Extended Essay

Subject: Mathematics

Topic: Mathematical Modeling of Population Growth

Research Question: To what extent can mathematical population growth models be used to simulate and describe the population growth patterns in China, as well as future predictions? Which model gives the most accurate approximation?

Word Count: 3215

1 Introduction

China is a country with a large population. In the past few decades, the population problem has consistently been one of the key factors restricting China's development. Worldwide, population forecasting has grown in importance. With the implementation and changes of China's economic development and population policy, what will China's population be in the future? The focus of this research raises to the following research question: *To what extent can mathematical population growth models be used to simulate and describe the population growth patterns in China, as well as future predictions? Which population gives the most accurate approximation?*

In this essay, the Malthusian model, logistic model and least squares polynomial approximation will be utilized to stimulate the population growth of China. The least square method will be introduced in section 4.1. Two ordinary differential equation (ODE) models will be constructed and applied. In particular, the logistic growth model will be the focus as it provides the most accurate estimation in the case of the Chinese population. Additionally, the estimated population will be compared to the actual population to find the model that gives the most accurate approximation. The total population of China provided in Figure 4.1 and Figure 4.2 were taken from the World Bank databases. All data processing is carried out with Google Sheets and Python.

2 Population Growth: Malthusian Growth Model

The Malthusian growth model, also known as the simple exponential model for population growth, was named after the British scholar Thomas Robert Malthus. Malthus proposed that the growth of a population will proceed exponentially if the growth goes unchecked (Malthus).

2.1 Assumptions of the Malthusian Model

The Malthusian model is one of the easiest population models, and like its other name, the so-called simple exponential model. The construction of this model is based on the following basic assumptions:

Assumption 1: The derivative of $P(t)$ is a continuous function of t for all $t > 0$ (Dreyer 30).

Assumption 2: The rate of change of the population is directly proportional to the population at any time (Dreyer 30).

The rate of change of the population is the derivative of $P(t)$. These two assumptions lead us to the next stage, which the model can be written in the form of a first order linear differential equation:

$$\frac{dP}{dt} = kP, \tag{2.1}$$

where k is a constant representing the rate of population growth and, as a continuous function of t (the independent variable), denotes the number of individuals in the population at time t. Hence, $P(t)$ must be a non-negative integer, and the changes in the function are caused by the births and deaths in the population. Noting that the population patterns of China are the focuses, therefore the unit for time t may be measured by years.

2.2 Analytical Solution

The two key assumptions of the Malthusian model bring us to the following initial value problem:

$$\frac{dP}{dt} = kP, \qquad P(0) = P_0$$

where P_0 represents the initial population size. From here, we may use the method of separation of variables to give:

$$\int \frac{1}{P} dP = \int k dt$$

$$ln\,|P| = kt + C$$

$$P = Ae^{kt},$$

where t is measured in years in the case of human populations, $A = \pm e^{C}$ or 0, and C is a constant. Substituting the initial value $P(0) = P_0$, the exact solution is:

$$P(t) = P_0 e^{kt}. \tag{2.2}$$

The Malthusian model expressed by Eq. (2.1) can be described as both exponential growth and exponential decline as results. Referring to Figure 2.1, exponential growth always occurs $\forall k > 0$. As $t \to \infty$, the population tends to approach positive infinity. Noting that there is a carrying capacity M for a specific environment. Thus, the exponential manner will always increase the population and approach the carrying capacity M of the population.

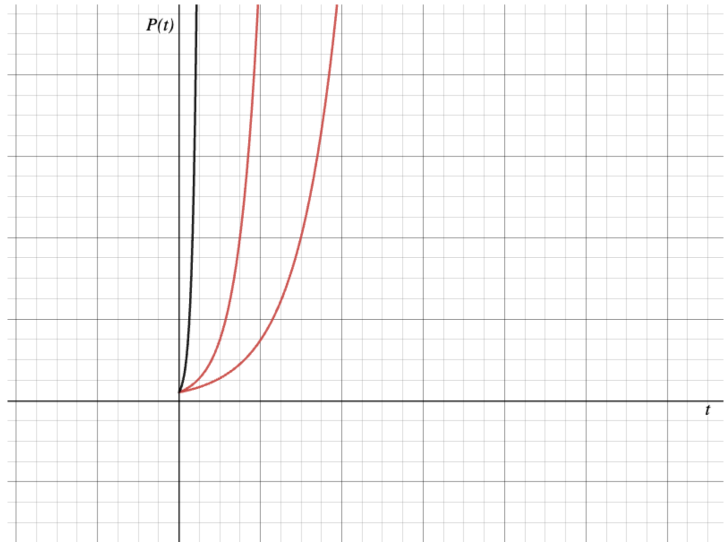

Figure 2.1 The family of solutions of Eq. (2.2) with $P_0 > 0$, $t \geq 0$ and a set of arbitrary $k > 0$.

However, if the sign of k becomes negative, the model describes an exponential decline with reference to Figure 2.2. In this case, as $t \to \infty$, the population will approach 0, indicating that the population is about to become extinct.

Figure 2.2 The family of solutions of Eq. (2.2) with $P_0 > 0$, $t \geq 0$ and a set of arbitrary $k < 0$.

3 Population Growth: Logistic Growth Model

The logistic differential equation was originally proposed by a Belgian mathematician Pierre François Verhulst in 1838 (Bacaër 35). Modeling the population growth is a very common application of the logistic equation.

