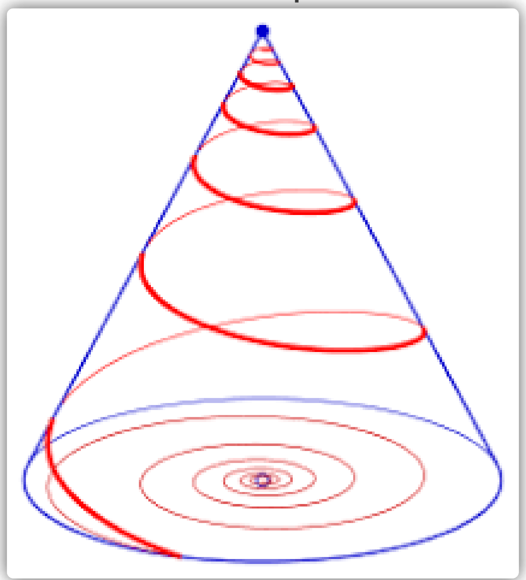
Motion Planning with Model Predictive Control

Introduction

In this project, a motion predictive controller is implemented in python and Matlab, to track conical spiral for a 3-axis triple integrator.

Concial Spiral



Algorithm Overview

MPC uses a system model to predict future states. Based on the prediction, MPC solves an online optimization algorithm to find the optimal control action that drives the predicted output to the reference.

A model predictive controller needs:

- a dynamic model of the process to predict states
- a cost function J over a planning horizon to incorporate future reference information
- an optimization algorithm to compute the control input u by minimizing the cost function J

Implementation Details (Python)

The source-code files are located in the directory /python/, they are:

- **get_PredictionMatrix.py**: Computes matrixes of the prediction model

```
def getPredictionMatrix(K, dt, p_0, v_0, a_0):

  Tp = np.zeros([K, K])
  Tv = np.zeros([K, K])
  Ta = np.zeros([K, K])

  for i in range(K):
      Ta[i, :i] = np.ones([1, i]) * dt
      for j in range(i):
          Tv[i, j] = (i - j + 0.5) * dt ** 2
          Tp[i, j] = ((i - j + 1) * (i - j) / 2 + 1 / 6) * dt ** 3
```

```python
    Ba = np.ones([K, 1]) * a_0
    Bv = np.ones([K, 1]) * v_0
    Bp = np.ones([K, 1]) * p_0
    for i in range(K):
        Bv[i] = Bv[i] + i * dt * a_0
        Bp[i] = Bp[i] + i * dt * v_0 + i ** 2 / 2 * a_0 * dt ** 2

    return Tp, Tv, Ta, Bp, Bv, Ba
```

- **get_ReferenceTrajectory.py**: Calculates a conical spiral as the tracking target. I duplicated the last reference points K times for the final MPC planning cycle.

```python
def get_ReferenceTrajectory(K,a,h,r,step_n):

    x = []
    y = []
    z = []
    c = 0

    verical = np.linspace(20, 0 , step_n);
    for t in verical:
        z.append(t)
        x.append((h - t)/h * r * cos(a * c * 0.2))
        y.append((h - t)/h * r * sin(a * c * 0.2))
        c = c+1
    ## duplicate the last reference point
    for i in range(K):
        z.append(z[-1])
        x.append(x[-1])
        y.append(y[-1])

    return x, y, z
```

