Ec 370

Money and Banking

Chapter 6: The Risk and Term Structure of Interest Rates - PART II

Xiang LI
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- **Previous lecture**: default risk, liquidity, income tax consideration explain why bonds with the **same maturity** have different interest rates
    - Risk structure of interest rates
- **Today's lecture**: three theories explaining why bonds with *identical* default risk, liquidity, tax consideration have *different* interest rates because their terms to **maturity are different**
    - Term structure of interest rates

Term structure of interest rates

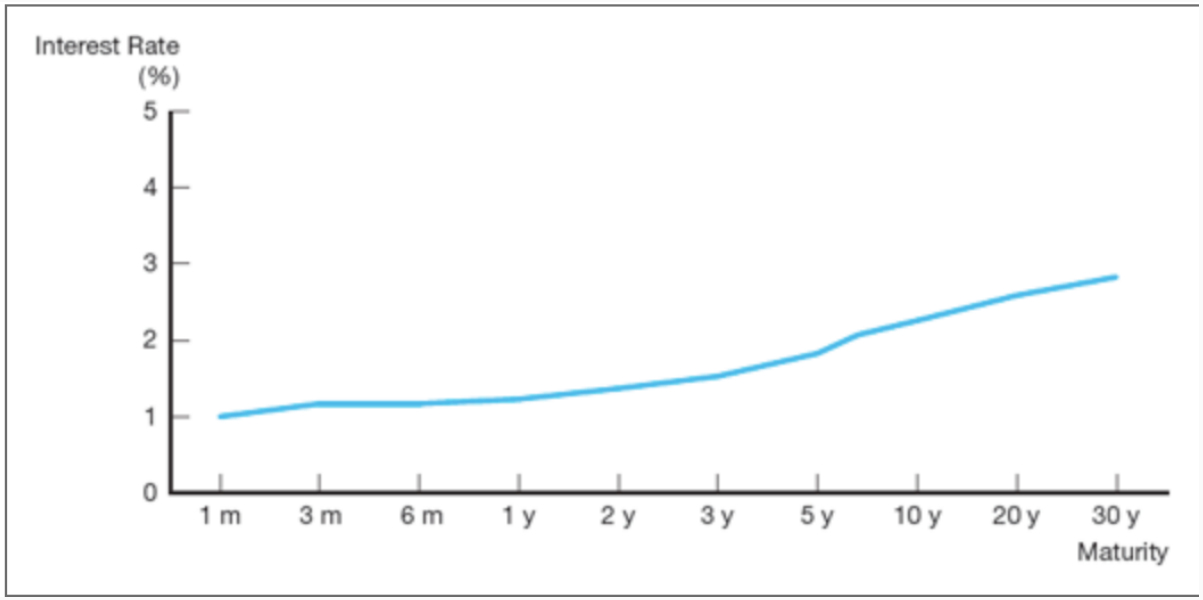
Yield Curve

- **Yield curve**: a plot of the yields on bonds with **differing** terms to maturity but the **same** risk, liquidity, and tax considerations
- Yield curves have three basic shapes
  - **upward-sloping** (most common): long-term interest rates > short-term interest rates
  - **flat**: short- and long-term interest rates are the same
  - **downward-sloping or inverted**: long-term interest rates < short-term interest rates

Yield Curve

Upward-sloping yield curve:

- **long-term interest rates is larger than short-term interest rates**



https://www.treasury.gov/resource-center/data-chart-center/interest-rates/pages/TextView.aspx?data=yieldYear&year=2020

Yield Curve

A good theory of the yield curve must explain these following **three important empirical facts**:

- **Fact #1**: interest rates on bonds of different maturities move together over time
- **Fact #2**: When short-term interest rates are low, yield curves are more likely to have an upward slope; when short-term interest rates are high, yield curves are more likely to slope downward
- **Fact #3**: Yield curves almost always slope upward