3.1 Assumptions of the Logistic Model

As discussed in the previous section, a population usually starts to grow exponentially in its earlier days. With limited resources, the population growth will eventually slow down as it approaches its carrying capacity. If $P(t)$ is the size of the population at time t, we assume that (Stewart et al. 632):

$$\frac{dP}{dt} \approx kP, \qquad \text{if } P \text{ is small.}$$

This shows the idea that the rate of population growth k is initially close to being proportional to the population size. The rate of population growth k will tend to decrease as it approaches its carrying capacity M, and will become negative if the population size ever exceeds M. Therefore, the simple expression for the logistic differential equation can be written as:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right). \tag{3.1}$$

Noticing that if the initial population size is very small compared to with carrying capacity, then $\frac{P}{M}$ is very close to zero and hence $\frac{dP}{dt} \approx kP$ (Stewart et al. 633). This means that the population will grow rapidly, similar to exponential growth. However, as the population size is approaching its carrying capacity M, $\frac{P}{M}$ is also growing to approach 1 as the carrying capacity M is constant.

Some useful information about the behavior of the logistic growth can be deduced from Eq. (3.1). Suppose that the population P exceeds the carrying capacity M, the right-hand side of Eq. (3.1) will become negative, thus, $\frac{dP}{dt} < 0$ and the population starts to decrease. If the population P lies between 0 and M, the right-hand side of Eq. (3.1) will be positive. Hence, $\frac{dP}{dt} > 0$ and the population keeps increasing. If the population P equals to its carrying capacity M, then the right-hand side of Eq. (3.1) will equal to 0, indicating that the population does not change. For the final case, it is quite obvious that if $P = 0$, the species is extinct, $\frac{dP}{dt}$ will be 0 as well. We can further analyze this by using the way of a phase line. **Phase line** is a visual representation of the behavior of solutions to a differential equation subject to various initial conditions (Strang and Herman). The equilibrium points are obtained by solving $\frac{dP}{dt} = 0$, where the solution is $P = 0 \; or \; M$. For the case of a carrying capacity M in the logistic equation, a phase line describing the behavior of the general solution to the differential equation is as shown in Figure 3.1 (Strang and Herman).

Figure 3.1 The phase line for the logistic differentiation equation (Strang and Herman).

This phase line demonstrates how the population shrinks over time when its carrying capacity M is exceeded or the population size P is less than zero (not available because it is assumed that $P \geq 0$). The population expands over time when P lies between 0 and M.

3.2 Analytical Solution

The general solution to the logistic differential equation can be obtained by using separation of variables. Eq. (3.1) can be rewritten as:

$$\frac{dP}{dt} = \frac{kP(M-P)}{M},$$

we have

$$\int \frac{M}{P(M-P)} dP = \int k dt .$$

The left-hand side of this equation can be integrated using the method of partial fraction decomposition, this enables us to rewrite the equation as (Strang and Herman):

$$\frac{M}{P(M-P)} = \frac{1}{P} + \frac{1}{M-P}$$

$$\int \left(\frac{1}{P} + \frac{1}{M-P}\right) dP = \int k dt$$

$$ln\,|P| - ln\,|M - P| = kt + C$$

$$ln\left|\frac{M-P}{P}\right| = - kt - C$$

$$\left|\frac{M-P}{P}\right| = e^{-kt-C} .$$

Then, exponentiate both sides of the equation to eliminate the natural logarithm, we get

$$\frac{M-P}{P} = Ae^{-kt}, \tag{3.2}$$

where we define $A = \pm e^{-C}$ (some constant). The value of A can be found by substituting $t = 0$ into Eq. (3.2), where $P = P_0$ (the initial population size):

$$A = \frac{M - P_0}{P_0}$$

Therefore, the general solution to the differential equation shown by Eq. (3.1) with an initial condition $P(0) = P_0$ is

$$P(t) = \frac{M}{1 + Ae^{-kt}}\,. \tag{3.3}$$

Referring to Figure 3.1, the logistic equation has exactly two equilibrium points at $P = 0 \; or \; M$, which are obtained by solving $\frac{dP}{dt} = 0$. A phase line portrait of logistic model can be illustrated as follows:

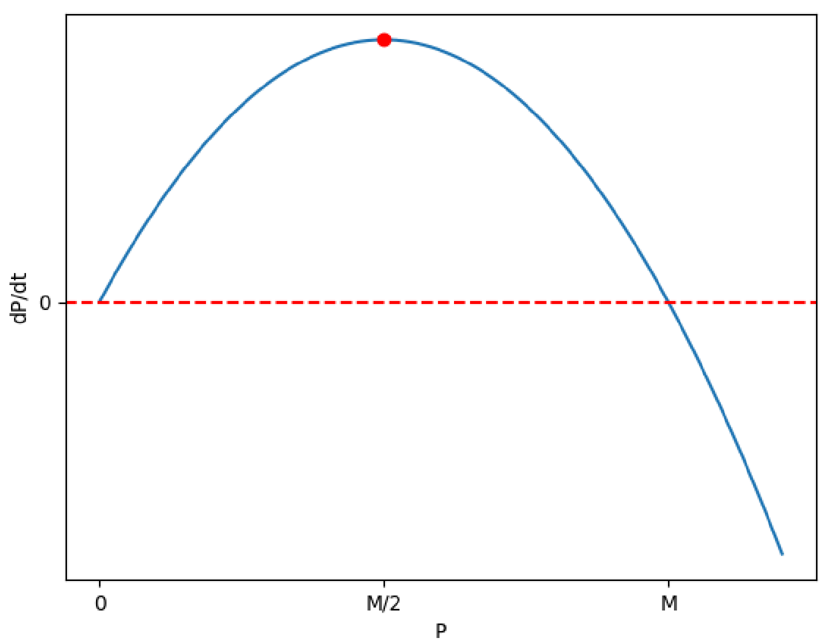

Figure 3.2 Phase line portrait of logistic model.

The behavior of logistic model shown by Figure 3.2 is summarized below:

P	dP/dt
$P > M$	$\frac{dP}{dt} < 0$
$0 < P < M$	$\frac{dP}{dt} > 0$
$P = M$	$\frac{dP}{dt} = 0$
$P = 0$	$\frac{dP}{dt} = 0$

Figure 3.3 Behavior of logistic model.

