Microeconomics. Problem set 2. Due date: tutorials till Dec, 21st.

Problem 1 (1.5p) *There are two consumer at the market Jim with preferences* $U(x,y) = xy$ *and Donna with* $U(x,y) = x^2y$. *Jim's income is* $I_J = 100$ *and Donna's* $I_D = 150$.

- *find optimal baskets for both if* $p_y = 1$ *and* $p_x = p$,
- *plot both demand functions,*
- *compute and plot the aggregate demand.*

Problem 2 (2.5p) *Suppose that a consumer's utility function is* $U(x,y) = xy + 10y$. *The marginal utilities* $MU_x = y$, $MU_y = x + 10$. *The prices are positive:* $p_x, p_y > 0$ *and income is* I.

- *Assume first we are at the interior optimum. Find a functional form for a demand curve for* x.
- *Suppose now that* $I = 100$. *Since* $x \geq 0$ *what is the maximum value of* p_x *for which this consumer would ever purchase any* x.
- *Suppose* $p_x = 20, p_y = 20$. *On a graph illustrating the optimal consumption bundle of* x, y *show that since* p_x *exceeds the one you calculated in the previous point, this corresponds to a corner optimal solution.*
- *Compare the* $MRS_{x,y}$ *with the ratio* $\frac{p_x}{p_y}$ *at the optimum in the previous point. Does this verify that the consumer would reduce utility if she purchased a positive amount of* x*?*
- *Assuming income remains at 100 draw the demand curve for* x. *Does its location depend on* p_y*?[0.5p]*

Problem 3 (1.5p) *Jack makes his consumption and saving decision two months at a time. His income this month is \$1000 and he knows that he will get a raise next month making his income \$1050. The current interest rate (at which he is free to borrow or lend) is %5. Denoting this month's consumption by* x *and next month's by* y *for each of the following utility functions state whether Jack would choose to borrow, lend or neither in the first month.*

- $U(x,y) = xy^2$, $MU_x = y^2$, $MU_y = 2xy$. *[1p]*
- $U(x,y) = x^2y$, $MU_x = 2xy$, $MU_y = x^2$. *[1p]*
- $U(x,y) = xy$, $MU_x = y$, $MU_y = x$. *[1p]*

Hint: in each case, start by assuming that Jack would simply spend his income in each month without borrowing and lending. Would doing so be optimal?

Problem 4 (1.5p) *Justin has a utility function* $U(x,y) = xy$ *with marginal utilities* $MU_x = y$, $MU_y = x$. *The price* $p_x = 2$ *and his income* 40. *When he maximizes utility subject to his budget constraint he purchases 5 units of* y. *What must be the price* p_y *and the amount* x *consumed.*

Problem 5 (1.5p) *Widgets are produced using two inputs, labor* L *and capital* K. *Table presents how many widgets can be produced using those inputs*

$\downarrow K \rightarrow L$	0	1	2	3	4
0	0	2	4	6	8
1	2	4	6	8	10
2	4	6	8	10	12
3	6	8	10	12	14
4	8	10	12	14	16

- *use the data to plot sets of inputs pairs that produce the same number of widgets. Then sketch the isoquants.*
- *Find the marginal product of* K *and* L *for each pair of inputs.*
- *Does the production exhibits IRS, DRS or CRS?*

Problem 6 (1p) *Two points* A, B *are on the isoquant drawn with labor on the horizontal axis. The capital-labor ratio at* B *is twice that of* A, *and the elasticity of substitution as we move from* A *to* B *is 2. What is the ratio of the* $MRTS_{L,K}$ *at* A *versus* B.

Problem 7 (1.5p) *What can you say about returns to scale of the Leontief production function:* $F(k,l) = \min\{ak, bl\}$, *where* a, b *are positive constants?*

Problem 8 (1p) *Suppose that production of airframes is characterized by* $f(K,L) = KL$. *The marginal products are* $MP_K = L$ *and* $MP_L = K$. *Suppose that the price of labor is* 10 *and capital* 1. *Find the cost-minimazing combination of capital and labor for producing* 121000 *airframes.*