Game Theory - Problem set 4
Due date: classes on June, 4th

Problem 1 (2.5p) *Analyze the example that was briefly sketched during class about a used car market (due to Akerlof). There is a continuum of used cars in the market, with quality* $q \sim U[0,1]$, *that is known only to the sellers. The sellers' valuation of a car of quality* q *is* $v_S(q) = q$, *while the buyers' valuation is* $v_B(q) = \frac{3}{2}q$.

- *If the quality was observable, what would be the total trade?*
- *Suppose the quality is seller's private infromation. Let* $\Omega(p)$ *denote the set of cars offered at price* $p \in [0,1]$ *(given sellers behave optimally). Plot* $E(v_B|p) = \frac{3}{2}E(q|q \in \Omega_p)$ *as a function of* p. *How do you interpret* F *and its fixed point?*
- *Assume market participants encounter price* $p_0 = \frac{1}{2}$. *How would sellers react? Calculate the buyers' maximal acceptable price* p_1 *that is a response to seller's adaptation to* p_0. *Construct in a similar manner* p_2,p_3. *What is* $\lim_{n\to\infty} p_n$*?*

Problem 2 (2.5p) *There is a single firm that offers a good and a single consumer. The good could be of high quality – bringing utility of* u_H *to the buyer – or of low quality with* u_L. *The buyer does not observe the product quality. Assume that the firm's cost of production depends on quality (note, however, that quality is not a choice variable!). The product price is set exogenously at* p, *and following inequliaties hold* $u_H > p > u_L > c_H > c > L$.

- *Given* p, *what is the buyer's condition for purchasing the product?*
- *Suppose the firm can advertise, i.e. spend some amount* A, *that is observable. Show that there does* not *exist a separating equilibrium, in which the firms with different qualities choose different levels of advertising.*

Problem 3 (2.5p) *Consider an infinitely repeated game, in which both players discount future payoffs with discount factor* $\frac{1}{2}$. *The strategic form game played every period is given by:*

	A	*D*
A	*2,3*	*1,5*
D	*0,1*	*0,1*

Show that $((A,A),(A,A),\ldots)$ *is not the decision path on any SPNE.*

Problem 4 (2.5p) *Consider the first price auction with 2 players. Valuations are private and independent. Before the auction each player observes random variable* t_i, *drawn independently from the uniform distribution on* $[0,1]$. *Then his/her valuation is* $v_i = t_i + 0.5$. *Then after betting* $b_i \geq 0$, *the payoffs are the following:*

$$u_i(b_1,b_2,t_1,t_2) = \begin{cases} (t_i + 0.5) - b_i & \textit{if} \quad b_i > b_{-i}, \\ 0 & \textit{if} \quad b_i < b_{-i}, \\ \frac{1}{2}(t_i - b_i + 0.5) & \textit{if} \quad b_i = b_{-i}. \end{cases}$$

Find NE of this game in linear strategies, i.e. find numbers α *and* β *such that* $b_i = \alpha t_i + \beta$ *is the equilibrium strategy. What is the expected payoff of player* i, *who draws* t_i*?*