

Homework 3
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Github link to Hw3 is: <https://github.com/lx1st/Courses/tree/main/Hw3>

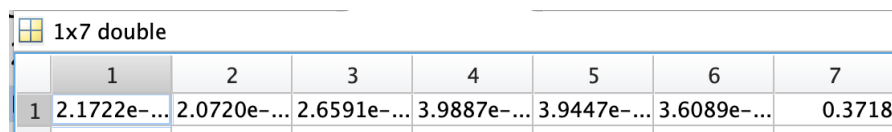
Problem 1.

Code is: qrfactor.m, modified_gram_schm.m and Hw3_main_problem1.m

- Generate a matrix A by randn(10, 7)
- $A(:,7) = A(:, 1)$;
- A_ill = A as below
- Then, $\text{cond}(A_ill) = 2.7e+16$
- $A_Noi = A + \text{randn}(10, 7) * 1e-12$

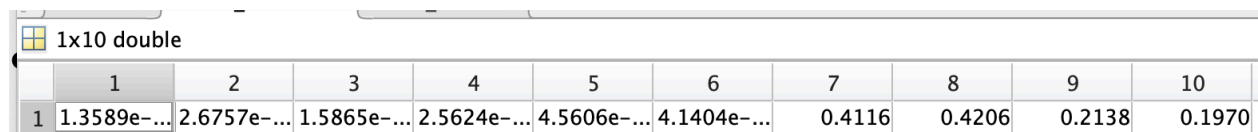
Comparison between Modified Gram-Schmidt algorithm & Household algorithm & Matlab algorithm

- The max mean absolute difference between the Q_ Modified_Gram_Schmidt matrix for A_ill and A_NoI is 0.3718:



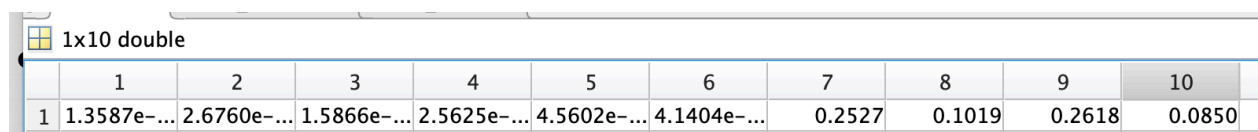
	1	2	3	4	5	6	7
1	2.1722e-...	2.0720e-...	2.6591e-...	3.9887e-...	3.9447e-...	3.6089e-...	0.3718

- The max mean absolute difference between the Q_ Household matrix for A_ill and A_NoI is 0.4116:



	1	2	3	4	5	6	7	8	9	10
1	1.3589e-...	2.6757e-...	1.5865e-...	2.5624e-...	4.5606e-...	4.1404e-...	0.4116	0.4206	0.2138	0.1970

- The max mean absolute difference between the Q_ Matlab matrix for A_ill and A_NoI is 0.2618:

