Looking for Evidence of Time-Inconsistent Preferences in Asset Market Data

`Narayana R. Kocherlakota(2001)`

Luis Maldonado Méndez

Outline

Features of the Model

Agents preferences in period 1

$$u(c_1) + \beta_{12}\left[u(c_2) + \beta_{23}u(c_3)\right]$$

Agents preferences in period 2

$$u(c_2) + \beta'_{23}u(c_3)$$

- Each agent is endowed with y_1 units of consumption in period 1.
- b_2 units of a short-term bond that pays off one unit of consumption in period 2.
- b_3 units of a long-term bond that pays off one unit of consumption in period 3 but can be retraded in period 2.
- The last asset is a commitment asset which cannot be retraded.It pays off one unit of consumption in period 3 and each agent is endowed with b_3^{com} units of it.
- Per capita endowment in periods 2 and 3 is given by $y_2 = b_2$ and $y_3 = b_3 + b_3^{com}$.
- For $\beta'_{23} \leq \beta_{23}$, where β'_{23} is the discount factor in period 2 between periods 2 and 3.

Decision Problems-Period 2 (DP2)

Period 2

$$\max_{c_2, c_3, b_{23}} \left[u\left(c_2^j\right) + \beta'_{23} u\left(c_3^j\right) \right]$$

subject to

$$c_2^j + q_{23} b_{23}^j = W_{llq}^j$$
$$c_3^j = b_{23}^j + \left(W_{com}^j / q_{23}\right)$$
$$c_2^j, b_{23}^j \geq 0$$

Decision Problems-Period 1 (DP1)

Period 1

$$\max_{(c^j,b^j,W^j)} \left\{ u\left(c_1^j\right) + \beta_{12} u\left(c_2^*\left(W_{\mathrm{llq}}^j W_{\mathrm{com}'}^j q_{23}\right)\right) \right.$$
$$\left. +\beta_{12}\beta_{23} u\left(c_3^*\left(W_{\mathrm{llq}}^j W_{\mathrm{com}}^j, q_{23}\right)\right)\right\}$$

subject to

$$c_1^j + p_{com} a_{com}^j + q_{12} b_{12}^j + q_{13} b_{13}^j$$
$$= y_1 + p_{com} \bar{b}_3^{com} + q_{13} \bar{b}_3 + q_{12} \bar{b}_2$$

$$W_{hq}^j = q_{23} b_{13}^j + b_{12}^j$$
$$W_{com}^j = q_{23} a_{com}^j$$
$$c_1^j, b_{12}^j, b_{13}^j, a_{com}^j \geq 0$$

Equilibrium

Eq, is a specification of consumption $\left(c_1^j, c_2^j, c_3^j\right)_{j\in[0,1]}$, asset holdings $\left(b_{12}^J\ b_{13}^j, b_{23}^j, a_{com}^j\right)_{j\in[0,1]}$ and prices $(q_{12}, q_{13}, q_{23}, p_{com})$ that satisfies three criteria. First, $\left(c_2^j, c_3^j\right)$solves DP2 given

$$
\begin{aligned}
&q_{23}\\
&W_{u_j}^j = q_{23} b_{13}^f + b_{12}^j\\
&W_{com}^j = q_{23} a_{com}^j
\end{aligned}
$$

Second, $\left(c_1^J, b_{12}^J, b_{13}', a_{com}^j\right)$ solves DP1 given q_{12}, q_{13}, q_{23} and p_{com}. And third market clears so that

$$
\begin{array}{lll}
\int c_1^j dj = y_1 & \int c_3^j dj = \bar{b}_3^{com} + \bar{b}_3 & \int b_{23}^j dj = \int b_{13}^j dj = \bar{b}_3\\
\int c_2^j dj = \bar{b}_2 & \int b_{12}^j dj = \bar{b}_2 & \int a_{\text{com}}^j dj = \bar{b}_3^{com}
\end{array}
$$

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Proposition 1. Suppose that $\beta'_{23} = \beta_{23}$. Then autarky is an equilibrium, and in any equilibrium,

$$c_t^i = y_t$$

for all t and

$$\begin{aligned} q_{12} &= \beta_{12} u'(y_2)/u'(y_1) \\ q_{13} &= \beta_{12}\beta_{23} u'(y_3)/u'(y_1) \\ q_{23} &= \beta_{23} u'(y_3)/u'(y_2) \\ p_{\text{com}} &= q_{13} \end{aligned}$$