- **mpc_solver.py**: solves the constructed optimization problem by using the OSQP library. To achieve this I divided mpc_solver into two parts.
  - First, I implemented the discrete-time model, setup constraints and defined objective function. So that the MPC problem is casted to a QP.

```python
for i in range(step_n-1):
    Tp, Tv, Ta, Bp, Bv, Ba = getPredictionMatrix(K, dt, p_0, v_0, a_0)

    Target_p_K = np.asarray(Target_p[count:count + K]).reshape(1, -1)

    H = scipy.linalg.block_diag(w4 * np.ones([K, K]) + w1 * (np.matmul(Tp.transpose(), Tp)),
                                w5 * np.ones([K, K]))

    F = np.concatenate([2 * w1 * (np.matmul(Bp.transpose(), Tp) - np.matmul(Target_p_K, Tp))
                        np.zeros([1, K])], axis=1)

    A = np.concatenate([conc_with_identity(Tv, 0),
                        conc_with_identity(-Tv, -1),
                        conc_with_identity(Ta, 0),
                        conc_with_identity(-Ta, -1),
                        conc_with_identity(np.ones([K, K]), 0),
                        conc_with_identity(-np  .ones([K, K]), -1),
                        conc_with_identity(np.zeros_like(Ta), -1)], axis=0)

    b = np.concatenate([np.ones([K, 1]) * max_v - Bv,
                        -np.ones([K, 1]) * min_v + Bv,
                        np.ones([K, 1]) * max_a - Ba,
                        -np.ones([K, 1]) * min_a + Ba,
                        np.ones([K, 1]) * max_j,
```

```
                        -np.ones([K, 1]) * min_j,
                        np.zeros([K, 1])], axis=0)
```

- Second, compressed sparse column matrix and solved the QP problem via OSQP

```
P = sparse.csc_matrix(H)
q = np.array(F.T)
A_ = sparse.csc_matrix(A)
l = np.array(-np.inf * np.ones_like(b))
# l = None
u = np.array(b)

# Create an OSQP object
prob = osqp.OSQP()

# Setup workspace and change alpha parameter
prob.setup(P, q, A_, l, u)

# Solve problem
res = prob.solve()

j = res.x[0]
```

- **main.py**: runs this project and visualizes the result

```
cd python
python3 main.py
```

```
-----------------------------------------------------------------
           OSQP v0.6.0  -  Operator Splitting QP Solver
              (c) Bartolomeo Stellato,  Goran Banjac
        University of Oxford  -  Stanford University 2019
-----------------------------------------------------------------
problem:  variables n = 40, constraints m = 140
          nnz(P) + nnz(A) = 3580
settings: linear system solver = qdldl,
          eps_abs = 1.0e-03, eps_rel = 1.0e-03,
          eps_prim_inf = 1.0e-04, eps_dual_inf = 1.0e-04,
          rho = 1.00e-01 (adaptive),
          sigma = 1.00e-06, alpha = 1.60, max_iter = 4000
          check_termination: on (interval 25),
          scaling: on, scaled_termination: off
          warm start: on, polish: off, time_limit: off

iter   objective    pri res    dua res    rho        time
   1  -1.1174e-02   5.13e-08   6.91e+00   1.00e-01   4.58e-04s
  50  -1.3757e-01   6.38e-09   1.08e-03   2.48e-04   9.94e-04s

status:               solved
number of iterations: 50
optimal objective:    -0.1376
run time:             1.00e-03s
optimal rho estimate: 4.66e-05
```

