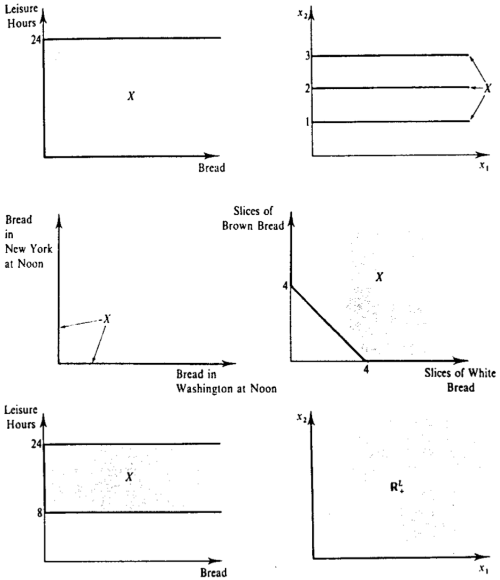
Consumer choice

Consumption set



Consumer's world

Assumption (Consumption set)

Let X be a closed and convex subset of $\mathbb{R}^n_+$ which contains the origin. We say that X is a consumption set.

- Let $x \in X$ be a consumption bundle.
- Let $B \subset X$ be a feasible set.
- Let $\succsim \subset X^2$ be a preference relation, i.e. it satisfies:
  - **Axiom 1:** $\succsim$ is complete (ability to make comparisons)
  - **Axiom 2:** $\succsim$ is transitive (consistency on different comparisons)
- Given any pair x^1 and x^2, exactly one of three possibilities hold:
  - $x^1 \succ x^2$
  - $x^2 \succ x^1$
  - $x^1 \sim x^2$

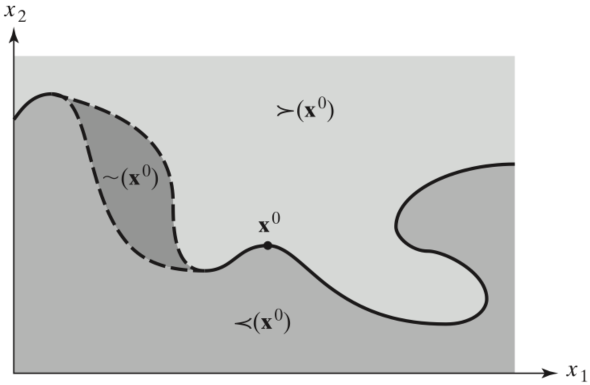
Consumer's world

Let x^0 be any point in the consumption set, X. We define the following subsets of X:

1. $\succsim (x^0) \equiv \{x | x \in X, x \succsim x^0\}$, the 'at least as good as' set.
2. $\precsim (x^0) \equiv \{x | x \in X, x^0 \succsim x\}$, the 'no better than' set.
3. $\prec (x^0) \equiv \{x | x \in X, x^0 \succ x\}$, called the 'worse than' set.
4. $\succ (x^0) \equiv \{x | x \in X, x \succ x^0\}$, called the 'preferred to' set.
5. $\sim (x^0) \equiv \{x | x \in X, x \sim x^0\}$, called the 'indifference' set.

Order only

Figure : A hypothetical set of preferences satisfying Axiom 1 and 2.



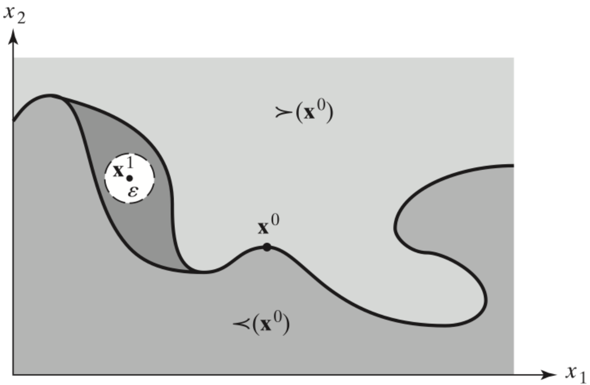
Order+Continuity

Additional axiom of purely mathematical motivation:

- **Axiom 3**: Continuity. For all $x \in \mathbb{R}^n_+$, the 'at least as good as' set, $\succsim(x)$, and the 'no better than' set, $\precsim(x)$, are closed in $\mathbb{R}^n_+$.

It guarantees that sudden preference reversals do not occur.

