Balancedness and Lindahl equilibrium

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Abstract

Condorcet winners, though they do not guarantee unanimity, are often a focal point for public policy. Can Lindahl prices and transfers, which ensure unanimity, be used to finance these policies? In settings where preferences are characterized by bliss points, we propose a condition, called *balancedness*, which requires that a policy lie in the interior of the convex hull of agents' bliss points. We show that any such policy (and not just Condorcet winners) can be decentralized as a Lindahl equilibrium. Our results provide first and second welfare theorems for Lindahl equilibria with satiated preferences.
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"The law of majority voting itself rests on an agreement and implies that there has been on at least one occasion unanimity." - J-J Rousseau, *The Social Contract*, Rousseau (1968).

1 Introduction

Condorcet winners (under some super-majority voting rule)[1], when they exist, are focal points in determining public choice outcomes (see, for instance, Sen (2020)). However, a minority of voters may disagree with the majority's decision. How, then, can a selected public policy be financed so as to ensure unanimity? One way to address this issue is to examine whether the policy can be supported by Lindahl prices and transfers (Lindahl, 1919). If so, financing a policy that enjoys majority support would ensure unanimity[2].

We address this issue in settings where agents' preferences are characterized by bliss points. Such preferences over policy alternatives allow agents to hold different conceptions of the "ideal" public policy. For example, transit users in a large city may prefer a high congestion charge,

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[1]In the remainder of the paper, we always refer to a Condorcet winner under some non-unanimity voting rule.
[2]Conversely, a Lindahl equilibrium that is also a Condorcet winner would have the desirable feature of being implementable by majority vote. We discuss the implementation of a Lindahl equilibrium below.

whereas outer-borough drivers may prefer a low or zero fee. Single-peaked preferences also arise in the large literature on spatial competition following Hotelling's seminal paper (Hotelling, 1929). Although our paper is motivated by the issue of decentralizing Condorcet winners via Lindahl equilibria, our analysis remains valid whenever the problem of decentralizing a Pareto-optimal allocation arises in settings where agents' preferences are characterized by bliss points.

Preferences over public policy or social states[3], characterized by bliss points, have been extensively studied in the public choice literature following Black (1958). This includes single-peaked preferences in one-dimensional settings and Euclidean preferences with bliss points in multidimensional settings. From the perspective of this paper, a key property of models with such preferences is that Condorcet winners under majority voting have been shown to exist when voters have these preferences (see, for example, Black (1958); Plott (1967); Caplin and Nalebuff (1988, 1991); Pivato (2015); Maskin (2023)).

Preferences with bliss points display satiation. However, while the existence of a Lindahl equilibrium has been generally established in settings where satiation is explicitly ruled out (see, for instance, Foley (1970), Roberts (1973), Mas-Colell (1980), Gul and Pesendorfer (2025), Carvajal and Song (2022), or Bonnisseau et al. (2023))[4], Ghosal and Polemarchakis (1999) shows that with satiated preferences[5], and for a fixed distribution of revenue, a Lindahl equilibrium may not exist. Moreover, beyond standard assumptions, Ghosal and Polemarchakis (1999) requires an additional sufficient condition to decentralize a Pareto-optimal state as a Lindahl equilibrium with transfers, namely irreducibility, i.e., for any partition of the set of individuals into two non-empty groups, each individual in one group can be made strictly better off in another feasible state. We show that when a selected policy or social state is irreducible, it can never be a Condorcet winner. Evidently, even in other settings where the objective is to decentralize a Pareto-optimal social state different from a Condorcet winner, irreducibility may not always be satisfied.

We propose a new balancedness condition that guarantees the existence of Lindahl prices and transfers[6]. Balancedness is satisfied, for example, when a chosen policy lies in the interior

[3]The terms "public policy" and "social states" will be used interchangeably throughout the paper.

[4]The existence of, or decentralization by, a competitive equilibrium requires assumptions that rule out certain prices as candidate equilibrium prices. For example, the assumption that preferences are (weakly) monotone implies that zero prices are ruled out as candidate equilibrium prices, so that (after normalization) only prices in the unit simplex need to be considered. When preferences display satiation, other assumptions are required, hence the need for existence and decentralization results different from those proved under weakly monotone preferences.

[5]Such a setting is consistent with Ruys (1974) (pages 79–88), who study Lindahl equilibria in a setting with only pure public goods.

[6]Our balancedness condition ensures the existence of a Lindahl equilibrium with transfers in spatial compe-

of the convex hull of the bliss points of voters with Euclidean preferences.

Balancedness is sometimes, but not always, consistent with the existence of a Condorcet winner. Specifically, there are settings in which irreducibility fails and no Condorcet winner exists, yet balancedness is satisfied, thereby ensuring the existence of a Lindahl equilibrium. Thus, markets with personalized prices and appropriately adjusted revenue distributions can ensure unanimity for a public policy even when it is not a Condorcet winner. However, when a Condorcet winner[7] is balanced, Lindahl prices and transfers can be used to ensure unanimity. When this condition fails, we provide an example showing that a Condorcet winner cannot be decentralized.

We interpret our balancedness condition as a compromise among the preferred policies of different voter or agent types. Returning to the congestion-pricing example, the adopted fee lies in the interior of the convex hull of agents' bliss points, and the city complements it with discounts and earmarked transit upgrades, which can be viewed as informal Lindahl prices and transfers designed to bring all groups on board. A similar logic may apply in international settings, such as EU burden-sharing for Ukraine's reconstruction or global climate funds. In such cases, a majority of small-GDP states may underfund a project because large-GDP members are relatively few. Lindahl prices and transfers - such as earmarked technological transfers or carbon-border adjustment rebates - could be designed to bring large-GDP members on board.

Our results can also be interpreted in relation to versions of the first and second welfare theorems for Lindahl equilibria with *satiated preferences* (see also Ruys (1974), Section 5.2, for early results on the existence of equilibrium in pure public goods economies). We also show that our *balancedness* condition is distinct from the pairwise balancedness condition proposed by Plott (1967). We provide a sufficient condition for the existence of a Condorcet winner that satisfies the balancedness condition. Under the assumption of Euclidean preferences, and building on Neligh (2025), we identify a mechanism that implements a Lindahl equilibrium. Finally, we demonstrate how our analysis can be extended to the provision of "social states" or "public policies" with quasi-linear preferences.