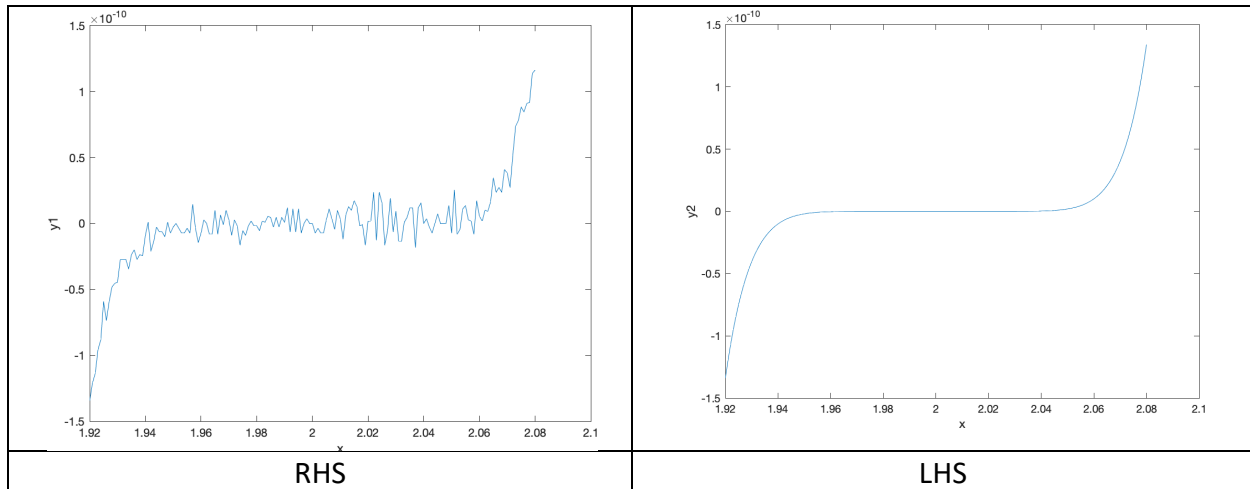


	1	2	3	4	5	6	7	8	9	10
1	1.3587e-...	2.6760e-...	1.5866e-...	2.5625e-...	4.5602e-...	4.1404e-...	0.2527	0.1019	0.2618	0.0850

- Conclusions:
 - The Householder algorithm is more stable than modified Gram-Schmidt algorithm
 - The MATLAB algorithm is more stable than modified Gram-Schmidt

Problem 2:

Code is: Main_Problem2.m



Problem 3:

Code is: main_problem3.m

3.1

10x10 double									
	1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	0	0	0	0
2	100	0	0	0	0	0	0	0	0
3	100	317.3017	0	0	0	0	0	0	0
4	100	257.4320	683.4026	0	0	0	0	0	0
5	100	201.2610	371.0391	725.3942	0	0	0	0	0
6	100	193.6671	348.4429	494.5295	1.0911e...	0	0	0	0
7	100	180.7223	277.6904	382.8949	678.8480	1.3276e...	0	0	0
8	100	168.4110	245.4045	332.6498	464.6810	756.4431	1.2858e...	0	0
9	100	177.5949	227.2688	322.2844	415.3682	599.0287	968.8693	1.8447e...	0
10	100	164.7234	210.1915	285.6196	383.3054	566.1293	672.9349	947.4664	1.7036e...

The iteration number is 100 and only the condition numbers of matrixes satisfying $m > n$ are calculated. The results are as above.

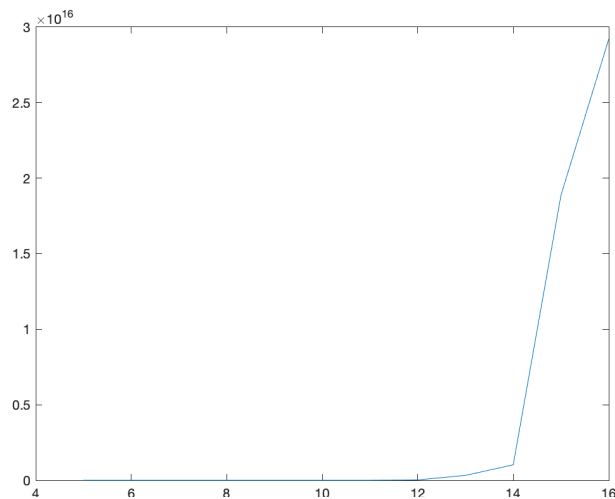
- When m is fixed, condition number increases with the increasement of n .
- When n is fixed, condition number decreases with the increasement of n .
- When $m-n$ is fixed, condition number increases with the increasement of n or m .

3.2

Setting A by `randn(5,4)`. The condition of original matrix A is 8.1953. After appending the 1st column after the last column of A, the appended condition number is 5.531×10^{16} and appended determinant is 7.3698×10^{-19} .

3.3

Setting the noise from 10^{-4} to 10^{-16} and condition numbers of matrix A with varying column appended are as below.



- It is clear that condition number increases with the negative power of noise level decreases.