Static Models of Oligopoly

Cournot and Bertrand Models

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Outline

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Oligopoly - definition

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- Cartels

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- Demand linear, decrising with p: $x(p)$

Mathematical representation

- Demand function

Mathematical representation

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k \end{cases} \tag{1}$$

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- Demand function

$$\pi_j(p_j, p_k) = x_j(p_j, p_k)(p_j - c) \quad (2)$$

- Profit function

Solution

Proposition 1

There is an unique Nash equilibrium (p_j^*, p_k^*) in the Bertrand duopoly model. In this equilibrium, both firms set their prices equal to cost: $p_j^* = p_k^* = c$.

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- Proof

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Corollary

In setting with $N > 2$ firms Bertrand model of oligopoly produces exactly same results as with $N = 2$ firms

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Bertrand with diversified product

- Two Players.
- Products are not perfect subsidies
- Example: spatial model
  - Reservation Price $V > c$
  - t - cost of 'traveling'
  - N - number of customers

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- **Inverse demand function** $p(q)$ **linear, decrising with** $q = \sum_{j=1}^{N} q_j$

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- Maximization problem

j-th player faces problem:

$$\underset{q_j \geq 0}{Max} \quad p(q_j + \bar{q}_k)q_j - cq_j \tag{3}$$

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- Let $b_j(\bar{q}_k)$ denote set of optimal responses of player j, given strategy (quantity) of player k

Nash Equilibrium

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$$p'\left(q_j^* + q_k^*\right) q_j^* + p\left(q_j^* + q_k^*\right) \leq c \tag{5}$$

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Let's add (5) and (6):

$$p'\left(q_j^* + q_k^*\right) \frac{\left(q_j^* + q_k^*\right)}{2} + p\left(q_j^* + q_k^*\right) = c \tag{7}$$

Proposition 2

In any Nash equilibria of the Cournot duopoly model with costs $c > 0$ per unit forthe two firms and an inverse demand function $p(\cdot)$ satisfying $p'(q) < 0$ for all $q \geq 0$ and $p(0) > c$, the market price is greater than c (the competitive and smaller than monopoly price.

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Let $q = \sum_{j=1}^{N} q_j$ and $N = 2$, inverse demand is given by:

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  - profit!

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which is monopolist solution

Summary

- The Bertrand Model
- The Cournot Model
- Strategical Subpbstitutes vs Strategical Complements

- Outlook
  - Dynamic Games