Proof. Let $\beta'_{23} = \beta_{23}$. Then solving DP1 is equivalent to solving

$$(c_1, c_2, c_3) \in \arg \max_{(c_1, c_2, c_3, a_{cm})} \left[u(c_1) + \beta_{12} u(c_2) + \beta_{12}\beta_{23} u(c_3)\right]$$

subject to

$$c_1 + q_{12}c_2 + (c_3 - a_{com})\, q_{13} + p_{com}a_{com} \leq y_1 + q_{12}\bar{b}_2 + q_{13}\bar{b}_3 + p_{com}\bar{b}_3^{com}$$

$$c_3 \geq a_{com}$$

$$c_1, c_2, c_3, a_{com} \geq 0$$

Main Findings-Proposition 1

If $p_{\text{com}} < q_{13}$, it is optimal to set $c_3 = a_{\text{com}}$. If $p_{\text{com}} > q_{13}$, it is optimal to set $a_{com} = 0$. Thus, DP1 is equivalent to

$$(c_1, c_2, c_3) \in \arg \max_{(e_1, c_2, c_3)} \left[u(c_1) + \beta_{12} u(c_2) + \beta_{12}\beta_{23} u(c_3) \right]$$

subject to

$$c_1 + q_{12} c_2 + \min(p_{com}, q_{13})\, c_3 \leq y_1 + q_{12} \bar{b}_2 + q_{13} \bar{b}_3 + p_{com} \bar{b}_3^{com}$$

$$c_1, c_2, c_3 \geq 0$$

This problem has a convex constraint set and a strictly concave objective. All agents have the same solution, which means that in equilibrium they must consume autarky. The rest of the proposition follows from the fact that all assets are in positive supply.

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1. *Model: Time-Inconsistent Preferences in Asset Market*
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   - Decision Problems-Period 2 (DP2)
   - Decision Problems-Period 1 (DP1)
   - Equilibrium
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3. *Main Findings-Proposition 2*
4. *Main Findings-Proposition 3*
5. *Conclusion*

Proposition 2. If $\beta'_{23} < \beta_{23}$, then in any equilibrium, $p_{\text{com}} > q_{13}$

Proof. Suppose not, and let $\left(c^*, b_2^*, b_{13}^*, b_{23}^*, a^*\right), (q_{12}, q_{13}\ \ q_{23}, p_{com},$ be an equilibrium in which $p_{com} \leq q_{13}$. From market-clearing, we know that for some $b_{2,3}^{j*} > 0$. Then $\beta_{23} u'\left(c_3^{j*}\right) > \beta'_{23} u'\left(c_3^{j*}\right) = u'\left(c_2^{j*}\right) q_{23}$. Let $\left(c_2^{j\prime}, c_3^{j\prime}\right)$ be the solution to this problem:

$$\max_{c_2 j, e_3} \left[u\left(c_2^j\right) + \beta_{23} u\left(c_3^j\right)\right]$$

subject to

$$\begin{array}{l} c_2^j + q_{23} c_3^j \leq c_2^{j*} + q_{2s} s_3^{j*} \\ c_2^j, c_3^j \geq 0 \end{array}$$

since $\beta_{23} u'\left(c_3^{j*}\right) > u'\left(c_2^{j*}\right) q_{23}$, we know that $u\left(c_2^j\right) + \beta_{23} u\left(c_3^j\right) > u\left(c_2^{j*}\right) + \beta_{23} u\left(c_3^{j*}\right)$. We also know that $c_3^{j*} < c_3^{j\prime}$, which implies in tum that $a_{com}^{j*} < c_3^j$ Now I claim that this plan

$$\begin{aligned} c_1^{j'} &= c_1^{j*} \\ b_{12}^{j\prime} &= c_2^{j'} \\ b_{13}^{j\prime} &= 0 \\ a_{com}^{j'} &= c_3^{j'} \end{aligned}$$

lies in the constraint set of DP1, given equilibrium prices $(p_{\text{com}'}, q_{12}, q_{13}, q_{23})$. This is demonstrated by the following chain of logic:

Main Findings-Proposition 2

$$
\begin{aligned}
c_1^{j'} &+ q_{12} b_{12}^{j_{12}} + p_{com} a_{com}^{j'} \\
&= c_1^{j'} + q_{12} c_2^{j'} + p_{com} c_3^{j'} \\
&= c_1^{j'} + q_{12} c_2^{j'} + q_{13} s_3^{j'} + (p_{com} - q_{13})\, c_3^{j'} \\
&= c_1^{j'} + q_{12} c_2^{j'} + q_{13} c_3^{j*} + (p_{com} - q_{13})\, c_3^{j} \\
&= c_1^{j*} + q_{12} b_{12}^{j*} + q_{13} a_{0m}^{j*} + p_{20m} a_{13}^{j*}
\end{aligned}
$$

$$
\begin{aligned}
= c_1^{j*} + q_{12} b_{12}^{j*} + q_{13} b_{13}^{j*} + p_{com} a_{com}^{j*}& \\
&+ \left(p_{com}^{j,*} + q_{23} b_{13}^{j*}\right) + p_{com} a_{com'}^{j*} \\
&< c_1^{j*} + q_{12}\left(b_{12}^{j,*} + q_{23} b_{13}^{j*}\right) + p_{com} a_{com'}^{j*}
\end{aligned}
$$

