Advanced Micro II - Problem set 2
due date: June, 3rd

Problem 1 (2p) *A risk averse agent (entrapreneur) considers a project with investment I. Agents has no money and must get them from the principal. Return on investment is random and equal to $\bar{V}$ with probability $\pi(e)$ or $\underline{V}$ with probability $1-\pi(e)$, where $e \in \{0,1\}$ is agent's effort. Assume $\bar{V} > \underline{V}$ and $\pi(1) > \pi(0)$. A contract specifies $\{\bar{z}, \underline{z}\}$, i.e. an amount to give back as a function of project's outcome. Agents utility is $u(x) - e$, where x is his net income. Outside option is equal to $\tilde{u}$.*

- *write the principal's problem who wants to implement $e = 1$.*
- *characterize the optimal contract.*
- *now assume principal wants to implement $e = 1$ but under limited liability, i.e. such that only nonnegative payments are allowed (in all cases). What is the highest I, that he would like to lend? Compare with the FB.*

Problem 2 (2p) *For a continuum of actions prove that if $f(q|a)$ has MLRP then $\frac{f_a(q|a)}{f(q|a)}$ is increasing with q.*

Problem 3 (3p) *Consider the hidden information problem as analyzed during classes but with three types $\{\underline{\theta}, \hat{\theta}, \overline{\theta}\}$ with probabilities $\underline{\lambda}, \hat{\lambda}, \overline{\lambda}$ summing up to 1. For simplicity assume $\overline{\theta} - \hat{\theta} = \hat{\theta} - \underline{\theta} = \Delta\theta$. Redo the first and second best calculations. In the second best when analyzing (6) IC constraints think carefully which of them would be binding.*

Problem 4 (3p) *Consider the hidden information problem as analyzed during classes. Redo the first and second best calculations for the more general utility function of the agent: $u(w) - C(q,\theta)$, where u is strictly increasing, strictly concave and differentiable, while differentiable C satisfies: $C'_q > 0, C'_\theta > 0, C''_{qq} > 0, C''_{q,\theta} > 0, C'''_{q,q,\theta} > 0$. Check first, wether the Spence-Mirrlees conditions is satisfied.*