Let's introduce the following notation:

$$\begin{array}{cc} s_1 & \text{(sun, wind)} \\ s_2 & \text{(rain, wind)} \\ s_3 & \text{(sun, no wind)} \\ s_4 & \text{(rain, no wind)} \end{array}$$

Based on the information given, we will try to find a set-valued function $v(\cdot)$ which satisfies the following conditions:

- It is a capacity, i.e. it satisfies
  1. $v(\emptyset) = 0$;
  2. $A \subset B \;\Rightarrow\; v(A) \leq v(B)$, for $A, B \subset S$;
  3. $v(S) = 1$.
- It is a convex capacity, i.e. it satisfies

$$v(A) + v(B) \leq v(A \cap B) + v(A \cup B), \quad \text{for } A, B \subset S.$$

We first note that according to the problem statement the capacity satisfies:

states in the subset	capacity value
$1,3$	0.3
$2,4$	0.4
$1,2$	0.5
$3,4$	0.2
$2,3,4$	0.6
$1,2,3$	0.9

We first check the conditions for being a capacity. We concentrate only on the conditions relevant for our analysis, for example below we state conditions for one-element subsets of S only. It is rather straightforward to infer that the following needs to be satisfied:

$$\begin{aligned} 0 \leq v(s_1) &\leq 0.3 \\ 0 \leq v(s_2) &\leq 0.4 \\ 0 \leq v(s_3) &\leq 0.2 \\ 0 \leq v(s_4) &\leq 0.2 \\ v(s_1, s_2, s_3, s_4) &= 1 \end{aligned}$$

Now we check conditions for a convex capacity, also concentrating only on the conditions relevant for our analysis. There are plenty of these conditions which may be relevant, so I will only give two examples of convexity conditions here and after that immediately give the final set of conditions which are obtained by cross-checking all the conditions together. Here is an example for one pair of subsets of S:

$$\begin{aligned} v(s_2, s_4) + v(s_1, s_2, s_3) &\leq v(S) + v(s_2) \\ v(s_3, s_4) + v(s_1, s_2, s_3) &\leq v(S) + v(s_3) \end{aligned}$$

Which gives $v(s_2) \geq 0.3$ and $v(s_3) \geq 0.1$. We proceed the same way for all combinations of subsets. After observing that the conditions do not specify the unique convex capacity, we shall make one additional simplification in order to make the problem easier - we will try to find a minimal capacity that satisfies all of the convexity and capacity conditions (by minimal capacity we mean, the minimal values of the capacity). After somewhat tedious examination of all possible combinations of subsets, we get these capacity values for one-element subsets:

$$\begin{aligned} v(s_1) &= 0 \\ v(s_2) &= 0.3 \\ v(s_3) &= 0.1 \\ v(s_4) &= 0 \end{aligned}$$

Given this information we can now find a Choquet Expected Utility for the two acts which we consider:

$$\begin{aligned} CEU(\text{windsurf}) &= v(S)u(w_3) + v(s_1, s_2)[u(w_2) - u(w_3)] + v(s_1)[u(w_1) - u(w_2)] \\ &= 1 \times 0 + 0.5 \times 0.6 + 0 \times 0.4 = 0.3 \\ CEU(\text{beach}) &= v(S)u(b_3) + v(s_1, s_3)[u(b_2) - u(b_3)] + v(s_3)[u(b_1) - u(b_2)] \\ &= 1 \times 0 + 0.3 \times 0.9 + 0.1 \times 0.1 = 0.28 \end{aligned}$$

So according to Choquet expected utility, windsurf is preferred to beach. According to the Schmeidler (1963) theorem we could obtain the same by minimizing Expected Utility value over the set of probabilities that are in the core of a convex game defined by our convex capacity. Below we present the optimization problem for windsurf:

$$\begin{aligned} & \min_{\mathbf{p} \in Core(v)} \mathbb{E}u(\text{windsurf}) \\ = & \min_{\mathbf{p} \in Core(v)} p_1 u(w_1) + p_2 u(w_2) + (p_3 + p_4) u(w_3) \\ = & \min_{\mathbf{p} \in Core(v)} p_1 + 0.6 p_2 \end{aligned}$$

and for beach:

$$\begin{aligned} & \min_{\mathbf{p} \in Core(v)} \mathbb{E}u(\text{beach}) \\ = & \min_{\mathbf{p} \in Core(v)} p_3 u(b_1) + p_1 u(b_2) + (p_2 + p_4) u(b_3) \\ = & \min_{\mathbf{p} \in Core(v)} p_3 + 0.9 p_1 \end{aligned}$$

The constraint that $\mathbf{p}$ is in the core of v, can be written as:

$$\begin{aligned} p_1 &\geq 0 \\ p_2 &\geq 0.3 \\ p_3 &\geq 0.1 \\ p_4 &\geq 0 \\ p_1 + p_3 &\geq 0.3 \\ p_2 + p_4 &\geq 0.4 \\ p_1 + p_2 &\geq 0.5 \\ p_3 + p_4 &\geq 0.2 \\ p_2 + p_3 + p_4 &\geq 0.6 \\ p_1 + p_2 + p_3 &\geq 0.9 \\ p_1 + p_2 + p_3 + p_4 &= 1 \end{aligned}$$

Since both optimization problems are linear programming problems, we can solve it using linear programming tools. I used Excel Solver whoich produces the following results:

Problem	p_1	p_2	p_3	p_4	$\min \mathbf{E}u$
Windsurf	0	0.5	0.5	0	0.3
Beach	0.2	0.6	0.1	0.1	0.28

The solutions given above are not necessarily unique. Nevertheless it is not what is being asked for. Anyway we know that all possible solutions have the same objective function value, and that is what we wanted to verify. The results obtained are ineed the same which we got from Choquet Expected Utility. They are 0.3 for the windsurf and 0.28 for the beach.