Game Theory - Problem set 3
Due date: classes on May, 11th

Problem 1 (4p) *Consider the following game with two players meeting in a pub. Player A is "Wimpy" with probability* 0.2 *and "Surly" with probability* 0.8. *Player A knows its type and decides to order "beer" or "quiche". Then observing A's order B decides to "Concede" or "Fight". B does not know A's type. Payoffs are given by:*

Player's A type	*Players A action*	*Players B action*	*Payoff*
Surly	*Beer*	*Fight*	*(1,0)*
Surly	*Beer*	*Concede*	*(3,1)*
Surly	*Quiche*	*Fight*	*(0,0)*
Surly	*Quiche*	*Concede*	*(2,1)*
Wimpy	*Beer*	*Fight*	*(0,1)*
Wimpy	*Beer*	*Concede*	*(2,0)*
Wimpy	*Quiche*	*Fight*	*(1,1)*
Wimpy	*Quiche*	*Concede*	*(3,0)*

(i) *Write the game in its extensive form and find all Bayesian-Nash equilibria.*

(ii) *Write a game tree and find all WPBE.*

Problem 2 (3p) *In the problem you will verify whether the $n = 2$ players Folk's theorem for repeated games with perfect monitoring presented in class holds for $n > 2$. For this reason analyze $n = 3$ players game as depicted below:*

	l	*r*
U	*1,1,1*	*0,0,0*
D	*0,0,0*	*0,0,0*

L

	l	*r*
U	*0,0,0*	*0,0,0*
D	*0,0,0*	*1,1,1*

R

- *compute the* min max *payoffs for all players,*
- *argue that for any pure action profile in a stage game with* 0 *payoffs there is at least one player with a profitable deviation. And so there are no combinations of actions that simultaneously minmaxes all of the players,*
- *by $\alpha_i(1)$ denote players i prob. of choosing the first action (U, l or L for respective players). Argue why for any action profile we must have $a_j(1) \geq 0.5$ and $a_k(1) \geq 0.5$ or $(1-a_j(1)) \geq 0.5$ and $(1-a_k(1)) \geq 0.5$ for some 2 players say j, k,*
- *as a result the remaining player (say i) can guarantee himself a payoff of* 0.25 *(at least) in the stage game.*
- *argue the no matter what the discount factor is, the repeated game's payoff must be ≥ 0.25,*
- *how that observation violates statement of the Folk's theorem?*

Problem 3 (3p) *Consider an infinitely repeated prisoners dilemma as analyzed in the last class (with work and shirk actions) with imperfect public monitoring. In class we have analyzed a grim trigger strategy. You will now use the tools presented in class to analyze the forgiving strategy (with one period memory). Specifically, let players exert effort after signal $\overline{y}$ and shirk after $\underline{y}$ (in the previous period) and exert effort in the first period.*

- *write a automaton representation of that strategy,*

- *write formulas for values (in each state) and compute them,*
- *verify the set of parameters* (p, q, r, δ) *for which the described strategy profile constitutes a NE of the normal form game induced by the current payoffs and continuation values,*
- *show that under the current strategy the value in the good state is higher that in the grim trigger strategy analyzed in class,*
- *what is the other benefit from using such strategy? (think about absorbing states).*