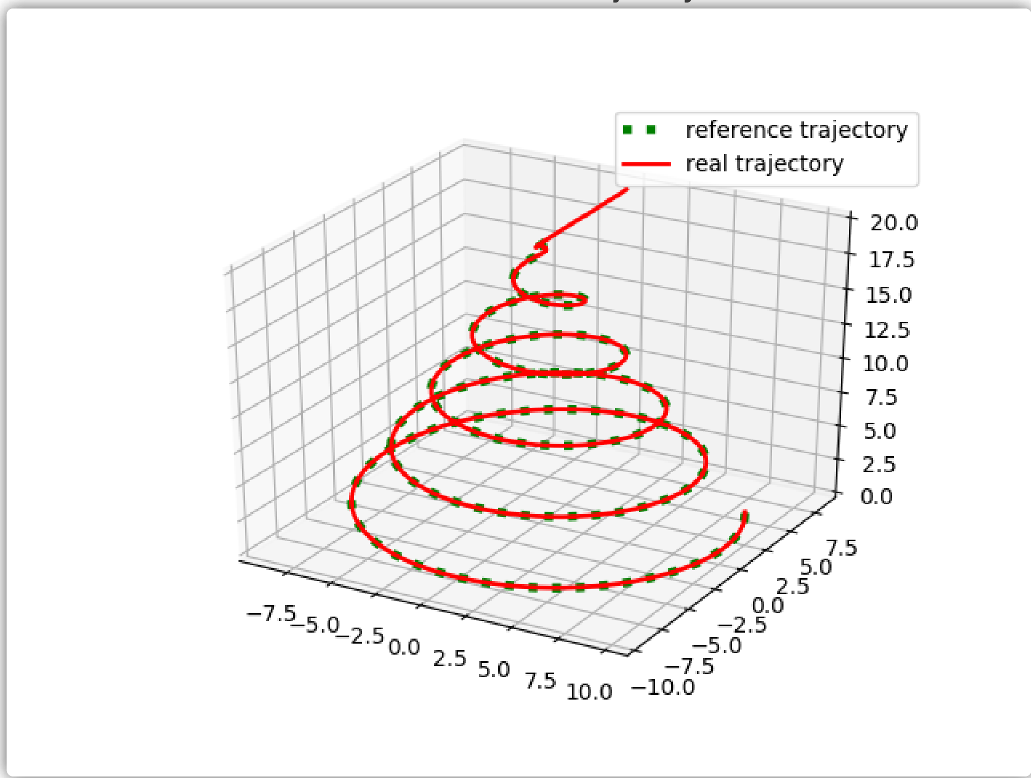
Dependenies

- scipy
- numpy
- matplotlib
- osqp

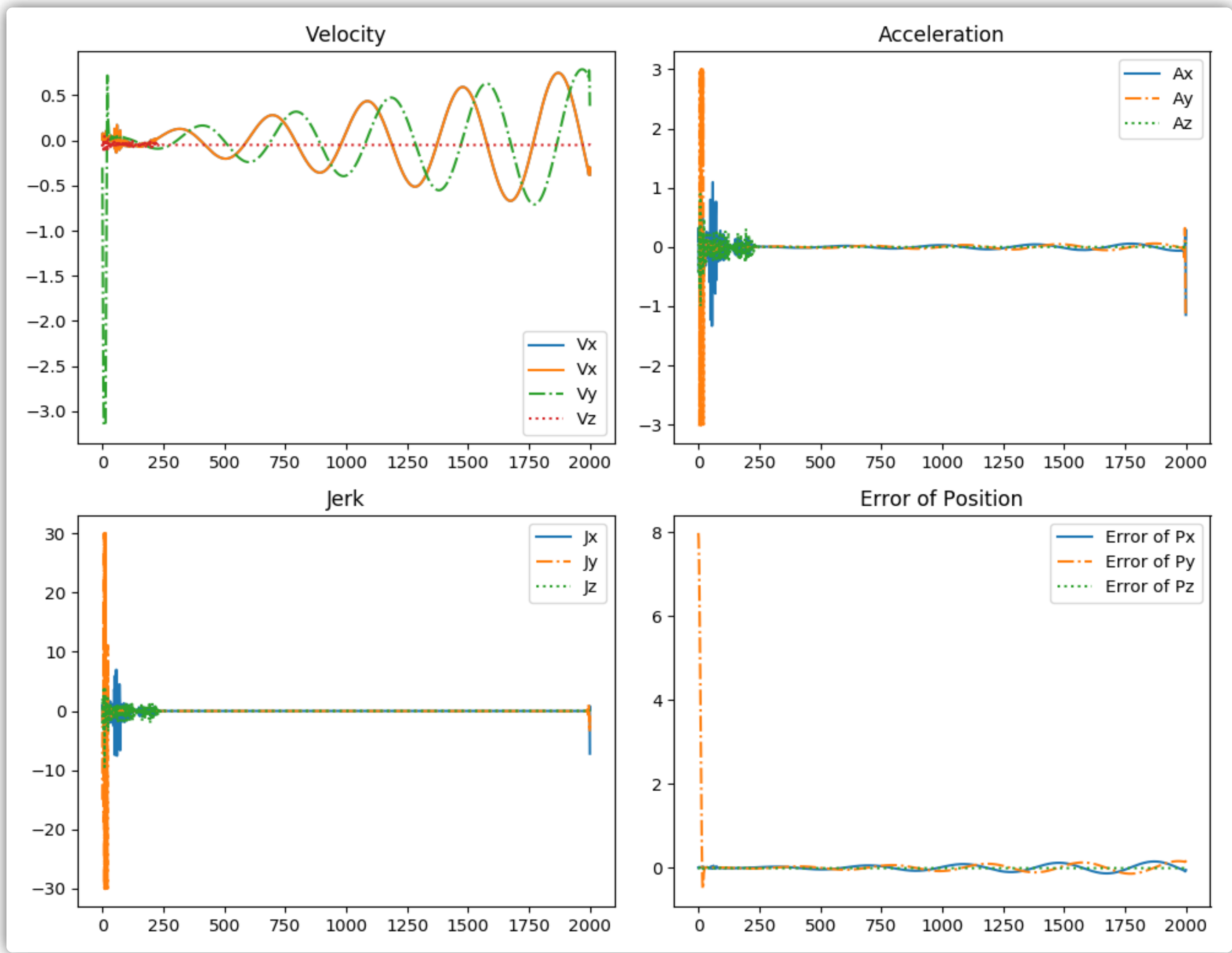
