Game Theory - Problem set 2
Due date: April, 21st (by e-mail to jfrana@sgh.waw.pl)

Problem 1 (2p) *Consider a modified game of partnership with independent investment profitabilities.*

		Player 2	
		Invest	Not
Player 1	Invest	(θ_1, θ_2)	$(\theta_1 - 1, 0)$
	Not	$(0, \theta_2 - 1)$	$(0, 0)$

Assume that player 1 has only one type $\theta_1 \in (0, 1)$*, while player 2 has two possible types* $\theta_2^H = \frac{3}{2}$ *and* $\theta_2^L = -\frac{1}{2}$ *with prior prob.* $1/2$.

- *model this game as a Bayesian game with ex-ante payoffs as expected values,*
- *determine pure-strategy BNE, depending on* θ_1.

Problem 2 (4p) *Consider the following modification to the classic Bach or Stravinsky (a.k.a. Battle of the Sexes) game: the two players still want to meet, but neither player knows whether the other party prefers Bach or Stravinsky (the prior probability of each of four outcomes is* $1/4$*).*

- *model this game as a Bayesian game with ex-ante payoffs as expected values,*
- *find all pure-strategy BNE of this game,*
- *verify that there are equilibria in which players do not go to the same concert with positive probability.*

Problem 3 (4p) *Consider a game of public good provision: each of the two players can contribute at a private cost* c_i *or not contribute (at zero cost). The good is provided if at least one of the players contribute and brings utility 1 (minus cost, if any) to everyone. The players cost is i.i.d. and drawn from* $Unif[0, 2]$ *distribution.*

- *Write the game* with observable (c_1, c_2) *in a matrix form and define a Bayesian Nash Equilibrium of the game*
- *Prove that in any BNE the strategies* $s_i(c_i)$ *must be cutoff strategies i.e.*

$$s_i(c_i) = \begin{cases} Contribute & \text{if } c < \bar{c} \\ Not & \text{if } c > \bar{c} \end{cases}$$

- *Find the cutoffs that define a BNE. Is the equlibrium unique?*
- *Can any of the strategies be removed as dominated? What would be the result if the process?*