Yield Curve

Three theories of the yield curve:

- Expectations theory: can only explain Fact #1 & #2
- Segmented markets theory: can only explain Fact #3
- Liquidity premium theory:
  - combination of expectations theory and segmented markets theory
  - can explain Fact #1 & #2 & #3

(1) Expectations Theory

For an investment of $1, there are two investment strategies for the choice of holding for 2 periods:

Option 1: purchase a 1-yr bond, and when it matures in 1 year, purchase another 1-yr bond

Option 2: purchase a 2-yr bond and hold it until maturity

- interest rate on a 1-yr bond at time t: i_t
- interest rate on a 1-yr bond *expected* for time t + 1: i^e_{t+1}
- interest rate on the 2-yr bond at time t: i_{2t}

(1) Expectations Theory

- expected return for Strategy 1:

$$(1+i_t)(1+i^e_{t+1})-1 = i_t + i^e_{t+1} + i_t(i^e_{t+1}) \approx i_t + i^e_{t+1}$$

- expected return for Strategy 2:

$$(1+i_{2t})(1+i_{2t})-1 = 2i_{2t} + (i_{2t})^2 \approx 2i_{2t}$$

- **Key assumption of Expectations Theory**: bonds with different maturities are perfect substitutes
  - the expected returns on bonds with different maturities are equal

(1) Expectations Theory

- equalize:

$$2i_{2t} = i_t + i^e_{t+1}$$

- rewrite:

$$i_{2t} = \frac{i_t + i^e_{t+1}}{2}$$

- generalize: interest rate on an n-yr bond is
  - $$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n}$$

Participation 7

- Q1 (1): the current interest rate on the one-year bond is 5%. If the 1-yr interest rates over the next four years are **expected** to be 6%, 7%, 8%, and 9%. Find out the interest rate on a 2-yr bond, a 3-yr bond, a 4-yr bond, and a 5-yr bond. Also draw the yield curve.

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n}$$

Participation 7

(1) Given $i_t = 5\%$, $i^e_{t+1} = 6\%$, $i^e_{t+2} = 7\%$, $i^e_{t+3} = 8\%$, $i^e_{t+4} = 9\%$:

- $i_{2t} = \frac{5\%+6\%}{2} = 5.5\%$
- $i_{3t} = \frac{5\%+6\%+7\%}{3} = 6\%$
- $i_{4t} = \frac{5\%+6\%+7\%+8\%}{4} = 6.5\%$
- $i_{5t} = \frac{5\%+6\%+7\%+8\%+9\%}{5} = 7\%$

Participation 7

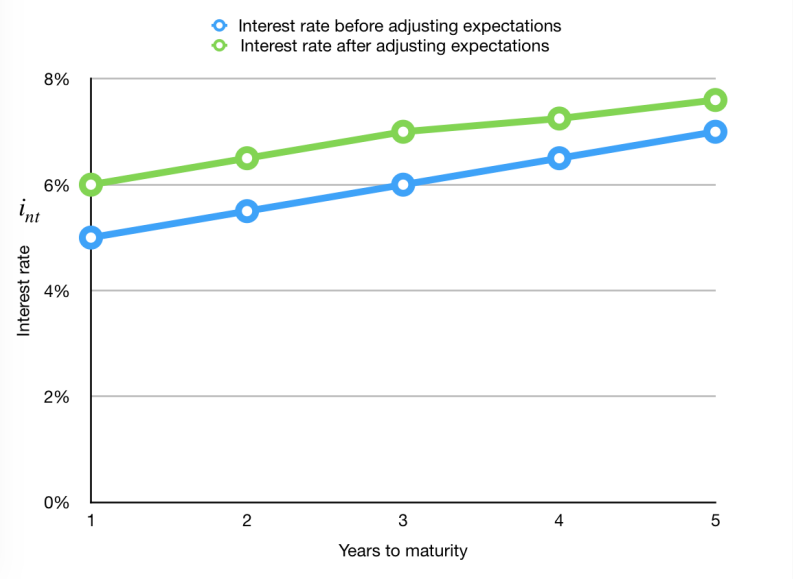
- Q1 (2): if i_t increases from 5% to 6%. Assume people expect interest rate on a 1-yr bond at time $t+1$ increases from 6% to 7%, and interest rate on a 1-yr bond at time $t+2$ increases from 7% to 8%, but do no adjust expectations further.