The population will slowly approach the equilibrium point $P = M$ over time, from both directions. The rate of population growth $\frac{dP}{dt}$ is at its maximum when $P = \frac{M}{2}$. In other words, the population is growing at its fastest rate at the instant it reaches half its carrying capacity. It is worth noting that $P = \frac{M}{2}$ is a point of inflection. The concavity of the logistic model changes. When $P > \frac{M}{2}$, the population will grow even slower towards its carrying capacity as shown in Figure 3.4.

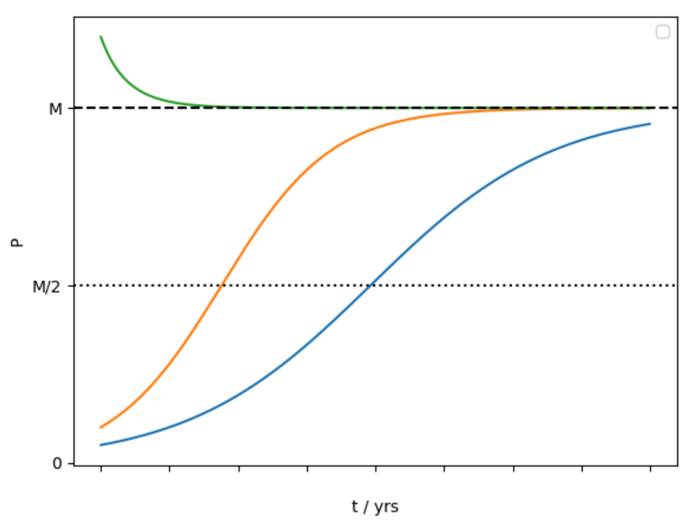

Figure 3.4 Logistic model starting with various initial states.

4 Modeling the Population

In the following section, the population data for China from 1960 to 2021 will be given as provided by the World Bank. However, only the population data from 1960 to 2001 will be modeled. The actual population from 2002 to 2021 will be compared with the estimated population from the mathematical models mentioned earlier, and a projection of the population growth until year 2032 will be made.

4.1 Least Square Method

The method of least squares is the most commonly used approach for solving curve-fitting problems; it involves determining the best approximations (Burden et al. 507). It is also a mathematical optimization modeling method used to predict the behaviour of the population size of China. The main principles of this method are explained in this section.

First of all, the aim of the least square method is to find a function $p(x)$ that approximates the original function $f(x)$. The function $p(x)$ is obtained from some discrete point sets. We let the value of $p(x)$ at each corresponding discrete point equal to $S(x_i)$, which we are finding a function $S(x)$ such that the sum of squares of errors from the points to the curve is a minimum:

$$\sum_{i=0}^{m} \delta_i^2 = \sum_{i=0}^{m} [S(x_i) - y_i]^2, \tag{4.1}$$

where y_i, for each $i = 0, 1, \cdots, m$, denotes the i th actual y-value, and $S(x_i)$ denotes the i th observed value on the approximating line (Burden et al. 507). δ_i is the difference between $S(x_i)$

and y_i. Here, we let the function be an algebraic polynomial (Burden et al. 509):

$$S(x) = a_0\varphi_0(x) + a_1\varphi_1(x) + \cdots + a_n\varphi_n(x) \quad (n < m). \tag{4.2}$$

Therefore, we obtain the following equivalent (Note that $y_i = f(x_i)$):

$$\sum_{i=0}^{m} \delta_i^2 = \sum_{i=0}^{m} [S(x_i) - y_i]^2 = \sum_{i=0}^{m} (\sum_{j=0}^{n} a_j\varphi_j(x_i) - f(x_i))^2. \tag{4.3}$$

In order to find the best approximation, it is amazingly obvious that we want the sum of squares of errors to be as small as possible, that is, we want Eq. (4.3) to be the smallest. The criterion is to minimize the sum of squares of δ_i.

In order to make the fitting situation more suitable for the real-life situation, a weight function $w(x)$ is introduced, and each term of the sum of squares will be weighted. The weight function $w(x)$ can be any values greater than or equal to 0 on the closed interval $[a, b]$ (Burden et al. 521). In the actual situation, the proportion and importance of each data point are different. It indicates that the weight of data at different points $(x_i, f(x_i))$ is different. However, this essay does not explore the weighted sum of squares of errors in depth. Instead, the curve-fitting equations will be calculated and approximated using the libraries of the Python programming language.

4.2 Raw data

To ensure sufficient data for analysis and comparison, the total population of China for each year from 1960 to 2021 are obtained from the World Bank databases, which are provided in Figure 4.1 and Figure 4.2.

Year	Population	Year	Population	Year	Population
1960	667070000	1974	900350000	1988	1101630000
1961	660330000	1975	916395000	1989	1118650000
1962	665770000	1976	930685000	1990	1135185000
1963	682335000	1977	943455000	1991	1150780000
1964	698355000	1978	956165000	1992	1164970000
1965	715185000	1979	969005000	1993	1178440000
1966	735400000	1980	981235000	1994	1191835000
1967	754550000	1981	993885000	1995	1204855000
1968	774510000	1982	1008630000	1996	1217550000
1969	796025000	1983	1023310000	1997	1230075000
1970	818315000	1984	1036825000	1998	1241935000
1971	841105000	1985	1051040000	1999	1252735000
1972	862030000	1986	1066790000	2000	1262645000
1973	881940000	1987	1084035000	2001	1271850000

Figure 4.1 Raw quantitative data showing the total population of China from 1960 to 2001.