Experiment result

Python Part

Reference and Real Trajectory:

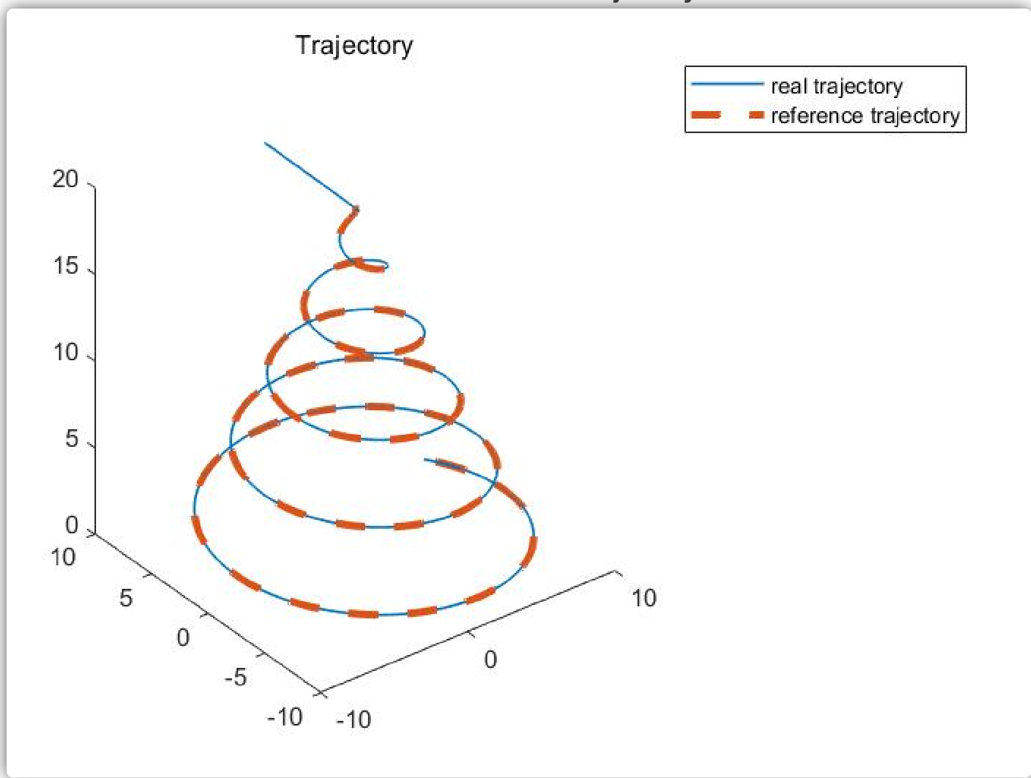


Visualization of States:

