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Homework 2

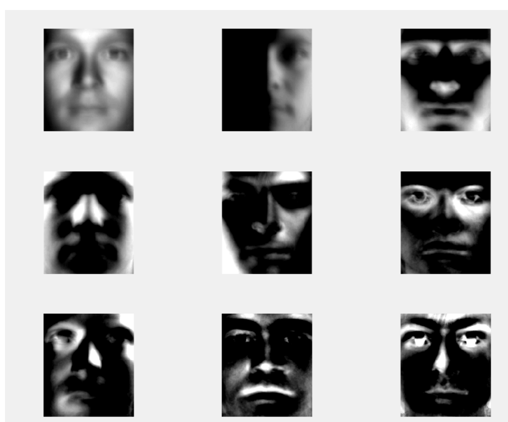
1. Do an SVD analysis of the images (where each image is reshaped into a column vector and each column is a new image).

   The link to the code of the Homework 2:
   https://github.com/lx1st/Courses/tree/main/Hw2

2. What is the interpretation of the U, Σ and V matrices? (Plot the first few reshaped columns of U)

   The interpretation of U: columns of U are the eigenfaces basis or rotation matrix.

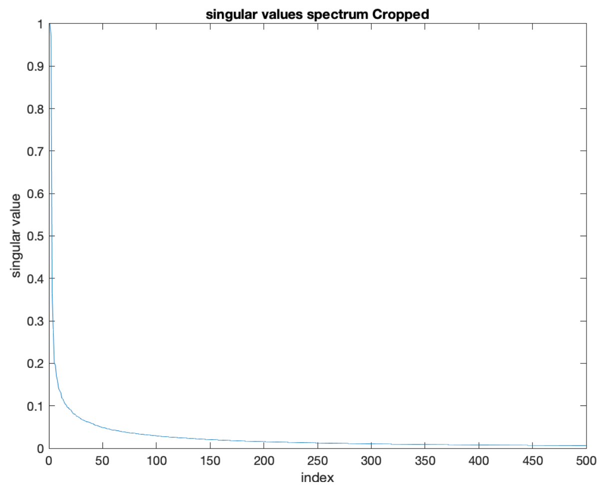
   The pictures of the first 9 columns of U are as below:



   The interpretation of Σ: the stretching factor or the degree of contribution from the columns of U.

   The interpretation of V: rows of V are the projection from the eigenfaces to the actual images.

3. What does the singular value spectrum look like and how many modes are necessary for good image reconstructions using the PCA basis? (i.e., what is the rank r of the face space?)



The above is the spectrum of the singular value for cropped images.

According to the above singular value spectrum for cropped images, 50 is enough for the rank of the face space to make PCA analysis.

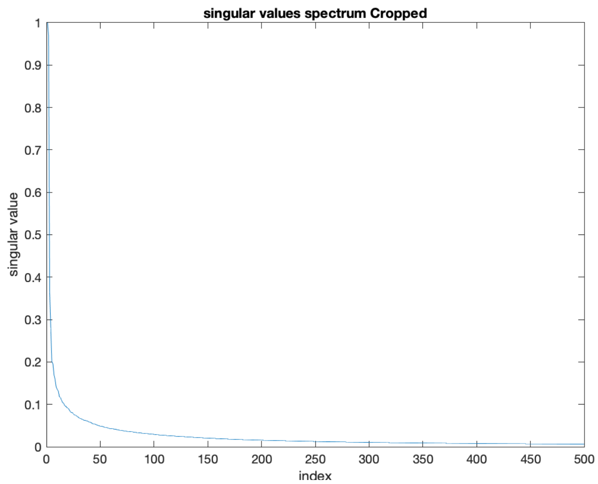
R = 10

R = 50

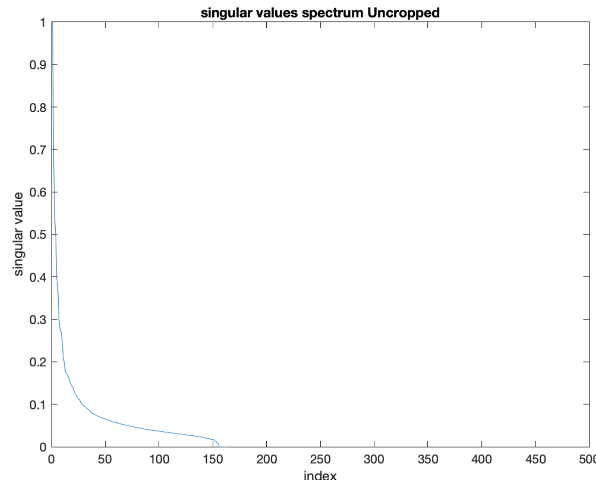
R = 200

R = Number of the columns of U

4. Compare the difference between the cropped (and aligned) versus uncropped images in terms of singular value decay and reconstruction capabilities.



cropped



uncropped images

According to the comparison of the singular value spectrum of cropped and uncropped picures, after aligning the picture, using of fewer columns of U & Σ can represent the characters of pictures.

R = 50

R = Image Number

R = 50

R = Image Number

The aligned pictures have a better ability to reconstruct as above.

2.1 The SVD of Matrix A :

$$A = U\Sigma V^* \ , \ \Sigma$$

$$\therefore AA^* = U\Sigma V^* (U\Sigma V^*)^*$$
$$= U\Sigma V^* \cdot V\Sigma U^*$$

$\because$ U, V are unitary matrix

$\therefore$ $V^* \cdot V = I$

$\therefore$ $AA^* = U\Sigma^2 V^*$.

$\therefore$ Σ^2 is the diagonal matrix with AA^* singular values.

$\therefore$ The singular values of AA^* are the square of A

The proof of $A^* \cdot A$ is similar.

2.2. $\because$ A is hermitian Matrix

$\therefore$ The eigenvalue decomposition of A : $A = U\Lambda U^* = A^*$

$$\because AA = (U\Lambda U^*)(U\Lambda U^*)$$
$$= U\Lambda^2 U^*$$

sing is [eigen].

$$AA^* = AA = U\Lambda^2 U^*$$

$\therefore$ eigenvalues of A are $\sqrt{\Lambda^2} = |\Lambda|$.

matrix of

2.3. $\det(A) = \det(U \Sigma V^*)$

$= \det(U)\det(\Sigma)\det(V^*)$ $\quad V^* = \frac{V^{-1}}{|V|}$

$\because U, V$ are unitary $\therefore V^*$ is also unitary

$\therefore \det(U) = \det(V) = \det(V^*) = 1$

$\because \Sigma$ is diagonal

$\therefore \det(A) = \det(\Sigma) = \prod_{j=1}^{m} \sigma_j.$