Errata for
An Introduction to Manifolds, Second Edition

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- p. 6, Proof of Lemma 1.4: For clarity, the point should be called y, instead of x. Use x only for the argument of f. Thus, in the first three lines of the proof, change the three instances of x to y. In Figure 1.3, change the two instances of x to y. Add to the beginning of the second paragraph "By the chain rule, ..." the sentences

  Let $x^1, \dots, x^n$ be the variables of f. Then in $f\big(p + t(y-p)\big)$,
  $$x^i = p^i + t(y^i - p^i).$$
  In the rest of the proof, change the twelve instances of x to y, but of course $\partial f/\partial x^i$ should not be changed.

- p. 8, Problem 1.3(b): Change "$]-1,1[$" to "$]-\pi/2,\pi/2[$".
- p. 9, Problem 1.5(b): Add at the end "(*Hint*: To show that a map is C^∞, you may use the fact that the sum, product, quotient, and composition of C^∞ functions are C^∞ whenever they are defined.)"
- p. 9, Problem 1.6: Replace g_{12} by h_{12} in two places. In the solution on p. 368, replace "gives the result" by "and setting $h_{12} = g_{12} + g_{21}$ give the result".
- p. 12, insert after the paragraph defining an algebra:

  Example. The set $C^\infty(U)$ of all C^∞ functions on an open set $U \subset \mathbb{R}^n$ is an algebra over $\mathbb{R}$.

- p. 20, line 5: Delete parentheses around a_r in its first occurrence.
- p. 20, line 6 of the Example 3.4: "$4 \to 1$" should be "$4 \mapsto 1$".
- p. 27, Remove the $*$ after Example 3.19 and place it after Exercise 3.20.
- p. 31, proof of Lemma 3.28: Replace the second displayed equation by
  $$\begin{array}{ccccccccccc} i_1 & < & i_2 & < & \cdots & < & i_{\ell-1} & < & i_\ell & & \\ \| & & \| & & & & \| & & \wedge & & \\ j_1 & < & j_2 & < & \cdots & < & j_{\ell-1} & < & j_\ell & < j_{\ell+1} & < \cdots . \end{array}$$

- p. 31, lines $-1, -2$, and -4 in the proof of Lemma 3.28: Replace a by α in "$\det[a^i(e_j)] = 0$" and "the matrix $[a^i(e_j)]$". Also "$i_1, \dots, i_l$" should be "$i_1, \dots, i_{l-1}$".
- p. 32, Problem 3.3, line 3: $A_k(L)$ should be $A_k(V)$.
- p. 33, Problem 3.9, line 2: zero covector $\to$ zero n-covector.
- p. 37, display -1: Replace $\omega(X)_p$ by $\omega(X)(p)$.

- p. 37, insert between display -1 and "Written out in ...":

  This function $\omega(X)$ is linear in X over the ring $C^\infty(U)$; i.e., if $f \in C^\infty(U)$, then $\omega(fX) = f\omega(X)$. To show this, it suffices to evaluate $\omega(fX)$ at an arbitrary point $p \in U$:

$$\begin{aligned}
(\omega(fX))(p) &= \omega_p(f(p)X_p) \quad &&\text{(definition of } \omega(fX))\\
&= f(p)\omega_p(X_p) \quad &&(\omega_p \text{ is } \mathbb{R}\text{-linear})\\
&= (f\omega(X))(p) \quad &&\text{(definition of } f\omega(X)).
\end{aligned}$$

