Game Theory - Problem set 3
Due date: classes on December, 19th

Problem 1 (2.5p) *Consider the first game from the previous problem set. Assume now that the second player observes the first player true move with probability p, while with probability $1-p$ he/she gets the false information (i.e.: if 1. player chooses T, then the second player observes T with probability p and B with probability $1-p$). Assume that both players know p. Write this game in its extensive form and assuming both players aim to maximize their expected payoffs find all WPBE.*

Problem 2 (2.5p) *Two firms simultaniously decide on entry/stay-out. Entry costs amount to $\theta_i \in [0,\infty)$ but are private information of each company and are independently drawn from a distribution with density $p(\cdot)$ (always strictly positive). Payoff of the i-th company is $\Pi^m - \theta_i$, if it enters alone; $\Pi^d - \theta_i$, if both enter or 0, if it stays out. Let $\Pi^m > \Pi^d > 0$, where Π^m denotes the monopoly profit, while Π^d profit of each firm in the duopoly. Find the Bayesian-Nash equilibrium and show it is unique.*

Problem 3 (2.5p) *Consider an infinitely repeated game, in which both players discount future payoffs with discount factor $\frac{1}{2}$. The strategic form game played every period is given by:*

	A	*D*
A	*2,3*	*1,5*
D	*0,1*	*0,1*

Show that $((A,A),(A,A),\ldots)$ is not the decision path on any SPNE.

Problem 4 (2.5p) *Consider the first price auction with 2 players. Valuations are private and independent. Before the auction each player observes random variable t_i, drawn independently from the uniform distribution on $[0,1]$. Then his/her valuation is $v_i = t_i + 0.5$. Then after betting $b_i \geq 0$, the payoffs are the following:*

$$u_i(b_1,b_2,t_1,t_2) = \begin{cases} (t_i+0.5)-b_i & \textit{if} \quad b_i > b_{-i}, \\ 0 & \textit{if} \quad b_i < b_{-i}, \\ \frac{1}{2}(t_i - b_i + 0.5) & \textit{if} \quad b_i = b_{-i}. \end{cases}$$

Find NE of this game in linear strategies, i.e. find numbers α and β such that $b_i = \alpha t_i + \beta$ is the equilibrium strategy. What is the expected payoff of player i, who draws t_i?