# Sep 3, 2024 (Due: 08:00 Sep 10, 2024)

**1.** Let $\hat{x}$ be an approximation to $x$. In practice it is often much easier to estimate $\tilde{E}_{\text{rel}}(\hat{x}) = |x - \hat{x}|/|\hat{x}|$ compared to $E_{\text{rel}}(\hat{x}) = |x - \hat{x}|/|x|$. What is the relationship between $E_{\text{rel}}$ and $\tilde{E}_{\text{rel}}$?

**2.** How to evaluate $f(x) = \tan x - \sin x$ for $x \approx 0$ so that numerical cancellation is avoided?

**3.** Let $A$ be a square banded matrix with half-bandwidth $\beta$ (i.e., $a_{ij} = 0$ if $|i-j| > \beta$). Suppose that the LU factorization of $A$ (without pivoting) is $A = LU$. Show that both $L$ and $U$ are banded matrices with half-bandwidth $\beta$.

**4.** Find the exact LU factorization of the $n \times n$ matrix

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 1 \\ -1 & 1 & 0 & \cdots & 0 & 1 \\ -1 & -1 & 1 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \cdots & 1 & 1 \\ -1 & -1 & -1 & \cdots & -1 & 1 \end{bmatrix}.$$

**5.** Implement Gaussian elimination (without pivoting) for solving nonsingular linear systems. You may assume that no divide-by-zero error is encountered. Measure the execution time of your program in terms of matrix dimensions and visualize the result by a log–log scale plot. (You may generate your test matrices with with normally distributed random elements.)

**6.** (optional) Write a program to solve the quadratic equation $ax^2 + bx + c = 0$ with real coefficients. Describe how to avoid cancellation when the equation has a tiny root.

**7.** (optional) Suppose that you are evaluating the harmonic series using IEEE double precision floating-point numbers and obtained a "converged" result. Make an estimate on when the computation converges, and what is the final result.