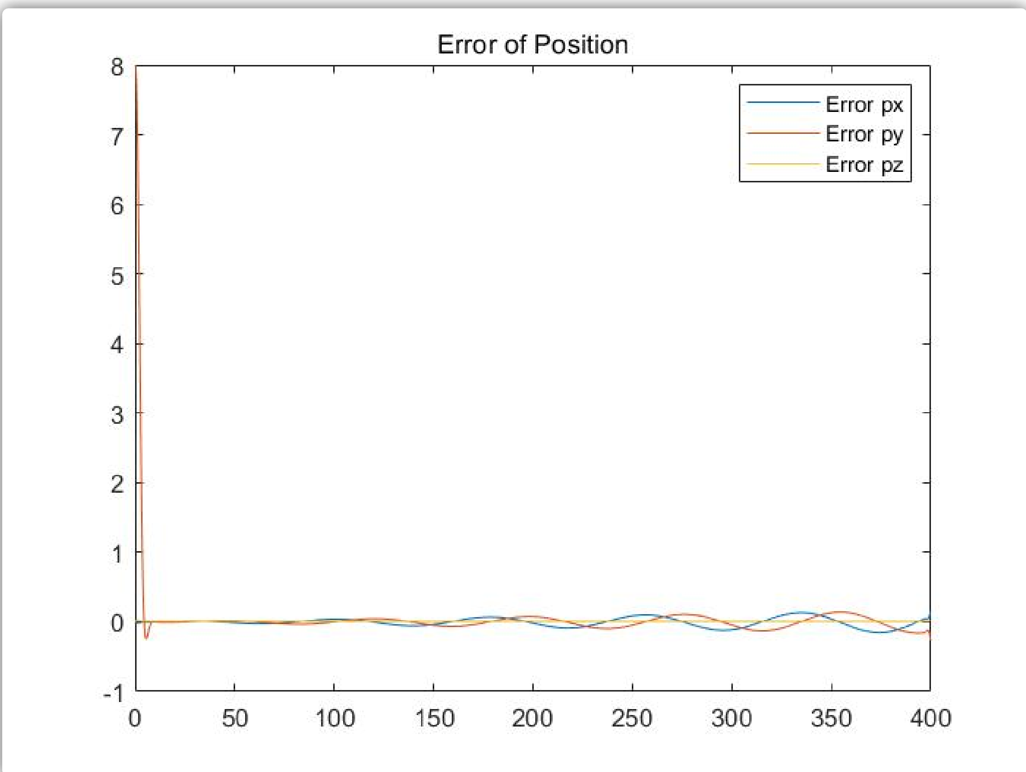


Matlab Part

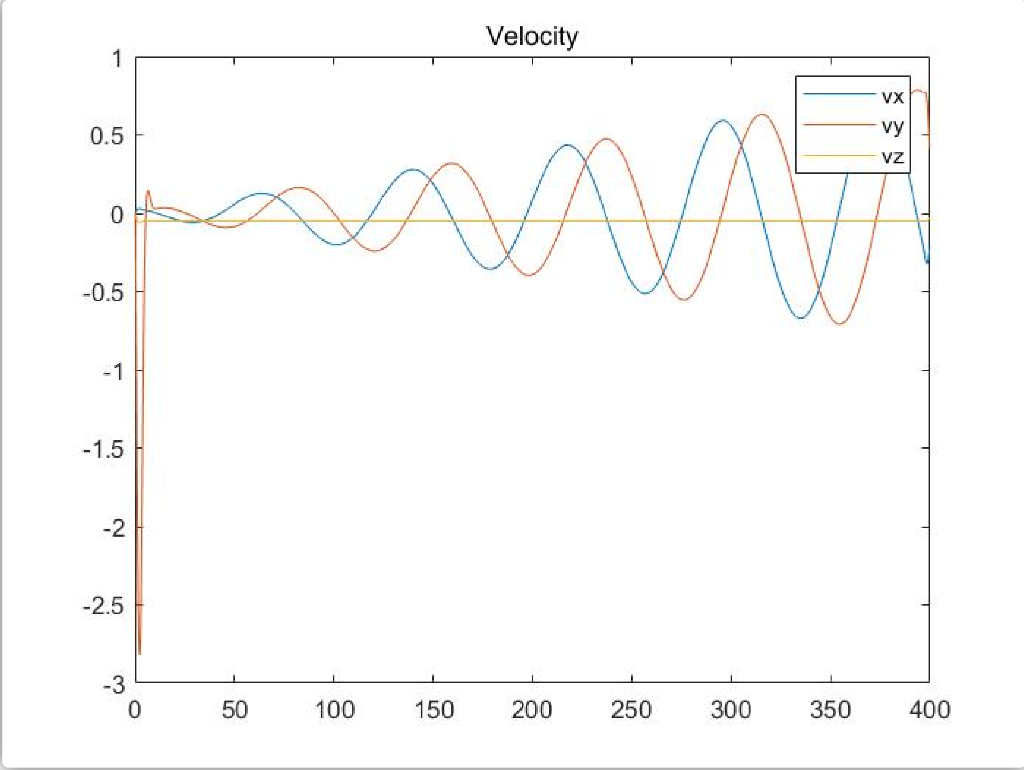
Reference and Real Trajectory:



