Aggregate demand, Competitive markets

Advanced Microeconomics

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Aggregate demand

Competitve markets

- Pareto optimality
- Partial equilibrium
- Comparative statics
- The Fundamental Welfare Theorems
- Welfare analysis
- Short run and long run

Questions

- When can aggregate demand be expressed as a function of prices and aggregate wealth?
- When does aggregate demand satisfy the weak axiom?
- When does aggregate demand have welfare significance?

Aggregate Demand and Aggregate Wealth I

$$x(p, w_1, \ldots w_I) = \sum_{i=1}^{I} x_i(p, w_i) =^? x(p, \sum_i w_i)$$

The aggregate demand must be identical for any two distributions of the same total amount of wealth across consumers.

For any p and commodity ℓ the wealth effect at p must be the same, i.e. all consumers' wealth expansion paths are parallel

Proposition

A necessary and sufficient condition for the set of consumers to exhibit parallel, straight wealth expansion paths at any price vector p in that preferences admit indirect utility functions of the form

$$v_i(p, w_i) = a_i + b(p) w_i$$

with the coefficients on w_i the same for every consumer i.

Aggregate Demand and Aggregate Wealth II

- The special form of indirect utility function is very restrictive
- Individual level of wealth may be generated by some underlying process that restricts the set of individual wealth levels (wealth distribution rule), e.g.
  - individual wealth level is generated by individuals' shareholding of firms and by their ownership of fixed stock of commodities
  - individual wealth level is partially determined by various government programs of redistribution

Definition

A family of functions $(w_1(p;w);..;w_I(p;w))$ assigning to each individual i a wealth level $w_i(p;w)$, fulfilling $\sum_{i=1}^{I} w_i = w$ is called **wealth distribution rule**.

Aggregate Demand and the Weak Axiom

Properties of individual demand which carry over to the aggregate demand function: continuity, homogeneity of degree zero and Walras' law ($p \cdot x(p, w_1, .., w_I) = \sum_I w_i$ for all $(p, w_1, .., w_I)$)
With a wealth distribution rule aggregate demand can be written as

$$x(p, w) = \sum_i x_i(p, w_i(p, w))$$

Definition

The aggregate demand function $x(p, w)$ satisfies the weak axiom (WA) if $p \cdot x(p', w') \leqslant w$ and $x(p, w) \neq x(p', w')$ imply $p' \cdot x(p, w) > w'$ for any (p, w) and (p', w')

Uncompensated law of demand

Definition

The individual demand function $x_i(p, w_i)$ satisfies the uncompensated law of demand (ULD) property if

$$(p' - p) \cdot [x_i(p', w_i) - x_i(p, w_i)] \leqslant 0$$

for any p, p' and w_i, with strict inequality if $x_i(p', w_i) \neq x_i(p, w_i)$.

- If $x_i(p, w_i)$ satisfies the ULD property, than $D_p x_i(p, w_i)$ is negative semidefinite, i.e. $dp \cdot D_p x_i(p, w_i) dp \leqslant 0$ for all dp
- If $D_p x_i(p, w_i)$ is negative definite for all p, then $x_i(p, w)$ satisfies the ULD property

Uncompensated law of demand and aggregation

Proposition

If every consumer's Walrasian demand function $x_i(p, w_i)$ satisfies the ULD property, so does the aggregate demand $x(p, w) = \sum_i x_i(p, w_i)$. This implies that the aggregate demand satisfies WA.

Proposition

If $\succeq_i$ is homothetic, then $x_i(p, w_i)$ satisfies ULC

Positive representative consumer

Definition

A **positive representative consumer** exists if there is a rational preference relation $\succeq$ on R^L_+ such that the aggregate demand function $x(p, w)$ is precisely the Walrasian demand function generated by this preference relation. That is, $x(p, w) \succeq x$ whenever $x \neq x(p, w)$ and $p \cdot x \leqslant w$

This is a fictional individual whose utility maximization problem when facing society's budget set would generate the economy's aggregate demand function

Social Welfare

Definition

A **social welfare function** is $W : R^I \rightarrow R$ that assigns a utility value to each possible vector $(u_1, .., u_I) \in R^I$ of utility levels for the i consumers in the economy.