Figure : A hypothetical preferences satisfying Axioms 1,2 and 3.

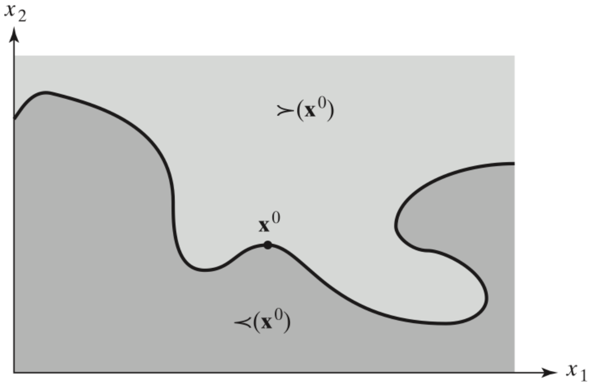


Order+continuity+improvement always possible

Let $B_\epsilon(x^0)$ denotes open ball of radius ϵ centred at x^0.

- **Axiom 4'**: Local Non-satiation. For all $x^0 \in \mathbb{R}^n_+$, and for all $\epsilon > 0$, there exists some $x \in B_\epsilon(x^0) \cap \mathbb{R}^n_+$ such that $x \succ x^0$

Figure : A hypothetical preferences satisfying Axioms 1,2,3 and 4'.



Order+continuity+strict monotonicity

If bundle x^0 contains at least as much of every good as does x^1 we write $x^0 \geqslant x^1$, while if x^0 contains strictly more of every good than x^1 we write $x^0 \gg x^1$.

- **Axiom 4**: Strict monotonicity. For all $x^0, x^1 \in \mathbb{R}^n_+$, if $x^0 \geqslant x^1$ then $x^0 \succsim x^1$, while if $x^0 \gg x^1$, then $x^0 \succ x^1$.

Figure : Satisfying Axioms 1,2,3 and 4'.

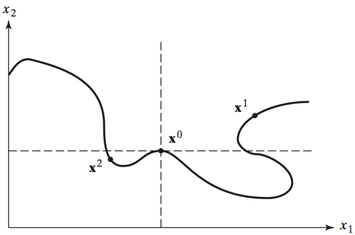
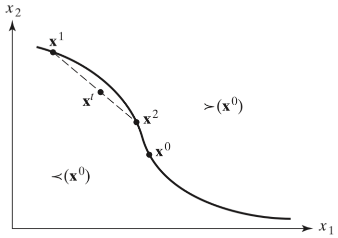


Figure : Satisfying Axioms 1,2,3 and 4.



Order+continuity+strict monotonicity+convexity

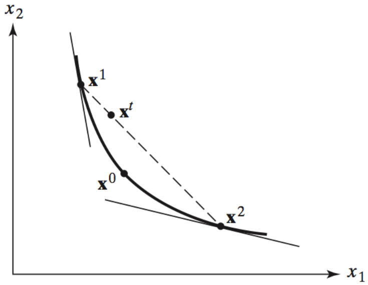
Two versions of the final axiom:

- **Axiom 5'**: Convexity. If $x^1 \succsim x^0$, then $tx^1 + (1-t)x^0) \succsim x^0$ for all $t \in [0,1]$.
- **Axiom 5**: Strict Convexity. If $x^1 \neq x^0$ and $x^1 \succsim x^0$, then $tx^1 + (1-t)x^0) \succsim x^0$ for all $t \in (0,1)$.

Two ways to understand:

- The consumer prefers more balanced bundles to more 'extreme' ones.
- Diminishing marginal rate of substitution in consumption.

Figure : Satisfying Axioms 1,2,3,4 and 5 or 5'



Summary of axioms

- Completeness and transitivity describe a consumer who can make consistent comparisons among alternatives.
- Continuity is intended to guarantee the existence of topologically nice ’at least as good as’ and ’no better than’ sets
- The rest is about tastes:
  - Some form of non-satiation, either weak or strong
  - Some bias in favour of balance in consumption, either weak or strong

Utility representation

Definition (Utility representation)

A real-valued function $u : \mathbb{R}^n_+ \to \mathbb{R}$ is called a utility function representing the preference relation $\succsim$, if for all $x^0, x^1 \in \mathbb{R}^n_+, u(x^0) \geqslant u(x^1) \iff x^0 \succsim x^1$.

