Nov 19, 2024 (Due: 08:00 Nov 26, 2024)

1. Let A be a Hermitian matrix. Describe how to transform A to a *real* tridiagonal matrix by unitary similarity before applying the implicit QR algorithm.

2. Let A and B be $n \times n$ real symmetric matrices. Suppose that B is positive definite. Show that AB is diagonalizable, and design an algorithm to compute all eigenvalues and eigenvectors of AB.

3. Implement the scaling-and-squaring algorithm (combined with truncated Taylor series) for computing the matrix exponential. Test the accuracy of your algorithm by a few diagonalizable matrices with known spectral decomposition.

(optional) Implement the Schur–Parlett algorithm and compare the accuracy.

(H, optional) Replace truncated Taylor series by Padé approximants in the scaling-and-squaring algorithm and compare the accuracy.

4. Let A and E be Hermitian matrices with $AE = EA$. Try to give an upper bound on

$$\|\exp(A + E) - \exp(A)\|_2.$$

Make sure your upper bound tends to zero when $\|E\|_2 \to 0$.

5. Have a quick glance at the paper "From Random Polygon to Ellipse: An Eigenanalysis" by A. N. Elmachtoub and C. F. Van Loan (available on eLearning). Reproduce the experiments in this paper.

6. (optional) In the lecture we have discussed how to solve Sylvester matrix equation $AX - XB = C$ using the Bartels–Stewart algorithm (through Schur decompositions). What happens the matrices are real, and if only *real* Schur decompositions are permitted?

7. (optional) Use truncated SVD to compress some grayscale images. If you only have colored images, you can convert them to grayscale using

$$\text{gray} = \alpha \cdot \text{red} + \beta \cdot \text{green} + \gamma \cdot \text{blue}.$$

Common choices of the constants are $(\alpha, \beta, \gamma) = (0.299, 0.587, 0.114)$ and $(\alpha, \beta, \gamma) = (0.2126, 0.7152, 0.0722)$.

You are also encouraged to think about how to compress colored images.