Social welfare maximization problem

$$\max_{w_1,\ldots,w_I} W(v_1(p, w_1), .., v_I(p, w_I))$$

$$s.t. \sum_{i=1}^{I} w_i \leqslant w$$

where $v_i(p, w)$ is consumer i's indirect utility function

Normative representative consumer

Definition

The positive representative consumer $\succeq$ for the aggregate demand $x(p,w) = \sum_i w_i(p, w_i(p,w))$ is a **normative representative consumer** relative to the social welfare function $W(\cdot)$ if for every (p,w) the distribution of wealth $(w_1(p,w), .., w_I(p,w))$ solves social welfare maximization problem and therefore the value function of this problem is an indirect utility function for $\succeq$

If there is a normative representative consumer, the preferences of this consumer have welfare significance and the aggregate demand function can be used to make welfare judgments

Notation

- We suppose that consumer i initially owns $\omega_{\ell i}$ of good ℓ (endowment)
- $\omega_i = (\omega_{1i}, .., \omega_{Li})$ – vector of endowments
- Additionally consumer i owns a share θ_{ij} of firm j, giving him a claim to fraction θ_{ij} of firm j's profit, $\sum_i \theta_{ij} = 1$

Definitions

Definition (feasible allocation)

An economic allocation $(x_1, ..., x_I, y_1, ..., y_J)$ is a specification of a consumption vector $x_i \in X_i$ for each consumer $i = 1, .., I$ and a production vector $y_j \in Y_j$ for each firm $j = 1, 2, .., J$. The allocation $(x_1, ..., x_I, y_1, ..., y_J)$ is feasible if $\sum_{i=1}^{I} x_{\ell i} \leqslant \omega_\ell + \sum_{j=1}^{J} y_{\ell j}$ for $\ell = 1, ..., L$.

Definition (Pareto optimality)

A feasible allocation $(x_1, ..., x_I, y_1, ..., y_J)$ is Pareto optimal (or Pareto efficient) if there is no other feasible allocation $(x'_1, ..., x'_I, y'_1, ..., y'_J)$ such that $u_i(x'_i) \geqslant u_i(x_i)$ for all $i = 1, .., I$ and $u_i(x'_i) > u_i(x_i)$ for some i.

Competitive equilibrium

Definition (competitive equilibrium)

The allocation $(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*)$ and price vector $p^* \in R^L$ constitute a competitive (or Walrasian) equilibrium if the following conditions are satisfied

1. **Profit maximization**: For each firm j, y_j^* solves $\max_{y_j \in Y_j} p^* \cdot y_J$
2. **Utility maximization**: For each consumer i, x_i^* solves $\max_{x_i \in X_i} u_i(x_i)$ s.t.$p^* \cdot x_i \leqslant p^* \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij}(p^* \cdot y_j^*)$
3. **Market clearing**: For each good $\ell = 1, ..., L$ $\sum_{i=1}^{I} x_{\ell i}^* = \omega_\ell + \sum_{j=1}^{J} y_{\ell j}^*$

Identifying competitive equilibria

Lemma

If $(x_1^*, ..., x_I^*, y_1^*, ..., y_J^*)$ and $p^* \gg 0$ satisfy the market clearing condition for all goods $\ell \neq K$, and if every consumer's budget constraint is satisfied with the equality, so that $p \cdot x_i = p \cdot \omega_i + \sum_j \theta_{ij} p \cdot y_j$ for all i, then the market for good k also clears.

Proof

Adding up the consumers' budget constrains over the I consumers and rearranging terms we get $\sum_{\ell \neq k} p_\ell (\sum_{i=1}^I x_{\ell i} - \omega_\ell - \sum_{j=1}^J y_{\ell j}) = -p_k (\sum_{i=1}^I x_{ki} - \omega_k - \sum_{j=1}^J y_{kj})$. By market clearing in goods $\ell \neq k$, the left-hand side of this equation is equal to zero. Thus the right-hand side must be equal to zero as well. This and $p_k > 0$ implies that we have market clearing in good k.