The remainder of the paper is organized as follows. The next section introduces the model

tition models as well (see Khan and Vohra (1987), which develops a notion of equilibrium, termed a Lindahl-Hotelling equilibrium, in which producers' prices are marginal cost prices and public goods are financed through Lindahl prices).

[7]The existence of a Condorcet winner under majority rule requires that the size of the largest coalition opposing the policy chosen by it be a minority, a point that applies to both finite (as in this paper) and infinite voter settings (as in Caplin and Nalebuff (1988); Caplin and Nalebuff (1991)).

and some preliminary results, followed by a section focused on the use of the balancedness condition in settings with Condorcet winners. The penultimate section concludes with a discussion of our model and concepts. The final section concludes.

2 The model and initial results

2.1 The Economy and Lindahl equilibria

The economy consists of a set of agents and a firm. Agents/voters have preferences over a set of feasible public policies/social states $Q \subset \mathbb{R}^L$, where L corresponds to different dimensions of social states.

Agents/voters are characterized by types from the finite set T. Let $\mathbf{t}$ denote the cardinality of T. The preferences of an agent of type $t \in T$ are represented by a continuous utility function $u_t : Q \to \mathbb{R}$. The set of agents/voters (society) can be summarized by a distribution of types μ in T, with μ_t denoting the measure/fraction of the set of agents of type t. Clearly, $\mu(T) = 1$. The actual number of agents can be finite or infinite. If the number of agents is finite and its set is N, we can partition N, so that each element of the partition is denoted by a type t such that whenever $i, j \in t$, $u_i = u_j$. In what follows, we restrict ourselves to type symmetric allocations and equilibria.

An individual of type t is satiated at $q \in Q$ if and only if $u_t(q) \geq u_t(q'), \forall q' \in Q$. An individual of type t is locally satiated in $q \in Q$ if there exists a neighborhood of q, $\upsilon_q \subset Q$, such that $u_t(q) \geq u_t(q'), \forall q' \in \upsilon_q$. A society is summarized by $S = (T, Q, \{u_t, \mu_t : t \in T\})$.

Commodities are indexed by $(l, t), l = 1, .., L, t \in T$; this is necessary for the variables that affect the preferences of each type to be the objects of choice of that type. A bundle of commodities is $x = (..., x_{l,t}, ...)$. The domain of bundles of commodities is $\mathbb{R}^{L\mathbf{t}}$. An individual of type t is described by the pair (X_t, u_t) where X_t is the consumption set (also interpreted as a private production set) of that individual, with $X_t = Q$ and $u_t : X_t \to \mathbb{R}$ is their ordinal utility function.

The firm is characterized by its technology (production set), $Y \subset \mathbb{R}^{L\mathbf{t}}$ consisting of production bundles $y = (..., y_t, ...)$ such that there exist $q \in Q$ such that $y_t = q$ for each $t \in T$. That is, technology of the firm is specified so that the commodities chosen by different types correspond to a specific social state at a feasible allocation in the economy. The economy associated with a society S is $E = \{T, Y, (X_t, u_t, \mu_t : t \in T)\}$.

A state of the economy is an array (y, x), consisting of a production plan for the firm $y \in Y$, and, for each type, a consumption vector $x_t \in X_t$. A state of the economy is feasible if and only if for all $t \in T$, $x_t = y_t = q$. Hence, there is an unambiguous association of a feasible social state with a feasible state of the economy, and vise-versa.

Given $q \in Q$, for a collection of types $T' \subseteq T$, denote $u^{T'}(q) = \{u_t(q) : t \in T'\}$. A feasible q is Pareto dominated by a feasible q' if $u^T(q') > u^T(q)$ i.e. $u_t(q') \geq u_t(q)$ for all $t \in T$ with a strict inequality for at least one $t \in T$. A feasible q is Pareto optimal if it is not Pareto dominated by any feasible q'.

Note that in our setting, preferences can be satisfied. The existence of a Pareto optimal q with satiated preferences is not evident: it is established in Lemma 1 in Ghosal and Polemarchakis (1999) under the assumption of compactness of Q.

The prices of commodities are $p = (..., p_t, ...)$. The prices $p_t \in \mathbb{R}^L$ are therefore type-dependent. The value of a bundle of commodities, x at prices p is $\sum_{t \in T} \mu_t p_t \cdot x_t$, that is: a vector of type-weighted prices $(\mu_t p_t)_{t \in T}$ (dot product) multiplied by a vector x. A revenue vector is $w = (..., w_t, ...)$. At prices p and an allocation x, a transfer of revenues is a revenue vector w such that $\sum_{t \in T} \mu_t w_t = \sum_{t \in T} \mu_t p_t \cdot x_t$.

Definition 1 (Lindahl equilibrium with transfers). *A list $(y^*, (x_t^*, p_t^*)_{t \in T})$ is a type symmetric*[8] *Lindahl equilibrium with transfers, if there exists a transfer of revenue $w^* = (w_t^*)_{t \in T}$ such that:*

1. *for each $t \in T$, x_t^* solves* $\max u_t(x)$ *over* $\{x \in Q : p_t^* \cdot x \leq w_t^*\}$,
2. *y^* solves* $\max_{y \in Y} \sum_{t \in T} \mu_t p_t^* \cdot y_t$,
3. *$x_t^* = y_t^*$, for each $t \in T$,*
4. *$\sum_{t \in T} \mu_t w_t^* = \sum_{t \in T} \mu_t p_t^* \cdot x_t^*$.*

In a type symmetric Lindahl equilibrium, prices and transfers are tailored to a type and not to each individual. From now on, we use the term Lindahl equilibrium and the type symmetric Lindahl equilibrium interchangeably. Since in any equilibrium, by market clearing (point 3), there exists a $q^* \in Q$ such that $x_t^* = y_t^* = q^*$ for each t, in what follows, we often use a simpler notation $(q^*, (p_t^*)_{t \in T})$ to denote a Lindahl equilibrium with transfers. Some comments.

[8]Focusing on type-symmeytric equilibria allows for a direct extension to models with a continuum of agents with m-majority voting Caplin and Nalebuff (1988, 1991). Also, Sen (2020) studies type-symmetric voting with a finite number of voters.