- Find out the interest rate on a 2-yr bond, a 3-yr bond, a 4-yr bond, a 5-yr bond, and draw the yield curve

Participation 7

(2) Given $i_t = 6\%$, $i^e_{t+1} = 7\%$, $i^e_{t+2} = 8\%$, $i^e_{t+3} = 8\%$, $i^e_{t+4} = 9\%$:

- $i_{2t} = \frac{6\%+7\%}{2} = 6.5\%$
- $i_{3t} = \frac{6\%+7\%+8\%}{3} = 7\%$
- $i_{4t} = \frac{6\%+7\%+8\%+8\%}{4} = 7.25\%$
- $i_{5t} = \frac{6\%+7\%+8\%+8\%+9\%}{5} = 7.6\%$



(1) Expectations Theory

- In summary, **A rise in actual short-term rates raises people's expectation on future short-term rates**. Therefore, increasing actual and expected short-term rates jointly drives up longer-term rates

- This is exactly why we observe **Fact #1** : interest rates on bonds of different maturities move together over time

  - $i_t \uparrow \Rightarrow i_{2t} \uparrow, \ldots, i_{nt} \uparrow$
  - $i_t \downarrow \Rightarrow i_{2t} \downarrow, \ldots, i_{nt} \downarrow$

- How does Expectations Theory explains **Fact #2**?

Participation 7

Post the 2007-2009 Great Recession, interest rate on 1-yr T-bills is usually less than 2%. So the current 5% 1-yr interest rate is way too high. Therefore, people expect the rates to decline in the future.

- Q1 (3): Assume people expect interest rate on a 1-yr bond at time $t+1$ decreases from 6% to 4%, and interest rate on a 1-yr bond at time $t+2$ increases from 7% to 3%, and interest rate on a 1-yr bond at time $t+3$ and $t+4$ are back to 2%.

- Find out the interest rate on a 2-yr bond, a 3-yr bond, a 4-yr bond, a 5-yr bond, and draw the yield curve

Participation 7

(3) Given $i_t = 5\%$, $i^e_{t+1} = 4\%$, $i^e_{t+2} = 3\%$, $i^e_{t+3} = 2\%$, $i^e_{t+4} = 2\%$:

- $i_{2t} = \frac{5\%+4\%}{2} = 4.5\%$
- $i_{3t} = \frac{5\%+4\%+3\%}{3} = 4\%$
- $i_{4t} = \frac{5\%+4\%+3\%+2\%}{4} = 3.5\%$
- $i_{5t} = \frac{5\%+4\%+3\%+2\%+2\%}{5} = 3.2\%$