$\square$

Two-good quasilinear model

- Two commodities: good ℓ and the *numeraire*
- x_i and m_i denote consumer i's consumption of good ℓ and the *numeraire*
- Utility function of consumer i's

$$u_i(m_i, x_i) = m_i + \phi_i(x_i)$$

- For convenience assume that the consumption of the *numeraire* can be negative
- $\phi_i(\cdot)$ is bounded above and twice differentiable, with $\phi_i'(x_i) > 0$ and $\phi_i''(x_i) < 0$ for all $x_i \geqslant 0$
- $\phi_i(0) = 0$
- Price of the *numeraire* is 1, price of good ℓ denote p

Firms

- Firm j produces good ℓ from good m according to a function $c_j(q_j)$
- Denote z_j firm j's use of good m as an input, its production set is

$$Y_j = \{(-z_j, q_j) : q_j \geqslant 0 \wedge z_j \geqslant c_j(q_j)\}.$$

- $c_j(\cdot)$ is twice differentiable, with $c_j'(q_j) > 0$ and $c_j''(q_j) \geqslant 0$ for all $q_j \geqslant 0$
- No initial endowment of good ℓ
- Consumer i's initial endowment of the *numeraire* is $\omega_{mi} > 0$ and $\omega_m = \sum_i \omega_{mi}$

Competitive equilibrium conditions I

Firms

q_j^* must solve $\max_{q_j \geqslant 0} p^* q_j - c_j(q_j)$. The necessary and sufficient first order condition: $p^* \leqslant c_j'(q_j^*)$, with equality if $q_j^* > 0$.

Consumers

Consumption vector (m_i^*, x_i^*) must solve $\max_{m_i \in R, x_i \in R_+} m_i + \phi_i(x_i)$ such that $m_i + p^* x_i \leqslant \omega_{mi} + \sum_{j=1}^{J} \theta_{ij}(p^* q_j^* - c_j(q_j^*))$.

Competitive equilibrium conditions II

The allocation $(x_1^*, ..., x_I^*, q_1^*, ..., q_J^*)$ and price p^* constitute a competitive equilibrium if and only if

$$p^* \leqslant c_j'(q_j^*)$$

with equality if $q_j^* > 0, j = 1, .., J$.

$$\phi_i'(x_i^*) \leqslant p^*$$

with equality if $x_i^* > 0, i = 1, .., I$

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

The equilibrium allocation and price are independent of the distribution of endowments and ownership shares! (with quasilinear preference)

General framework

- Each consumer preferences are affected by a vector of exogenous parameters $\alpha \in R^M \Rightarrow \phi_i(\cdot)$ becomes $\phi_i(x_i, \alpha)$
- Similarly in case of firms, we have exogenous parameters $\beta \in R^S$ and $c_j(\cdot)$ becomes $c_j(q_j, \beta)$
- Effective price (net of taxes/subsidies) can differ form the market price p: denote $\hat{p}_i(p,t)$ and $\hat{p}_j(p,t)$ effective prices paid by consumer i's and received by firm j's
- p^* is determined as a solution of

$$\phi_i'(x_i^*, \alpha) = \hat{p}_i(p^*, t), i = 1, .., I.$$

$$c_j'(q_j^*, \beta) = \hat{p}_j(p^*, t), j = 1, .., J.$$

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

Example – Sales tax

- Consumer must pay an amount of $t \geqslant 0$ (in units of *numeraire*) for each of good ℓ consumed
- Hence $\hat{p}_i(p,t) = p + t$ and $\hat{p}_j(p,t) = p$ for all j
- The new equilibrium market price $p^*(t)$ must satisfy $x(p^*(t)+t) = q(p^*(t))$
- Effect on prices of a marginal increase in t is

$$p^{*'}(t) = -\frac{x'(p^*(t)+t)}{x'(p^*(t)+t) - q'(p^*(t))}$$

- Hence $-1 \leqslant p^{*'}(t) < 0$ at any t
- It means that the price $p^*(t)$ falls as t increases while the consumers price $p^*(t)+t$ rises (weakly)
- The total quantities produced and consumed fall

Two-good quasilinear economy

- The boundary of the economy's utility possibility set is linear
- All the points in this boundary are associated with consumption allocations that differ only in the distribution of the *numeraire* among consumers
- The optimal consumption and production levels can be obtained as a solution to **aggregate surplus** maximization problem

$$\max_{(x_1,..,x_I)\geqslant 0,(q_1,..,q_J)\geqslant 0} \sum_{i=1}^{I} \phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j) + \omega_m$$

s.t.