Theorem (Existence of utility representation)

If the binary relation $\succsim$ is complete, transitive, continuous, and strictly monotonic, there exists a continuous real-valued function $u : \mathbb{R}^n_+ \to \mathbb{R}$, which represents $\succsim$.

Theorem (Uniqueness of utility representation)

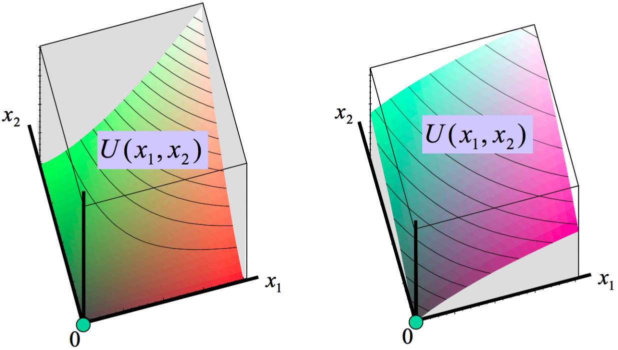
Let $\succsim$ be a preference relation on $\mathbb{R}^n_+$ and suppose $u(x)$ is a utility function that represents it. Then $v(x)$ also represents $\succsim$ if and only if $v(x) = f(u(x))$ for every x, where $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing on the set of values taken on by u.

Properties

Theorem (Properties of Preferences and Utility Functions)

Let $\succsim$ be represented by $u : \mathbb{R}^n_+ \to \mathbb{R}$. Then:

1. *$u(x)$ is strictly increasing if and only if $\succsim$ is strictly monotonic.*
2. *$u(x)$ is (strictly) quasiconcave if and only if $\succsim$ is (strictly) convex.*



MRS

We will henceforth assume even more: that the utility function is differentiable whenever necessary.
Consider any bundle $x^1 = (x_1^1, x_2^1)$. Let $(x_1, x_2) = (x_1, f(x_1))$ trace the indifference curve through x^1:

$$u(x_1, f(x_1)) = \text{const}$$

Let's differentiate it wrt. x_1:

$$\frac{\partial u(x_1, x_2)}{\partial x_1} + \frac{\partial u(x_1, x_2)}{\partial x_2} f'(x_1) = 0$$

So after defining the slope of the indifference curve $MRS_{12}(x_1^1, x_2^1) \equiv |f'(x_1^1)|$:

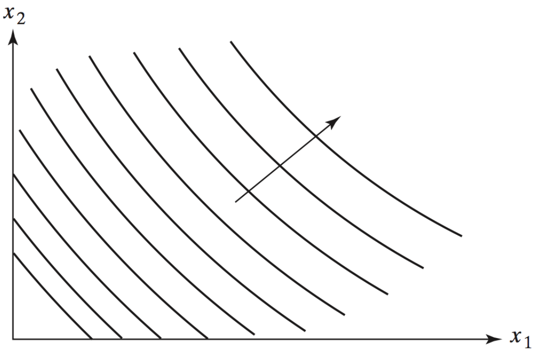
$$MRS_{12}(x^1) = \frac{\partial u(x^1)/\partial x_1}{\partial u(x^1)/\partial x_2}$$

.

Assumption (Consumer preferences)

The consumer preference relation $\succsim$ is complete, transitive, continuous, strictly monotonic, and strictly convex on $\mathbb{R}^n_+$. Therefore, by the above theorems it can be represented by a real-valued utility function, u, that is continuous, strictly increasing, and strictly quasiconcave on $\mathbb{R}^n_+$.

Figure : Indifference map for preferences satisfying the assumption.



Competitive Budget

- We suppose that n commodities are all traded in the market at dollar prices that are publicly quoted. Formally, prices are represented by a price vector:

$$p = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix} \in \mathbb{R}^n_+$$

- Assumption of price-taking

Definition

The Walrasian, or competitive budget set, $B_{p,w} = \{x \in \mathbb{R}^n_+ : p \cdot x \leqslant w\}$ *is the set of all feasible consumption bundles for the consumer who faces market prices* p *and has wealth* w*.*

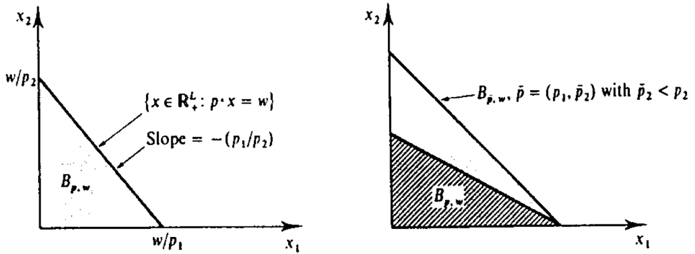


Figure : LHS: Walrasian budget set, RHS: the effect of the price change on Walrasian budget set.

