Advanced Micro II - Problem set 3
due date: June, 6th

Problem 1 (4p) *Suppose that there are 3 periods (dates). In each of periods* 0 *and* 1*, a student can either study (denoted* s*) or play (denoted* p*). In period* 2 *he makes no choice, but he takes an exam and receives a score that equals the number of periods he had studied. That is, the maximum score is* 2*: if he does not study at all, he gets* 0*; if he studies one period, he gets* 1 *etc.*

Now suppose the student's preferences in periods 0 *and* 1 *respect a dynamic* $\beta - \delta$ *model with the following specifications:* $\beta = 0.5, \delta = 0.5$ *and the within period utility of studying is* -4 *while that if play* 4*. The final date (within period) utility of score is* $u(0) = -18, u(1) = 0, u(2) = 40$.

- *write all feasible plans (triples of possible decision (date 0 and 1) and scores (date 3))*
- *find the naive solution (realized decision plan and utility value)*
- *write all time-consistent plans*
- *find the sophisticated solution. Compare it with the naive solution.*

Problem 2 (4p) *The couple consumes two commodities: a private good* C_i *and a public good* Q*, which is domestically produced from parental time according to a Cobb-Douglas (CD) function* $Q = (t_1 t_2)^\alpha$ *- a typical example being children's welfare or their human capital (HC). Individual preferences are CD with*

$$u_i(C_i, Q) = C_i Q.$$

Agents only differ by their HC: H_i*; the time not devoted to children,* $1 - t_i$*, is spent on market work for a wage* $w_i = W H_i$*. The couple budget constraint is therefore given by:*

$$C_1 + C_2 + w_1 t_1 + w_2 t_2 = w_1 + w_2.$$

Consider this as a matching model with TU, i.e. any (interior) efficient solution household allocation must maximize the sum of utilities.

- *find such (interior) efficient household allocation, i.e.* C_1*,* C_2 *and* t_1 *and* t_2 *(HINT: there is some indeterminacy in individual consumption levels but aggregate consumption is fixed)*
- *compute the surplus function* $f(H_1, H_1)$*, i.e. efficient sum of utilities (HINT: this is a value to the above program)*
- *verify whether the stable (but also optimal) matching is PAM or NAM (or it perhaps depends on the parameters of the model).*

Problem 3 (5p) *Here we consider an important variation of the above TU matching model. Let* $u_i(L_i, Q) = L_i Q$ *be given with* L_i *denoting leisure. Now the agents only differ in their wages (not income but income-generation capacity)* w_i*. We assume the efficient solution maximize the* weighted *sum of utilities i.e.*

$$u_1 + \mu u_2$$

under budget constraint $Q + w_1 L_1 + w_1 L_2 = (w_1 + w_2)T$*, where* T *is a total time available.* μ *is a Pareto weight that will be determined (that would generally depend on wages).*

- *write the FOC for* interior*, efficient household allocation. Determine* μ*.*
- *using this* μ *find the optimal allocation, i.e.* Q *as well as* L_1 *and* L_2 *(again there is some indeterminacy in* L_1 *and* L_2*).*
- *compute the value of the above problem (ie. the surplus function* $f(w_1, w_2)$*).*
- *write the efficiency frontier* $u_1 = \phi(w_1, w_2, u_2)$*, i.e. pairs of* (u_1, u_2) *s.t.* $u_1 + \mu u_2 = f(w_1, w_2)$*.*
- *determine appropriate partial derivatives of* ϕ *to verify generalized ID condition.*
- *is the stable matching PAM /NAM / it depends?*