# Chapter_5
# 光线 球体 是否相交

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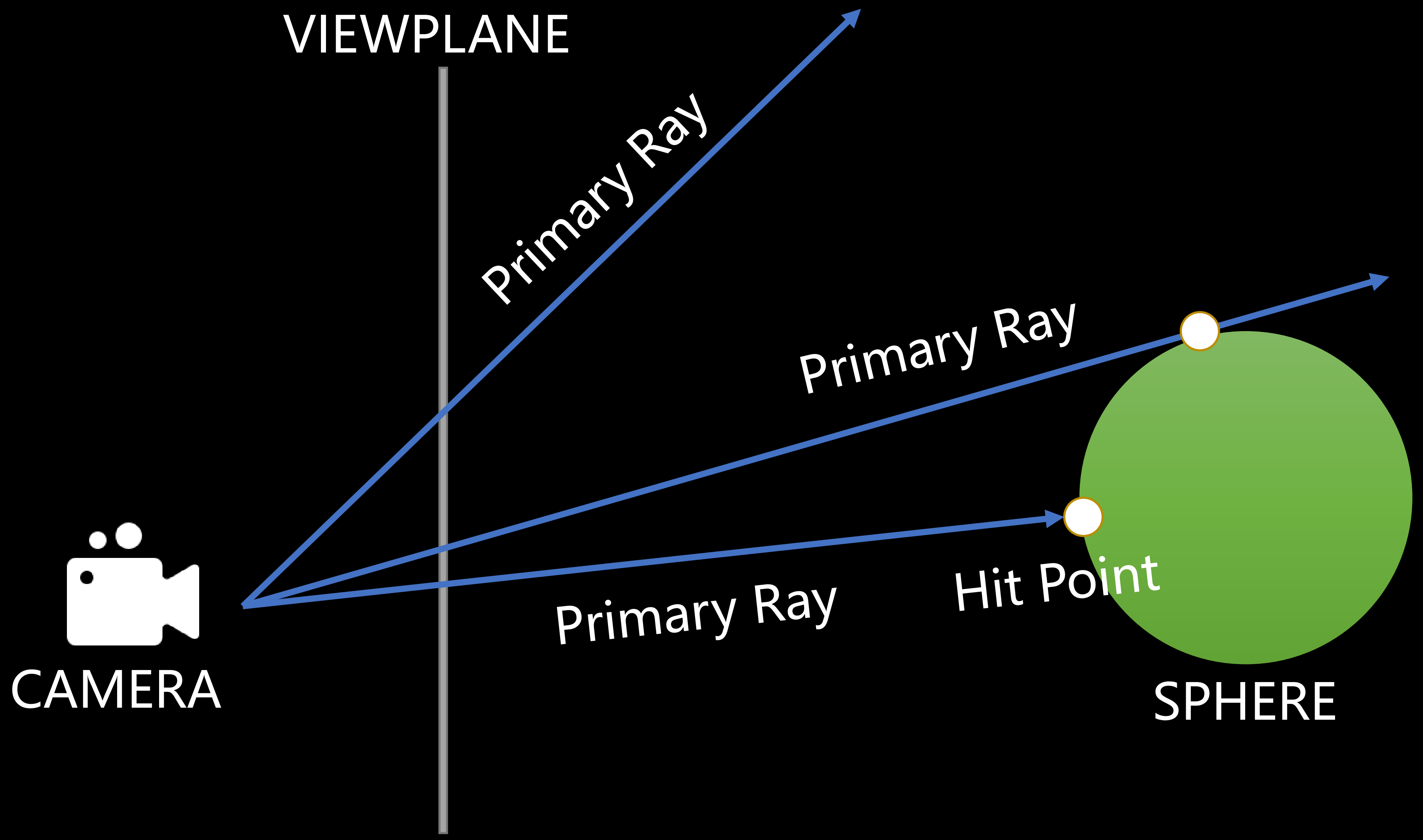
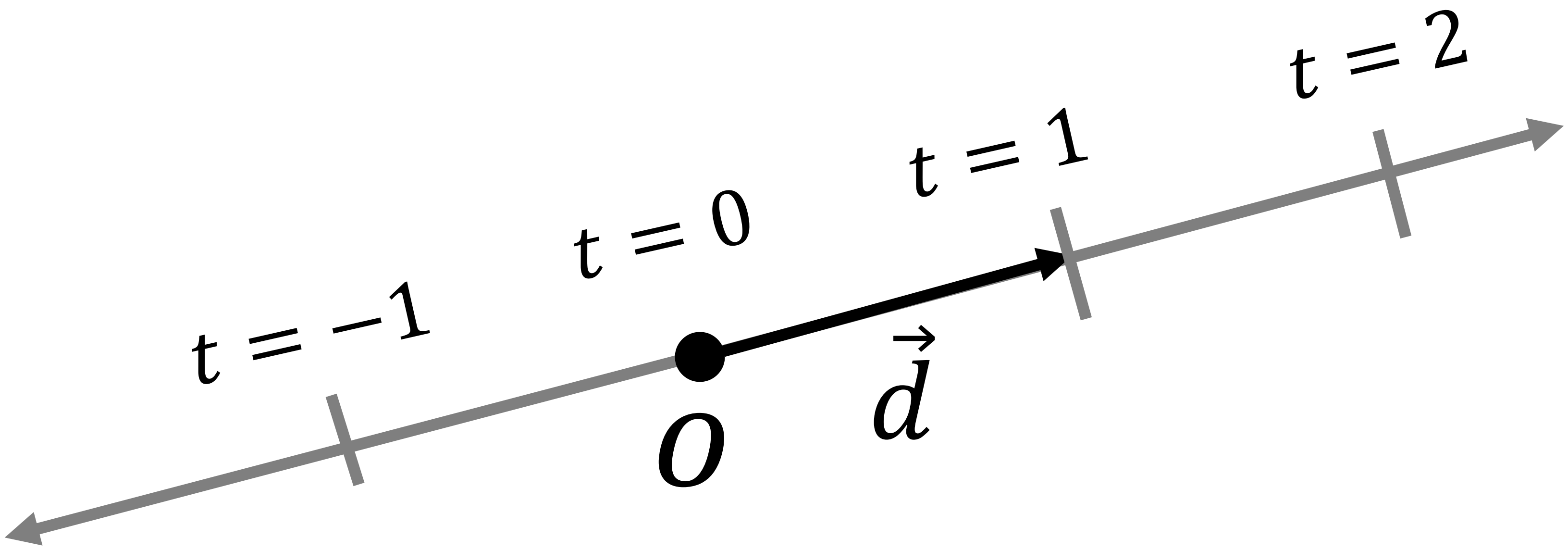