- p. 38, delete the second paragraph starting with "This function is actually ...".
- p. 38, Exercise 4.4, line 2: M should be $\mathbb{R}^3$.
- p. 47, line -2: Replace "finds" by "found".
- p. 53, Proposition 5.10, lines 1–2 of proof: "Proposition 5.8" should be "Lemma 5.8".
- p. 54, line 11: $f\colon U \to \mathbb{R}^n$ should be $f\colon U \to \mathbb{R}^m$.
- pp. 56–57, Remark: This remark uses the concept of a diffeomorphism, which is not defined until the next section. Move the entire remark consisting of four paragraphs to p. 63, right before Section 6.4.
- p. 61, Definition 6.5, line 3: Insert "with $F(U) \subset V$" before "such that".
- p. 61, Definition 6.5, line 4: Replace $\phi(F^{-1}(V) \cap U)$ by $\phi(U)$.
- p. 67**, Definition 6.23, display: Change F to $(F|_U)$.
- p. 70, Problem 6.1(b) Hint: The identity map $\mathbb{R}' \to \mathbb{R}$.
- p. 71, line 1 of paragraph 2: Insert "usually" between "is" and "a process".
- p. 72, line -3: "$f := f \circ \pi$" should be "$f := \bar{f} \circ \pi$".
- p. 81, Problem 7.6, line 2: R should be $\mathbb{R}$.
- p. 82, Problem 7.8 (c), (d): Move the hint for (d) to the end of the hint for (c).
- p. 83, line -9: $F(k,n)$ should be $G(k,n)$.
- p. 94, Figure 8.3: The i in a_i should be a superscript. This occurs in two places.
- p. 105, Figure 9.4: The rightmost $\mathbb{R}^n$ should be $\mathbb{R}^m$.
- p. 106, line 5: "$S := f^{-1}(c)$" should be "$S := F^{-1}(c)$".
- p. 109, Problem 9.10 should be starred.
- p. 112, line 5: Replace 1 by $\mathbb{1}$.
- p. 117, line -2: "$\psi(f(q)) = (y^1(f(q)), \dots, y^n(f(q))$" should be "$\psi(f(q)) = (y^1(f(q)), \dots, y^m(f(q)))$".
- p. 118, lines 1 and 3: "$\psi(f(q)) = (y^1(f(q)), \dots, y^m(f(q))$" should be "$\psi(f(q)) = (y^1(f(q)), \dots, y^m(f(q)))$". Also "$(x^1(q), \dots, x^k(q)), 0 \dots, 0)$" should be "$(x^1(q), \dots, x^k(q), 0 \dots, 0)$".
- p. 134, line -3: Change "$M \times R$" to "$M \times \mathbb{R}$". In fact, in harmony with Example 12.6, one may want to change all occurrences of "$M \times \mathbb{R}$" on line -3 to "$M \times \mathbb{R}^r$".
- p. 135, display 2: $U \times \mathbb{R}^n$ should be $U \times \mathbb{R}^r$. ("n" should be "r".)
- p. 138, line 4: "$\mathbb{R}^n$" should be "$\mathbb{R}^r$".
- p. 139, Problem 12.2, line 1: "about p" on a manifold M.
- p. 139, Problem 12.2 (a): "at $\phi(p)$" $\longrightarrow$ "at $\tilde{\phi}(p)$"