(2) Segmented Markets Theory

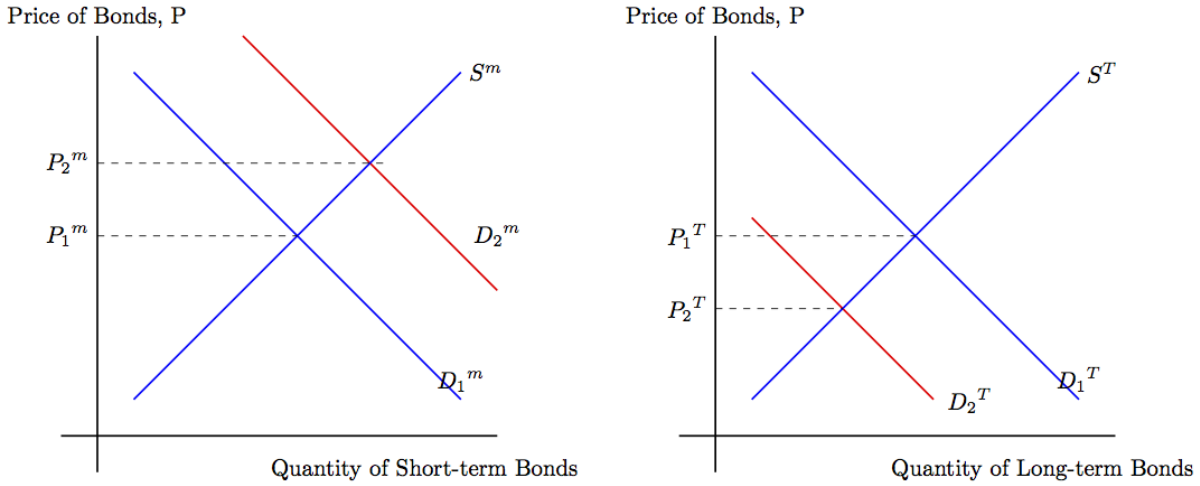
- Expectations Theory assumes that investors do not prefer bonds of one maturity over another
- However, people may prefer short-term bonds to long-term bonds
  - people who are limited by a short holding period will prefer to hold short-term bonds
- And people may prefer long-term bonds to short-term bonds
  - someone who was putting funds away for their young child to go to college would want to hold longer-term bonds
- So when expectations theory does not hold and fails to explain Fact #3, we need another theory

(2) Segmented Markets Theory

- **Key assumption of segmented markets theory**: bonds of different maturities are NOT substitutes
- demand/supply for different maturity bonds are independent and completely separated
- interest rates of bonds of different maturity are determined by the demand and supply of that bond only are independent
- Recall Chapter 4: shorter-term bonds have less interest rate risk, and shorter-term bonds are typically more liquid
- Therefore, investors typically prefer shorter-term bonds

(2) Segmented Markets Theory

Hence, **demand** for long-term bonds is **typically** lower than that for short-term bonds



- In summary: under segemented markets theory, **long-term bonds typically have higher interest rates** (this is exactly Fact #3)

(3) Liquidity Premium Theory

- Let's call the additional interest that investors require as compensation for buying longer-term bonds: **liquidity premium**
- l_{nt}: liquidity premium for the n-year bond at time t
  - $l_{nt} > 0$, for all n
  - l_{nt} grows with the term to maturity n (longer-term bonds have greater liquidity premium)

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n} + l_{nt}$$

(3) Liquidity Premium Theory

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n} + l_{nt}$$

- As a combination of expectation theory and segmented markets theory, liquidity premium theory assumes bonds of different maturity are partial (not perfect) substitutes
- Fact #1 and #2 are explained by the first term in the equation
- Fact #3 is explained by the second term in the equation - the liquidity premium term

Participation 7

- Let's return to the exercise
- Q1 (4): the current 1-yr interest rate is 5%, and 1-yr interest rates over the next four years are expected to be 6%, 7%, 8%, and 9%. Assume liquidity premiums for 2- to 5-yr bonds are 0.25%, 0.5%, 0.75%, and 1.0%, respectively.
- Find out the interest rate on a 2-yr bond, a 3-yr bond, a 4-yr bond, a 5-yr bond, and draw the yield curve

