasible in practice.

The 'no pivotality' condition limits the scenarios in which truthful reporting is the unique Nash equilibrium to scenarios where no expert can individually change the chosen action. This condition largely depends on the number of agents and the decision threshold chosen by the principal. While this condition limits equilibrium existence it is worth pointing out that the principal is most interested in a truthful aggregate report if the true mean belief is further away from the threshold. In the extreme case of $\tilde{\mu} = \alpha$ the principal is indifferent between the two actions and receiving an aggregate report $\bar{r} \neq \tilde{\mu}$ does not affect the principal. The condition further indicates that if the principal had some prior about the true mean belief she should choose a decision threshold, α further away from it.

A final point to mention is that the size of the scoring rule incentives do not matter with this mechanism. The principal can choose any strictly proper scoring rule, even with arbitrarily small payoffs. This is a clear difference to the setting where the principal can only consult a single expert. In that case the agent has a clear trade-off between scoring rule incentives and utility from the action choice. The higher the payoffs from the scoring rule the smaller the range of beliefs the agent would misreport. This is not the case with multiple agents. The precise form of the scoring rule incentives are of lesser importance. The main factor leading agents to report their belief truthfully is that each agent individually must not be able to change the decision of the principal.

6. Discussion of Alternative Mechanisms

The previous sections focused on a direct mechanism in which all agents make one simultaneous report and the principal committed to an intuitive decision rule. This section discusses some alternative mechanisms.

6.1. Delegation

Delegating the decision to the group of agents is a natural alternative to the direct mechanism analyzed above. In fact, with just one agent the principal may not benefit at all from eliciting a precise belief instead of an action recommendation. Restricting the message space such that the agent can only communicate a_1 or a_2 is likely equivalent for the principal. This is not the case with multiple agents. If the principal delegates the decision to the group of agents rather than asking each agent to report a belief individually could lead to drastic misreporting. The agents may aim to maximize their total expected utility from any mechanism, essentially acting as a larger single agent. A truthful report, $\bar{r} = \tilde{\mu}$, mainly depends on the average action preferences from all agents, $\bar{u}$. If $\bar{u} = 0$, a truthful report could be guaranteed. However, as the principal does not know $\bar{u}$ any reported belief may be misreported. Nonetheless, with few agents and/or $\bar{u} \approx 0$ delegating the decision to the group of agents may be better than a simultaneous reporting mechanism.

6.2. Repeated Simultaneous Reporting Mechanism

Letting agents report their beliefs repeatedly could serve as a practical implementation of the mechanism analyzed in the previous section. In such a mechanism the principal can announce the aggregate report of all agents after each round and before agents submit another report. This allows agents to learn about the average report of all other agents, $\tilde{r}_{-i}$, which is unknown in the beginning. Depending on the expectation of $\tilde{r}_{-i}$, agents may expect to be pivotal for the decision and thus expect to have an incentive to misreport their true belief. However, with a larger number of agents it is increasingly unlikely that any agent is in fact pivotal. Letting agents report in multiple rounds may correct agents expectations about pivotality and thus lead to more accurate results.

How agents report their belief precisely depends partly on what agents infer from the announced aggregate reported belief. Suppose that agents are myopic, i.e. they assume that other agents do not change their behavior from one round to the next. In this case, and assuming $\tilde{\mu} \notin [\alpha - \frac{1}{n}, \alpha + \frac{1}{n}]$, the aggregate reported belief always converges to $\bar{r} = \tilde{\mu}$. With a sufficiently large number of agents, convergence often only takes two rounds and also in most other cases should be achieved after just a few more rounds. Other assumptions on how agents learn from the announced aggregate reported belief could be made. These complicate the analysis but in many cases share the same result.

A mechanism with repeated simultaneous reporting and feedback from the principal seems to deliver highly accurate belief reports. At the same time it is feasible to implement in practice even if agents do not perfectly know the average true belief of all other agents, $\tilde{\mu}$.