- The Walrasian budget set $B_{p,w}$ is a convex set.
- Convexity of $B_{p,w}$ depends on the convexity of $X = \mathbb{R}^n_+$.

We exlude realistic possibilities such as:

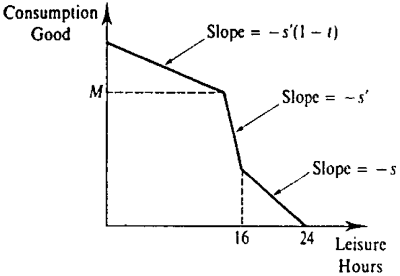


Figure : More realistic budget set that we exclude.

The consumer's Walrasian demand ~~correspondence~~ function $x(p,w)$ assigns a set of chosen consumption bundles for each (p,w)

Def. $x(p,w)$ is homogeneous of degree zero if $x(\alpha p, \alpha w) = x(p,w)$ $\forall p, w$ and $\alpha > 0$

Def. $x(p,w)$ satisfies Walras law if for every $p >> 0$, and $w > 0$, we have $p \cdot x = w$ $\forall x \in x(p,w)$

Ex. $n = 3$ $x(p,w)$ is given by

$$x_1(p,w) = \frac{p_2}{p_1+p_2+p_3} \cdot \frac{w}{p_1}$$

$$x_2(p,w) = \frac{p_3}{p_1+p_2+p_3} \cdot \frac{w}{p_2}$$

$$x_3(p,w) = \frac{\beta p_1}{p_1+p_2+p_3} \cdot \frac{w}{p_3}$$

Comparative statics

wealth effects

for fixed prices $\bar{p}$, the function $x(\bar{p}, w)$ the consumer Engel's function.

It's image in $\mathbb{R}^n_+$ $E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ is known as wealth expansion path

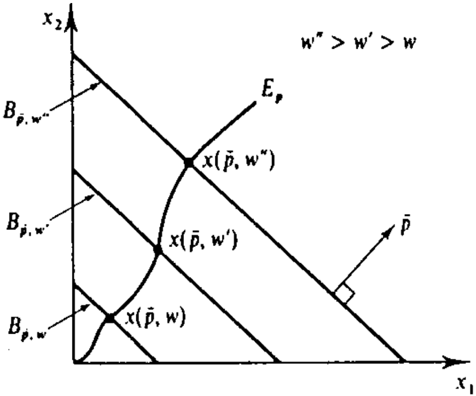


Figure : The wealth expansion path at prices $\bar{p}$.

~~At~~ At any (p, w) the derivative $\frac{\partial x_i(p,w)}{\partial w}$ is known as wealth effect

Commodity is normal at (p,w) ≥ 0

~~at~~ it is inferior at (p,w) < 0

if every good is normal at all (p, w) then we say demand is normal

$$D_w\, x(p,w) = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial w} \\ \vdots \\ \frac{\partial x_n(p,w)}{\partial w} \end{bmatrix} \in R^n$$

<u>price effects</u>

$\frac{\partial x_i(p,w)}{\partial p_j}$ the price effect of p_j on the demand for good i

< 0 - normall good at (p,w)

> 0 Giffen good at (p,w)

$$D_p\, x(p,w) = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} & \cdots & \frac{\partial x_1(p,w)}{\partial p_n} \\ \vdots & & \\ \frac{\partial x_n(p,w)}{\partial p_1} & \cdots & \frac{\partial x_n(p,w)}{\partial p_n} \end{bmatrix}$$

