Advanced Microeconomics (QEM) - Problem set 2
Due date: Wednesday, November 9th (in class)

Problem 1 (1.5p) *There are $L = 3$ commodities. A firm produces two outputs, namely commodities 1 and 2, by using commodity 3 as an input. The transformation function of the firm is*

$$t(y_1, y_2, y_3) = \alpha y_1 + \beta y_2 - 2\sqrt{-y_3} \text{ with } y_3 \leq 0$$

where the parameters α and β are strictly positive. Write the production set determined by this transformation function and show that this production set satisfies the three properties: impossibility of free production (i.e., no free lunch), free-disposal, decreasing returns to scale.

Problem 2 (1.5p) *Exercise 3.3.1 from the Lecture Notes (subpoints 4, 8, 10).*

Problem 3 (2p) *There are $L = 3$ commodities. The firm produces commodity 3 by using commodities 1 and 2 as inputs. The production function of the firm is given by*

$$f(z_1, z_2) = \sqrt{z_1 + z_2} \text{ with } z_1 \geq 0 \text{ and } z_2 \geq 0$$

1. *Write the cost minimization problem (CMP) of this firm.*
2. *Explain why the demand of inputs of this firm must be non-empty for every $(w_1, w_2) \in \mathbb{R}^2_{++}$ and for every output level.*
3. *Compute the demand of inputs and the cost function of this firm.*
4. *Determine the supply of this firm for every $(p, w_2, w_3) \in \mathbb{R}^3_{++}$.*

Problem 4 (1p) *$L = 3$ is the number of commodities. The firm produces commodity 3 using commodities 1 and 2 as inputs. The production function is given by*

$$f(z_1, z_2) = (z_1)^{\alpha}(z_2)^{\beta} \text{ with } \alpha > 0, \beta > 0, z_1 \geq 0 \text{ and } z_2 \geq 0$$

with $\alpha + \beta \leq 1$. Determine the demand of inputs and the cost function of the firm [Suggestion: Distinguish the two cases $\alpha + \beta < 1$ and $\alpha + \beta = 1$].

Problem 5 (1p) *Exercise 3.3.4 from the Lecture Notes (subpoint 2).*

Problem 6 (2p) *Exercise 3.3.5 from the Lecture Notes (subpoint 4 and 7).*

Problem 7 (2p) *Exercise 5.E.5 from MWG.*