First, condition 1 in the preceding definition implies that in a Lindahl equilibrium with transfers if $u_i(x_t) > u_t(x_t^*)$ for some $x_t \in X_t$, then $p_t^* \cdot x_t > w_t^*$. Suppose that point 1 in the definition of a Lindahl equilibrium with transfers is replaced by the following auxillary condition: for each $t \in T$ if $u_t(x_t) \geq u_t(x_t^*)$ with $x_t \in X_t$, then $p_t^* \cdot x_t \geq w_t^*$; moreover, there exists at least one $t \in T$ such that if $u_t(x_t) > u_t(x_t^*)$ for some $x_t \in X_t$, then $p_t^* \cdot x_t > w_t^*$. Then, we say that $(y^*, (x_t^*, p_t^*)_{t\in T})$ is a *Lindahl quasi-equilibrium with transfers* w^*.

Second, if $y^* \in Y$ is in the interior of Y, then the existence of a Lindahl equilibrium requires $\sum_{t\in T} \mu_t p_t^* \cdot y_t^* = 0$. This results from the linearity of the firm's objective and the zero-marginal costs condition.

Third, for a fixed distribution of transfers, a Lindahl equilibrium need not exist, as demonstrated by the following example.

Example 1 (Equilibrium non-existence without transfers)**.** *The set of feasible social states is* $Q = [-1, 1]$. *Individuals are indexed by* $t \in T = \{1, 2\}$, $\mu_t = \frac{1}{2}, t \in T$ *and have utility functions* $u_1(q) = -(q-1)^2$ *and* $u_2(q) = -(q-\frac{1}{2})^2$ *with domain over social states* $X_t = Q$. *The distribution of transfers is fixed with* $w_t = 0$, $t \in T$. *At prices* $p = (p_1, p_2)$, *the budget constraint for a type* t *individual is* $p_t x_t \leq 0$. *Suppose first that* $p_1 + p_2 = 0$. *If* $p_1 = 0$, *then* $x_1 = 1$ *while* $x_2 = \frac{1}{2}$; *if* $p_1 > 0$, *then* $x_1 = 0$ *while* $x_2 = \frac{1}{2}$; *if* $p_1 < 0$, $x_1 = 1$ *while* $x_2 = 0$; *all are contradictions to market clearing. Alternatively,* $p_1 + p_2 > 0$, *from the maximization of the firm's profits,* $y = (1, 1)$. *If* $x_1 = x_2 = 1$, *then* $p_1 + p_2 \leq 0$, *a contradiction to market clearing. By similar logic,* $p_1 + p_2 < 0$ *leads to a contradiction. However, note that there are transfer distributions for which Lindahl equilibria exist. Specifically, if* $w_1 + w_2 = 0$, $w_1 \in [\frac{1}{2}, 1]$ *support* $q \in [\frac{1}{2}, 1]$ *as Lindahl equilibrium outcomes.*

2.2 Condorcet winners, irreducibility and Lindahl equilibria

We say, a feasible q is dominated by a feasible q' for a collection of agents $T' \subseteq T$, if $u^{T'}(q') > u^{T'}(q)$. A feasible social state q is dominated by a feasible social state q' under a rule of m-majority voting, if there is a collection of agents $T' \subseteq T$ such that $u^{T'}(q') > u^{T'}(q)$ and $\mu(T') > m$, where $\mu(T') = \sum_{t\in T'} \mu_t$.

We are now in a position to state the formal definition of a Condorcet winner:

Definition 2 (Condorcet winner)**.** *A feasible social state* $q \in Q$ *is a Condorcet winner of* m*-majority if it is not dominated by any feasible social state under a* m*-majority voting.*

Proposition 1 in Ghosal and Polemarchakis (1999) established that the existence of a Lindahl equilibrium with transfers follows if the economy is irreducible. It will be convenient to recall the definition of irreducibility in our type-symmetric setting.

Definition 3 (Irreducibility). *A feasible social state $q \in Q$ is irreducible if and only if, for any partition of T into two nonempty sets T_1, T_2, there exists a feasible social state $q' \in Q$ such that for each $t \in T_1$ we have $u_t(q') > u_t(q)$.*

Let $Q^m \subset Q$ denote the set of $m - majority$ Condorcet winners and let Q^{IR} denote the set of irreducible social states. The following proposition shows that these sets cannot have common elements under a minimal assumption.

Proposition 1. *Suppose that there exists a proper subset of T denoted by T_1 with $\mu(T_1) > m$. Then $Q^m \cap Q^{IR} = \emptyset$.*

Proof. The argument is based on contradiction. Suppose $Q^m \cap Q^{IR} \neq \emptyset$. Take $q \in Q^m \cap Q^{IR}$. By assumption, there exists a proper subset of T, say T_1 with $\mu(T_1) > m$. Take any such set. Since q is irreducible, there exists $q' \in Q$ such that every individual in T_1 strictly prefers q' to q. But since $\mu(T_1) > m$, q cannot be a m-majority Condorcet winner. □

An implication of the preceding proposition is that when the social state satisfies irreducibility, it cannot be a Condorcet winner. On the face of it, this result would seem to imply that the set of Lindahl equilibria that are simultaneously Condorcet winners is empty. However, irreducibility is only a sufficient condition for the existence of a Lindahl equilibrium with transfers. The next example illustrates that the Lindal equilibria with transfers may exist even without irreducibility being satisfied, or even when a Condorcert winner does not exist.