- p. 143, line -1: g should be evaluated at "$\frac{\|x\|^2-a^2}{b^2-a^2}$".
- p. 146, line 4, insert after W_q: "only finitely many of the f_α's can be nonzero and"
- p. 147, Problem 13.3 (b): After "a manifold.", insert the sentence "Assume that $A \subset U$."
- p. 150, lines 4 and 5 in the proof of Lemma 14.1: Change "$\tilde{\phi}\colon TU \xrightarrow{\sim} U \times \mathbb{R}^n$" to "$\tilde{\phi}\colon TU \xrightarrow{\sim} \phi(U) \times \mathbb{R}^n$", and "$\tilde{\phi} \circ X\colon U \to U \times \mathbb{R}^n$" to "$\tilde{\phi} \circ X\colon U \to \phi(U) \times \mathbb{R}^n$".
- p. 152, first display: Replace $\tilde{X}(q)$ by $\tilde{X}_q$.
- p. 160, Definition 14.14: Change "A vector field X on N is *F-related* to a vector field $\tilde{X}$ on M" to "A vector field X on N and a vector field $\tilde{X}$ on M are *F-related* to each other"
- pp. 171–174: On these four pages, change "AXA^{-1}" to "$A^{-1}XA$", and "$A(\cdots)A^{-1}$" to "$A^{-1}(\cdots)A$".
- p. 172, Part (ii) of the Proof of Lemma 15.18 uses the notation from edition one. Replace it by "Apply part (i) to the matrices $A^{-1}X$ and A."
- p. 178, line 8: "identity" should be "identify".
- p. 179, Problem 15.9 (b), lines 5 and 6: "elements of order 2" $\to$ "elements of order at most 2"
- p. 186, lines -4 and -3: After "If a line has rational slope..." insert "or ∞".
- p. 191, line -2 of the Proof of Proposition 17.2: Apply both sides of (17.1) to t.
- p. 191, heading of 17.2: Change to "Local Expression for the Differential of a Function".
- p. 194, line 4 in the proof of Lemma 17.5: Replace T^*M by T^*U.
- p. 194, lines 4 and 5 of the Proof of Lemma 17.5: "$\tilde{\phi}\colon T^*U \to U \times \mathbb{R}^n$" and "$\tilde{\phi} \circ \omega\colon U \to U \times \mathbb{R}^n$" should be "$\tilde{\phi}\colon T^*U \to \phi(U) \times \mathbb{R}^n$" and "$\tilde{\phi} \circ \omega\colon U \to \phi(U) \times \mathbb{R}^n$", respectively.
- p. 197, line 9: $(V, y^1, \ldots, y^n)$ should be $(V, y^1, \ldots, y^m)$.
- p. 198, line -3: Insert "and 17.10" after "by Proposition 17.11".
- p. 201, line -7: Both "$\mathbb{R}^n$" should be "U".
- p. 202, proof of Proposition 18.3, 2nd display: "by Lemma 18.2" $\longrightarrow$ "by (18.2)"
- p. 206, line 4: "for C^∞ function" should be "for C^∞ functions".
- p. 207, line 4 of the Proof of Proposition 18.12: "the C^∞ inverse" should be "a C^∞ inverse".
- p. 208, line 1: By Proposition 18.7(iv)
- p. 209, Problem 18.9(d): Replace the initial phrase by "As the image of a compact, connected set G under a continuous map".
- p. 214, third line of top display: Change $DD\tilde{x}$ to $DD\tilde{x}^I$.
- p. 215, proof of Propositions 19.7: In second line of the last display, change "(Proposition 19.5)" to "(Proposition 17.10)". Then move Proposition 19.7 before Proposition 19.5.
- p. 216, In analogy with with the title of Subsection 17.6, change the title of Subsection 19.6 to "...an Immersed Submanifold". Also change "a regular" to "an immersed" on line 2 of Subsection 19.6.
- p. 218, Problem 19.3, last line: Change $i \circ c$ to $i \circ h$.
- p. 220, Problem 19.12, (c): Replace by
  "If D is a derivation of $C^\infty(M)$ and $p \in M$, define $D_p\colon C_p^\infty(M) \to \mathbb{R}$ by
  $$D_p[f] = (D\tilde{f})(p) \in \mathbb{R},$$
  where $[f]$ is the germ of f at p and $\tilde{f}$ is a global extension of f, such as those given by Proposition 18.8. Show that $D_p[f]$ is well defined. (*Hint*: Apply Problem 19.7.)"

- p. 220, Problem 19.12, (d): Change "derivation" to "point-derivation".
- p. 223, line -2 of the Proof of Proposition 20.2: "$d\left(\frac{\partial}{\partial t}\Big|_{t_0}\omega_t\right)$" should be "$d\left(\frac{d}{dt}\Big|_{t_0}\omega_t\right)$".
- p. 225, (20.6), (20.7), and the two lines above (20.8): $(-t, p)$ in the formula should be $(-t, \varphi_t(p))$.
- p. 228, 4th line of 2nd display: Change $\sum_{i=1}^{k}$ to $\sum_{i=1}^{\ell}$.
- p. 228, line 6 after the proof of Proposition 20.8: Change "Proposition 18.7(iii)⇒(i)" to "Proposition 18.7 (iv)⇒(i)".
- p. 232, Proof of Theorem 20.12: Add to the end of the proof:

