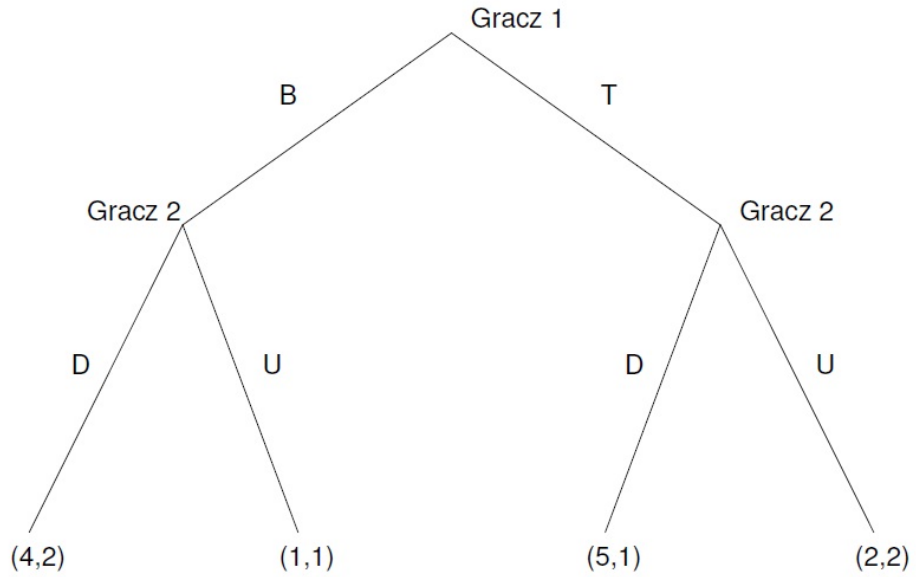
Game Theory - Problem set 2
Due date: classes on November, 21th

Zadanie 1 (2p) *Consider the following game:*



(i) Find all SPNE. Is it unique? Are there other NE (pure or mixed)?

(ii) Assume player 2 does not observe player's 1 moves. Write the new extensive form game. What are the NE of the modified game (mixed or pure)?

Zadanie 2 (2p) *Consider an oligopolistic market with strategic investments. There are two firms both with constant unit costs 2. Firm 1 can invest f in the new technology, that allows to produce at zero unit costs. Firm 2 observes the first firm decision. After investment is taken (or not) by the first firm both firms decide on their outputs: q_1 and q_2 like in standard Cournot model. So it is a two stage game. Assume that demand is given by $p(Q) = 14 - Q$, and each firm aim is to maximize own profit.*

(i) Write the profit function of the first firm for both cases: with and without investment.

(ii) Find Cournot-Nash equilibrium in the second stage for both cases. Write equilibrium profits in both cases.

(iii) Then find SPNE. How does it depend on cost f? When the first firm decides to investment in the SPNE?

Zadanie 3 (2p) *There is a oligopolistic market with 7 firms. The first one with costs $TC_1(q_1) = \frac{1}{3}q_1^3 + 2q_1 + 5$, dominated the market and become a leader. Costs of other firms are: $TC_i(q_i) = \frac{1}{2}q_i^2 + 20$, $i = 2, \ldots, 7$. demand is given by $Q(p) = 800 - 4p$. Find productions levels of each firm in the SPNE, assuming that they compete by setting quantities.*

Zadanie 4 (2p) *Consider a game with two players: the government and representative household. Household can choose two action $a_h \in \{0, 1\}$ and get transfer $t \in \{0, 1\}$. Household's aim is to maximize transfer minus cost of action (0 for $a_l = 0$, and $\frac{1}{2}$ for $a_h = 1$). The government's aim is to max $2(a_h - 1)^2 + t$. Before household takes its decision the government declares a transfer scheme $t(a_h)$, i.e. transfer as a function of action taken, e.g. a_h.*

(i) Draw a game-tree for the extensive form game assuming the government's decisions are binding, i.e. declared transfer scheme cannot be changed.

(ii) Draw a game-tree for the extensive form game assuming the government's decisions are not binding and can be changed after household takes its action.

(iii) Find all SPNE for both games.

Zadanie 5 (2p) *There are two players: seller and buyer that interact for two periods. In period* 1 *seller decides on investment level* I*, which cost* $I \geq 0$*. In period* 2*, seller can sell one unit of good to the buyer, but the transportation cost is* $c(I)$*, where* $c'(0) = -\infty$*,* $c' < 0$*,* $c'' > 0$*, and* $c(0) < v$*, with* v *denoting buyer's valution. There is no discounting, and socially optimal level of investment* I^* *satysfies* $1 + c'(I^*) = 0$*.*

(i) Assume that in period 2 *buyer observes* I *and offers* take-it-or-leave-it *offer to the seller. What is the buyer's offer? What is the SPNE of this game?*

(ii) Propose a contract, that can be signed in period 1*, such that the optimal allocation is reached (assume that contract's value cannot depend on* I*).*