Advanced Micro II - Problem set 3
due date: May, 9th

Problem 1 (3p) *Suppose that there are 3 periods, and preferences* $\succeq_0, \succeq_1, \succeq_2$ *in each period are respectively represented by the following utility functions:*

$$\begin{aligned} U_0(c_0, c_1, c_2) &= u(c_0) + \beta\delta u(c_1) + \beta\delta^2 u(c_2), && (1)\\ U_1(c_1, c_2) &= u(c_1) + \beta\delta u(c_2), && (2)\\ U_2(c_2) &= u(c_2). && (3) \end{aligned}$$

where $u(c) = 10 \ln 1 + c$, $\delta = 0.8$ *and* $\beta = 0.7$. *In period 0 the agent has available 1 apple, which he can either consume (and thus get an immediate utility* $u(1)$*) or save (and get the immediate utility* $u(0)$*). If he saves, the apple will magically turn into 2 apples, which will become available to him in period 1. In period 1, he can decide whether to consume 1 apple and save 1, or to consume both, or to save both. Each apple saved is available for consumption in period 2 (no magic here). In period 2 he can either consume or throw away what is available to him (we assume for simplicity, that he cannot throw away anything in previous periods). Hint: use Excell for calculations.*

1. *What are the consumption plans available to the agent in period 0? (e.g. (1,0,0) is a possible consumption plan)*
2. *If the agent is naive, then which plan does he adopt in period 0?*
3. *Which plan does he adopt in period 1?*
4. *In period 2, does he go through with the plan adopted in period 1?*
5. *If the agent is sophisticated, what is the set of consistent plans in period 1, given that he saves the apple in period 0?*
6. *If the agent is sophisticated, what is the set of consistent plans in period 0?*
7. *Which plan does the agent choose in period 0?*
8. *Does he go through with this plan?*
9. *If the agent had the opportunity to commit to a plan in period 0, which plan would he commit to?*

Problem 2 (4p) *Consider a society with a continuum of agents* i *uniformly distributen on* $(0, 1)$, *each living infinite time horizon and (exponentially) discounting future with* $\delta_i = i$. *Each agent poses the same instantaneous utility function* $u(c_t)$ *and each consume the same consumption path* $c = c_1, c_2, \ldots, c_t$.

- *write the collective utility functions, i.e. the sum of agents' lifetime utilities (with equal weights) over stream c;*
- *calculate its value integrating over* i *(assume you can replace the integral with the summation operation);*
- *is the resulting "collective" discount factor exponential? how do we name such a discount factor?*
- *does the resulting "collective" utility function represent time consistent (aggregate) preferences?*

Problem 3 (3p) *Consider a three period exchange economy with time inconsistent preferences as consdered by Gabrieli, Ghosal (Non-Existence of Competitive Equilibria with Dynamically Inconsistent Preferences, Economic Theory 52, p. 299-313, 2013). Consider their counterexample to existence of Walrasian equilibrium for a sophisticated decision maker. Explain intuitively, why does the equilibrium do not exists in their example. Next, consider a model proposed by Kocherlakota (Looking for Evidence of Time-Inconsistent Preferences in Asset Market Data, Quarterly Review 25, p. 13-24, 2001). Explain intuitively, why does the symmetric equilibrium do not exists in his example.*