$$\sum_{i=1}^{I} x_i - \sum_{j=1}^{J} q_j = 0$$

Fundamental Theorems of Welfare Economics

The First Fundamental Theorem

If the price p^* and allocation $(x_1^*, ..., x_I^*, q_1^*, ..., q_j^*)$ constitute a competitive equilibrium, then this allocation is Pareto optimal

The Second Fundamental Theorem

For any Pareto optimal levels of utility $(u_1^*, ..., u_I^*)$, there are transfers of the *numeraire* commodity $(T_1, .., T_I)$ satisfying $\sum_i T_i = 0$, such that a competitive equilibrium reached from the endowments $(\omega_{m1} + T_1, ...\omega_{mI} + T_I)$ yields precisely the utilities $(u_1^*, ..., u_I^*)$.

Social welfare function and the aggregate surplus

- Social welfare function $W(u_1, .., u_I)$ assigns a social welfare value to every utility vector
- Assume there is a central authority who redistributes wealth by means of transfers of the *numeraire* commodity in order to maximize social welfare
- Quasilinear utility function $\Rightarrow$ *changes in social welfare can be measured by changes in the aggregate surplus for* **any** *social welfare that society may have*
    - No transfer necessary if the social welfare function is "utilitarian", i.e. $\sum_i u_i$

Assumptions

Let $x = \sum_i x_i$ be the aggregate consumption of good ℓ and $q = \sum_j q_j$ be aggregate output of good ℓ

Assumption 1

For any x the individual consumptions of good ℓ are distributed optimally across consumers, i.e. $\phi_i'(x_i) = P(x)$ for every i

This is satisfied if, for example, consumers are price-takers and all face the same price

Assumption 2

The production of any total amount q is distributed optimally across firms, i.e. $c_j'(q_j) = C'(q)$ for every j

This is satisfied if, for example, firms are price-takers and all face the same price

We do not require that both consumers and firms face the same prices!

The impact of change in quantities

- Consider a differential change in consumption $(dx_1, ..., dx_I, dq_1, ..., dq_J)$, such that $\sum_i dx_i = \sum_j dq_j$
- The change in the aggregate surplus is

$$dS = \sum_{i=1}^{I} \phi_i'(x_i)dx_i - \sum_{j=1}^{J} c_j'(q_j)dq_j$$

- Using Assumption 1 and 2 we get

$$dS = P(x) \sum_{i=1}^{I} dx_i - C'(q) \sum_{j=1}^{J} dq_j$$

- Denote $\sum_i dx_i = \sum_j dq_j = dx$ and using the fact that $x = q$ (by market feasibility) we obtain

$$dS = [P(x) - C'(x)]dx$$

Total value of the aggregate surplus

- $$S(x) = S_0 + \int_0^x [P(s) - C'(s)]ds$$
  where s_0 is a constant, and represents the value of the surplus when there is no consumption or production of good ℓ
- $S(x)$ is maximized at x^* such that $P(x^*) - C'(x^*)$, which is exactly the competitive equilibrium aggregate consumption level (compare with the first welfare theorem)

Example – a distortionary tax

- The central authority collects a tax and returns the tax revenue raised to the consumers by means of lump-sum transfer
- We study impact of this tax-and-transfer scheme on welfare
- Note that:
  - $\phi_i'(x_i^*(t)) = p^*(t) + t$
  - $c_j'(q_J^*(t)) = p^*(t)$
- Then the change in the aggregate surplus is

$$S^*(t) - S^*(0) = \int_{x^*(0)}^{x^*(t)} [P(s) - C'(s)]ds$$

  This is negative since $x^*(t) < x^*(0)$ and $P(x) \geqslant C'(x)$ for all $x \leqslant x^*(0)$
- **There is a deadweight loss of distortionary taxation!**

Consumer surplus

$$CS(\hat{p}) = \sum_{i=1}^{I} \phi_i(x_i(\hat{p})) - \hat{p}x(\hat{p})$$

From Assumption 1 we have

$$CS(\hat{p}) = \int_0^{x(\hat{p})} P(s)ds - \hat{p}x(\hat{p}) = \int_0^{x(\hat{p})} [P(s) - \hat{p}]ds$$