Example 2 (Motivating example). *Consider 3 types of voters of equal measure, each with Euclidian preferences over 3 candidates/policies represented as points in $\mathbb{R}^2$. The bliss point of a voter of type 1 is $t_1 = (-1, -2)$, a voter of type 2 is $t_2 = (4, 0)$, and a voter of type 3 is $t_3 = (-1, 4)$. The policies are given by $A = (0, 0)$, $B = (3, -2)$, and $C = (2, 3)$ as illustrated in Figure 1. Observe that there is a Condorcet cycle. In fact, none of the policies can be sustained by a majority vote. Indeed, suppose that voters start with a proposal B, then the types $\{1, 3\}$ prefer A, but then the types $\{2, 3\}$ prefer C, and then the types $\{1, 2\}$ prefer B. Moreover, neither of the three proposals is irreducible: in the proposal A the subset of types $\{1, 3\}$ cannot be improved by some social state, in C the subset of types $\{2, 3\}$ cannot be improved, while for B the*

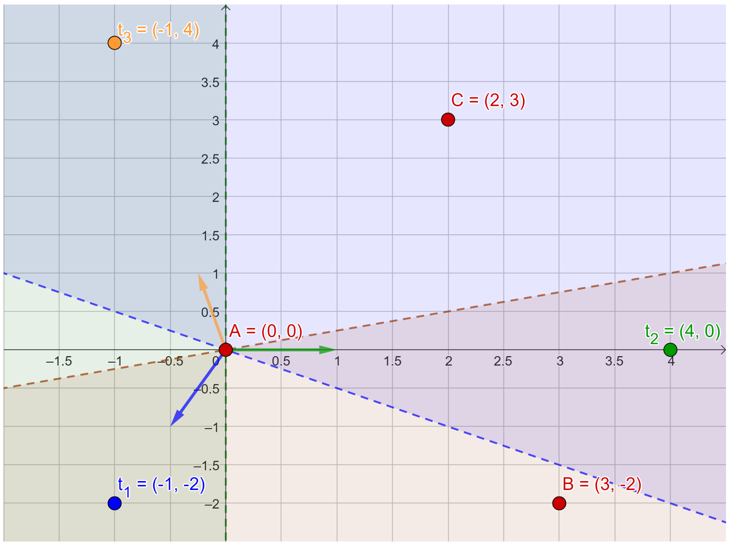

Figure 1: Example of the Condorcet cycle with 3 voters (with bliss points: t_1, t_2, t_3). None of the three policies (A, B, C) is irreducible, but A can be decentralized with personalized prices and transfers. Shaded areas denote voters' budget sets (using color codes: blue for t_1, green for t_2 and orange for t_3) in the Lindahl equilibrium with tansfers.

subset of types $\{1,2\}$ *cannot be improved. However, proposal* A *can be decentralized in a Lindahl equilibrium, with personalized prices, e.g.* $p_1 = (-8,-16)$, $p_2 = (12,0)$ *and* $p_3 = (-4,16)$ *satisfying* $p_1 + p_2 + p_3 = 0$ *and transfers* $w_1 = w_2 = w_3 = 0$.

Notice that proposal A is a "political compromise", i.e. is in the interior of the convex hull of the voter's bliss points t_1, t_2 and t_3. This observation is the focus of the next section.

3 Bliss points, balancedness and Lindahl decentralization

3.1 Single-Peaked preferences with bliss points over policies

To begin with, we study the case where $Q \subset \mathbb{R}$. As before, the preferences of the agents on Q are anonymous and depend only on their type $t \in T \subseteq Q$, which denotes their bliss point with respect to policy q. Formally:

Assumption 1 (Single-peaked preferences)**.** *For each* $t \in T$ *a preference is represented by a utility* $u_t : Q \to \mathbb{R}$ *that is single-peaked, that is, it increases strictly in* $\{q \in Q : q < t\}$ *and decreases strictly in* $\{q \in Q : q > t\}$.

A typical example of such preferences is given by the utility $u_t(q) = -(q-t)^2$ or a Euclidean norm utility. We examine whether, for single-peaked preferences, a symmetric Lindahl equilibrium with transfers can decentralize a m-majority Condorcet winner. We start with the case where $m = \frac{1}{2}$, and the types are distributed symmetrically around the median voter.

Example 3 (Symmetric distribution around the median voter)**.** *Consider a set of $N+1$ (N even) voters indexed by t from $T = \{0, \frac{1}{N}, \frac{2}{N}, \ldots, 1\}$ with $\mu_t = \frac{1}{N+1}$ and preferences over $q \in Q = [0,1] = T$ are given by $u_t(q) = -(q-t)^2$.*

By the median voter theorem, if agents vote on policies q_1, q_2 proposed by 2 candidates, each candidate will propose the same policy $q_1 = q_2 = 0.5$, that is, a bliss point for the median voter. We now analyze the possibility of decentralizing $q = 0.5$ in the Lindahl equilibrium with transfers. Observe that $MU_{q,t} = -2q + 2t$ for $t \in T$ and therefore for personalized prices $p_t = 2t - 2q$ with fixed q each agent will choose the same policy, if affordable. The sum of such prices is:

$$\frac{1}{N+1}\sum_{t\in T} p_t = \frac{2}{N+1}\sum_{n=0}^{N}(\frac{n}{N} - q) = \frac{2}{N+1}\left[\frac{N+1}{2} - (N+1)q\right] = 1 - 2q.$$

Observe that this equals zero for $q = 0.5$.

As a result of the aggregate price being equal to 0 (and only for this price), the firm's profit is zero for any q, the firm is therefore indifferent and therefore can supply $q = 0.5$. To finance the purchase of $q = 0.5$, we propose incomes $w_t = t - 0.5$. Observe that $\sum_{t\in T}\frac{1}{N+1}w_t = 0$, and hence such transfers are budget-balanced.

In the above example, we decentralize the median voter outcome (0.5) with personalized prices and transfers. Some comments follow.

First, in fact, without appropriate transfers or profit allocation, a Lindahl equilibrium may not exist or decentralize the desired social outcome. Second, the above example may suggest that the symmetry of the voters' (bliss points) distribution around the median voter is necessary for prices to sum up to zero. We will demonstrate that this is not the case. In fact, a Lindahl equilibrium can decentralize the median voter outcome for more general distributions of voters' bliss points. Third, if preferences over social states are single-dimensional, we can, in fact, decentralize the social outcome (here the median voter) with two (non-zero) levels of prices only[9]. Fourth, the Lindahl equilibrium can be used to decentralize not only the median voter outcome but also other Pareto optimal allocations, in fact, any allocation that is contained in the interior of the convex hull of bliss points of the individuals/voters.

We formalize all of this in the following proposition.

[9] See also Carvajal and Song (2022).

