General equilibrium - Problem set 1
due date: classes on November, 5th

Problem 1 (2p) *Consider an exchange economy with two consumers and two goods. Find the Pareto-optimal set of allocations and the set of Walrasian equilibria and depict them in the Edgeworth box. Preferences and initial endowments are given by:*

(i) $u_1(x, y) = x + y$, $u_2(x, y) = 2x + y$, $w_1 = w_2 = (1, 1)$,

(ii) $u_1(x, y) = \ln(x) + y$, $u_2(x, y) = \ln(x) + 2y$, $w_1 = (2, 1)$, $w_2 = (1, 2)$.

Problem 2 (2p) *Consider an exchange economy with two goods and two consumers. Goods are indivisible and can be consumed only in integer numbers. Let both consumers have continuous and strictly monotone and strictly convex preferences.*

(i) It is so, that allocation in any Walrasian equilibrium is Pareto-optimal? Prove of give a counterexample.

(ii) Now, let one of the goods be perfectly divisible and the other not. It is still so, that allocation in any Walrasian equilibrium is Pareto-optimal? Prove of give a counterexample.

Problem 3 (2p) *Consider an exchange economy:* $u_1(x_A, x_B) = \sqrt{x_A x_B}$, $u_2(x_A, x_B) = 2\ln(x_A) + \ln(x_B)$, $e_1 = (1, 0)$, $e_2 = (0, 4)$.

- *Find prices in the Walrasian equilibrium with* p_A *normalized to* 1 *and find Lagrange multipliers for budget constraints of each consumer,*
- *Find* λ_i*, so that the WE allocation (from the previous point) maximizes the social welfare function over the feasible set. Comment referring to Negishi theorem and check if you are right.*

Problem 4 (4p) *Proof the second part of the Negishi theorem as stated in class. That is proof that: if* (x, y) *solves problem (2) (for some weights* $\lambda >> 0$*) then there exits a price vector* $p \neq 0$ *and* $p \geq 0$ *such that* (x, y, p) *is a WE with transfers, for which* $\frac{1}{\lambda_i}$ *is a marginal utility of wealth in the equilibrium. Recall, that* u_i *are all concave and strictly increasing,* Y_j *are all closed and convex with* $0 \in Y_j$ *and the economy is productive, i.e. there exists a feasible allocation* x', y' *such that* $\sum_i x'_i >> 0$.