Year	Population	Year	Population	Year	Population
2002	1280400000	2009	1331260000	2016	1387790000
2003	1288400000	2010	1337705000	2017	1396215000
2004	1296075000	2011	1345035000	2018	1402760000
2005	1303720000	2012	1354190000	2019	1407745000
2006	1311020000	2013	1363240000	2020	1411100000
2007	1317885000	2014	1371860000	2021	1412360000
2008	1324655000	2015	1379860000		

Figure 4.2 Raw quantitative data showing the total population of China from 2002 to 2021.

For modeling purposes, the year 1960 will be treated as the base year. Therefore, we have the following initial condition: $P(0) = 667070000$. The tables above can thus be adjusted.

Time / yrs	Population	Time / yrs	Population	Time / yrs	Population
0	667070000	14	900350000	28	1101630000
1	660330000	15	916395000	29	1118650000
2	665770000	16	930685000	30	1135185000
3	682335000	17	943455000	31	1150780000
4	698355000	18	956165000	32	1164970000
5	715185000	19	969005000	33	1178440000
6	735400000	20	981235000	34	1191835000
7	754550000	21	993885000	35	1204855000
8	774510000	22	1008630000	36	1217550000
9	796025000	23	1023310000	37	1230075000
10	818315000	24	1036825000	38	1241935000
11	841105000	25	1051040000	39	1252735000
12	862030000	26	1066790000	40	1262645000
13	881940000	27	1084035000	41	1271850000

Figure 4.3 Adjusted total population of China from 1960 to 2001.

Time / yrs	Population	Time / yrs	Population	Time / yrs	Population
42	1280400000	49	1331260000	56	1387790000
43	1288400000	50	1337705000	57	1396215000
44	1296075000	51	1345035000	58	1402760000
45	1303720000	52	1354190000	59	1407745000
46	1311020000	53	1363240000	60	1411100000
47	1317885000	54	1371860000	61	1412360000
48	1324655000	55	1379860000		

Figure 4.4 Adjusted total population of China from 2002 to 2021.

The general trend of the population growth of China from 1960 to 2001 can be seen from Figure 4.5, which is formed using the raw data in Figure 4.3.

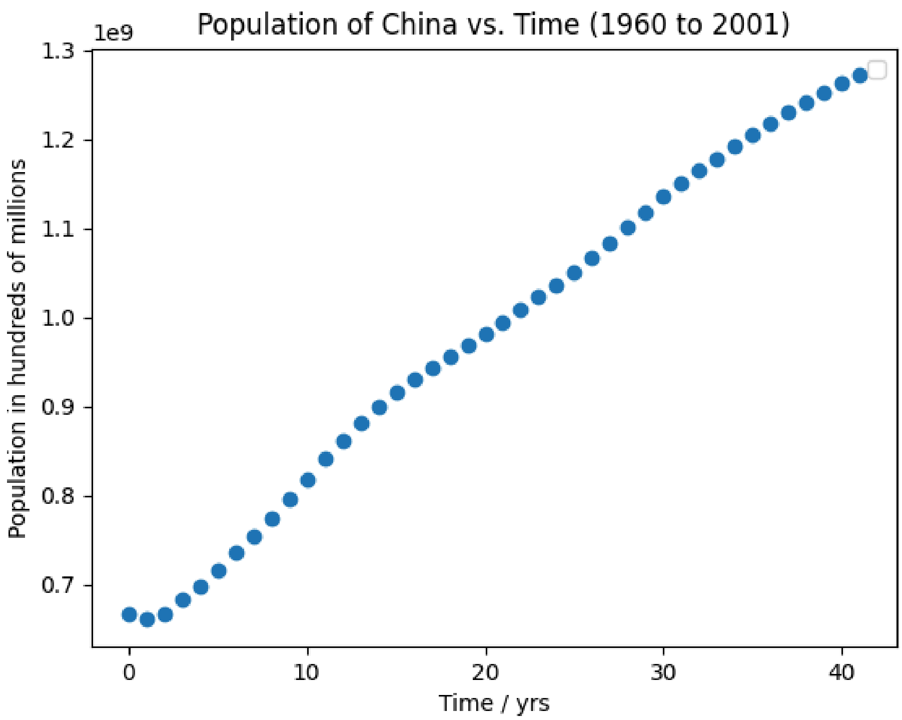

Figure 4.5 Scatter plot of Population of China vs. Time (1960 to 2001).

4.3 Estimation of Population by Malthusian Model

Again, the Malthusian model is also known as the exponential model for population growth. Recall the Eq. (2.2), the model is in the form of $P(t) = P_0 e^{kt}$. The modeling process can be implemented using the libraries Matplotlib, SciPy, and NumPy of the Python programming language.

It is worth noting that I have imported numpy as ‘np’ and matplotlib.pyplot as ‘plt’ at the beginning of the code. After importing all three libraries, we define the function of best-fit curve (noting the presence of an initial condition $P(0) = 66707000$):

```
def func(x, k):
    return 667070000 * np.exp(k * x)
```

For simplicity, the variable x is used in place of t. Next, we define the data points to be fit as two numpy arrays separated by x (time) and y (population). We can use the curve_fit function to fit any form function and estimate the parameters of it (Kong et al. 289):

```
k = optimize.curve_fit(func, xdata = x, ydata = y)[0]
print(f'k={k}')
```

Afterwards, we use numpy.linspace to generate a set of x values for the best-fit curve to create a sequence of evenly spaced values across the minimum and maximum x-values, and hence, compute the y values.

```
x_curve = np.linspace(min(x), max(x), 100)
y_curve = func(x_curve, k)
```

Finally, we are able to plot the population data and to see what the best-fit curve looks like using the Matplotlib library. The proper title and axis labels are given and the colors of the data and the fitted curve are distinguished shown in Figure 4.6.

```
plt.plot(x, y, 'bo', label='Data')
plt.plot(x_curve, y_curve, 'r', label='Best-fit curve')
plt.legend()
plt.xlabel('Time / yrs')
plt.ylabel('Population in hundreds of millions')
plt.title('Malthus Model')
plt.show()
```

