Game Theory - Problem set 4 (final)
Due date: classes on June, 4th

Problem 1 (3p) *Consider a Cournot duopoly with linear inverse demand $p = a - bQ$, where $Q = q_1 + q_2$. Assume now that each company know its own marginal costs, but does not know marginal costs of the other. Marginal cost can be c_h or c_l with probabilities μ and $1 - \mu$. Let $c_h > c_l$. Find all Bayesian Nash equilibria of this game.*

Problem 2 (4p) *Consider the following game with two players meeting in a pub. Player A is "Wimpy" with probability 0.2 and "Surly" with probability 0.8. Player A knows its type and decides to order "beer" or "quiche". Then observing A's order B decides to "Concede" or "Fight". B does not know A's type. Payoffs are given by:*

Player's A type	*Players A action*	*Players B action*	*Payoff*
Surly	*Beer*	*Fight*	*(1,0)*
Surly	*Beer*	*Concede*	*(3,1)*
Surly	*Quiche*	*Fight*	*(0,0)*
Surly	*Quiche*	*Concede*	*(2,1)*
Wimpy	*Beer*	*Fight*	*(0,1)*
Wimpy	*Beer*	*Concede*	*(2,0)*
Wimpy	*Quiche*	*Fight*	*(1,1)*
Wimpy	*Quiche*	*Concede*	*(3,0)*

(i) Final all Bayesian-Nash equilibria.

(ii) Final all WPBE.

Problem 3 (3p) *Similarly like in the problem set 3, consider the first price sealed auction with two bidders but common value of the object being sold. As before each player observes a random variable t_i that is independently drawn from uniform distribution on $[0, 1]$ but the actual value of the object is now $v = t_1 + t_2$. Thus given only his only information the expected value of the object is $t_i + 0.5$ as in problem from the previous list. Then after betting $b_i \geq 0$, the payoffs are the following:*

$$u_i(b_1, b_2, t_1, t_2) = \begin{cases} (t_i + t_2) - b_i & \text{if} \quad b_i > b_{-i}, \\ 0 & \text{if} \quad b_i < b_{-i}, \\ \frac{1}{2}(t_i + t_2 - b_i) & \text{if} \quad b_i = b_{-i}. \end{cases}$$

Find NE of this game in linear strategies, i.e. find numbers α and β such that $b_i = \alpha t_i + \beta$ is the equilibrium strategy. Show that for each value of t_i the equilibrium bid is lower in this auction than in the private-value auction from the previous list.