Errata for *Differential Geometry: Connections, Curvature, and Characteristic Classes*

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- p. 14, Problem 2.4, second display: $(f'(x))^2$
- p. 24, last display: $D_Z < X, Y >$ should be $Z < X, Y >$.
- p. 30, line 1: on $\to$ along
- p. 40, Problem 5.4: Need to assume that the Gauss map is one-to-one and the Gaussian curvature K is of one sign (either ≥ 0 or ≤ 0) on M.
- p. 62, (8.5): The second term $-\langle L(X), Z\rangle L(X)$ should be $-\langle L(Y), Z\rangle L(X)$.
- p. 62, last display: $R(X, Y)$ should be $R(X, Y)Z$
- p. 67, second equation in the last dispay: $(\nabla_X Y)_{\text{nor}}$ should be $(D_X Z)_{\text{nor}}$.
- p. 67, last line: E should be M.
- p. 69, first display: $R(X, Y)$ should be $R(X, Y)Z$.
- p. 81, Section 11.2, second paragraph: "$X, Y \in \mathfrak{X}(U)$" should be "$X \in \mathfrak{X}(U), Y \in \Gamma(U, E)$".
- p. 105, Prop. 14.5: Replace "manifold with a connection" with "Riemannian manifold".
- p. 140, below Equation (17.2): $c(t)$ should be $\gamma(s)$.
- p. 140, the first line of Proposition 17.2: "an affine connection" should be "the Riemannian connection" (because the connection should be compatible with the metric in order to use Proposition 11.4 in the proof).
- p. 141, line 1: $e_{i,c(t)}$ and $De_{i,c(t)}/ds$ should be $e_{i,\gamma(s)}$ and $De_{i,\gamma(s)}/ds$, respectively.
- p. 159, line 3: $\text{Hom}(R, W)$ should be $\text{Hom}_R(V, W)$.
- p. 167, the first line of the proof of Lemma 19.7: $\bigwedge^n V \to V$ should be $\bigwedge^n V \to R$
- p. 179, second display: $f^*(E|_{U_\alpha}) = (f^*E)|_{f^{-1}(U_\alpha)} \xrightarrow{\sim} f^{-1}(U_\alpha) \times V$
- p. 189, the second paragraph of the proof of Proposition 21.1: "To show that $\alpha \wedge \beta$ is smooth" should be "To show that $\alpha \cdot \beta$ is smooth."
- p. 199, line after second display: Replace "this may be an infinite sum" by "the Betti numbers b_i may be infinite".
- p. 200, second display: $b^i = b^{n-i}$ should be $b_i = b_{n-i}$.
- p. 205, bottom display: In the second term, $R(X, W)Y$ should be $R(Z, W)Y$.
- p. 209, Section 22.8, first display: $T_pM \times T_pM \to T_pM$ should be $T_pM \times T_pM \to \mathbb{R}$.

- p. 212, lines 7–8: this global form $P(\Omega)$ is closed and its cohomology class is independent of the connection.
- p. 221, second display: The two vertical arrows should be pointing up instead of down.
- p. 235, Section 25.8: "As before, one shows that $[Q(\Omega)]$ is closed and ..." should be "As before, one shows that $Q(\Omega)$ is closed and ..."
- p. 266, Proposition 29.6, second line: Replace "a manifold M of dimension n" by "a manifold M of dimension n, and let $\pi : \mathrm{Fr}(E) \to M$ be its frame bundle".
- p. 279, below Example 31.10: $\Omega^0_p(P, V)$ should be $\Omega^0_\rho(P, V)$.
- p. 289, proof of (ii) and (iii): The sentence starting with "Since the Ω^i are right-equivariant ..." is incorrect so that the rest of the proof is invalid. One way to fix this is to follow Kobayashi and Nomizu [12 in the References], vol. 2, p. 295, by first showing that there is a one-to-one correspondence between degree n homogeneous polynomials and n-multilinear functions and then continuing with their proof.
- p. 303, line after first display: T_pM should be T_pN.
- p. 322, solution 2.5: the denominator should have an exponent $3/2$.
- p. 322, last line: $dN_{c(t)}/dt$ should be $N_{c(t)}$.
- p. 324, line -3: $2\ddot{y}$ should be $\ddot{y}$.