VIEWPLANE
Primary Ray
Primary Ray
Primary Ray
Hit Point
CAMERA
SPHERE

# 光线参数方程

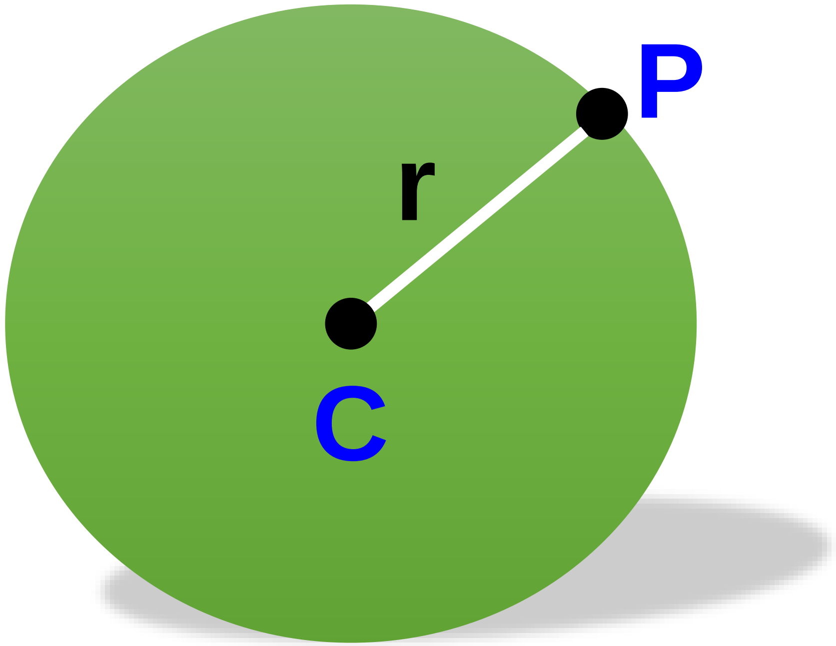
**Ray**

$$P(t) = o + t * d$$

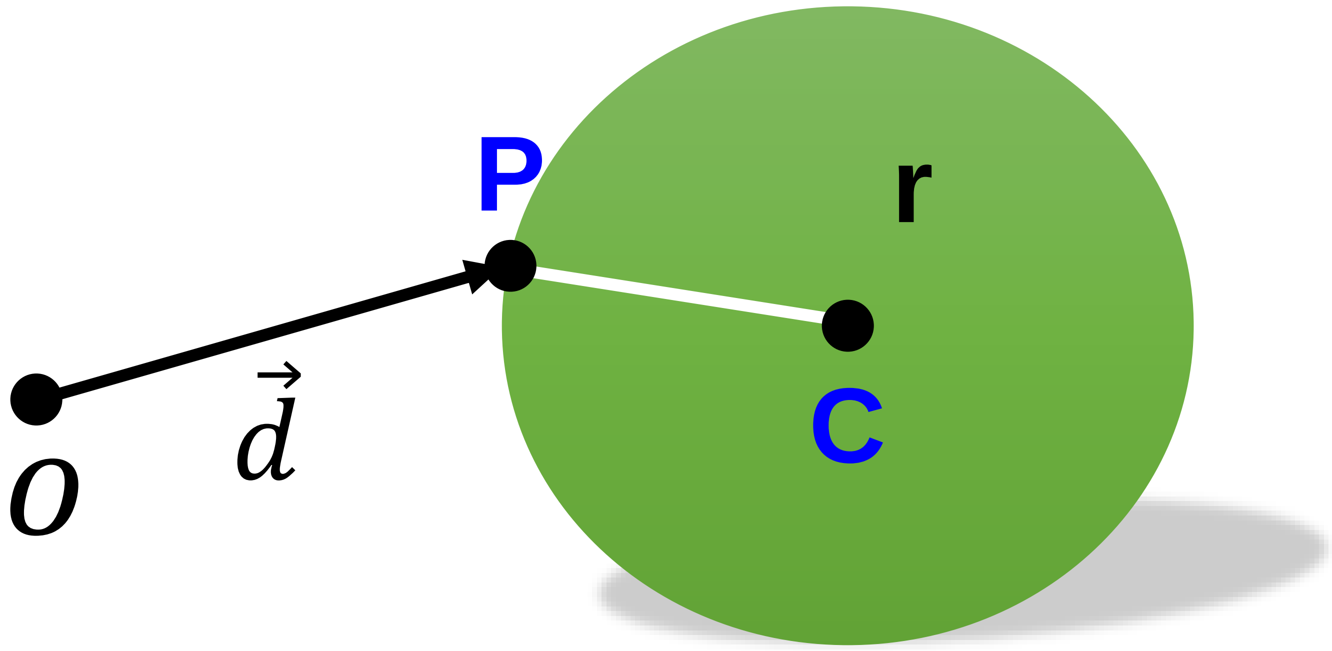


# 球体参数方程

**Sphere** $(P-C)*(P-C)-r^2=0$



# 光线 球体求交点




将射线表示 带入球体方程

$$(P(t) - C) * (P(t) - C) - r^2 = 0$$

# 光线 球体求交点

$$(P(t)-C)*(P(t)-C)-r^2=0$$

$$(O+t*\vec{d}-C)*(O+t*\vec{d}-C)-r^2=0$$

射线起点

射线方向

向量点乘

未知参数

球心

半径

# 光线 球体求交点

$$(O + t * \vec{d} - C) * (O + t * \vec{d} - C) - r^2 = 0$$

$$t^2 \vec{d} * \vec{d} + t2\left(\vec{d} * (O - C)\right) + (O - C) * (O - C) - r^2 = 0$$

一元二次方程
$$\begin{cases} a: & \vec{d} * \vec{d} \\ b: & 2\left(\vec{d} * (O - C)\right) \\ c: & (O - C) * (O - C) - r^2 \end{cases}$$

# 光线 球体 求交 点

$$\begin{cases} a: & d * d \\ b: & 2\left(d * (O - C)\right) \\ c: & (O - C) * (O - C) - r^2 \end{cases}$$

$$\begin{cases} \Delta = b^2 - 4ac \\ t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{cases}$$

