Game Theory - Problem set 1
Due date: classes on March, 24th

Problem 1 (2p) *Find all pure strategy Nash equilibria in the game between three players:* 1 *(choosing:* U, D*),* 2 *(choosing:* L, R*) and* 3 *(choosing matrix* A, B, C*) with payoffs given by:*

A

	L	R
U	0,0,3	0,0,0
D	1,0,0	0,0,0

B

	L	R
U	2,2,2	0,0,0
D	0,0,0	2,2,2

C

	L	R
U	0,0,0	0,0,0
D	0,1,0	0,0,3

Problem 2 (2p) *Consider the Inspection game between worker and principal. Worker can* S *shirk or* W *work, while principal inspect* I *or not* NI*. Cost of inspection is* h*, cost of work is* g*.* v *is the value of work to the principal.* w *stands for wage (transfer).*

	I	NI
S	$0, -h$	$w, -w$
W	$w-g, v-w-h$	$w-g, v-w$

- *Assume* $w > g > h > 0$.
- *Find all Nash Equilibria in mixed (or pure) strategies.*

Problem 3 (3p) *Army* A *has a single plane, that can be sent to destroy one of the targets. Army* B *has a single gun, that can be used to protect one of the targets. The value of the target is* v_i*, where* $v_1 > v_2 > v_3 > 0$*. Army* A *can destroy a target only that is not protected by* B*. Army* A*'s aim is to maximize expected loss of army* B*, and army* B*'s aim is to minimize such a loss. Write it as a (strictly competitive) strategic form game and find Nash equilibria in mixed strategies.*

Problem 4 (2p) *Consider a first price, sealed-bid auction as a strategic game. Suppose we have* $\{1, \ldots, n\}$ *players, where* i*-th player valuation is* v_i*. Let* $v_1 > v_2 > \ldots > v_n > 0$*. Each player bets (some nonnegative amount) in a closed envelope and the winner is the one that gives the highest bid. If few players gives the same highest bid then the winner is the one with the smallest* i *among those that bid the highest. It is a first price auction, i.e. the winner pays the amount he/she bids. Show that in any PS Nash equilibrium player* 1 *wins.*

Problem 5 (1p) *Diamond introduced the search model, in which player* i *expands effort* $a_i \in [0, 1]$ *searching for trading partners, and has a payoff function given by (with* $\theta > 0$ *being parameter characterizing search environment):*

$$u_i(a_i, a_{-i}) = \theta a_i \sum_{j \neq i} a_j - c_i(a_i)$$

Under what conditions placed on c_i*, is this a supermnodular game? Then, using theorems from the class, what can be said about the NE of this game? How do they depend on parameter* θ*?*