

Problem 1 (2p) Consider a modified game of partnership with independent investment profitabilities.

		Player 2	
		Invest	Not
Player 1	Invest	(θ_1, θ_2)	$(\theta_1 - 1, 0)$
	Not	$(0, \theta_2 - 1)$	$(0, 0)$

Assume that player 1 has only one type $\theta_1 \in (0, 1)$, while player 2 has two possible types $\theta_2^H = \frac{3}{2}$ and $\theta_2^L = -\frac{1}{2}$ with prior prob. $1/2$.

- model this game as a Bayesian game with ex-ante payoffs as expected values,
- determine pure-strategy BNE, depending on θ_1 .

Problem 2 (4p) Consider the following modification to the classic Bach or Stravinsky (a.k.a. Battle of the Sexes) game: the two players still want to meet, but neither player knows whether the other party prefers Bach or Stravinsky (the prior probability of each of four outcomes is $1/4$).

- model this game as a Bayesian game with ex-ante payoffs as expected values,
- find all pure-strategy BNE of this game,
- verify that there are equilibria in which players do not go to the same concert with positive probability.

Problem 3 (4p) Consider a game of public good provision: each of the two players can contribute at a private cost c_i or not contribute (at zero cost). The good is provided if at least one of the players contribute and brings utility 1 (minus cost, if any) to everyone. The players cost is i.i.d. and drawn from $Unif[0, 2]$ distribution.

- Write the game in a matrix form and define a Bayesian Nash Equilibrium of the game
- Prove that in any BNE the strategies $s_i(c_i)$ must be cutoff strategies i.e.

$$s_i(c_i) = \begin{cases} \text{Contribute} & \text{if } c < \bar{c} \\ \text{Not} & \text{if } c > \bar{c} \end{cases}$$

- Find the cutoffs that define a BNE. Is the equilibrium unique?
- Can any of the strategies be removed as dominated? What would be the result if the process?