  "Thus,
  $$X(\omega(Y_1,\dots,Y_k)) = (\mathcal{L}_X\,\omega)(Y_1,\dots,Y_k) + \sum_{i=1}^{k}\omega(Y_1,\dots,[X,Y_i],\dots,Y_k).$$
  Solving for $(\mathcal{L}_X\,\omega)(Y_1,\dots,Y_k)$ gives the formula in the theorem."
- p. 234, Problem 20.10: The second term "$-y\,dx\wedge dy$" should be "$-y\,dx\wedge dz$"
- p. 239, line after 4th display: "orientation $(v_1,\dots,v_n)$" $\longrightarrow$ "orientation $[(v_1,\dots,v_n)]$"
- p. 241, line 5: Replace the sentence "But under the identification ... at $(0,0)$." by "Under the identification (21.1), the curve $c(t) = (0,t)$ for $t\in]-\epsilon,\epsilon[$ maps to $\bar{c}(t) = (1,-t)$. Hence, the tangent vector $c'(0) = e_2$ at p maps to $\bar{c}'(0) = -e_2$ at q, and the ordered basis e_1, e_2 at $p = (0,0)$ maps to $e_1, -e_2$ at $q = (1,0)$."
- p. 241, line 7 in the first paragraph: Change "Thus, at $(0,0)$" to "Thus, at $(1,0)$".
- p. 245, line -8: "(⇒)" should be "(⇐)".
- p. 248, Fig. 22.1: Change "$\mathrm{int}(\mathcal{H}^n)$" to "$(\mathcal{H}^n)^\circ$" for consistency with the text above the figure.
- p. 249, line 18: "there are" should be "there is".
- p. 251, line -4: Change "$p\in U\subset S$" to "$p\in U\subset A$".
- p. 254, line 3 under the Subsection 22.5: "$c((0,\varepsilon[)\subset M^\circ$" should be "$c(]0,\varepsilon[)\subset M^\circ$".
- p. 254, second paragraph of Section 22.5: Replace the second and third sentences by

  "In a coordinate neighborhood $(U, x^1,\dots,x^n)$ in M, such a vector field X can be written as a linear combination
  $$X_p = \sum_i a^i(p)\,\frac{\partial}{\partial x^i}\bigg|_p \quad \text{for } p\in U\cap\partial M.$$
  The vector field X along ∂M is said to be *smooth at* $p\in\partial M$ if there exists a coordinate chart U containing p such that the functions a^i on $U\cap\partial M$ are C^∞ at p; it is said to be *smooth* if it is smooth at every point $p\in\partial M$."
- p. 256, 3rd line of last example: Chaneg T_pC to $T_{c(p)}C$.
- p. 261, display above Definition 23.1: "$\inf_P L(f,P)$" $\longrightarrow$ "$\inf_P U(f,P)$"
- p. 265, line 9: (U,ϕ) instead of $\{(U,\phi)\}$. (Remove the braces.)
- p. 266, line 2: Replace "$\phi_\alpha|_{U_\alpha\cap U_\beta}$" and "$\psi_\alpha|_{U_\alpha\cap U_\beta}$" by "$\phi_\alpha|_{U_\alpha\cap V_\beta}$" and "$\psi_\alpha|_{U_\alpha\cap V_\beta}$", respectively.
- p. 267, lines 3 and 6: For consistency with equations (23.4, 23.5) on p. 264, put "det" before both occurrences of J, the Jacobian.
- p. 272, Problem 23.3, line 2: "$\Omega_c^k(M)$" $\longrightarrow$ "$\Omega_c^n(M)$"
- p. 273, line 9: "smooth a" should read "a smooth".