Proposition 2. *Let Assumption 1 be satisfied. Let $q \in Q$ be such that there exists a strictly positive fraction of types with bliss points $t < q$ and a strictly positive fraction of types with bliss points $t > q$. Then there exists a Lindahl equilibrium with transfers decentralizing q.*

Proof. Denote a fraction of the population with bliss points t strictly below q by μ_-; a fraction with bliss points at $t = q$ by μ_0 and a fraction μ_+0 of the population with bliss points $t > q$. Clearly: $\mu_- + \mu_0 + \mu_+ = 1$. By assumption: $\mu_- > 0$ and $\mu_+ > 0$. To decentralize q, we set $p^*_- < 0$, $p^*_0 = 0$ and $p^*_+ > 0$ such that $\mu_- p^*_- + \mu_+ p^*_+ = 0$. Clearly, such prices exist. Then for each agent with $t < q$ we set $w^*_- := qp^*_-$ and for each agent with $t > q$ we set $w^*_+ := qp^*_+$. Finally, $w^*_0 := 0$. Observe that for types $t < q$, the budget inequality implies that the set of affordable policies is $Q \cap [q, \infty)$. In that set, the optimal choice for this fraction is q. Similarly, for the types $t > q$, the budget inequality implies that the set of affordable policies is $Q \cap (-\infty, q]$. In that set, the optimal choice for this fraction is q. Ultimately, the optimal choice for a fraction μ_0 is q.

Clearly $\mu_- p^*_- + \mu_0 p^*_0 + \mu_+ p^*_+ = 0$. The firm is therefore indifferent to Q and can choose q. Finally, we need to show that the transfers are budget balanced. Indeed: $w^*_- \mu_- + w^*_0 \mu_0 + w^*_+ \mu_+ = qp^*_- \mu_- + qp^*_+ \mu_+ = q(\mu_- p^*_- + \mu_+ p^*_+) = 0$ for each q. □

A corner/boundary policy may not be possible to be decentralized in a Lindahl equilibrium unless there is a (strictly) positive fraction of types with that policy as their bliss point.

Example 4 (Corner bliss-points)**.** *Consider a set of bliss points $[0, 1] = Q$ with a positive mass of agents at $t = 0$ (with mass $\mu_0 \in (0, 1)$) and the remaining measure of agents' ($\mu_+ = 1 - \mu_0$) distributed over $(0, 1]$. Suppose that we want to decentralize $q = 0$ as an equilibrium. Observe that 0 is the corner in Q. We set prices $p_+ > 0$ for agents with $t > 0$ but for $t = 0$ we set up $p_0 = -\frac{p_+ \mu_+}{\mu_0}$. Again, with incomes $w_t = 0$ for each t, agents with $t > 0$ can afford only nonpositive q while agents $t = 0$ can afford any nonnegative q, and therefore both groups choose $q = 0$. The prices sum up to 0 and therefore the firm is indifferent and chooses $q = 0$ making zero profit. The sum of transfers is trivially zero. Alternatively, setting $p_0 < -\frac{p_+ \mu_+}{\mu_0}$, agents in $t = 0$ choose their bliss points. Prices do not sum up to 0 but to some negative number. The firm chooses a lower bound of Q, $q = 0$.*

3.2 Multi-dimmensional policies and Euclidian preferences

We start with the following definition.

Definition 4 (Balancedness)**.** *An $n \times m$ matrix A of real numbers is balanced if there exists a column vector $\alpha \in \mathbb{R}^m_{++}$ such that $A\alpha = 0 \in \mathbb{R}^n$.*

As we show below, the results for a single dimmentional Q cannot be extended in a straightforward way to a multi-dimentional Q, unless a certain matrix we define below is *balanced.* In fact, this *balancedness* condition is trivially satisfied for interior allocations in a convex hull of agents' bliss points in $Q \subset \mathbb{R}$. As a consequence, our results for a multidimentional Q extend these for a single dimmentional Q.

We impose the following assumption:

Assumption 2. *Suppose Q is open and convex and u_t is quasi-concave for each $t \in T$.*

We are now in a position to state our two main results, which establish the key existence result.

Theorem 1. *Assume 2 and suppose $q \in Q$, and let $(p_t)_{t\in T} \in \mathbb{R}^L$ be a vector of marginal utilities supporting q, i.e. $p_t \in \partial u_t(q)$. If the $L \times T$ matrix $[p_t]_{t\in T}$ is balanced, then there exists a Lindahl equilibrium with transfers decentralizing q.*

Proof. Since $[p_t]_{t\in T}$ is balanced, there exists a vector $(\alpha_t)_{t\in T}$ such that $\sum_{t\in T} \alpha_t p_t = 0$. Defining $p'_t = p_t \frac{\alpha_t}{\mu_t}$, we obtain $\sum_{t\in T} \mu_t p'_t = 0$. Setting $w'_t = p'_t \cdot q$, we obtain a Lindahl equilibrium $q, (p'_t)_{t\in T}$ with transfers $(w'_t)_{t\in T}$. Indeed, as $\sum_{t\in T} \mu_t p'_t = 0$, the firm is indifferent and can choose $y^*_t = q, t \in T$. Transfers are also balanced. By construction, each t at prices p'_t and income w'_t can afford q. The FOC for the interior optimal choice, say q^*_t, is $\lambda_t p'_t \in \partial u_t(q^*_t)$. By quasi-convexity, it is also sufficient. Taking the Lagrange multiplier $\lambda_t := \frac{\mu_t}{\alpha_t}$ this implies that $p_t \in \partial u_t(q^*_t)$, and therefore we can take $q^*_t = q$ as the optimal choice of each t. □

For a finite set $A \subset \mathbb{R}^L$ define $\mathrm{con}^\circ(A)$ as the strict convex hull of $A \subset \mathbb{R}^L$, that is

$$\mathrm{con}^\circ(A) := \left\{ z \in \mathbb{R}^L : \forall a \in A\, \exists \alpha_a > 0 \text{ s.t. } \sum_{a\in A} \alpha_a = 1 \text{ and } z = \sum_{a\in A} a\alpha_a \right\}.$$

We interpret a policy contained in the interior of the strict convex hull of the voter's bliss points as a compromise between the most preferred policy choices of different voter types. We say that the type t has Euclidian preferences $\succeq_t$ with t denoting its bliss point, whenever $x' \succeq_t x$ if and only if $d(x,t) \leq d(x',t)$ and $d(\cdot,\cdot)$ is the Euclidian distance in $\mathbb{R}^L$.

