Game Theory - Problem set 2
Due date: classes on April, 16th

Problem 1 (2p) *Consider a symmetric 2x2 game as analyzed in class:*

	A	B
A	$0,0$	$2,1$
B	$1,2$	$0,0$

We have argued that there is a evolutionary stable mixed strategy $p = (\frac{2}{3}, \frac{1}{3})$. Show formally that it is so.

Problem 2 (3p) *Consider a Cournot model with homogeneous product and n firms, with the inverse demand $P(Q)$ and total production cost $C(q_i)$. We have argued in class that typically (e.g. $P(Q) = A - BQ$) it is a submodular game, as the best responses map is decreasing.*

In this exercise you will realize that, when looking for symmetric Nash equilibria in Cournot game, with a particular change of variables, we can defined increasing best responses and apply Tarski fixed point theorem for existence of NE.

So let $n = 2, 3, \dots, n$ firms produce y each. We can rewrite the optimization problem of firm 1 as choosing the total supply z, i.e. if 1 produces $z - (n-1)y$ then:

$$\Pi(z, y) = (z - (n-1)y)P(z) - C(z - (n-1)y).$$

- *State conditions on P and C, such that Π has increasing differences in (z, y).*
- *Denote a best response, i.e. the argument maximizing profit with $z^*(y)$. Now consider a function $F(y) = \frac{n-1}{n} z^*(y)$. Show how fixed points of F, say y^* correspond to the symmetric Nash equilibrium production levels of the original Cournot game (i.e. without change of variables)?*

Problem 3 (2p) *Consider a second-price auction for a single good: given bids profile, the good is assigned to the individual with the highest bid, however, the price paid by that individual is the second-highest bid. Assume players have private information about their valuations. The prior distribution of each v_i is Unif $[0, V]$ and they are i.i.d.*

- *Propose a symmetric (meaning all the players choose ex-ante the same strategy: $s(v_i)$) BNE of the game*
- *Are there any other BNE?*
- *Suppose there is a minimum bid $\underline{b}$ that the players must submit in order to participate (note: this is not a participation fee, no cost is borne if the player's bid is not maximal). If only one player participates, the second-price is $\underline{b}$. What is the BNE now?*

Problem 4 (3p) *Consider a game of public good provision: each of the two players can contribute at a private cost c_i or not contribute (at zero cost). The good is provided if at least one of the players contribute and brings utility 1 (minus cost, if any) to everyone. The players cost (=type) is i.i.d. and drawn from Unif $[0, 2]$ distribution.*

- *Write the game in a matrix form and define a Bayesian Nash Equilibrium of the game*
- *Prove that in any BNE the strategies $s_i(c_i)$ must be cutoff strategies i.e.*

$$s_i(c_i) = \begin{cases} Contribute & \text{if } c < \bar{c} \\ Not & \text{if } c > \bar{c} \end{cases}$$

- *Find the cutoffs $\bar{c}$ that define a BNE. Is the equlibrium unique?*
- *Can any of the strategies $s_i(c_i)$ be removed as dominated? What would be the result of the process?*