Advanced Micro II - Problem set 3
due date: May, 29th

Problem 1 (2p) *Consider the monopolist facing demand $D(p)$ and constant marginal costs c, solving:*

$$\max_{p \in [c,\infty)} (p-c)D(p).$$

State the weakest conditions on D, such that:

- *optimal price is weakly increasing in c.*
- *margin for optimal price $m = p - c$ is weakly increasing / decreasing in c.*
- *% margin for an optimal price $\hat{m} = \frac{p-c}{p}$ is weakly increasing / decreasing in c*
- *Discuss the taxation pass-through problem, i.e.: how the incident of a sales tax t, changes $m, \hat{m}$? Is that possible that the monopolists passes more than 100% of the tax change on clients? (interpret the above mentioned changes as a cost change, i.e.: $c' = c + t$.*

Problem 2 (2p) *Consider a Cournot duopoly with homogenuous product, where: $\pi_i(q_i, q_j) = q_i P(q_i + q_j) - C(q_i)$, where C is the total costs function, and P is inverse demand. State conditions on P and C, such that it is a submodular game, i.e. BR-ses are strong set order decreasing.*

Problem 3 (2p) *Consider a QSM game as analyzed during classes. Assume that each $u_i(s_i, s_{-i})$ is additionally increasing in s_{-i}. Prove that the greatest Nash equilibrium Pareto dominates all other NE. Hint: use characterization of the fixed points of a monotone function used in the proof of Tarski fixed point theorem for the greatest selection of then best response map. What is the relation between set Y and the set of Nash Equilibria?*

Problem 4 (2p) *Let C be a subset of R^l, and T a subset of R. Consider function $F : R^l \times T \to R$, for which $F(x,t) = \bar{F}(x) + f(x_i, t)$, where $f : R \times T \to R$ is supermodular. Let $x'' \in \arg\max_{x\in C} F(x, t'')$ and $x' \in \arg\max_{x\in C} F(x, t')$ for any $t'' > t'$. Show that, if $x'_i > x''_i$ then $x'' \in \arg\max_{x\in C} F(x, t')$ and $x' \in \arg\max_{x\in C} F(x, t'')$.*

Problem 5 (2p) *Let $\{f(s,t)\}_{t\in T}$ be a family of density functions on $S \subset R$. T is a poset. Consider*

$$v(x,t) = \int_S u(x,s) f(s,t) ds.$$

Prove the following statement. Suppose u has increasing differences and that $\{f(\cdot, t)\}_{t\in T}$ are ordered with t by first order stochastic dominance. Then v has increasing differences in (x,t).