6.3. Sequential Reporting Mechanism

Another alternative to the simultaneous reporting mechanism analyzed in the previous section is a mechanism with sequential reporting of beliefs. In such a mechanism the principal asks each agent to report a belief after each other and agents know the aggregate reported belief of all previous agents. One can distinguish two cases: Each agent knows how many other agents will report a belief after him or, the principal randomly stops asking agents for a belief and takes the current aggregate reported belief to make a decision. In both cases the truth-telling equilibrium, $\bar{r} = \tilde{\mu}$, does not necessarily exist even if $\tilde{\mu} \notin [\alpha - \frac{1}{n}, \alpha + \frac{1}{n}]$ and not all agents prefer the same action to be chosen. The existence of a truth-telling equilibrium depends on the profile of action preferences of the agents. It may exist if the number of agents preferring each action is roughly evenly distributed but not otherwise.

A popular mechanism frequently used to elicit beliefs from a group of agents is a prediction market. Prediction markets also work with sequential belief reports and a random stopping point.[8] Unlike the setting in this paper, the aggregation mechanism does not work by taking

[8]The stopping point may be announced but no agents knows the number of other agents entering the

a simple average. Therefore, a direct comparison cannot be made easily. Nonetheless, it seems plausible that if agents have vested interests in the action chosen by the principal, this mechanism will not deliver an accurate aggregate belief in the end. As with a simpler sequential mechanism truth-telling may depend largely on the number of agents preferring each action. Additionally, to make matters worse, in a market setting each agent can choose the 'weight' of his belief report himself.

6.4. Other mechanisms

Possibly one of the simplest mechanisms that is frequently used in practice is a voting mechanism. In fact, this mechanism can also be analyzed in the framework introduced in this paper by restricting the message space of the agents to two binary signals (a_1 and a_2). It should be clear that while this method is often used in practice it leads to a worse outcome for the principal whenever she can consult multiple agents.

Undoubtedly one can imagine many other mechanisms and variations of mechanisms. It seems impossible to create a perfect mechanism that guarantees truth-telling in all cases. Nonetheless, the question remains, what is the best imperfect mechanism a principal could implement in practice?

7. Conclusion

This paper considered a setting in which an uninformed principal is faced with a decision. The principal can consult multiple experts for advice, however, the experts are affected the principal's decision. I show that with just one agent, truth-telling can only be achieved if the agent's preferences happen to be aligned with the ones of the principal or if the principal pays substantial incentives for truthful reporting (relative to how much the agent prefers one action over the other). With multiple agents, the principal can implement a direct reporting mechanism, similar to a 'prediction poll', commit to an intuitive decision rule and pay minimal incentives in the form of a scoring rule. All agents reporting their belief truthfully is the unique Nash equilibrium if no agent is individually pivotal and not all agents prefer the same action to be chosen by the principal. In practice the principal should therefore

mechanism after him.

focus on consulting as many agents as possible rather than offering high incentives to few agents for their advice.

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A. Proofs

A.1. Theorem 1

Suppose $S^*(r,\omega)$ is given as in Lemma 1. Then G^{net} is given by

$$G^{net}(\mu) = \begin{cases} G(\mu) + u & \text{if } \mu \geq \alpha \\ G(\mu) + u & \text{if } \mu < \alpha \end{cases}$$

This implies that G^{net} is strictly convex and hence proper. Therefore, truth-telling is the dominant strategy, as shown in section 3.2.

Conversely, suppose that the agent reports his belief truthfully for every belief, μ, and some fixed outside preferences, u. Hence, $t_r^{net}(\mu) \leq G^{net}(\mu) \quad \forall \mu \in [0,1]$. Therefore, $G^{net}(\mu)$ must be convex, as shown for example by Gneiting and Raftery (2007). Hence, the scoring rule is given as stated in 1.

A.2. Proposition 1

The proof consists of two parts. Part 1 shows that given some $\tilde{r}_{-i}$, if agent i is not pivotal, for any (strictly) proper scoring rule S it is (strictly) optimal for the agent to report his belief truthfully, $r_i = \mu_i$. Part 2 shows that given some $\tilde{r}_{-i}$, such that agent i is pivotal, the only report $r_i \neq \mu_i$ that could be optimal is given by $r_i = c_{i,+}$ if $u > 0$ or $r_i = c_{i,+} - \epsilon$ if $u < 0$. Taking together part 1 and part 2 leads to the optimal report as stated in Proposition 1.