Finally

$$CS(\hat{p}) = \int_{\hat{p}}^{\infty} x(s)ds$$

Hence the change in the consumer surplus is

$$CS(p^*(t) + t) - CS(p^*(0)) = -\int_{p^*(0)}^{p^*(t)+t} x(s)ds$$

Producer surplus

$$\Pi(\hat{p}) = \hat{p}q(\hat{p}) - \sum_{j=1}^{J} c_j(q_j(\hat{p}))$$

From Assumption 2 we have

$$\Pi(\hat{p}) = \Pi_0 + \int_0^{q(\hat{p})} [\hat{p} - C'(s)]ds = \Pi_0 + \int_0^{\hat{p}} q(s)ds$$

Finally, the change in producer surplus is

$$\Pi(p^*(t)) - \Pi(p^*(0)) = -\int_{p^*(t)}^{p^*(0)} q(s)ds$$

Long run competitive equilibrium

Definition

Given an aggregate demand function $x(p)$ and a cost function $c(p)$ for each potentially active firm having $c(0) = 0$, a triple (p^*, q^*, J^*) is a log-run competitive equilibrium if

1. q^* solves $\max_{q \geqslant 0} p^* q - c(q)$ (Profit maximization)
2. $x(p^*) = J^* q^*$ (Demand=supply)
3. $p^* q^* - c(q^*) = 0$ (Free entry condition)

The equilibrium price comes form equation demand with long-run supply, which takes into account firms' entry and exit decisions

Long run aggregate supply correspondence

$$Q(p) = \begin{cases} \infty & \pi(p) > 0 \\ \{Q \geqslant 0 : Q = Jq, J \geqslant 0 \wedge J \text{integer} \wedge & \pi(p) = 0 \\ q \in q(p)\} & \end{cases}$$

p^* is a long-run competitive equilibrium price if and only if $x(p^*) \in Q(p^*)$

Constant returns to scale

- $c(q) = cq$, assume $x(c) > 0$
- From Profit maximization we have $p^* \leqslant c$ (otherwise there is no profit-maximizing production)
- $x(c) > 0$ and Demand=Supply condition requires $q^* > 0$
- Free entry condition: $(p^* - c)q^* = 0$
- Hence, $p^* = c$ and aggregate consumption is $x(c)$
- J^* and q^* are indeterminate: any J^* and q^* such that $J^* q^* = x(c)$ satisfies Profit maximization and Demand-Supply

$$Q(p) = \begin{cases} \infty & p > c \\ [0, \infty) & p = c \\ 0 & p < c \end{cases}$$

Decreasing returns to scale

- Assume $c(\cdot)$ is increasing and strictly convex, also $x(c'(0)) > 0$
- **No long-run equilibrium can exist!**
  - If $p > c'(0)$ then $\pi(p) > 0$ and supply is infinite
  - If $p \leqslant c'(0)$ then the long-run supply is zero while $x(p) > 0$

Efficient scale

- There must exist a strictly positive output level $\overline{q}$ at which a firm's average costs of production are minimized!
- Suppose there is unique $\overline{q} > 0$ and $\overline{c} = c(\overline{q})/\overline{q}$ and $x(\overline{c}) > 0$
- At any long-run equilibrium we have $p^* = \overline{c}$ and each active firm's supply is $q^* = \overline{q}$, therefore $J^* = x(\overline{c})/\overline{q}$
- The equilibrium price and output are exactly the same as in case of constant returns to scale with unit cost $\overline{c}$

Implications

- If the efficient scale is large (in relation to market demand) equilibrium number of firms is small, what about the assumption of price-taking?
- If the demand at price $\overline{c}$ is not integer multiple of $\overline{q}$ $\Rightarrow$ no equilibrium (nonconvexity of technology)
- The equilibrium outcome maximizes the aggregate surplus and is therefore Pareto optimal (the first welfare theorem)

Long-run comparative statics

- The short run comparative statics effects of a demand shock: solve for competitive equilibrium given J^* firms, each with cost function $c_s(\cdot)$ and the new demand function
- The long-run comparative statics effects: solve for the long-run (i.e. free entry) equilibrium given the new demand and long-run cost function $c(\cdot)$