Advanced Microeconomics (QEM) - Problem set 1
Due date: Monday, October 12th (end of day) by email

Problem 1 (1p) *Problem 3.B.2 from MGW*

Problem 2 (1p) *Problem 3.B.3 from MGW*

Problem 3 (1p) *Problem 2.4.12 (points 1,2 and 5) from the Lecture Notes.*

Problem 4 (1p) *Problem 2.4.14 from the Lecture Notes.*

Problem 5 (3p) *Consider household with preferences given by:*

$$u(x_1, x_2, \ldots, x_T) = \sum_{t=1}^{T} \beta^{t-1} \ln(x_t),$$

with $\beta \in (0,1)$ facing prices $p_1, p_2, \ldots, p_T$ and income w (that has to be used during the whole T-period lifetime).

1. *find demand (optimal consumptions path)*
2. *is consumption increasing, decreasing or constant with time? how does parameter β influence demand x_1?*
3. *find the value function (indirect utility function)*
4. *write expenditure minimization problem and find Hicksian demand*
5. *is Hicksian demand increasing, decreasing or constant with time? how does parameter β influence Hicksian demand h_1?*
6. *find the expenditure function*

Problem 6 (5.5p) *Consider the following utility function, called CES (constant elasticity of substitution function):*

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}, \quad \text{where } 0 \neq \rho < 1.$$

This function is obviously strictly increasing. For the above CES function:

1. *Show that u is strictly quasiconcave.*
2. *Formulate the Utility Maximization Problem and solve it to derive demand $d_1(p, w)$ and $d_2(p, w)$ [You may assume interior solution and Walras law which follows from strict monotonicity of $u(\cdot)$].*
3. *Form the indirect utility function $v(p, w)$.*
4. *Show that the indirect utility function is:*
   (a) *homogeneous of degree zero in (p, w),*
   (b) *increasing in w and decreasing in p.*
5. *Verify the Roy's identity.*
6. *Formulate the Expenditure Minimization Problem and solve it to derive Hicksian demand $h(p, \underline{u})$ [You may assume interior solution and no excess utility which follows from continuity of $u(\cdot)$]*
7. *Form the expenditure function $e(p, \underline{u})$.*
8. *Verify that the expenditure function is concave in p.*
9. *Verify the Shephard's lemma.*