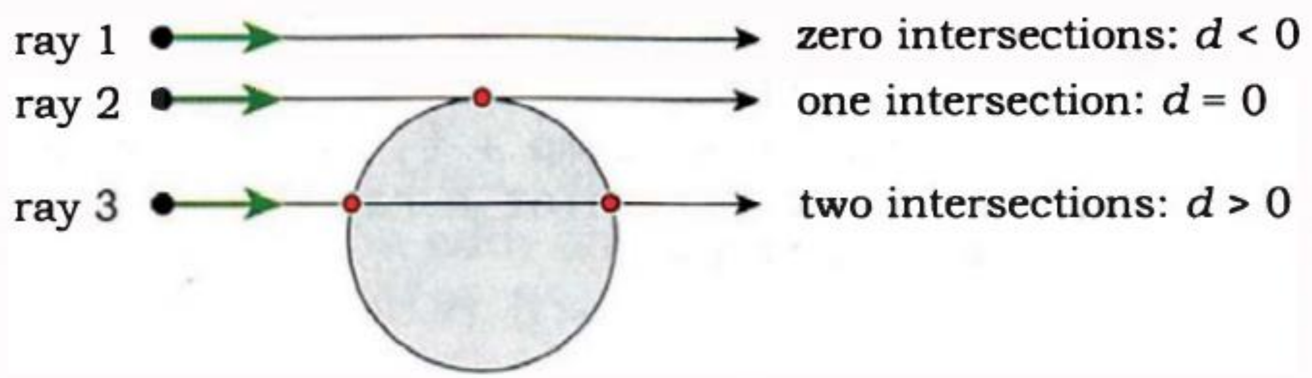


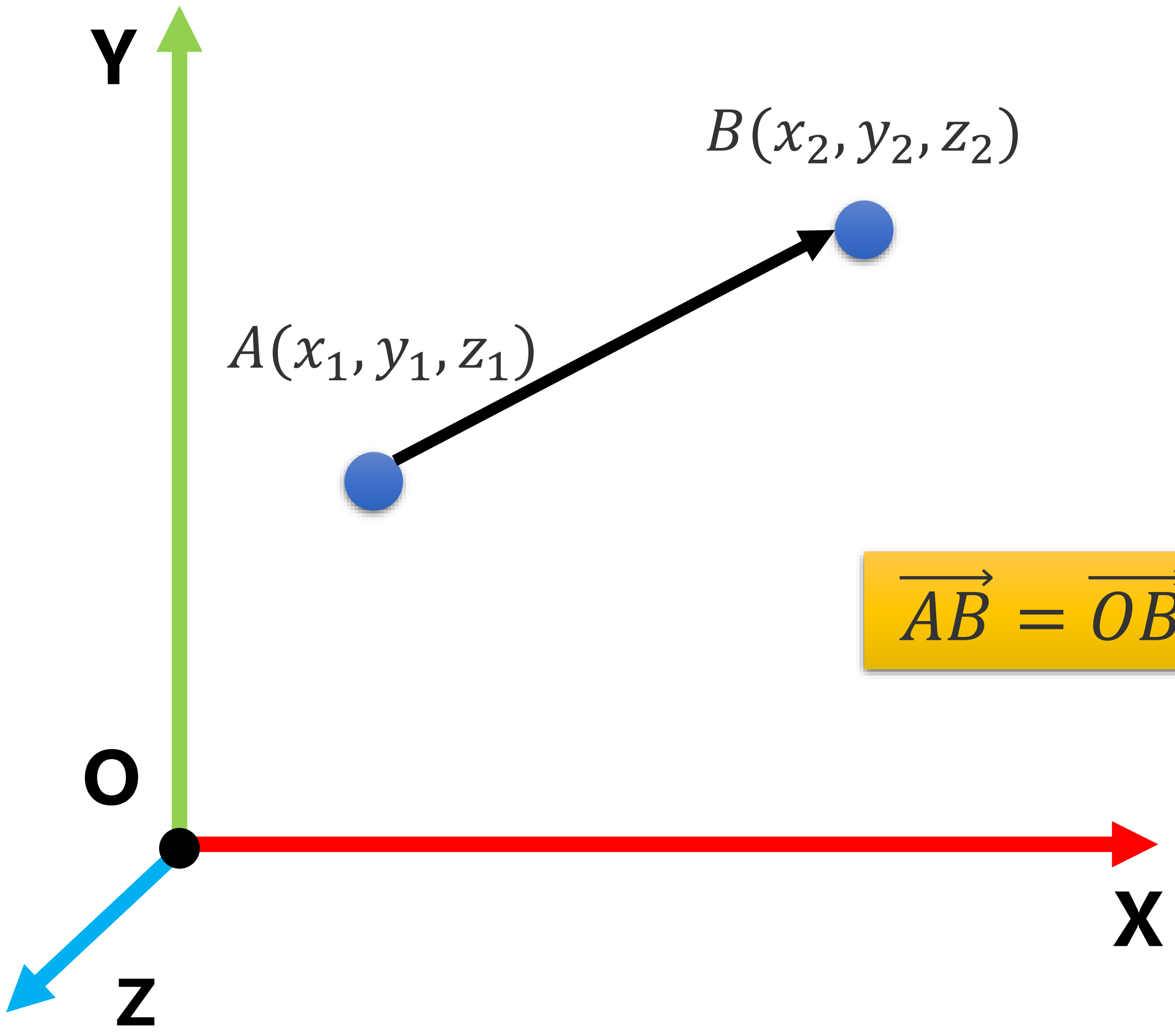

**Figure 3.7.** Ray-sphere intersections.

# 光 线 球 体 求 交 点

$$a:\quad \underbrace{d}_{\text{射线方向}} \underbrace{*}_{\text{点乘}} \underbrace{d}_{\text{射线方向}}$$

$$b:\quad 2\left(\underbrace{d}_{\text{射线方向}} \underbrace{*}_{\text{点乘}} (\underbrace{O}_{\text{射线起点}} - \underbrace{C}_{\text{球心位置}})\right)$$

# 两 个 点 减 法------得到从A指向B的向量




$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

# 两 个 点 减 法------得到从A指向B的向量

$$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