Theorem 2. *Assume 2 and suppose that for each $t \in T$ utility u_t represents Euclidian preferences (with t denoting a bliss point of type t). Let $q \in Q$ be such that $q \in con^{\circ}(T)$. Then there exists a Lindahl equilibrium with transfers decentralizing q.*

Proof. For each type in T define $p_t = t - q$. By the definition of Euclidian preferences for each t there exists a Lagrange multiplier $\lambda_t \geq 0$ such that $\lambda_t p_t \in \partial u_t(q)$. In fact $\lambda_t > 0$ whenever $t \neq q$. For $t = q$ we can take, in particual, any $\lambda_t > 0$. We will show that the matrix $[\lambda_t p_t]_{t \in T}$ is balanced. In fact, the weight vector required to balance this price matrix is $\frac{1}{\overline{\lambda}}[\frac{\alpha_t}{\lambda_t}]_{t \in T}$, where $\overline{\lambda} = \sum_t \frac{\alpha_t}{\lambda_t}$ is a normalization scalar. To see that, recall that q is an element of $\mathrm{con}^{\circ}(T)$, so we have $q = \sum_{t \in T} \alpha_t t$ with all $\alpha_t > 0$. Then

$$\sum_{t \in T} \frac{1}{\overline{\lambda}} \frac{\alpha_t}{\lambda_t} \lambda_t p_t = \frac{1}{\overline{\lambda}} \sum_{t \in T} \alpha_t [t - q] = \frac{1}{\overline{\lambda}} [\sum_{t \in T} \alpha_t t - \sum_{t \in T} \alpha_t q] = \frac{1}{\overline{\lambda}} [q - q] = 0,$$

observing in the third equality that $\sum_{t \in T} \alpha_t = 1$. Hence $[\lambda_t p_t]_{t \in T}$ is balanced, therefore, by Theorem 1 $(y_t = q, \frac{\alpha_t}{\mu_t} p_t : t \in T)$ is a Lindahl equilibrium with transfers $w_t := \frac{\alpha_t}{\mu_t} p_t \cdot q$, $t \in T$. $\square$

Hence, any Condorcet winner in $\mathrm{con}^{\circ}(T)$ (so that it is a compromise policy) can be decentralized as a Lindahl equilibrium with transfers. Clearly, the set $\mathrm{con}^{\circ}(T)$ can be characterized by the interior of the convex hull of the extreme subset of $\mathrm{con}(T)$. So, a Condorcet winner in $\mathrm{con}^{\circ}(T)$ is a strict compromise between the preferred policies of extremal types.

The next example illustrates that the interiority assumption in the statement of Theorem 2 is critical.

Example 5 (Nonexistence at the boundary)**.** *Consider $Q \in \mathbb{R}^2$ and three types of agents of equal measure, one with bliss points: one in $t_1 = (0, 1)$, one in $t_2 = (-1, 0)$, and one in $t_3 = (1, 0)$, respectively. The policy $q = (0, 0)$ belongs to the convex hull of the bliss points. However, a matrix of supporting prices $[p_t]_{t=1,2,3} = [(0, 1), (-1, 0), (1, 0)]$ is not balanced and there is no Lindahl equilibrium decentralization q. However, there exists a Lindahl quasi-equilibrium decentralizing q with $(p_t)_{t=1,2,3} = ((0, 0), (-1, 0), (1, 0))$ and incomes $w_1 = w_2 = w_3 = 0$.*

3.3 Welfare theorems for Lindahl equilibria under satiated preferences

We conclude this section with comments relating our results to the 1st and 2nd welfare theorems for production economies. The 1st welfare theorem says the following:

Proposition 3. *Suppose assumption 2 holds and u_t is strict quasi-concave. If $q^*, (p_t^*)_{t\in T}$ is a Lindahl equilibrium with transfers, then q^* is Pareto optimal.*

The assumptions of the proposition are, in particular, satisfied for the Euclidean preferences.

Proof. To see that, argue by contradiction. Suppose that there exists $q' \in Q$ such that $u_t(q') \geq u_t(q^*)$ with strict inequality for at least one t. Then, by maximization of each agent's utility, at a Lindahl equilibrium: $p_t^* \cdot q' \geq w_t$ with strict inequalities for at least some t. Then

$$\sum_t \mu_t q' \cdot p_t^* > \sum_t \mu_t w_t = (\sum_t \mu_t p_t^*) \cdot q^* \geq (\sum_t \mu_t p_t^*) \cdot q',$$

where we first use conditions 4 and 2 from the definition of a symmetric Lindahl equilibrium. This gives a contradiction[10]. □

We next characterize the Pareto optimal allocations for Euclidean preferences.

Proposition 4. *Suppose Q is open, $con(T) \subset Q$ and the preferences are Euclidean. Then $q^* \in Q$ is Pareto optimal if and only if $q \in con(T)$.*

Proof. Indeed, for Euclidian preferences, Pareto optimality is equivalent to maximization of the social welfare function for some weights $(\alpha_t)_{t\in T}$ such that $\alpha_t \geq 0$ and $\sum_t \alpha_t = 1$. Taking a utility representation of Euclidian preferences $u_t(q) = -0.5 \sum_{l=1}^{L}(q_l - t_l)^2$ we obtain the social welfare function (for a public policy $q \in Q$) equal to $-0.5 \sum_{t\in T} \alpha_t \sum_{l=1}^{L}(q_l - t_l)^2$. The first order conditions for maximization yield $\sum_{t\in T} \alpha_t(t_l - q_l) = 0$ for each l. This is equivalent to $\sum_{t\in T} \alpha_t t = q$, as required. □

The convex hull of the set of types' bliss points analyzed in Theorem 2 is hence the set of Pareto optimal allocations.Theorem 2 is hence our version of the second welfare theorem. Every "interior" q can be decentralized as an equilibrium with transfers. In fact a slightly stronger result can be shown that allows one to decentralize "boundary" allocations.

Proposition 5. *Assume 2 and suppose that there exists $t \in T$ such that $q = t$, then there exists a Lindahl equilibrium with transfers decentralizing q. If additionally utility u_t represents Euclidian preferences (with t denoting a bliss point of type t) and if $q \in con(T)$ then there exists a Lindahl quasi-equilibrium with transfers decentralizing q.*

[10]See also Debreu (1975) Theorem 6.3 for a related result.

