Advanced Microeconomics (QEM) - Problem set 1
Due date: classes on October 7th

Problem 1 (1p) *Using Kuhn-Tucker theorem solve:*

$$\max_{x,y} \quad 3xy - y^3$$
$$2x - y = -5$$
$$5x + 2y \geq 37$$
$$x \geq 0, y \geq 0.$$

Consider the following utility function, called CES (constant elasticity of substitution function):

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}, \quad \text{where } 0 \neq \rho < 1.$$

This function is obviously strictly increasing.

Problem 2 (2.5p) *For the above CES function:*

1. *Show that u is strictly quasiconcave.*
2. *Formulate the Utility Maximization Problem and solve it to derive Walrasian demand functions $x_1(p, w)$ and $x_2(p, w)$ [You may assume interior solution and Walras law which follows from strict monotonicity of $u(\cdot)$].*
3. *Form the indirect utility function $v(p, w)$.*
4. *Show that the indirect utility function is:*
   - (a) *homogeneous of degree zero in (p, w),*
   - (b) *increasing in w and decreasing in p.*
5. *Verify the Roy's identity.*

Problem 3 (2.5p) *For the above CES function:*

1. *Formulate the Expenditure Minimization Problem and solve it to derive Hicksian demand function $\Delta(p, \alpha^0)$ [You may assume interior solution and no excess utility which follows from continuity of $u(\cdot)$]*
2. *Form the expenditure function $c(p, \alpha^0)$.*
3. *Show that the expenditure function is concave in p.*
4. *Verify the Shephard's lemma.*
5. *Write Slutsky equation for the above functions.*

Problem 4 (2p) *Problem 2.4.12 from the Lecture Notes.*

Problem 5 (2p) *Problem 2.4.14 from the Lecture Notes.*