**Point3D.cs**

```
//两个点之间的减法，得到向量
0 个引用
public static Vector3D operator -(Point3D p1, Point3D p2)
{
    return new Vector3D(p1.X-p2.X, p1.Y - p2.Y, p1.Z - p2.Z);
}
```

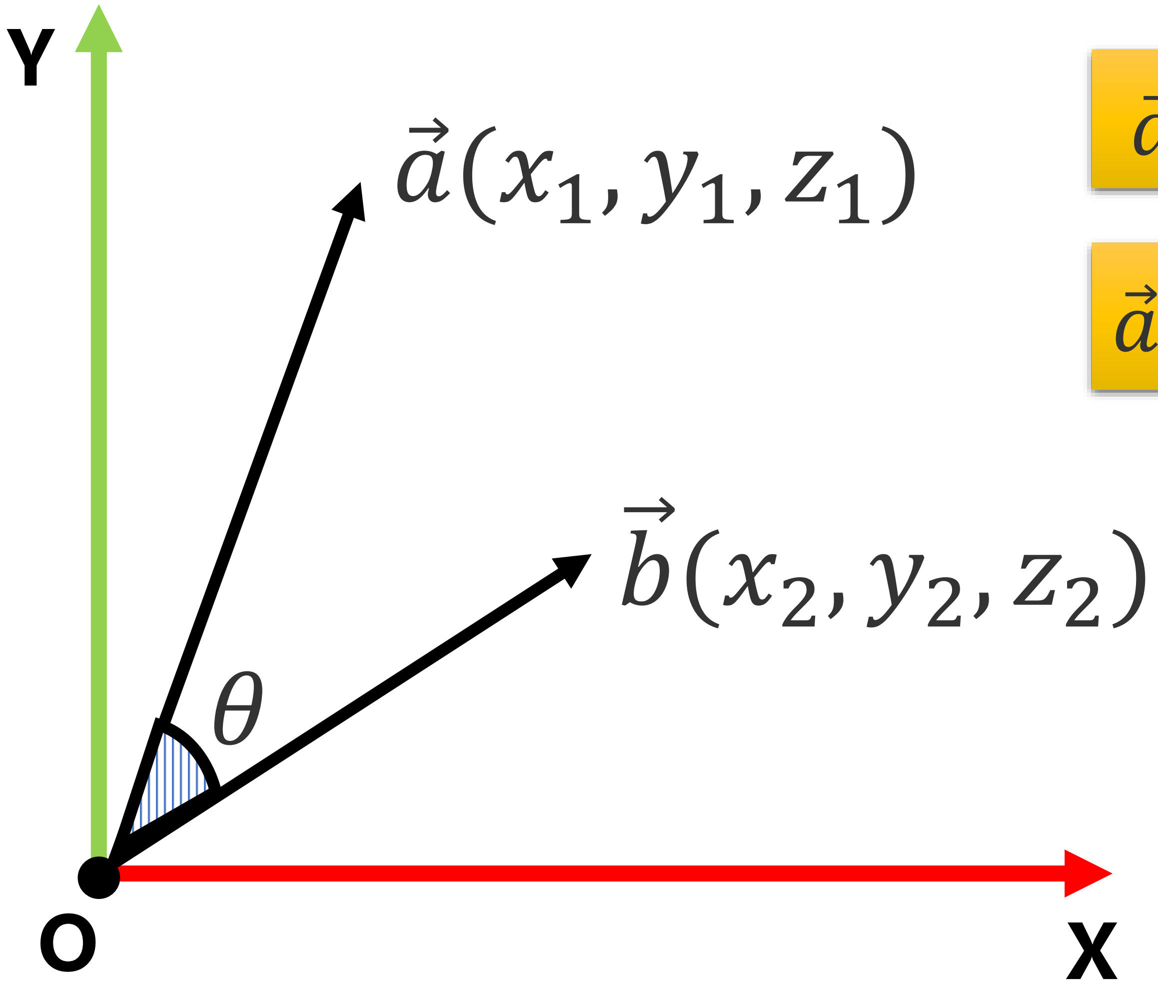
**Form1.cs（测试）**

```
        //光线类测试
        1 个引用
        private void btnTest_Click(object sender, EventArgs e)
        {
            Point3D p1 = new Point3D(1, 2, 10);
            Point3D p2 = new Point3D(-10, 1, 3);

            Vector3D v = p1 - p2;
        }  已用时间 <= 2ms
    }
}
```

| v | {SyRayTracer.Vector3D} |
|---|---|
| X | 11 |
| Y | 1 |
| Z | 7 |
| _x | 11 |
| _y | 1 |
| _z | 7 |

# 向量点乘




$$\vec{a} * \vec{b} = (x_1 * x_2 + y_1 * y_2 + z_1 * z_2)$$

$$\vec{a} * \vec{b} = |\vec{a}| * |\vec{b}| * cos\theta$$

# 向量点乘

**Vector3D.cs**

```
//向量点乘
0 个引用
public static double operator*(Vector3D v1, Vector3D v2)
{
    return v1.X * v2.X + v1.Y * v2.Y + v1.Z * v2.Z;
}
```

**Form1.cs**
**(测试)**

```
        //光线类测试
        1 个引用
        private void btnTest_Click(object sender, EventArgs e)
        {
            Vector3D v1 = new Vector3D(1, 2, 2);
            Vector3D v2 = new Vector3D(-1, 3, 0);

            double d = v1 * v2;
        }  已用时间 <= 1ms   d 5
    }
```

# 向量的其它运算

**Vector3D.cs**

```
//数 * 向量
0 个引用
public static Vector3D operator *(double d, Vector3D v)
{
    return new Vector3D(d * v.X, d * v.Y, d * v.Z);
}

//向量 * 数
0 个引用
public static Vector3D operator *( Vector3D v, double d)
{
    return new Vector3D(d * v.X, d * v.Y, d * v.Z);
}

//向量加法
0 个引用
public static Vector3D operator +(Vector3D v1, Vector3D v2)
{
    return new Vector3D(v1.X + v2.X, v1.Y * v2.Y, v1.Z * v2.Z);
}

//向量减法
0 个引用
public static Vector3D operator -(Vector3D v1, Vector3D v2)
{
    return new Vector3D(v1.X - v2.X, v1.Y - v2.Y, v1.Z - v2.Z);
}
```

# 向量的模，及归一化

**Vector3D.cs**

```
//向量的模，及向量的大小
2 个引用
public double Magnitude()
{
    return Math.Sqrt(X * X + Y * Y + Z * Z);
}

//对本向量进行归一化，本向量被改变
0 个引用
public void Normalize()
{
    double d = Magnitude();
    X = X / d;
    Y = Y / d;
    Z = Z / d;
}

//返回本向量的归一化向量，本向量不变
0 个引用
public Vector3D GetNormalizeVector()
{
    double d = Magnitude();
    return new Vector3D(X/d, Y/d, Z/d);
}
```

# 光 线 球 体 求 交 点

$$
\begin{cases}
a: \quad d * d \\
b: \quad 2\left(d * (O - C)\right) \\
c: \quad (O - C) * (O - C) - r^2
\end{cases}
$$

$$
\begin{cases}
\Delta = b^2 - 4\,ac \\
t = \dfrac{-b \pm \sqrt{b^2 - 4\,ac}}{2a}
\end{cases}
$$

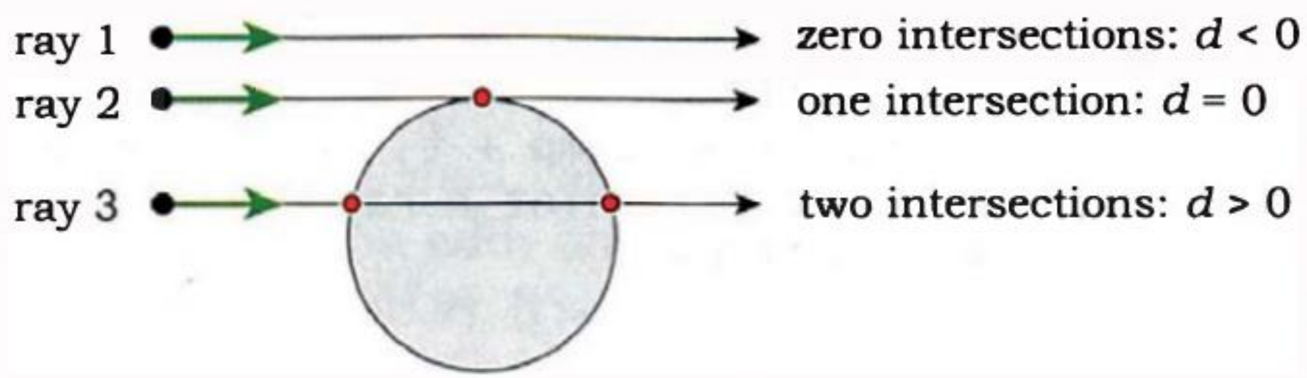



**Figure 3.7.** Ray-sphere intersections.

# 参考

```cpp
bool hit_sphere(const vec3& center, float radius, const ray& r) {
    vec3 oc = r.origin() - center;
    float a = dot(r.direction(), r.direction());
    float b = 2.0 * dot(oc, r.direction());
    float c = dot(oc, oc) - radius*radius;
    float discriminant = b*b - 4*a*c;
    return (discriminant > 0);
}
```

```cpp
bool
Sphere::hit(const Ray& ray, double& tmin, ShadeRec& sr) const {
     double      t;
     Vector3D    temp   = ray.o - center;
     double      a      = ray.d * ray.d;
     double      b      = 2.0 * temp * ray.d;
     double      c      = temp * temp - radius * radius;
     double      disc   = b * b - 4.0 * a * c;

     if (disc < 0.0)
          return(false);
     else {
```

# 我们的第一个版本

**Ray.cs**
**(后续扩展)**

```
//射线是否与球体相交
0 个引用
public bool isHit(Sphere sphere)
{
    Vector3D oc = Origin - sphere.Center;

    double a = Direction * Direction;
    double b = 2.0 * (Direction * oc);
    double c = oc * oc - sphere.Radius * sphere.Radius;

    double delta = b * b - 4.0 * a * c;

    return delta > 0;
}
```

# 测试它是否工作正常

**Example**

Given a ray with an origin at $[1\ \ -2\ \ -1]$ and a direction vector of $[1\ 2\ 4]$, find the nearest intersection point with a sphere of radius $S_r = 3$ centered at $[3\ 0\ 5]$.

# 加入断点，逐句调试，是否正确?

First normalize the direction vector, which yields:

$$\text{direction vector magnitude} = \sqrt{(1*1+2*2+4*4)} = \sqrt{21}$$
$$\mathbf{R}_d = [1/\sqrt{21} \quad 2/\sqrt{21} \quad 4/\sqrt{21}]$$
$$= [0.218\ 0.436\ 0.873].$$

Now find $A$, $B$, and $C$, using equation (A5):

$$A = 1 \text{ (because the ray direction is normalized)}$$
$$B = 2*(0.218*(1-3) + 0.436*(-2-0) + 0.873*(-1-5))$$
$$= -13.092$$
$$C = (1-3)^2 + (-2-0)^2 + (-1-5)^2 - 3^2$$
$$= 35.$$