On complementary symmetry and reference dependence*

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Abstract

In this paper we reevaluate the validity of complementary symmetry and the supporting experimental evidence. We first show how to generalize the complementary symmetry hypothesis for arbitrary prospects in $\mathbb{R}^N$. Second, we show how to elicit selling prices that are consistent with the complementary symmetry hypothesis. This task is new and we refer to it as a short selling price elicitation. Third, we run an experiment to verify the complementary symmetry hypothesis in this new setting. Our arguments are important as they question validity of previous experimental findings concerning complementary symmetry as well as shed some new light on various models of setting the appropriate reference acts.

Keywords: complementary symmetry; short selling price; buying price; reference dependence.

1 Introduction

Birnbaum and Zimmermann (1998) and Birnbaum et al. (2016) formulated a complementary symmetry (CS) hypothesis for binary monetary prospects. This hypothesis holds if the sum of the buying price of the prospect in which $\$x$ is paid out with probability p and $\$y$ otherwise and the selling price of the complementary prospect in which $\$x$ is

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paid out with probability $1-p$ and $\$y$ otherwise equals $x+y$. In this paper, we first extend this hypothesis to arbitrary (monetary) prospects in $\mathbb{R}^N$, i.e. with multiple instead of binary outcomes and possibly without a well-defined probability distribution. For this reason we will henceforth refer to a prospect as the real-valued random variable defined on some finite state space N. Now, let L be a prospect and suppose $-L$ is the same as L but with prizes being the negative of those in L. Note that "relinquishing L" is consequentially equivalent as "acquiring $-L$", just as losing 100 dollars is the same as receiving -100 dollars. We will refer to this basic fact as the symmetry. In this context, our general question is how to use this symmetry in eliciting appropriate buying and selling prices, next test CS experimentally, and finally validate some of the rules of setting the relevant reference points. When doing so we need to address two challenges that we discuss now.

First, we need to relate the actual elicitation tasks with the appropriately chosen buying and selling prices. The propensity to acquire a prospect is captured by the task of eliciting its buying price i.e. the maximal monetary amount the decision maker is willing to pay to acquire the prospect. Our first question is: what is the corresponding task that would capture the propensity to relinquish a prospect in a way that would allow us to use the symmetry. The first natural candidate is elicitation of a selling price, i.e. the minimal monetary amount the decision maker is willing to accept to relinquish a prospect. Note however, that selling, as opposed to buying, implicitly assumes that the decision maker initially owns (the right to) the prospect. This difference in initial position may impact prospect valuation, especially when it is uncertain, and thus destroy the symmetry between the two tasks. We thus borrow the notion of short-selling, well known in finance in the context of financial assets. Short-selling a prospect, also known as taking a short position in it, means selling the right to the prospect without actually having it at the time of transaction. In our context, selling and short-selling can be also distinguished by the person pay out the prizes. In case of selling, you sell the right to the prospect whose prizes will paid out by a third party. In case of short-selling, on the other hand, the right is sold to the prospect whose prizes will be paid out by yourself.

Second, since we are interested in experimental testing, we should design the price elicitation tasks in a way that is understood by human subjects. Specifically, we want to ensure the prospects we ask people to consider buying or short-selling present positive value to them (such that they prefer having it to not having it). Otherwise they might have difficulty in grasping the idea of paying or receiving a negative amount which would correspond to the value of a undesirable prospect. While it is safe to assume that a prospect L containing only nonnegative prizes presents a nonnegative value to almost anybody, it is just as certain that almost anybody would find its negative – prospect $-L$ – containing only nonpositive values desirable. So before we ask people to consider buying or short-selling $-L$ we will shift all of its prizes upwards to make sure they find it attractive. We will do it in a way that controls for possible framing effects to minimize their possible impact on the symmetry.