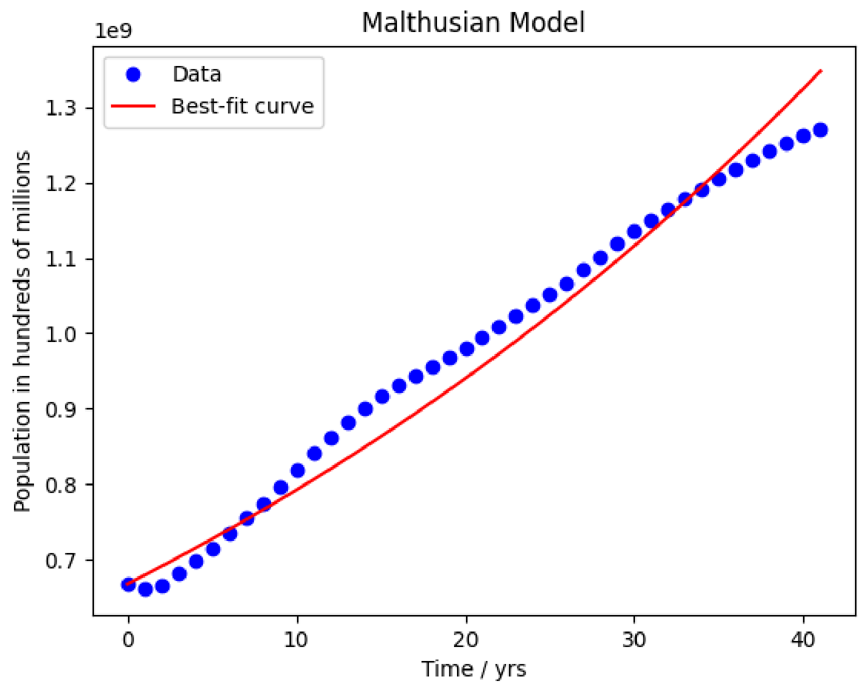

Figure 4.6 Malthusian growth curve fitted to the Chinese population data points from 1960 to 2001 with the rate of population growth $k = 0.01715696$.

The parameters computed by Python lead us to the equation of this 'Best-fit curve':

$$P(t) = 667070000e^{0.01715696t}. \tag{4.4}$$

The estimated population for the year 2002 to 2021 are calculated using the formulas and functions in Google Sheets by substituting the years into Eq. (4.4). Figure 4.7 below shows a comparison of theoretical and actual population values. Noting that the population data used to create Figure 4.7, Figure 4.9 and Figure 4.11 to compare actual population sizes with the different models are in the Appendix.

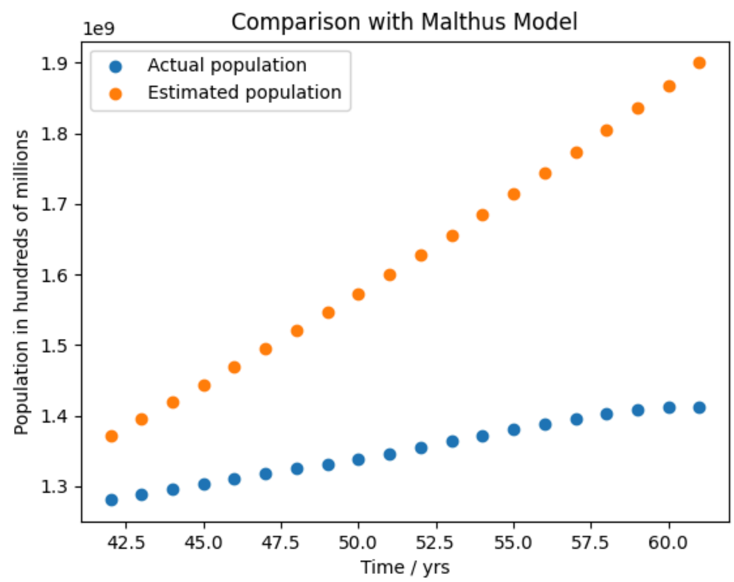

Figure 4.7 Scatter plot of the actual population and Malthusian model.

It can be seen that there is a relatively large error between the theoretical population and the actual population, which becomes larger and larger with the passage of time. The major limitations of the Malthusian model will be discussed in section 4.7.

4.4 Estimation of Population by Logistic Model

Recall the Eq. (3.3), the curve fitted with logistic model will be in the form of $P(t) = \frac{M}{1 + Ae^{-kt}}$. The modeling process is implemented using the same method as the Malthusian model described in the previous section. The best-fit curve is defined as follows:

```
def func(x, M, A, k):
    return M / (1 + A * np.exp(-k * x))
```

where the variable x is been used in place of t. Other parameters are explained in sections 3.1 and 3.2.