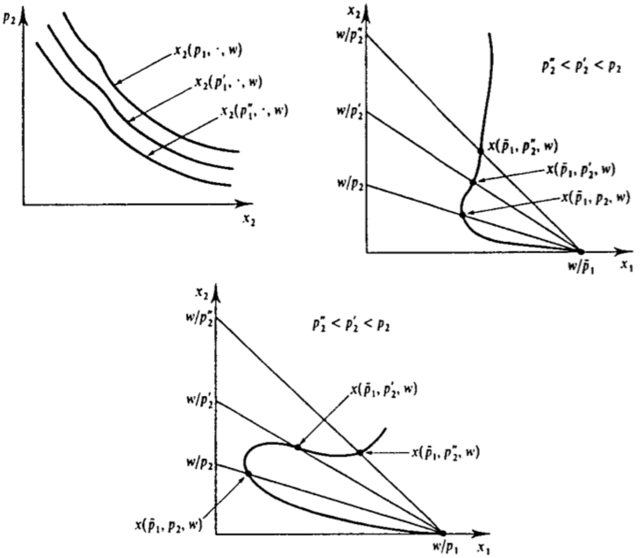


Figure : Top left: The demand for good 2 as a function of its price (for various levels of p_1). Top right: An offer curve. Bottom: An offer curve where good 2 is a Giffen good at $(\bar{p}, p_2', w)$.

Implications of homogeneity of degree zero

$$\boxed{x(\alpha p, \alpha w) - x(p, w) = 0} \quad \forall \alpha > 0$$

differentiate it w.r.t. α, $\alpha = 1$

$$\forall p, w \quad \boxed{\sum_{j=1}^{n} \left(\frac{\partial x_i(p,w)}{\partial p_j} \cdot p_j \right) + \frac{\partial x_i(p,w)}{\partial w} \cdot w = 0 \Big/ \overline{x_i(p,w)}}$$

for $i = 1, \dots, n$

$$\cancel{\frac{\partial x_i(\alpha p, \alpha w)}{\partial \alpha}} =$$

$$\varepsilon_{ij}(p,w) = \frac{\partial x_i(p,w)}{\partial p_j} \cdot \frac{p_j}{x_i(p,w)} \left(\frac{\Delta x_i}{x} \Big/ \frac{\Delta p_j}{p_j} \right.$$

$$\varepsilon_{iw}(p,w) = \frac{\partial x_i(p,w)}{\partial w} \frac{w}{x_i(p,w)}$$

$$\boxed{\sum_{j=1}^{n} \varepsilon_{ij}(p,w) + \varepsilon_{iw}(p,w) = 0} \quad \forall i = 1, \dots, n$$

An equal % change in all prices and wealth leads to no change in demand

Implication of Walras law

$$\boxed{p \cdot x(p,w) = w} \quad \forall p, w$$

Differentiate it w.r.t. p

Cournot aggregation

$$\forall p, w \quad \boxed{\sum_{i=1}^{n} p_i \frac{\partial x_i(p,w)}{\partial p_j} + x_j(p,w) = 0}$$

for $j = 1, \dots, n$

$$\sum_{i=1}^{n} p_i \cdot x_i(p,w) = w \quad \Big| \frac{\partial}{\partial p_j}$$

$$\cancel{x_j(p,w) + \sum_{i=1}^{n} p_i \frac{\partial x_i(p,w)}{\partial p_j} = 0} \quad \forall j$$

$$p_1 x_1(p_1, p_2, \ldots, p_n, w) + \ldots + p_n x_n(p_1, p_2, \ldots, p_n, w) = w$$

$$p_1 \frac{\partial x_1}{\partial p_j} + \ldots \underline{\left| p_j \frac{\partial x_j}{\partial p_j} + x_j \right|}$$

$$\sum_{i=1}^{n} p_i \cdot x_i(p,w) = w \quad \forall p, w$$

differentiate wrt. w

$$\forall p,w \quad \boxed{\sum_{i=1}^{n} p_i \frac{\partial x_i(p,w)}{\partial w} = 1}$$

Engel aggregation

WARP

if $x, y \in A \cap B$

and $x \in C(A)$, $y \in C(B)$

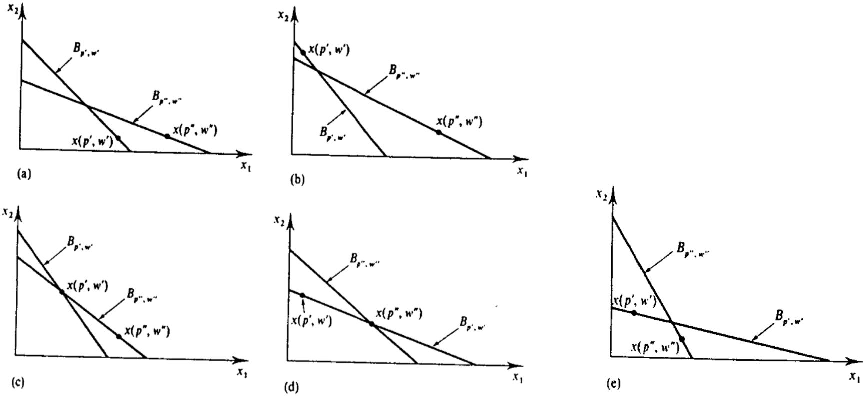
then $x \in C(B)$

Def. The Walrasian demand function $x(p,w)$ satisfies WARP if the following holds for any two (p,w) and (p',w')