Studies on the complementary symmetry hypothesis were started by Birnbaum and Zimmermann (1998). Specifically, they defined an extension of cumulative prospect theory in which the reference point is allowed to be random (the idea later adopted by Schmidt et al., 2008 in their third-generation prospect theory, TGPT), defined buying and selling prices in this model and obtained the complementary symmetry property relating these prices. Birnbaum et al. (2016) has shown that the CS property holds for binary prospects under the extended version of prospect theory using some popular parametric assumptions. On the other hand, the property has been shown to fail in experimental settings (Birnbaum and Sutton, 1992; Birnbaum and Zimmermann, 1998; Birnbaum et al., 2016), the fact used by Birnbaum (2018) to question TGPT. Since then in a number of subsequent theoretical work (Lewandowski, 2018; Chudziak, 2020; Wakker, 2020), the CS property has been shown to hold much more generally than implied by Birnbaum et al. (2016). In particular, Wakker (2020) has shown that it holds for any binary relation on any subset of binary prospects, implying that the CS property is not a property of TGPT but a property of buying and selling prices as defined by Birnbaum and Zimmermann (1998), irrespectively of preferences.

Taking this background into account, the contribution of our paper is fourfold. First, the original CS property was only defined for probability distributions of binary prospects. In this paper we generalize it to *multiple outcome prospects*, with or without probability distribution. Thus our approach is suitable for decisions under risk (preferences over exogenously given probability distributions), but also naturally extends to the context of uncertainty or ambiguity, in which case probability distributions may either be unknown or not even well-defined. Second, the Birnbaum and Zimmermann (1998) and Schmidt et al. (2008) version of prospect theory defines the selling price of a prospect L as a scalar S satisfying the following indifference $S - L \sim 0$, where $\sim$ denotes the symmetric part of a preference relation over prospects.[1] We claim that this condition does not represent the *selling price*, but rather the *short-selling price* of a prospect. This parallels our previous discussion on defining a complementary (symmetric) task to buying. Consequently, one may question the validity of previous experimental tests of the CS property. In this paper we design the task of short-selling and experimentally test the CS property in this new setting. This is our third contribution. Fourth and finally, in order to relate formulas such as $S - L \sim 0$ with the corresponding testable task, we need to show how the choice alternatives such as $S - L$ or 0, defined over wealth changes, are derived from a more basic choice alternatives over wealth levels by applying an explicit reference point rule. As a consequence, we show how the CS property can be used to test various rules of setting reference points in such richer reference dependent models.

2 Generalized complementary symmetry

N is a a finite set of states. Object of choice are prospects – state contingent real-valued outcomes denoted by $L \in \mathbb{R}^N$. If there is an objective probability measure on N, then objects of the form $(x, y; p)$ would denote the probability distribution of a prospect with values x, y.[2] For a scalar λ and prospect L, we write $L + \lambda$ to denote $L + \lambda\mathbb{1}$, where $\mathbb{1}$ is a unit vector in $\mathbb{R}^N$. Assume that preferences are given by some binary relation $\succsim$ on the set of prospects.

Following the literature, the buying price of prospect L, denoted by $B(L)$, and the

[1] $S - L$ and 0 denote prospects with prizes equal to, respectively, $S - L(n)$ and 0 for each state $n \in N$.

[2] Given prospect $L : N \to \mathbb{R}$ and a probability measure π on 2^N, one derives the probability distribution P_L of L by setting $P_L(x) = \sum_{n \in N : L(n) = x} \pi(n)$ for any $x \in \mathbb{R}$. Decisions are under risk if the decision maker is indifferent between any two prospects that have identical probability distributions. Otherwise, i.e. either if there is no exogenously given probability measure π, or preferences depend on states, we refer to decisions under uncertainty or ambiguity.

complementary selling price of L, denoted by $B^*(L)$, are scalars satisfying:

$$L - B(L) \sim 0, \tag{1}$$

$$B^*(L) - L \sim 0. \tag{2}$$

In what follows we will assume that for any prospect L, $B(L)$ and $B^*(L)$ exist and are unique[3]. Then, it is immediate to observe that both B and B^* satisfy the translation invariance property, i.e. for any prospect L and $\lambda \in \mathbb{R}$, $B(L + \lambda) = B(L) + \lambda$ (same for B^*) as well as the symmetry[4]: $B^*(-L) = -B(L)$. We say that a pair of prospects (L', L'') are perfect hedges if they satisfy $L' + L'' = \theta$, for some scalar θ. Note that L' and L'' exhibit maximal negative correlation between each other: accepting a portfolio of L' and L'' removes the risk completely. We now state our first result.