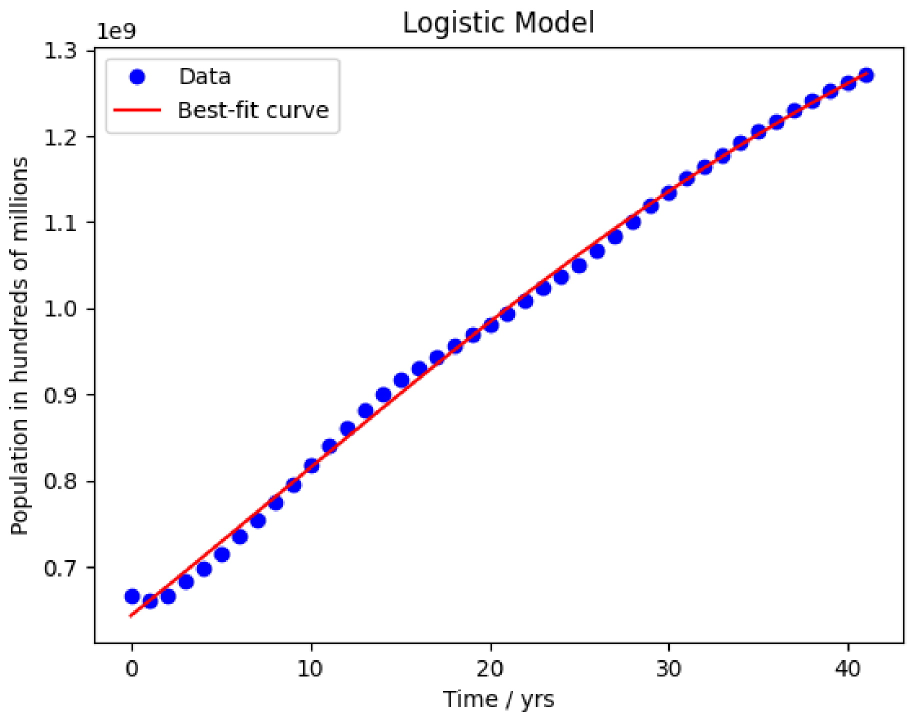

Figure 4.8 Logistic growth curve fitted to the Chinese population data points from 1960 to 2001 with parameters $M = 1577600000$, $A = 1.45287$, and $k = 0.0439275$.

Therefore, the equation of the logistic growth curve can be expressed as:

$$P(t) = \frac{1577600000}{1 + 1.45287e^{-0.0439275t}} . \tag{4.5}$$

A comparison of the estimated population with the actual population is shown in Figure 4.9 below.

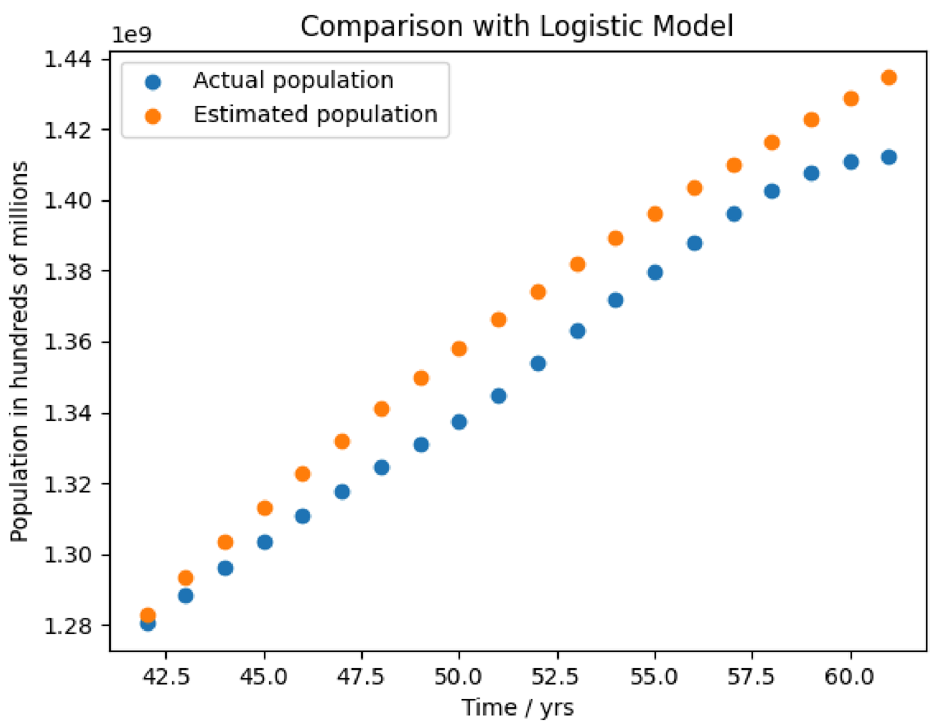

Figure 4.9 Scatter plot of the actual population and logistic model.

In comparison to the data generated by the Malthusian model in Figure 4.8, the logistic model gives much more precise data. Some limitations and effectiveness of the model are discussed in section 4.8.

4.5 Polynomial Least Squares

By looking at the scatter plot in Figure 4.5, the data points seem to follow a quadratic trend. With reference to Eq. (4.2), we can use polynomial and least squares to model the quadratic best-fit curve in the following form:

$$P(t) = at^2 + bt + c\,, \tag{4.6}$$

where a, b, and c are the parameters of this model, which will be estimated using another programming method. Recalling Eq. (4.3), we are minimizing the error between actual data points and estimated fitted values.

After importing the numpy and matplotlib libraries, and defining the data points as two numpy arrays separated by x (time) and y (population). We can use numpy.polyfit and matplotlib to make a scatter plot, then visualize the quadratic best-fit curve (Kong et al. 288).

```
y_est = np.polyfit(x, y, 2)
plt.plot(x, y, 'o', label='Data')
plt.plot(x, np.polyval(y_est, x), label='Least-squares fit')
plt.legend()
plt.xlabel('Time / yrs')
plt.ylabel('Population in hundreds of millions')
plt.title('Quadratic Model')
```

The estimated parameters can be obtained by simply printing the output of '`y_est`'.

