Advanced Micro II - Problem set 3
due date: April, 8th

Problem 1 (1p) *Consider the monopolist facing demand $D(p)$ and constant marginal costs c, solving:*

$$\max_{p\in[c,\infty)} (p-c)D(p).$$

State the weakest conditions on D, such that % margin for an optimal price $\hat{m} = \frac{p-c}{p}$ is weakly increasing / decreasing in c.

Problem 2 (2p) *Consider the optimal growth model with linear production function $f(k) = k$ and full depreciation, i.e. consumption c solving:*

$$V(k) = \max_{c\in[0,k]} u(c) + \beta V(k-c).$$

State the weakest conditions on u, such that

- *optimal consumption policy is weakly increasing in k,*
- *optimal investment policy $i = k - c$ is weakly increasing in k,*
- *both the optimal investment and consumption policies are weakly increasing and Lipschitz continuous [1].*

Note: what conditions on u do you need to impose to make sure that function $(c, k) \to V(k-s)$ solving the Bellman equation has desired SPM/ID conditions in k, c? Recall that V is a fixed point (exists by Banach contraction principle) of the T operator defined on a Bellman equation.

Problem 3 (1p) *Consider a Cournot duopoly with homogenuous product, where: $\pi_i(q_i, q_j) = q_i P(q_i + q_j) - C(q_i)$, where C is the total costs function, and P is inverse demand. State conditions on P and C, such that it is a submodular game, i.e. BR-ses are strong set order decreasing.*

Problem 4 (1p) *Consider a supermodular game as analyzed during online class. Assume that each $u_i(a_i, a_{-i})$ is additionally increasing in s_{-i}. Prove that the greatest Nash equilibrium Pareto dominates all other NE.*

Problem 5 (3p) *Let C be a subset of R^l, and T a subset of R. Consider function $F : R^l \times T \to R$, for which $F(x,t) = \bar{F}(x) + f(x_i, t)$, where $f : R \times T \to R$ is supermodular. Let $x'' \in \arg\max_{x\in C} F(x, t'')$ and $x' \in \arg\max_{x\in C} F(x, t')$ for any $t'' > t'$. Show that, if $x'_i > x''_i$ then $x'' \in \arg\max_{x\in C} F(x, t')$ and $x' \in \arg\max_{x\in C} F(x, t'')$.*

Problem 6 (2p) *Let $\{f(s,t)\}_{t\in T}$ be a family of density functions on $S \subset R$. T is a poset. Consider*

$$v(x,t) = \int_S u(x,s) f(s,t) ds.$$

Prove the following statement. Suppose u has increasing differences and that $\{f(\cdot,t)\}_{t\in T}$ are ordered with t by first order stochastic dominance. Then v has increasing differences in (x, t).

[1] Function $f : R \to R$ is Lipschitz continuous iff $|f(k_2) - f(k_1)| \le |k_2 - k_1|$.