Proposition 1. *For a pair of perfect hedges (L', L'') such that $L' + L'' = \theta$ for some scalar θ the following holds:*

$$B(L') + B^*(L'') = \theta. \tag{3}$$

Proof. Using the symmetry property of B and B^* and the translation invariance of B^*, we have $0 = B(L) - B(L) = B(L) + B^*(-L) = B(L) + B^*[\theta - L] - \theta$, thus obtaining (3). □

Next observe that, if (L', L'') are perfect hedges, they can be written as $L' = L + \lambda$, $L'' = \theta - \lambda - L$ for any $\lambda \in \mathbb{R}$ and any prospect L. This observation allows us to state our main result on generalized complementary symmetry property.

Corollary 1 (Generalized Complementary Symmetry)**.** *Let L be a prospect. The following holds for any pair of scalars λ, θ:*

$$B(L + \lambda) + B^*(\theta - \lambda - L) = \theta. \tag{4}$$

[3]Generalizations including non-uniqueness are possible. See Wakker (2020) for relevant results and ideas that can be extended to prospects in $\mathbb{R}^N$.

[4]Both follow from similar reasoning as in Chudziak (2020).

Proof. Note that the prospects $L + \lambda$ and $\theta - \lambda - L$ are perfect hedges for any pair of scalars θ, λ and hence the result is true by a direct application of Proposition 1. $\square$

This result is important for numerous reasons. First, it generalizes the previous results on CS in the literature from probability distributions of binary prospects to prospects over $\mathbb{R}^N$. Indeed, for $N = 2$ taking for example $L = (x, y)$ and letting $\theta = x + y$ with $\lambda = 0$ we obtain the standard CS. Second, the corollary shows that the complementary symmetry property follows immediately from general properties of translation invariance and symmetry. Third, as it is clear from the construction, complementary symmetry does not depend on the (existence of) the underlying probability distribution. This suggest its validity for arbitrary situations involving uncertainty or ambiguity. In what follows we will propose a way of selecting λ and θ so that complementary symmetry can be appropriately tested.

2.1 Defining the right task

To use (3) or (4) for testing, we need to propose an experimental task for eliciting prices that are represented by B and B^*. Put differently, we ask a question which observable choice tasks correspond to (3) or (4).

Birnbaum and Zimmermann (1998) and later Schmidt et al. (2008) argued that $B(L)$ in (1) represents the buying and $B^*(L)$ in (2) the selling price of L. While we agree with the first part concerning $B(L)$, we believe that (2) represents the task of *short-selling* rather than *selling*. $B^*(L)$ is thus the short-selling (rather than the selling) price of L, i.e. the minimal price that the decision maker would accept to take the short-position in L. We will now provide theoretical (this section) as well as experimental (next section) arguments in favor of our hypothesis.

The intuition is simple. Consider a (gain) prospect L'. You like its positive prizes but dislike the risk (of getting less than its maximal value). The way to eliminate this risk is to accept L' jointly with another gain prospect L'' such that $L' + L'' = \theta$ for some scalar θ, i.e. (L', L'') are perfect hedges. Suppose that we agree that the task of determining the buying price is represented well by (1) but are not sure which observable (and testable) task corresponds to (2). Suppose we are looking for the complementary task to buying L' that would eliminate the risk just as hedging, i.e. we are looking for B^* satisfying $\theta = B(L' + L'') = B(L') + B^*$. We will derive it from generalized complementary symmetry. For this reason consider buying $-L''$ and observe this task is the same as short-selling L'', and so we set $B^* := -B(-L'')$. Substituting it into (1) we get $0 \sim -L'' - B(-L'') = B^* - L''$. This is precisely the definition of $B^*(L'')$ in (2), and so by Proposition 1 we obtain the desired $\theta = B(L) + B^*$.

