Invertibility Criteria

Let $A \in M_{n\times n}(\mathbb{F})$, and let $T_A : \mathbb{F}^n \to \mathbb{F}^m$ be the linear transformation determined by the matrix A. The following conditions are equivalent.

- A is invertible
- $\operatorname{rank}(A) = n$
- $\operatorname{RREF}(A) = I_n$
- T_A is one-to-one
- T_A is onto
- $\operatorname{Null}(A) = \{\vec{0}\}$
- $\operatorname{Col}(A) = \mathbb{F}^n$. That is, $\forall \vec{b} \in \mathbb{F}^n$, the system $A\vec{x} = \vec{b}$ is consistent.
- $\operatorname{nullity}(A) = 0$

More...

- A is invertible $\iff \det(A) \neq 0$
- $A^{-1} = \dfrac{1}{\det(A)} \operatorname{adj}(A)$
- $\lambda = 0$ is not an eigenvalue of A