Advanced Microeconomics (QEM) - Problem set 1
Due date: Wednesday, October 20th (in class)

Problem 1 (2p) *For the quasi-linear utility function $u(x) = x_1 + \sqrt{x_2}$ analyzed in class compute the Hicksian demand and the expenditure function.*

Problem 2 (1.5p) *Prove proposition 2.3.9 from the Lecture notes.*

Problem 3 (0.5p) *Problem 3.B.2 from MWG*

Problem 4 (0.5p) *Problem 3.B.3 from MWG*

Problem 5 (2p) *Consider household with preferences given by:*

$$u(x_1, x_2, \dots, x_T) = \sum_{t=1}^{T} \beta^{t-1} \frac{x_t^{1-\sigma}}{1-\sigma},$$

with $\beta \in (0, 1]$ and $\sigma > 0, \neq 1$ facing prices $p_1, p_2, \dots, p_T$ and income w (that has to be used during the whole T-period lifetime).

1. *find demand*
2. *find the value function (indirect utility function)*
3. *write expenditure minimization problem and find Hicksian demand*
4. *find the expenditure function*

Problem 6 (2p) *Consider the following utility function:*

$$u(x) = \sum_{i=1}^{2} \alpha_i \log(x_i), \text{ where } \alpha_1 = 1/3, \alpha_2 = 2/3.$$

Let $w = 5, p = (1, 1)$. Assume prices have changed to $p' = (1, 2)$.

1. *Compute income and substitution effect of a price change on demand of good 2 (use Hicksian decomposition).*
2. *Compute income and substitution effect of a price change on demand of good 2 (use Slutzky decomposition).*

Problem 7 (4.5p) *Consider the following utility function, called CES (constant elasticity of substitution function):*

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}, \text{ where } 0 \neq \rho < 1.$$

This function is obviously strictly increasing. For the above CES function:

1. *Show that u is strictly quasiconcave.*
2. *Formulate the Utility Maximization Problem and solve it to derive demand $d_1(p, w)$ and $d_2(p, w)$ [You may assume interior solution and Walras law which follows from strict monotonicity of $u(\cdot)$].*
3. *Form the indirect utility function $v(p, w)$.*
4. *Show that the indirect utility function is:*
   (a) *homogeneous of degree zero in (p, w),*

(b) *increasing in w and decreasing in p.*

5. *Verify the Roy's identity.*

6. *Formulate the Expenditure Minimization Problem and solve it to derive Hicksian demand $h(p, \underline{u})$ [You may assume interior solution and no excess utility which follows from continuity of $u(\cdot)$]*

7. *Form the expenditure function $e(p, \underline{u})$.*

8. *Verify that the expenditure function is concave in p.*

9. *Verify the Shephard's lemma.*