Proof. Suppose first there exists $t' \in T$ with $t' = q$. Then for all $t \neq q$ take any marginal $p_t \in \partial u_t(q)$ and define prices $p'_t := \frac{p_t}{\mu_t}$. Finally, take $p'_{t'} := -\frac{\sum_{t\in T} p_t}{\mu_{t'}}$. Setting $w_t = p'_t \cdot q$, we obtain a Lindahl equilibrium $q, (p'_t)_{t\in T}$ with transfers $(w_t)_{t\in T}$. Indeed, as $\sum_{t\in T} \mu_t p'_t = 0$, the firm is indifferent and can choose $y^*_t = q, t \in T$. Transfers are also balanced. By construction, each t at prices p'_t and income w_t can afford and chooses q. This is clear by construction for any $t \neq q$. For t' policy q is its global maximum, hence is also chosen by t'.

Now suppose that there is no $t \in T$ with $t = q$. Under Euclidean preferences, from the point of view of Theorem 2, we only need to consider q from the boundary of $\mathrm{con}(T)$. By $T' \subset T$ denote the set of $t \in T$ that forms a face of the (L-dimensional) polyhedron that contains q. In particular, there exists a vector of $[\alpha_t]_{t\in T'}$ such that $q = \sum_{t\in T'} \alpha_t t$. Hence, by an analogous construction as in the proof of Theorem 2, the matrix of prices $[\lambda_t p_t]_{t\in T'}$ (with $\lambda_t p_t \in \partial u_t(q)$ and $p_t = t - q$) is balanced. For the remaining $t \notin T'$ we take $p_t := 0$. Setting $w_t := \frac{\alpha_t}{\mu_t} p_t \cdot q$ for $t \in T'$ and $w_t := 0$ for the remaining types ($t \notin T'$) we obtain a Lindahl quasi-equilibrium with transfers. □

4 Discussion

Can any Condorcet be balanced? Suppose that a Condorcet winner $q \in \mathbb{R}^L$ exists for a m-majority rule. Consider the vector of supporting prices (marginal utilities) $(p_t)_{t\in T}$ such that $p_t \in \partial u_t(q)$. Is any such matrix $[p_t]_{t\in T}$ balanced? The answer is no, as demonstrated by the following example.

Example 6. *Similarly to example 5, consider $Q \subset \mathbb{R}^2$ with $\mu_{t_1} = 0.1$ of agents in $t_1 = (0,1)$ and $\mu_{t_2} = \mu_{t_3} = 0.45$ of agents in $t_2 = (-1,0)$ and $t_3 = (1,0)$, respectively. The policy $q = (0,0)$ belonging to the convex hull of the bliss points $\{t_1, t_2, t_3\}$ is a m-majority winner for $m > 0.55$, i.e., any counterproposal will not collect a m-majority of the voters to beat q. However, the matrix of supporting prices $[p_t]_{t=1,2,3} = [(0,1), (-1,0), (1,0)]$ is not balanced. Nevertheless, there exists a quasi-equilibrium of Lindahl, with $(p_t)_{t=1,2,3} = ((0,0), (-1,0), (1,0))$ with income $w_1 = w_2 = w_3 = 0$.*

Condorcet winner and Plott (1967) In the following example, we investigate the relations between our balancedness condition and the condition C proposed by Plott (1967), which we call pairwise balancedness. Plott (1967) used pairwise balancedness to show the existence of a majority winner.

Example 7 (Balancedness vs. Pairwise Balancedness)**.** *Consider 5 individuals with Euclidian preferences over $Q \subset \mathbb{R}^2$. Assume that at a given point $q \in Q$ one individual is satisfied, while the gradients of the remaining (even number of) individual preferences at q form a matrix:*

$$\nabla U(q) = \begin{bmatrix} -1 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 \end{bmatrix}.$$

Observe that $\nabla U(q)$ is balanced. Indeed, a vector of weights $\alpha = [1\,3\,2\,1]^T$ satisfies: $\nabla U(q)\alpha = 0$. However, $\nabla U(q)$ cannot be divided into balanced pairs, as required by Plott (1967). Indeed, neither of the pairs including the first individual is balanced:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}.$$

And hence balancedness does not imply condition C. However, the reverse implication is satisfied. If the number of agents is even and $\nabla U(q)$ is pairwise balanced, then it is trivially balanced. Our balancedness condition is therefore weaker than the condition C of Plott (1967). As a result, if the Plott (1967) conditions for the existence of a majority winner are satisfied, then so is balancedness, and there exists a Lindahl equilibrium with transfers.

Existence of a Condorcet winner of m-majority Plott (1967) condition C guaranties the existence of a Condorcet winner with majority votes. Here we discuss the existence of a Condorcet winner of a m-majority in $\text{con}^\circ(T)$. Suppose that there exists a pair of types t, t' s.t. there is a $q \in Q$ which is at the point of tangency between the Euclidian preferences of t and t'. We call q a pairwise tangent between the types t, t'. Consider a subset of types $\hat{T}$ of T such that $\mu(\hat{T}) > m$. If $\hat{T}$ can be partitioned into two non-empty subsets $(\hat{T}^1, \hat{T}^2)$ such that for every $t \in \hat{T}^1$, there is a $t' \in \hat{T}^2$ for which $q \in \text{con}^\circ(T)$ is a pairwise tangent between t, t'. Then, it follows that q is a m-Condorcet winner as no $t \in \hat{T}$ can be better off by any other $q' \in Q$ and $\mu(T/\hat{T}) \leq m$ and, by construction, $q \in \text{con}^\circ(T)$. Note that this is only one possible set of sufficient conditions for the existence of a Condorcet winner.