2.2 Experimental design

Note that properties (3) and (4) hold irrespective of the value of θ or/and λ. However, it has been demonstrated that framing effects may have an impact on the valuation of a prospect (Sayman and Öncüler, 2005). In particular, the relative attractiveness of prospects may change depending on the choice task being presented either in the loss or in the gain frame (McClelland and Schulze, McClelland and Schulze; Irwin, 1994). As argued in the introduction it is semantically awkward to ask for a buying or short-selling price of something unwanted, i.e. the decision maker would prefer to opt out even if the object was for free. This is the case of a loss prospect (having non-positive outcomes only): experimental results document many subjects choose corner solution in this case, to reflect their lack of acceptance of such a task *per se*. Mixed prospects are also problematic as judging their attractiveness depends on the decision maker attitude to gains vs. losses (a given mixed prospect may seem better than the status quo for one decision maker and worse for another). On the other hand, it is relatively safe to assume people like money so that they would not turn down the offer in which they cannot lose. These two observations a.o. suggest we shall use gain prospects when designing the experimental tasks of eliciting prices. We also want to control for possible range[5] effects that are known to have potential effects on the evaluation of prospects (Mellers et al., 1992; Kontek and Lewandowski, 2018).

Recall that B and B^* are shift-invariant so shifting per se of all the outcomes of the

[5]By range we mean in this paper the minimal and the maximal element of a prospect rather than the image of the domain of the prospect.

evaluated prospects does not destroy the symmetry. However, we want to choose the values by which we shift the outcomes in a way that controls for the framing effects discussed above. For this reason we choose the values of θ and λ such that $L' = L + \lambda$ and $L'' = \theta - \lambda - L$ are both gain prospects and their ranges coincide, i.e. $\max(L') = \max(L'')$, $\min(L') = \min(L'') \geq 0$.[6] It is easily verified that the latter two conditions imply $\theta - 2\lambda = \max(L) + \min(L)$ and the former imply $\lambda \geq -\min(L)$ and $\theta - \lambda \geq \max(L)$.

For example, suppose L is a gain prospect, then setting $\lambda = 0$ gives $\theta = \max(L) + \min(L)$ and $L' = L$, $L'' = \max(L) + \min(L) - L$, precisely the choice of prospects used by Birnbaum and Zimmermann (1998) and others. If L is not a gain prospect, one may set $\lambda = -\min(L)$ and hence $\theta = \max(L) - \min(L)$ and $L' = L - \min(L)$ and $L'' = \max(L) - L$. To see that follow the example. For $N = 2$ (like in Birnbaum et al. (2016)) and $L' = (48, 60)$ we have $\theta = 108$ and $L'' := \theta - L' = (60, 48)$. For $N = 4$ with $L' = (80, 100, 120, 200)$ we should impose $\theta = 280$ and $L'' := \theta - L' = (200, 180, 160, 80)$.

3 Implications for reference point rules

So far we have argued that the difference between selling and short selling prices is the initial position (possession). To observe and analyze that explicitly in formulas (1) and (2) we need to consider *wealth levels* and not only their changes. Specifically, until now preferences were defined on prospects, i.e. state contingent acts where prizes are interpreted as *changes of wealth* relative to *some* reference point wealth. Clearly, wealth changes depend critically on such reference wealth levels. In this section, we analyze more formally, how different tasks (e.g. of eliciting selling or short selling prices) differ with respect to various reference point setting rules. In particular, we focus on reference point rules that imply preferences satisfying the complementary symmetry hypothesis.

Formal models of reference dependence have been developed theoretically by Sugden (2003), used by Schmidt et al. (2008) and tested by Baillon et al. (2020), a.o. Even though Sugden (2003) defines a more general model, when it comes to applications involving monetary prizes authors usually assume a special case of this general model in which

[6] Here $\max(\cdot)$ and $\min(\cdot)$ operations denote the maximal (resp. the minimal) outcome of a prospect.

prospects are determined as *differences* between considered acts and some reference acts.[7] Following that approach, we now consider various rules of setting the reference point, that are empirically grounded but also relevant for testing the CS property.

Specifically, Baillon et al. (2020) analyze six popular reference point setting rules and find empirical support mainly for two of them. Here we take best four out of their six rules[8], namely: Status Quo, MaxMin, MinMax and Prospect Itself. In the context of buying and selling task that we analyze, it is important to *specify* the status quo and the prospect itself rules: the status quo rule takes the *deterministic part of the current wealth* as the reference point whereas the prospect itself rule takes the *whole current wealth* (deterministic or random) as the reference point.