Part 1. Agent i not being pivotal implies that $\tilde{r}_{-i}$ is such that $\alpha \notin (\frac{n-1}{n}\tilde{r}_{-i}, \frac{n-1}{n}\tilde{r}_{-i} + \frac{1}{n}]$. Consider two cases. 1) $\tilde{r}_{-i} < \frac{n}{n-1}\alpha - \frac{1}{n-1}$: Then for any r_i it is the case that $\bar{r} = \frac{n-1}{n}\tilde{r}_{-i} + \frac{1}{n}r_i < \alpha$. This implies that $EU_i(r_i) = E_{\mu_i}S(r_i) + u_i$. Hence, $r_i = \mu_i$ is optimal.

2) $\tilde{r}_{-i} \geq \frac{n}{n-1}\alpha$: Then for any $r_i \in$ it is the case that $\bar{r} = \frac{n-1}{n}\tilde{r}_{-i} + \frac{1}{n}r_i \geq \alpha$. This implies that $EU_i(r_i) = E_{\mu_i}S(r_i) + u_i$. Hence, $r_i = \mu_i$ is optimal.

Part 2. Consider agent i with true belief $\mu_i < c_{i,+}$ and $u > 0$. A truthful report would thus lead to $\bar{r} < \alpha$ and a_1 being chosen by the principal. Suppose the agent reports $r_i' > c_{i,+}$ such that $\bar{r}' > \alpha$. Both reports, $c_{i,+}$ and r_i', lead to the same action being chosen by the principal, a_2. Hence, the agent would have a higher expected utility from reporting $r_i = r_i' - \epsilon < r_i'$. Thus r_i' is never optimal. The same holds true vice versa for an agent with $\mu_i > c_{i,+}$ and $u < 0$.

A.3. Theorem 2

Existence: Let $\tilde{\mu}_{-i}$ denote the average truthful report of all agents excluding agent i, i.e. $\tilde{\mu}_{-i} := \frac{1}{n-1}\sum_{j\neq i}\mu_j$. The no pivotality condition implies that for each agent i, $\frac{1}{n}r_i + \frac{n-1}{n}\tilde{\mu}_{-i} \geq \alpha$ or $\frac{1}{n}r_i + \frac{n-1}{n}\tilde{\mu}_{-i} < \alpha \quad \forall\ r_i$. Then for any strictly proper scoring rule S, $EU_i(\mu_i) > EU_i(r_i)\ \forall\ r_i \neq \mu_i$. Hence, each agent i reporting $r_i = \mu_i$ is a Nash Equilibrium.

Uniqueness: Suppose $\bar{r} \neq \tilde{\mu}$ was also a Nash equilibrium. Consider 3 cases for $\bar{r}$. Case 1) $\bar{r}$ is not near the decision threshold α, i.e. $\bar{r} > \alpha$ or $\bar{r} < \alpha - \epsilon$: Any agent that reported $r_i \neq \mu_i$ would be strictly better off by reporting $r_i' - \epsilon$ if $r_i > \mu_i$ or $r_i' + \epsilon$ if $r_i < \mu_i$. The reason is as follows. $\bar{r} \neq \alpha$ and $\bar{r} \neq \alpha - \epsilon$ implies that reporting r_i' does not change which action is selected by the principal. Hence, $EU_i(r_i') > EU_i(r_i)$. Therefore, $\bar{r} \neq \tilde{\mu}$ with $\bar{r} < \alpha - \epsilon$ or $\bar{r} > \alpha$ is not a Nash equilibrium. Case 2) $\bar{r} = \alpha$: Diversity implies that at least one agent i has action preferences such that $u_i < 0$. In this case, $EU_i(\mu_i - \epsilon) > EU_i(\mu_i)$ as it would lead to $\bar{r} = \alpha - \epsilon'$ and the choice of a_1 by the principal. Hence, $\bar{r} = \alpha$ is not a Nash equilibrium. Case 3) $\bar{r} = \alpha - \epsilon$: Diversity implies that at least one agent i has action preferences such that $u_i > 0$. In this case, $EU_i(\mu_i + \epsilon) > EU_i(\mu_i)$ as it would lead to $\bar{r} = \alpha$ and the choice of a_2 by the principal. Hence, $\bar{r} = \alpha - \epsilon$ is not a Nash equilibrium.

Therefore, $\bar{r} \neq \tilde{\mu}$ cannot be a Nash equilibrium. Taken together, this implies that all agents reporting $r_i = \mu_i$ is the unique Nash equilibrium for any profile of beliefs and action preferences that satisfy (a) no pivotality and (b) diversity.