Solution to 6.4.5.

Demand $x_a = x_b = d_a(1, p_b, w) = \frac{w}{1+p_b}$. Solving profit maximization problem $p_b\sqrt{-y_a - 1} + y_a$ we get: supply $(y_a, y_b) = s(1, p_b) = (-\frac{p_b^2}{4} - 1, \frac{p_b}{2})$ for any price $p_b \geq 4$ and profit $\pi(1, p_b) = \frac{p_b^2}{4} - 1$. If $p_b < 4$ then $(y_a, y_b) = (0, 0)$ and $\pi() = 0$.

Case 1. Assume interior solution $p_b \geq 4$, then $w = 3 + p_b + \pi(1, p_b) = 2 + p_b + \frac{p_b^2}{4}$ Hence by market clearing $(x_a, x_b) = (e_a, e_b) + (y_a, y_b)$

$$(\frac{2 + p_b + \frac{p_b^2}{4}}{1 + p_b}, \frac{2 + p_b + \frac{p_b^2}{4}}{1 + p_b}) = (3, 1) + (-\frac{p_b^2}{4} - 1, \frac{p_b}{2})$$

This implies a.o. $2 + p_b + \frac{p_b^2}{4} = (1 + p_b)(1 + \frac{p_b}{2})$ hence $2 + p_b + \frac{p_b^2}{4} = 1 + \frac{3}{2}p_b + \frac{p_b^2}{2}$ and $\frac{p_b^2}{4} + \frac{p_b}{2} - 1 = 0$. This equation has no solution.

Case 2. Assume corner solution $p_b < 4$. Then $w = 3 + p_b + \pi(1, p_b) = 3 + p_b$ Hence by market clearing $(x_a, x_b) = (e_a, e_b) + (y_a, y_b)$

$$(\frac{3 + p_b}{1 + p_b}, \frac{3 + p_b}{1 + p_b}) = (3, 1) + (0, 0).$$

Hence there is no solution as well.

The reason we have non existence of WE results from non-convexity of the production set.