Observe, this distinction is important: in the buying price task, for example, the reference point stays at the decision maker's initial wealth that does not contain the prospect.[9] In the selling price task, however, on top of usual deterministic part the decision maker's initial wealth contains the prospect itself. For this reason we shall henceforth refer to the prospect itself rule as the "random status quo" rule. We also combine the Maxmin and the Minmax rule into one since they are the same in the context of buying and selling prices.

According to the reference model (Sugden, 2003) for each task we must specify at least three acts, each of them defined over the wealth levels: the two acts among which the choice is made and the reference act that is used to define the reference dependent preferences. In what follows we will consider four tasks, i.e. to elicit: buying price, selling price, short-selling and, for completeness, the price to compensate for the loss. Table 1 lists these acts for some gain prospect L and three reference point rules: (i) SQ: deterministic status quo wealth; (ii) RSQ: status quo wealth allowed to be random; and (iii) Maxmin/Minmax: the maxmin (or minmax) of the two acts. Note that the SQ rule

[7] That is, prospects were derived from state-contingent acts where prizes are interpreted as *wealth levels*. Although theoretically indistinguishable for monetary lotteries, here we use *acts* for wealth levels and *prospects* to denote acts on wealth changes.

[8] Excluding those that performed particularly bad as measured either by marginal posterior distribution or by a proportion of sharply classified respondents satisfying a particular reference point rule.

[9] We assume that all the uncertainty that is not part of the decision problem has been previously resolved. We do not allow for any additional background risk either.

Table 1: Decision problems with different reference rules.

Tasks	Act 1	Act 2	SQ	RSQ	Maxmin/Minmax
Buying price	$W+L-B$	W	W	W	W
Selling price	$W+S$	$W+L$	W	$W+L$	$W+S$
Short-selling price	$W-L+B^*$	W	W	W	W

Table 2: Evaluated prospects (wealth changes) under different reference point rules.

Tasks	SQ		RSQ		Maxmin/Minmax	
Buying price	$L-B$	0	$L-B$	0	$L-B$	0
Selling price	S	L	$S-L$	0	0	$L-S$
Short-selling price	B^*-L	0	B^*-L	0	B^*-L	0

does not differ in all considered tasks from a fixed reference point with no re-framing.[10]

Table 2 gives the corresponding prospects. Interestingly, note that buying price and short-selling prices each have the same prospect representation under all three rules. On the other hand, selling price and loss compensation each have a different prospect representation under each of these rules. We now present the testable predictions of each of these rules and discuss complementary symmetry properties between the four tasks.

Observe that under the SQ rule all of B, S, B^* are different in general. Moreover, applying the same reasoning as in section 2, it is clear that the complementary symmetry property holds only for buying and short-selling prices, i.e. for the pair (B, B^*). The situation is different under Random SQ. In such case, not only $S = B^*$ but also complementary symmetry can be stated for *both pairs*: (B, S), (B, B^*). This is a stark difference to the SQ reference dependent model. Finally, under Maxmin/Minmax rule we obtain that $S = B$ and the complementary symmetry can be tested for pairs: (B, B^*) and (S, B^*).

Observe, that the generalized complementary symmetry as defined in this paper is implied by all three models of setting the reference point rules, while the symmetry between buying and selling only for the RSQ model. As a result, evidence on violations of the symmetry between buying and selling prices can be interpreted as a violation of the RSQ model. Similarly, symmetry between selling prices and loss compensation under

[10]Fixed reference point is in fact equivalent to the Expected Utility of wealth model in which wealth is taken to be equal to 0. Such as model, unlike the reference-dependent models that allow for reframing, shares strong normative properties with the standard EU model.

SQ model cannot be expected to hold generally, unless some more restrictive assumptions on the shape of preferences are imposed.

4 Experimental results

In a series of experiments, Birnbaum and others (Birnbaum and Sutton, 1992; Birnbaum and Zimmermann, 1998; Birnbaum et al., 2016) have tested complementary symmetry prediction and found it to be systematically refuted. That is, the sum of the median reported buying price of $(x, y; p)$ and the median reported selling price of $(x, y; 1 - p)$ was found to be always below $x + y$ and the deviation from this benchmark increased with the range, i.e. $|x - y|$, of prospects. For example for $x = 48, y = 60, p = 0.5$ the sum of buying and selling prices was 104, being slightly below $x + y = 108$, while for $x = 12, y = 96, p = 0.5$, the sum dropped to 75, significantly below $x + y = 108$.