The one inequality comes from the assumption that $p_{\text{com}} \leq q_{13}$ and the result that $c_3^{j\prime} > a_{com}^{j*}$ We also know that $\beta_{23}^{\prime} u^{\prime}\left(c_3^{j\prime}\right) q_{23}^{-1} < u^{\prime}\left(c_2^{j\prime}\right)$
This implies that $c_2^{*}\left(b_{12}^{j}, c_3^{j}/q_{23}\right) = c_2^{j\prime}$ and $c_3^{*}\left(b_{12}^{j\prime}, c_3^{j\prime}/q_{23}\right) = c_3^{j\prime}$. We have a contradiction: the plan $\left(c_1^{j}, b_{12}^{j}, b_{13}^{j\prime}, a_{com}^{j}\right)$ lies in the constraint set of DP 1 and delivers a higher value of the objective in DP1 than $\left(c_1^{j*}, b_{12}^{j*}, b_{13}^{j*}, a_{com}^{j*}\right)$

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Main Findings-Proposition 3

Proposition 3. Suppose that $\beta'_{23} < \beta_{23}$. Then in any equilibrium, $b_{23} a^j_{com} = 0$ for all j

Proof. Consider an arbitrary equilibrium $\left(c^*, b^*_{12}, b^*_{13}, b^*_{23} \quad a^*\right), \left(q_{12}, q_{13}, q_{23}, p_{com}\right)$. We know from Proposition 2 that $p_{com} > q_{13}$. Let $b^{j*}_{23} > 0$ and $a^{j*}_{com} > 0$ for some j. Then set $a^{j\prime}_{com} = a^{j*}_{com} - \varepsilon$, $b^{j}_{12}{}' = b^{j*}_{12} + \varepsilon q_{23}$, and $c^{j\prime}_1 = c^{j*}_1 + \varepsilon p_{com} = \varepsilon q_{13}$, with ε sufficiently small that all the primed variables are positive. Clearly, this new plan satisfies the budget constraint in DP1 and delivers more consumption in period 1. I claim that

$$c^*_t \left(b^{j\prime}_{12} + q_{23} b^{j*}_{13}, q_{23} a^j_{com} \right) = c^{j*}_t$$

for $t > 1$, so that this new plan also provides more utility to the agent in period 1 To prove my claim, we need to solve DP2:

$$\max_{c^j_2, b^j_{23}, c^j_3} \left[u\left(c^j_2\right) + \beta'_{23} u\left(c^j_3\right) \right]$$

subject to

$$\begin{aligned} & c^j_2 + q_{23} b^j_{23} = q_{23} b^{j*}_{13} + b^{j*}_{12} + \varepsilon q_{23} \\ & c^j_3 = b^j_{23} + a^{j*}_{com} - \varepsilon \\ & c^j_2, b^j_{23} \geq 0 \end{aligned}$$

If an element of the constraint set satisfies $\beta'_{23} u'\left(c^j_3\right) = u'\left(c^j_2\right) q_{23}$ then that element solves DP2. But because $b^{j*}_{23} > 0$, we know that $\left(c^{j*}_2, c^{j*}_3\right)$ satisfies this first-order condition. By setting $b^j_{23} = b^{j,*}_{23} + \varepsilon$, we can see that $\left(c^{j*}_2, c^{j*}_3\right)$ lies in the constraint set, too. It follows that $\left(c^j_1, b^j_{12}, b^{j*}_{13}, a^j_{com}\right)$ lies in the constraint set of DP1 and improves the period 1 utility of agent j

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Conclusion

- Because of dynamic arbitrage, there can be no evidence of time-inconsistency in the prices of one-period assets or in the prices of long-term,retradable assets.
- However,if people have time-inconsistent preferences,commitment assets are systematically overpriced, and people do not hold both them and retradable assets in equilibrium.
- If preferences are time-inconsistent. The government has a new role to play: it can improve current welfare by limiting future choices.
- The data analysis here demonstrates that this kind of strong evidence supporting time inconsistency is not currently available.