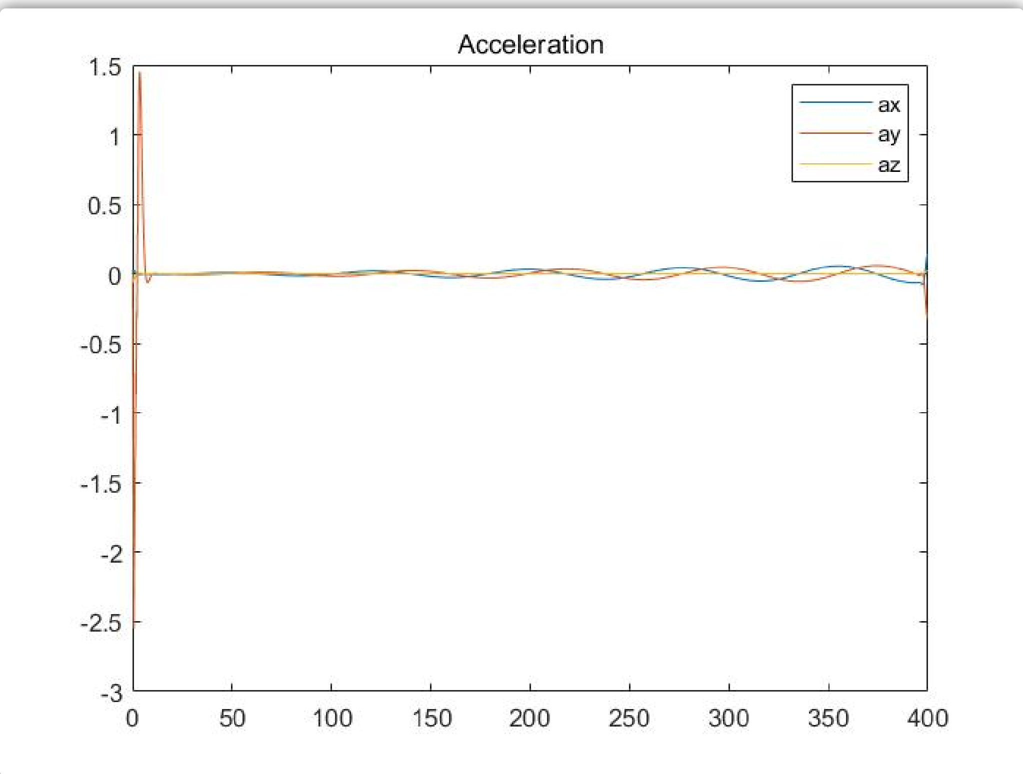
Visualization of States - Error of Position:



Visualization of States - Velocity:



Visualization of States - Acceleration:



Visualization of States - Jerk: