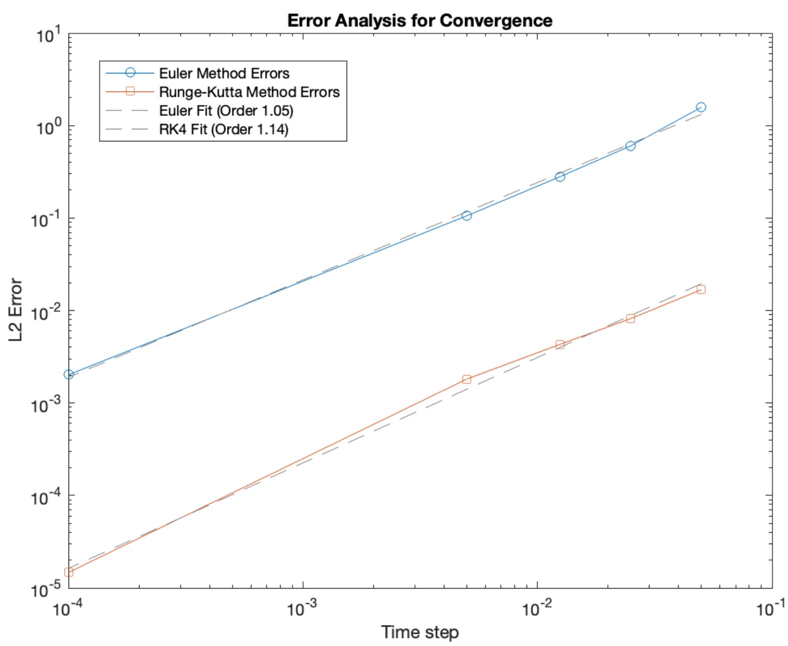
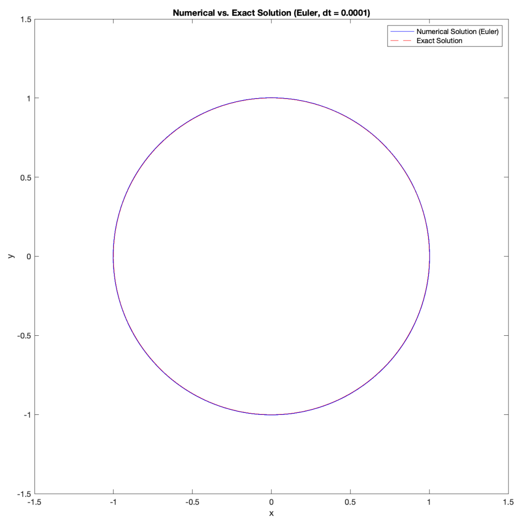
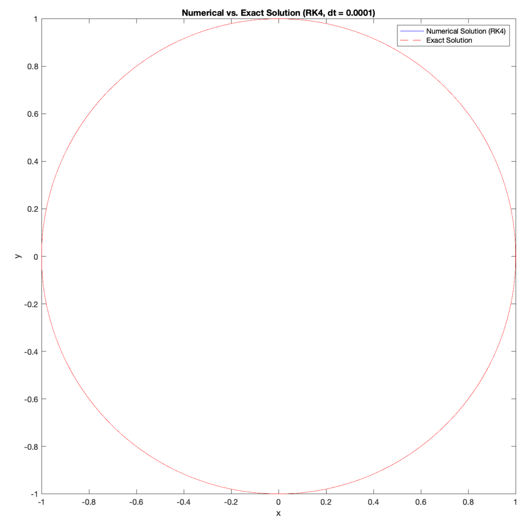
**Computing exercise:**
(Code is attached as .m files)

**Code validation:**
When I tried my solver on the two-component ODE given in the homework, taking $T = 2\pi$, I recorded the L2 error and plotted a loglog plot, then used linear regression to approximate the slope. For the Euler method, the slope is 1.05, which is expected as it is a first-order method. However, for the RK4 method, the slope is 1.14. Though it is a fourth-order method, it does not fit to a line with a slope of 4. As stated in the problem, it is a challenge to verify the fourth order accuracy for the RK4 method because of the roundoff in the ODE solver and possibly because of off-by-one errors in the number of time steps. However, the error of the RK4 method is still a lot less than the Euler method.

