Assume $x_l(p, w)$ does not depend on p_j where $j \neq l$ and is homogenous of degree one with w. x_l is also homogenous of degree 0 with prices and income. That is for any $\lambda_1 > 0$ and $\lambda_2 > 0$ we have:

$$x_l(p_l, w) = \frac{1}{\lambda_1} x_l(\lambda_2 p_l, \lambda_2 \lambda_1 w),$$

taking $\lambda_2 = \frac{1}{p_l}$ and $\lambda_1 = \frac{p_l}{w}$

$$x_l(p_l, w) = \frac{w}{p_l} x_l(1, 1),$$

denoting $x_l(1, 1) =: \alpha_l$ we obtain that

$$x_l(p_l, w) = \alpha_l \frac{w}{p_l}.$$