Lindahl mechanism Neligh (2025) proposed a minimum-offer Lindahl mechanism to decentralize efficient levels of public good (but see also Gul and Pesendorfer (2025)). In Neligh (2025), individuals have quasi-linear preferences over a private good and a public good. We can propose a similar mechanism in our setting with Euclidian preferences. Suppose that the mechanism designer knows the distribution of types and the set of feasible social states, i.e., the planner knows $S = (T, Q, \{u_t, \mu_t : t \in T\})$ but not which type of an individual agent. Given this information, the mechanism designer proposes a vector of type-specific Lindahl prices and a distribution of transfers, i.e. $(p_t^*, w_t^*)_{t \in T})$. Taking these prices and transfers as given, each agent i, $i = 1, ..., N$ must announce their type, i.e., their utility function $u^{t'} \in U$ where U is the support of the distribution μ and then propose a social state $q_i \in \{q \in Q : p_{t'} q \leq w_{t'}\}$. Note than an individual of t could announce that they are a different type $t' \neq t$. If a type t individual announces its true type t, they will always choose the Lindahl social state q^* as q^* solves max $u_t(q)$ over $\{q \in Q : p_t^* \cdot q \leq w_t^*\}$. If an individual of type t announces a different type $t' \neq t$, then choose $q \neq q*$ as they solve max $u_t(q)$ over $\{q \in Q : p_{t'}^* \cdot q \leq w_{t'}^*\}$. If $q_i = q$ for all $i = 1, ..., N$ then the mechanism implements q; otherwise, it implements q^* the Lindahl equilibrium social state. First, note that $q_i = q^*$ is a Lindahl equilibrium as no individual has an incentive to deviate. Indeed, given $q_j = q^*$ for $j \neq i$ by deviating, an agent i does not change the outcome of the mechanism: it remains q^*. Suppose, there exists another Nash equilibrium where $q_i = q'$ for all $i = 1.., N$ and $q' \neq q^*$. As a Lindahl equilibrium is Pareto optimal, it must be the case that there is at least one $t'' \in T$ such that $u_{t''}(q') < u_{t''}(q^*)$: any agent j belonging to the type t'' can propose any other social state satisfying $q_j \in \{q \in Q : p_{t''} q \leq w_{t''}\}$ (in particular, q^*) which will ensure that q^* is implemented. Hence, the mecahnism implements the Lindahl equilibrium at any Nash equilibrium.

Quasi-linear utility and non-zero marginal costs Our results can be extended to incorporate quasi-linear preferences and non-zero marginal costs of production. This extension can be useful if the production of "policies" or "social states" q is costly. We illustrate it for $L = 2$. Suppose that the preferences are given by $U_t : Q \times \mathbb{R} \to \mathbb{R}$ with $U_t(q, x_t) = u_t(q) + x_t$. The production function transforms $z \in \mathbb{R}$ to $q \in Q$ via $q = f(z)$. In this example, we assume $f(z) = \beta z$ with $\beta > 0$. Suppose that the economy is endowed with ω (distributed according to $(\omega_t)_{t \in T}$) units of the quasi-linear good while the endowment of q is zero.

Observe that for our results to hold in this economy we have to modify our balancedness condition to account for a quasilinear component (that is, that the preferences of all agents are monotone with a quasilinear good). The Lindahl equilibrium with transfers $(w_t)_{t \in T}$ is

$(q^*, z^*, (p_t^*, p_t')_{t\in T})$ such that for each type t, (q^*, z^*) solves $\max_{q,z} u_t(q) + \omega_t - z$ under $p_t^* q \le p_t' z + w_t$, while (q^*, z^*) also solves $\max_{q,z}(q \sum_t \mu_t p_t^* - z \sum_t \mu_t p_t')$ and markets clear $\frac{q^*}{\beta} = z^* \le \omega$ with $\sum_t \mu_t w_t = 0$. For concave and differentiable u_t to decentralize $q^* \le \beta\omega$, one can set $p_t := u_t'(q^*)$. For a matrix of such prices $[p_t]_{t\in T}$, the appropriately adopted version of the balancedness conditions means that there exists a vector of strictly positive weights $(\alpha_t)_{t\in T}$ such that $\sum_t \alpha_t p_t = \frac{1}{\beta}\sum_t \alpha_t$. Under this condition, setting prices $p_t^* = \frac{\alpha_t}{\mu_t} p_t$ and $p_t' = \frac{\alpha_t}{\mu_t}$ implies a zero profit condition, and setting $w_t = z^*(\beta p_t^* - p_t')$ guaranties that agents can afford (q^*, z^*). Such transfers are balanced, i.e. $\sum_t \mu_t w_t = 0$, again, by a zero-profit condition.

Note that, to obtain the above result, we have allowed two personalized prices, that of q and that of a quasi-linear good. When normalizing all prices of the quasi-linear good to the same number, e.g. setting $p_t' = 1$ for all $t \in T$ the equilibrium may not exist, as it may not be possible to balance $[p_t]_{t\in T}$.

5 Conclusion

This paper has proposed a new condition, balancedness, to decentralize Condorcet winners as Lindahl equilibria when voters' preferences are characterized by bliss points over social states.

When our balancedness condition is satisfied, in various contexts such as the "ideal" provision or location of public goods (e.g., a local park funded by subscription) or the "optimal" avoidance of public bads (e.g., the location of a municipal garbage dump), as well as the older literature on the market for votes (see Casella et al. (2012)), the prices and transfers derived from this condition can be interpreted as the subsidies required for minority voters who disagree with the majority choice. Our results can therefore be applied in many public economic models. However, when a policy lies outside the interior of the convex hull of (transformed) bliss points, meaning that it is not a compromise between the preferred policies of all the different agent types, our findings suggest a limit to the use of market mechanisms to manage disagreements.

Open questions include generalizing our results to non-linear production sets by introducing cost share equilibria. As in the congestion example, in empirical/policy settings where compromise between the different preferred policies of voter/agent types is the norm, we could use our balancedness condition to compute Lindahl prices and transfers and compare them to those actually observed. In addition, in certain settings, not all agents may be present to vote on policy alternatives. For example, policies related to emissions mitigation at a point in time affect not only the current generation but also future generations, who are not present today when such policies are voted on. When emissions mitigation is costly, the current generation may prefer a higher level of current emissions than a different future generation, who will have to bear the cost of not cutting current emissions in the future. In such a setting, understanding how Lindahl prices and transfers can be used to decentralize emissions mitigation policies, which sit in the convex hull of these bliss points, is of interest in order to determine intergenerational subsidies or transfers needed to decentralize Pareto-optimal climate policies. We leave these issues open for further research.

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