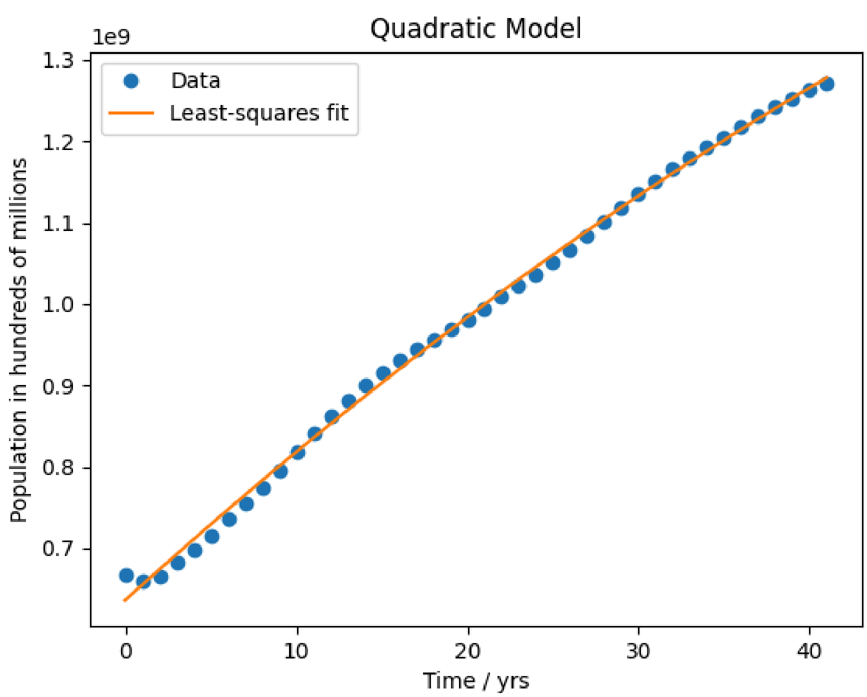

Figure 4.10 Quadratic model fitted to the Chinese population data points from 1960 to 2001 with parameters $a = -82456.4644$, $b = 19034906.8$ and $c = 636023827$.

The parameters give us the equation of the quadratic best-fit curve shown in Figure 4.10:

$$P(t) = -82456.4644t^2 + 19034906.8t + 636023827\,, \tag{4.7}$$

Figure 4.11 below demonstrates a comparison of the population estimated by the quadratic model and the actual population from 1960 to 2001.

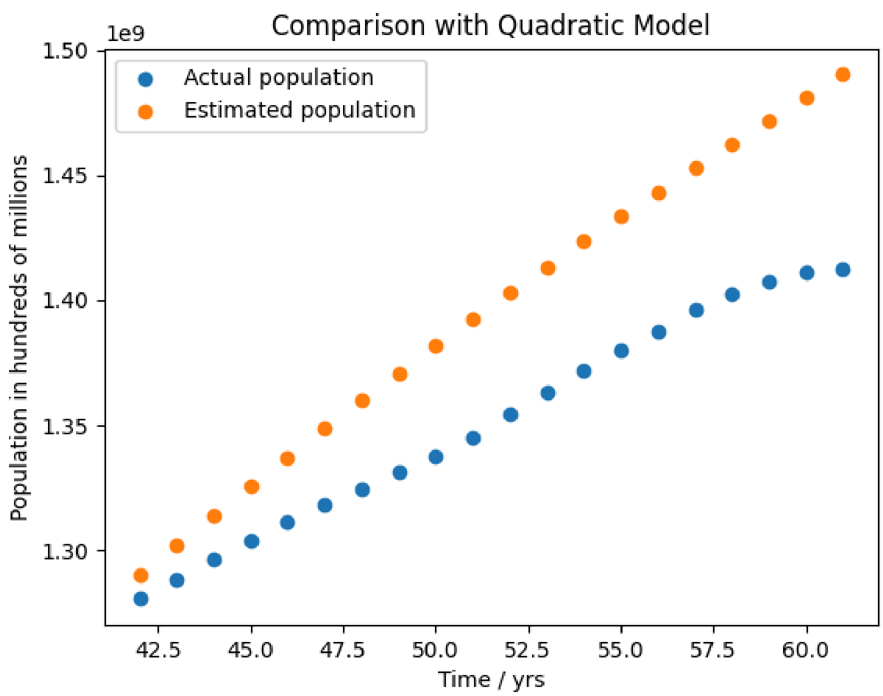

Figure 4.11 Scatter plot of the actual population and quadratic model.

As shown, a larger error than the logistic model is seen. Taken all together, the logistic model most accurately simulates the population from 2002 to 2021. Consequently, the logistic model will be used to project the future population of China.

4.6 Predicting the Chinese Population Size

For the main purpose of predicting the population of China from 2022 to 2032, the population data from 1960 to 2021 in Figure 4.3 and Figure 4.4 will be used.

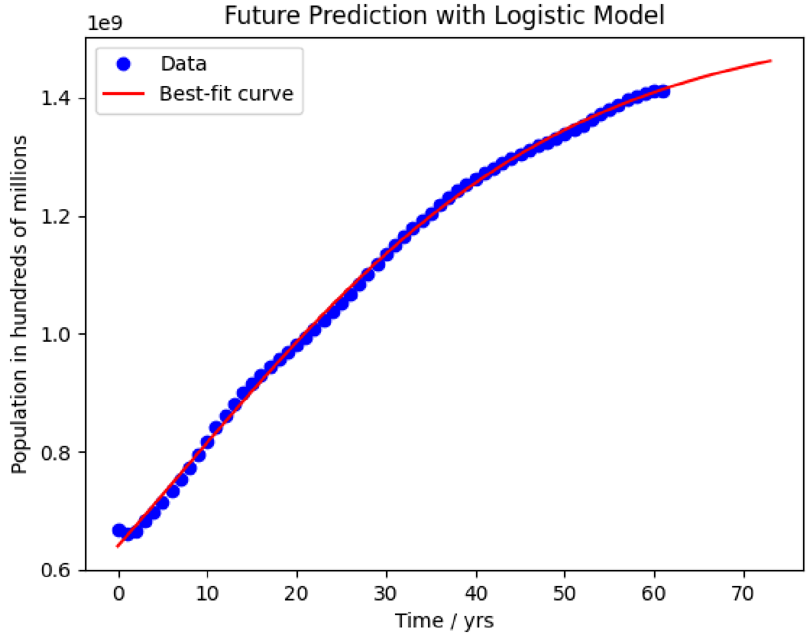

Figure 4.12 The projected Chinese population from 2022 to 2032 is fitted by a logistic growth curve with parameters $M = 1532903924.2236981$, $A = 1.394056690803845$, and $k = 0.046055636541949455$.