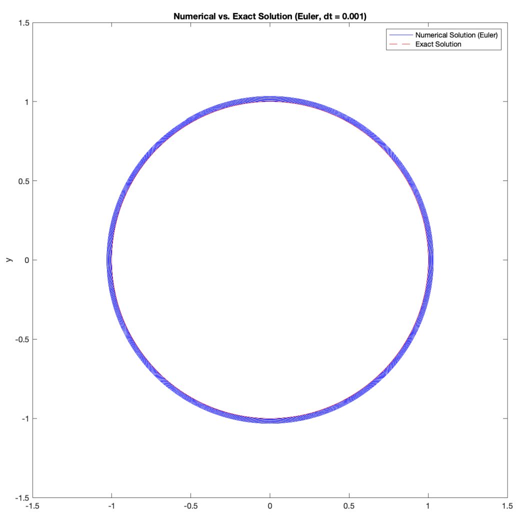


**Code play:**
This is my plot of the solution curve in 2D with a longer time of $T = 20\pi$, $dt = 0.0001$.



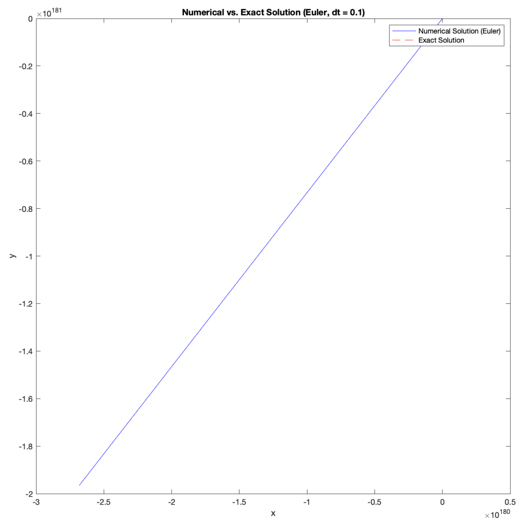



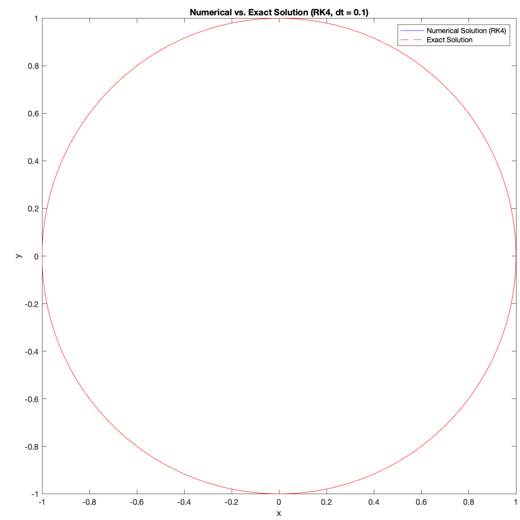

This is my plot of the solution curve in 2D with $T = 20\pi$, $dt = 0.01$, which is too large so that the first order "solution" deviates significantly from the exact solution.





This is my plot of the solution curve in 2D with $T = 20\pi$, $dt = 0.1$, for the solution to blow up.




From the plots, we can see that with a small enough time step, both of the methods will produce a close enough trajectory visually, although the RK4 method produces the trajectory that is a lot closer to the exact solution. It may not be as obvious when the time step is chosen to be small enough (dt $= 0.0001$), but when we increase the time step (dt $= 0.01$), though the Euler solution still looks like the unit circle, we can see that it deviates significantly from the exact solution. Then, when we choose the time step to be too large (dt $= 0.1$), the solution will blow up.