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n} + l_{nt}$$

Participation 7

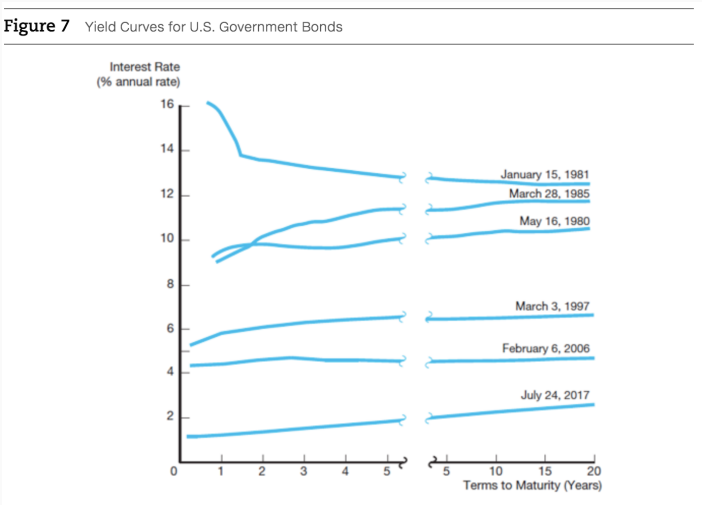
(4) In part (1), we have solved the first term for each bond to be: $5\%, 5.5\%, 6\%, 6.5\%, 7\%$. Given $l_t = 0\%$, $l_{2t} = 0.25\%$, $l_{3t} = 0.5\%$, $l_{4t} = 0.75\%$, $l_{5t} = 1\%$:

- $i_t = 5\% + 0\% = 5\%$
- $i_{2t} = 5.5\% + 0.25\% = 5.75\%$
- $i_{3t} = 6\% + 0.5\% = 6.5\%$
- $i_{4t} = 6.5\% + 0.75\% = 7.25\%$
- $i_{5t} = 7\% + 1\% = 8\%$

Interpreting Yield Curve

- Recall Chapter 5: falling interest rates are associated with recessions (procyclical)
- when inverted yield curve is observed, short-term interest rates are expected to fall sharply in the future, and the economy is expected to be more likely to enter a recession
- Yield curve is a powerful predictor of an economic downturn

Interpreting Yield Curve



The steep inverted yield curve occurred on January 15, 1981

- This indicated that short-term interest rates were expected to decline sharply in the future
- Indeed, by March 1981, 3-month Treasury bill rates had declined from the 16% to 13%
- Soon afterwards, a recession started in July 1981

Interpreting Yield Curve

Who discovered the predictive power of the yield curve?

- The relationship between inverted yield curve and recessions was discovered by Professor Campbell Harvey in his dissertation in 1986. He got his Ph.D. from University of Chicago

What measure to choose?

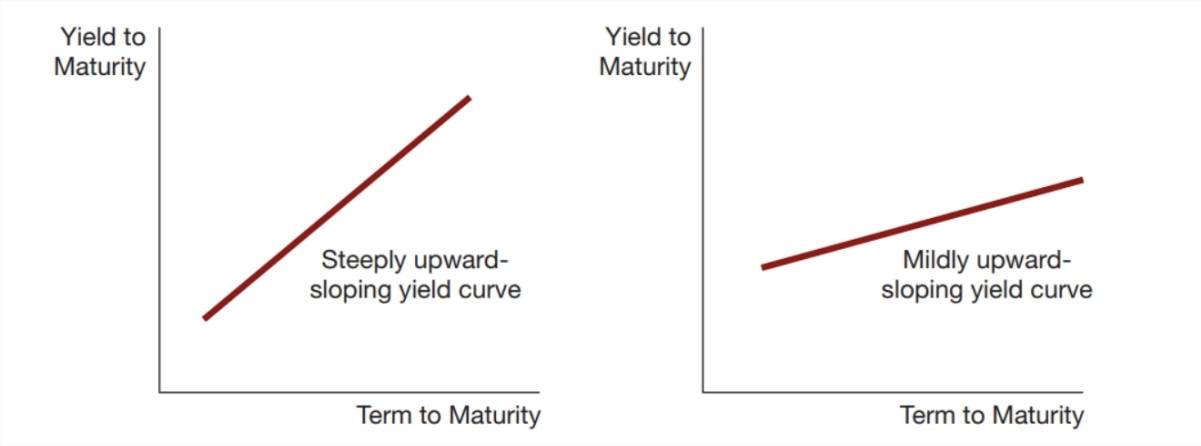
- Common measures are 2-year/10-year spread and 3-month/10-year spread
  - available from 1976-: https://fred.stlouisfed.org/series/T10Y2Y
  - available from 1982-: https://fred.stlouisfed.org/series/T10Y3M

Interpreting Yield Curve

If the yield curve flattens, when will it invert? If the yield curve inverts, when will the recession happen?