if $p \cdot x(p',w') \leq w$ and $x(p',w') \neq x(p,w)$

then $p' \cdot x(p,w) > w'$

Demand in panels (a) to (c) satisfy WARP; demand in panels (d) and (e) does not.



~~if~~ $p \Rightarrow q$

$\neg q \Rightarrow \neg p$

if $p \cdot x(p', w') \leq w$ and $p' \cdot x(p, w) \leq w'$

then $x(p', w') = x(p, w)$

Price changes affects the consumer in 2 ways:

1) - alter the relative cost of different commodities

2) - change the consumer real ~~l~~ wealth

a change in price is accompanied by a change in wealth s.t. an initial bundle is still affordable at new prices

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Consumer faces (p, w) and chooses $x(p, w)$

when price changes to p' wealth is adjusted

$$w' = p' \cdot x(p, w)$$

wealth adjustment: $\Delta w = \Delta p \cdot x(p, w)$

$$\Delta p = (p' - p)$$

Slutsky wealth compensation

compensated price change

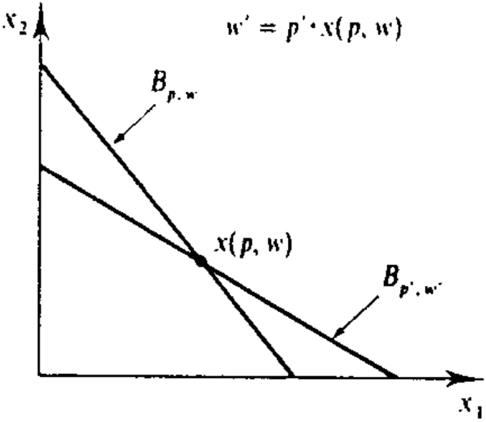

Figure : A compensated price change from (p, w) to (p', w').

Prop: Suppose that $x(p,w)$ is homogeneous of degree zero and satisfies Walras law. Then

$x(p,w)$ satisfies WARP

iff

for any compensated price change from an initial (p,w) to a new $(p',w') = (p', p' \cdot x(p,w))$

we have:

$$(p'-p)[x(p',w')-x(p,w)] \leq 0$$

with strict inequality if $x(p',w') \neq x(p,w)$

$A \Longrightarrow B$

$$\underbrace{p'[x(p',w')-x(p,w)]}_{1} - \underbrace{p[x(p',w')-x(p,w)]}_{2} \leq 0$$

1 $\quad p' \cdot x(p,w) = w'$

By Walras law $\quad p' \cdot x(p',w') = w' \qquad$ 1 $= 0$

2 Since $p' \cdot x(p,w) = w'$, $x(p,w)$ is affordable at (p',w').

By WARP $\quad p \cdot x(p',w') > w$

2 > 0

By Walras law $\quad p \cdot x(p,w) = w$

Example

$$\Delta p = (0, \dots, 0, \Delta p_i, 0, \dots, 0)$$

$$\Delta p \cdot \Delta x = \Delta p_i \cdot \Delta x_i \leq 0$$

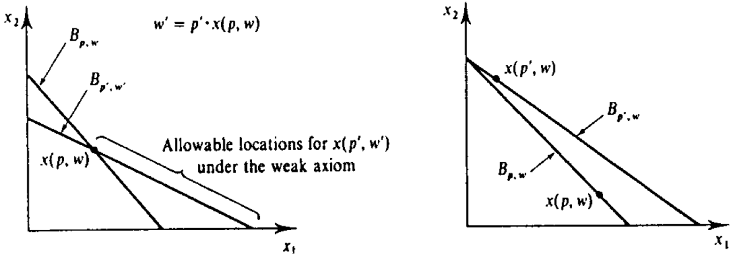

Figure : LHS: Demand must be nonincreasing in own price for a compensated price change. RHS: Demand for good 1 can fall when its price decreases for an uncompensated price change.