- p. 279, line -2: $A^{k\times\ell}$ should be $A^{k+\ell}$.
- p. 294, line -1: Delete one of the extra occurrences of "$\frac{\mathbb{R}\oplus\mathbb{R}}{\operatorname{im} j^*}$".
- p. 299: Replace the Example "The map F ..." just before Proposition 27.9 by two examples:

  Example. The map
  $$F(x,t) = \cos^2\left(\frac{\pi}{2}t\right)x + \sin^2\left(\frac{\pi}{2}t\right)\frac{x}{\|x\|}$$
  is a deformation retraction from the punctured plane $\mathbb{R}^2 - \{0\}$ to the unit circle S^1.

  Example. The map F in Example 27.6 is a deformation retraction from $\mathbb{R}^n$ to a singleton $\{p\}$.
- p. 318, line -11: $\mathcal{T}_1$ is *coarser* than $\mathcal{T}_2$
- p. 324, Example following Definition A.15: Change the radius from $1/n$ to $1/m$. The example should now read:

  "For $p \in \mathbb{R}^n$, let $B(p, 1/m)$ be the open ball of center p and radius $1/m$ in $\mathbb{R}^n$. Then $\{B(p,1/m)\}_{m=1}^{\infty}$ is a neighborhood basis at p. Thus, $\mathbb{R}^n$ is first countable."
- p. 328, proof of Prop. A.23, ($\Leftarrow$), line 3: Replace "$f(U) = f(f^{-1}(V)) \subset V$" by "$p \in U \subset f^{-1}(V)$. This means precisely that $f(U) \subset V$"
- p. 328, line -2: Change "Since both f and i are continuous" to "If $f\colon X \to Y$ is continuous, then since i is continuous"
- p. 334, last line of paragraph after Definition A.47: "$\mathrm{cl}_M(A)$" $\longrightarrow$ "$\mathrm{cl}_S(A)$"
- p. 337, line -1: Replace "$f \circ \pi_2$" by "$g \circ \pi_2$".
- p. 338, Problem A.21: Insert "nonempty" before "Zariski-open".
- p. 343, display 2 in Th. B.4: $(F \circ f \circ G^{-1})$ (The inverse should be inside the parentheses.)
- p. 343, display 2 in Th. B.4: "x^n" should be "x^m".
- p. 347, line -7: Replace "$i \geq 1$. by "$i \geq 0$, and define V_{-1} to be the empty set."
- p. 359, line 4: $f\colon \mathbb{C}^{2n} \times \mathbb{C}^{2n} \to \mathbb{C}$ should be $f\colon \mathbb{C}^{2n} \to \mathbb{C}^{2n}$.
- p. 362, Solution to 7.11, line 2: The first term should be $[\frac{tx}{\|tx\|}]$, with double bars in the denominator.
- p. 362, Solution to 7.11: In the commutative diagram "$\mathbb{R}^n - \{0\}$" should be "$\mathbb{R}^{n+1} - \{0\}$".
- p. 369, line 1: Remove the parenthesis around e_J.
- p. 369, Answer to Problem 4.3: Delete comma from "$dy = \sin\theta\, dr$"
- p. 371, Solution to 8.7, first display: $\bar{x}^n$ should be $\bar{x}^m$.
- p. 373, Solution to Problem 11.1: Replace the second and third sentences by "Let H be the plane $\{(a^1, \ldots, a^{n+1}) \in \mathbb{R}^{n+1} \mid \sum a^i p^i = 0\}$. Show that $T_p(S^n) \subset H$."
- p. 377, Problem 17.1: The numerator should be $-y\,dx + x\,dy$.
- p. 380, line 6 of the solution to Problem 22.3: Replace "On $U \cap M$" by "On U".
- p. 385, Solution to A.15*: The directions of the proof arrows should be reversed.
- p. 385, Solution to A.15*, first display: "$(U \cap V)$" should be "$(U \cup V)$".
- p. 389, line -2: rank of a matrix A (pp. 82, 344)