Advanced Microeconomics (QEM) - Problem set 1
Due date: classes on October 16th

Problem 1 (2p) *Consider the following utility function:*

$$u(x) = \sum_{i=1}^{2} \alpha_i \log(x_i), \text{ where } \alpha_1 = 1/3, \alpha_2 = 2/3.$$

Let $w = 5, p = (1, 1)$*. Assume prices have changed to* $p' = (1, 2)$*.*

1. *Compute income and substitution effect of a price change on demand of good 2 (use Hicksian decomposition).*
2. *Compute income and substitution effect of a price change on demand of good 2 (use Slutzky decomposition).*

Problem 2 (5.5p) *Consider the following utility function, called CES (constant elasticity of substitution function):*

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}, \quad \text{where } 0 \neq \rho < 1.$$

This function is obviously strictly increasing. For the above CES function:

1. *Show that* u *is strictly quasiconcave.*
2. *Formulate the Utility Maximization Problem and solve it to derive demand* $d_1(p, w)$ *and* $d_2(p, w)$ *[You may assume interior solution and Walras law which follows from strict monotonicity of* $u(\cdot)$*].*
3. *Form the indirect utility function* $v(p, w)$*.*
4. *Show that the indirect utility function is:*
   (a) *homogeneous of degree zero in* (p, w)*,*
   (b) *increasing in* w *and decreasing in* p*.*
5. *Verify the Roy's identity.*
6. *Formulate the Expenditure Minimization Problem and solve it to derive Hicksian demand* $h(p, \underline{u})$ *[You may assume interior solution and no excess utility which follows from continuity of* $u(\cdot)$*]*
7. *Form the expenditure function* $e(p, \underline{u})$*.*
8. *Verify that the expenditure function is concave in* p*.*
9. *Verify the Shephard's lemma.*

Problem 3 (2.5p) *Let* $(-\infty, \infty) \times \mathbf{R}^L+$ *be the consumption set. The consumer has strictly convex preferences which are represented by a utility function* $u(x) = x_1 + \phi(x_2, x_3, \ldots, x_L)$*. We assume* $p >> 0$ *and we normalize* $p_1 = 1$*.*

1. *Try to show that the demand for commodities* $\{2, 3, \ldots, L\}$ *must be independent of wealth. How does demand for commodity 1 react to changes in wealth* w*?*
2. *Using your previous result, define the indirect utility function. Show that* v *is linear in wealth, i.e.* $v(p, w) = w + \xi(p)$ *for some function* ξ*. You do not need to find this function.*
3. *Now let* $L = 2$ *and* $\phi(x_2) = \alpha \ln(x_2)$*. Solve the UMP as a function of* p, w*. Recall that we allow demand for commodity 1 to be negative.*