Advanced Microeconomics - Problem set 3
Due date: classes on November 14th

Problem 1 (2pt) *Consider a pure exchange economy with* $u_1(x, y) = \ln(x) + y$, $u_2(x, y) = \ln(x) + 2y$, $\omega_1 = (1, 0)$, $\omega_2 = (0, 1)$. *Find the set of Pareto-optimal allocations and Walrasian equilibria and depict them on the Edgeworth box.*

Problem 2 (3pt) *Consider a production economy with* $L = 2$ *commodities,* $I = 2$ *consumers and one firm. The firm produces the commodity* 2 *using the commodity* 1 *as an input with constant returns to scale. The production set of the firm is given by*

$$Y = \{(y_1, y_2) \in \mathbf{R}^2 : y_1 \leq 0, y_2 \leq -\alpha y_1\}$$

with $\alpha > 0$. *The two consumers have the same preferences represented by the utility function* $u_i(x_{i1}, x_{i2}) = x_{i1}x_{i2}$. *The initial endowments are* $\omega_1 = (1, 2), \omega_2 = (4, 1)$. *Price of commodity* 1 *is normalized to* 1.

- *Compute the demand of the consumers with respect to the price* p_2 *and* w_i
- *Compute the supply and the profit function of the producer with respect to the price* p_2 *and the marginal productivity* α.
- *The shares of the consumers on the profit of the firm have no influence on the competitive equilibria of this economy. Why so?*
- *Compute the unique competitive equilibrium of this economy with respect to* α.
- *Compute the utility level of the consumers with respect to the marginal productivity* α. *Show that the utility level of the second consumer is increasing. Show that the utility of the first consumer is constant, then decreasing and finally increasing.*
- *Can you explain the differences of the behavior of the utility levels of the consumers with respect to the marginal productivity?*

Problem 3 (3pt) *Consider a pure exchange economy with* $L = 2$ *commodities and* $I = 2$ *consumers. The initial endowments are* $\omega_1 = (1, 1) = \omega_2$. *The consumption set of both consumers* $\mathbf{R}_{++}$, *the utility functions are given by* $u_1(x_{11}, x_{12}) = \frac{1}{3}\ln(x_{11}) + \frac{2}{3}\ln(x_{12})$, $u_2(x_{21}, x_{22}) = \frac{1}{4}\ln(x_{21}) + \frac{3}{4}\ln(x_{22})$.

- *Find all Pareto optimal allocations.*
- *For equity of treatment, the planner wishes to obtain a Pareto optimal allocation which guarantees the same consumption in commodity 1 for both consumers. Determine this specific Pareto optimal allocation* $(\tilde{x}_1, \tilde{x}_2) \gg 0$.
- *in order to decentralize this Pareto optimal allocation* $(\tilde{x}_1, \tilde{x}_2)$ *the planner has the possibility to implement some transfer between the initial endowments of commodity 1. Determine the transfer which leads to a competitive equilibrium satisfying the equity of treatment.*

Problem 4 (2pt) *6.4.7 from our notes.*