These experimental findings, which were originally used by Birnbaum (2018) to question the third-generation prospect theory should be reevaluated in light of our findings. We have argued that (2) does not represent the selling but the short-selling price. This suggest the previous experimental results did not test the CS property. Indeed, as shown by Chudziak (2020) (theorem 2.2) after accepting definition of the buying price B, complementary symmetry hols if and only if one accepts definition of the (complementary) selling price.

Our experiment involved 60, mainly student, subjects from Warsaw School of Economics and University of Georgia. As our focus is on testing the CS property we elicit from each subject four prices: buying, selling and short-selling for one of four available equal chance binary prospects: $48 - 60$, $36 - 72$, $24 - 84$ and $12 - 96$, outcomes measured in dollars. Observe that the outcomes in each gamble sum up to 108 dollars.

Our experimental results are summarized in table 3 and clearly show that the CS tested with the use of short selling prices is confirmed in all ranges (in fact is exactly confirmed in three out of four). In contrast, the CS tested with the use of selling prices is violated. Observe, our results also replicate Birnbaum (2018) finding: that the sum of median buying and selling prices is below $x + y$ for each x, y and decreases with $|y - x|$,

Table 3: CS with selling vs. short-selling prices.

ranges	$96-12$	$84-24$	$72-36$	$60-48$
$B+S$	73	101	100	108
$B+B^*$	103	108	108	108
number of subjects	11	6	10	11

Table 4: Validity of CS in a within-subject experiment.

Cluster ID	1	2	3	4	5	6	7	8	Sum
CS: $B+S$	1	1	0	1	1	0	0	0	
CS: $B+B^*$	1	1	1	0	0	1	0	0	
CS: $S+B^*$	1	0	1	1	0	0	1	0	
no. of subjects	11	7	6	0	1	8	3	2	38

while the same is not true if selling price is replaced by a short selling price.

Next, in difference to Birnbaum (2018) we have elicited all three prices (B, S, B^*) for each subject. This allows us to test the CS property within subjects and not as in Birnbaum (2018) using the median prices only. The advantage is that we can capture the heterogeneity of subjects and check for each subject whether (s)he fulfills the CS property of a given kind or not.

For 20 (out of 38) of our subjects the sum of reported buying and short selling prices equals exactly 108. Nevertheless, to avoid testing such knife-edge predictions, we decided to classify all cases with the sum of reported prices in $[100, 116]$ as fulfillment of the tested CS property, otherwise as the violation. Specifically, since we have all three prices for each subject we can test each of the three possible CS properties, i.e. for pairs (B, B^*), (B, S), (S, B^*) for each person. We have then classified all observations into those satisfying the three complementary symmetry hypothesis: (B, B^*) only, (B, B^*), (B, S) only, and (B, B^*), (S, B^*) only; or none of them. These correspond to the three reference point setting rules developed in section 3. Indeed, SQ rule implies the (B, B^*) complementary symmetry; RSQ rule implies (B, B^*) and (B, S) CS while minmax/maxmin rules implies (B, B^*), (S, B^*) CS.[11]

Table 4 summarizes our results. There are three important conclusions in the view of

[11] Note that the support for the RSQ or the Minmax/maxmin rules is fully contained in the support of the SQ rule. This per se is an argument in favor of the latter rule.

the reference point setting rules. First, there is a significant number of subjects that give support only the SQ rule and not the others. Second, that there is 45-50%of subjects that support either the Minmax/Maxmin rule or to the RSQ rule, meaning that none of them is clearly better than another. Importantly, around 85% of subjects satisfies the $B + B^*$ CS property (and thus supports the SQ rule). And third, out of 12 subjects satisfying only one out of three CS properties, 8 subjects (or 2/3) satisfied the $B + B^*$ CS property.

Concluding, in this paper we have tested the validity of the CS for gambles directly comparable to those of Birnbaum and Sutton (1992) or Birnbaum (2018). Further experimental tests using short selling prices are necessary to validate the CS hypothesis in more general contexts. Specifically, the generalized CS property, as defined in our paper, could be tested for more general prospect types e.g. multiple outcome ones with well defined probability distribution but also including uncertainty or ambiguity contexts. We leave it for further studies.

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