The projected population are calculated by substituting the years into the equation below:

$$P(t) = \frac{1532903924.2236981}{1 + 1.3940566908038452e^{-0.046055636541949455t}}, \quad (4.8)$$ [1]

Year	t	$P(t)$
2022	62	1419092661
2023	63	1423850998
2024	64	1428425040

[1] **Figure 4.13** Continued.

2025	65	1432820717
2026	66	1437043868
2027	67	1441100235
2028	68	1444995455
2029	69	1448735054
2030	70	1452324443
2031	71	1455768912
2032	72	1459073628

Figure 4.13 Projected population from 2022 to 2032

4.7 Discussion of Malthusian Model

The Malthusian model works best for relatively small populations and short periods of time. It is definitely not the best choice for such a huge population base in China. By just looking at the differential equation, we can tell that for such a population in China, the time interval between the changes in the population may be fairly short. Referring to Figure 4.6, the model gives relatively accurate results for the first 35.5 years while after 35.5 years, the errors become disproportionately large.

In addition, the model also assumes some ideal conditions: all members of the population are identical, in this case humans. There is an unlimited environment, adequate food and immunity from disease (Stewart et al. 606).

In conclusion, the Malthusian model may be able to adequately capture the expansion of a small population over a finite time. It might be necessary to add extra components to the model to make it more accurate and realistic.

4.8 Discussion of Logistic Model

From Figure 4.12, it can be seen that the population of China has been showing an increasing trend since 1960. After the founding of the People's Republic of China in 1949, the Chinese government's policies encouraged families to have more children. However, due to the one-child policy implemented in 1979, the rate of population growth has slowed down, which can also be seen from the flatter slope after $t = 19$ in the same figure.

Overall, the logistic growth provides a much more accurate model. The population data fit the logistic growth curve very well as shown in Figure 4.12. At the beginning, the population seems to show an exponential growth trend, as time goes by, it converges to its carrying capacity 1532903924 at a progressively slower rate.

Both the Malthusian model and the logistic model have similar limitations, except that the logistic model takes into account the carrying capacity, which is affected by resources vital to human survival such as water, food and shelters.

5 Conclusion

Referring to Eq. (4.8), a logistic model that provided the most accurate approximation for the population of China from 2022 to 2032 was constructed using the population data from 1960 to 2021. Mathematical population growth models Models can simulate the population growth of a population to a very large extent and account for fluctuations in growth. This is well demonstrated by Figure 4.12. The limitations of the model were discussed in section 4.7 and 4.8.

Moreover, there are more factors that the logistic model does not take into account. For example, changes in the environment, emigration of people, natural disasters, epidemics and other random events that may occur. Each of these events has potentially large effects on population growth.

A population as large as China's and approaching the maximum capacity it can carry implies that the Chinese government needs to use effective policies to control the rate of population growth in order to feed the people and allow them to have a higher standard of living. Well-controlled population growth and effective policies will improve the well-being of the nation as a whole.

Appendix

Comparison of the actual population sizes with different models

Year	t	$P(t)$	Population
2002	42	1371262748	1280400000
2003	43	1394992430	1288400000
2004	44	1419132755	1296075000
2005	45	1443690827	1303720000
2006	46	1468673877	1311020000
2007	47	1494089258	1317885000
2008	48	1519944451	1324655000
2009	49	1546247069	1331260000
2010	50	1573004853	1337705000
2011	51	1600225679	1345035000
2012	52	1627917562	1354190000
2013	53	1656088653	1363240000
2014	54	1684747244	1371860000
2015	55	1713901771	1379860000
2016	56	1743560818	1387790000
2017	57	1773733113	1396215000
2018	58	1804427540	1402760000
2019	59	1835653133	1407745000
2020	60	1867419085	1411100000
2021	61	1899734746	1412360000

Figure 1 Comparison of the actual population and Malthusian model.

Year	t	$P(t)$	Population
2002	42	1283018263	1280400000
2003	43	1293397697	1288400000
2004	44	1303489550	1296075000
2005	45	1313296277	1303720000
2006	46	1322820730	1311020000
2007	47	1332066129	1317885000
2008	48	1341036029	1324655000
2009	49	1349734295	1331260000
2010	50	1358165066	1337705000
2011	51	1366332732	1345035000
2012	52	1374241907	1354190000
2013	53	1381897398	1363240000
2014	54	1389304183	1371860000
2015	55	1396467385	1379860000
2016	56	1403392253	1387790000
2017	57	1410084135	1396215000
2018	58	1416548460	1402760000
2019	59	1422790717	1407745000
2020	60	1428816441	1411100000
2021	61	1434631190	1412360000

Figure 2 Comparison of the actual population and logistic model.

Year	t	$P(t)$	Population
2002	42	1290036709	1280400000
2003	43	1302062817	1288400000
2004	44	1313924011	1296075000
2005	45	1325620293	1303720000
2006	46	1337151661	1311020000
2007	47	1348518117	1317885000
2008	48	1359719659	1324655000
2009	49	1370756289	1331260000
2010	50	1381628006	1337705000
2011	51	1392334810	1345035000
2012	52	1402876701	1354190000
2013	53	1413253679	1363240000
2014	54	1423465744	1371860000
2015	55	1433512896	1379860000
2016	56	1443395135	1387790000
2017	57	1453112462	1396215000
2018	58	1462664875	1402760000
2019	59	1472052376	1407745000
2020	60	1481274963	1411100000
2021	61	1490332638	1412360000

Figure 3 Comparison of the actual population and quadratic model.

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