Game Theory - Problem set 4
Due date: June, 2th, to `jfrana@sgh.waw.pl`

Problem 1 (2.5p) *Analyze the example that was briefly sketched during class about a used car market (due to Akerlof). There is a continuum of used cars in the market, with quality* $q \sim U[0,1]$*, that is known only to the sellers. The sellers' valuation of a car of quality* q *is* $v_S(q) = q$*, while the buyers' valuation is* $v_B(q) = \frac{3}{2}q$*.*

- *If the quality was observable, what would be the total trade?*
- *Suppose the quality is seller's private information. Let* $\Omega(p)$ *denote the set of cars offered at price* $p \in [0,1]$ *(given sellers behave optimally). Plot* $E(v_B|p) = \frac{3}{2}E(q|q \in \Omega_p) =: F(q)$ *as a function of* p*. How do you interpret* F *and its fixed point?*
- *Assume market participants encounter price* $p_0 = \frac{1}{2}$*. How would sellers react? Calculate the buyers' maximal acceptable price* p_1 *that is a response to seller's adaptation to* p_0*. Construct in a similar manner* p_2*,*p_3*. What is* $\lim_{n\to\infty} p_n$*?*

Problem 2 (2.5p) *There is a single firm that offers a good and a single consumer. The good could be of high quality – bringing utility of* u_H *to the buyer – or of low quality with* u_L*. The buyer does not observe the product quality. Assume that the firm's cost of production depends on quality (note, however, that quality is not a choice variable!). The product price is set exogenously at* p*, and following inequliaties hold* $u_H > p > u_L > c_H > c > L$*.*

- *Given* p*, what is the buyer's condition for purchasing the product?*
- *Suppose the firm can advertise, i.e. spend some amount* A*, that is observable. Show that there does* not *exist a separating equilibrium, in which the firms with different qualities choose different levels of advertising.*

Problem 3 (2.5p) *Consider a modified example from our class. One company offers three types of jobs for SGH alumni: actuaries, market analyst and clerks. The alumni have various levels of math skills: high, medium or low, each with frequency 1/3. The company's and alumnus' utility for a match from skill to job are summarized in the table below. Imagine SGH authorities (that*

		Job		
		Actuary	Analyst	Clerk
Alumni	High	$(4,6)$	$(3,5)$	$(1,2)$
math	Medium	$(2,2)$	$(3,4)$	$(1,2)$
skill	Low	$(0,-1)$	$(2,1)$	$(1,2)$

observe math skill perfectly) wish to maximize the expected utility of their alumni by appriopiate grade reports. You can assume there are at most three grades $5,4,3$

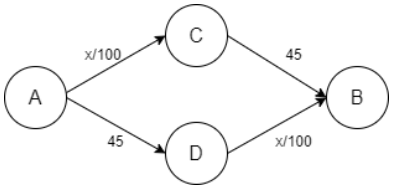
- *show that there exists a partially revealing BNE with two grades. Describe the school's strategy and the company's beliefs and response*
- *propose beliefs that could support an equilibrium with one grade only.*

Problem 4 (2.5p) *Consider a traffic game, pictured in the left panel of the figure below: each of 4000 drivers want to get from city (node) A to city B, minimizing their travel time. The car travelling time depends on choices of other drivers – in particular, if* x *players are choosing route AC, the travelling time is* $x/100$*.*

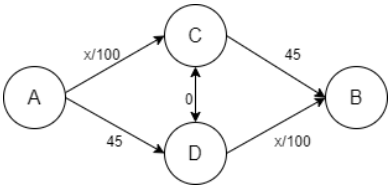
- *characterize all pure-strategy NE of the game. What are the possible travel times in PSNE?*

- *suppose the local government decides to improve the drivers' life, building a super-efficient route from C to D (as in the right figure). The travel time on the route is (normalized to) 0. Show that the new game has a unique NE. Compare the travel time with the one from previous point.*

Note: This is called Braess Paradox.



(a) Routes with travelling cost



(b) Modified routes