- Indeterminate ... The lags can be 12 months, 18 or 20 months. But the recession will not happen right away.
- Professor Harvey said the curve needs to stay inverted for three months to be reliable
- Although the yield curve has a strong track record, it is not a cause of a financial crisis, nor does it share any insight onto the timing and magnitude of a recession (there can be false positives)

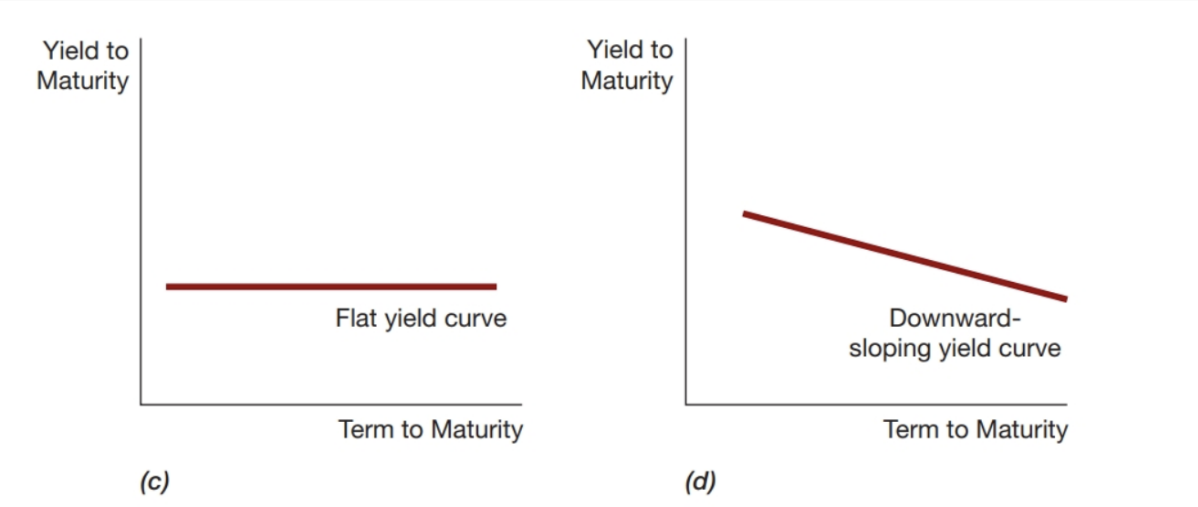
Interpreting Yield Curve



Steeply rising yield curve: short-term interest rates are expected to rise in the future

Moderately steep yield curve: short-term interest rates are not expected to rise or fall much in the future

Interpreting Yield Curve



Flat yield curve: short-term interest rates are expected fall moderately in the future

Inverted yield curve: short-term interest rates are expected to fall sharply in the future

Interpreting Yield Curve

What does yield curve say about inflation?

- Recall Chapter 4 Fisher Equation: $i = r + \pi^e$
  - when observing flat or inverted yield curve: short-term i are expected to fall
  - hence inflation is expected to decline in the near future

A 3-D View of a Chart That Predicts The Economic Future:
https://www.nytimes.com/interactive/2015/03/19/upshot/3d-yield-curve-economic-growth.html?fbclid=IwAR1y8BI1JOx3DMrZlOmStEeeeYOpnU_GJY_MO69Lap0HY8GR5jz0qvh1FZs