Accurate and efficient solution of electromagnetic scattering from randomly rough surface using MoM-SMCG with adaptive quadrature

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An adaptive quadrature method is proposed and implemented with the full-wave MoM-SMCG to accurately and efficiently solve the electromagnetic scattering from randomly rough surfaces. Without using high-order basis functions and dense surface discretisation, high numerical accuracy of computing the inner integrals for near-field impedance matrix elements is ensured by exploiting the orthogonal Legendre polynomials. Far-field interactions are calculated using the acceleration algorithms and the iterative solver to archive reasonable computational complexity and memory consumption. The proposed method is validated on solving the electromagnetic scattering and emission from rough ocean surfaces with fine-scale roughness and large dielectric constants. By comparing to the existing methods, simulation results indicate the proposed method has high computational efficiency and memory saving with also maintaining good accuracy.

Introduction: Numerical simulations of the electromagnetic (EM) scattering from 1D randomly rough surfaces have been extensively used in the microwave remote sensing of soil, ocean, ice and snow etc. [1]. By solving the dual surface integral equations (SIEs), the method of moments (MoM) using the pulse basis function with point matching is a typical method in these applications. However, due to the quadrature errors induced by inner integrals and singularities, the MoM with pulse/point matching suffers from low numerical accuracy in rough surface problems [2, 3]. Two commonly-used ways to address this accuracy issue are: (1) increasing the discretising density of surface profile [3, 4]; (2) applying high-order basis functions, e.g. the Nystrom method [5–8]. Both ways aim to improve the description accuracy of surface fields, but they would also significantly increase the number of unknowns to be solved, which further imposes extra burdens to the already large consumptions of memory and CPU power. Over the past decades, although various acceleration methods have been developed to reduce the computational costs in solving the dual SIEs with MoM, e.g. the sparse matrix canonical grid (SMCG) method [1] and the fast multipole method (FMM) [9], which can reduce the complexity from $O(N^2)$ to $O(N\log N)$ for 2D problems, the challenge still preserves for large-scale rough surfaces with also large permittivities [8]. Therefore, improving the quadrature accuracy in computing the impedance matrices to reduce the number of unknowns is of particular significance and interest for rough surface scattering problems.

In this letter, we propose an adaptive quadrature method to accurately compute the impedance matrices for solving 2D rough surface problems using MoM-SMCG (abbreviation of standard SMCG incorporated with MoM) with pulse/point matching. Inspired by the Nystrom method, the proposed method computes the inner integrals for near-field impedance matrix elements by exploiting the orthogonal Legendre polynomials. For the far-field interactions in SMCG expansions, the Gauss–Legendre (G–L) quadrature is implemented to compute the integration Kernels. Also, the neighbourhood impedance boundary condition (NIBC) technique is incorporated to accelerate the iterative solver by reducing the conditional number of matrix equation [3, 8]. Numerical results illustrate the proposed method has a significant accuracy improvement comparing to the original MoM or the standard MoM–SMCG. Without increasing the number of unknowns, high computational efficiency and memory saving are achieved.

Formulation: We start with the formulation based on the dual SIEs derived from the extinction theorem for 2D rough surface problems, which could be expressed in summation form after implementing MoM with pulse/point matching [1]

$$\begin{aligned}\sum_{p=1}^{N_p} Z_D u_p + \sum_{p=1}^{N_p} Z_N \psi_p &= \psi_{inc}(\mathbf{t}_m)\\ \frac{1}{\rho}\cdot\sum_{p=1}^{N_p} Z_{1D} u_p + \sum_{p=1}^{N_p} Z_{1N}\psi_p &= 0\end{aligned} \tag{1}$$

with the impedance matrix elements defined as

$$\begin{aligned}Z_D &= \int_{\sigma_p} \mathrm{d}t' K_D(\mathbf{t}_m;\mathbf{t}') = \int_{\sigma_p} \mathrm{d}t' g(x(t_m), z(t_m); x(t'), z(t'))\\ Z_{1D} &= \int_{\sigma_p} \mathrm{d}t' K_{1D}(\mathbf{t}_m;\mathbf{t}') = \int_{\sigma_p} \mathrm{d}t' g_1(x(t_m), z(t_m); x(t'), z(t'))\\ Z_N &= \int_{\sigma_p} \mathrm{d}t' K_N(\mathbf{t}_m;\mathbf{t}') = -\int_{\sigma_p} \mathrm{d}t' \sqrt{1+[z'(t')]^2}\,\widehat{n}'\cdot\nabla' g(\mathbf{t}_m;\mathbf{t}')\\ Z_{1N} &= \int_{\sigma_p} \mathrm{d}t' K_{1N}(\mathbf{t}_m;\mathbf{t}') = -\int_{\sigma_p} \mathrm{d}t' \sqrt{1+[z'(t')]^2}\,\widehat{n}'\cdot\nabla' g_1(\mathbf{t}_m;\mathbf{t}')\end{aligned} \tag{2}$$

where $\mathbf{t}_m$ and $\mathbf{t}'$ denote the position of testing field point $(x(t_m),z(t_m))$ and source point $(x(t'), z(t'))$, respectively. ψ and u are the unknown surface fields to be solved. σ_p represents the integral path of pth discretising interval. $g(\mathbf{t}_m;\mathbf{t}')$ and $g_1(\mathbf{t}_m;\mathbf{t}')$ represent the 2D free space Green's functions in upper and lower mediums, respectively. Other notations, e.g. ψ_{inc}, ρ, $\widehat{n}$, are defined in [8].

In the standard MoM using pulse basis function, the Newton–Cotes (N–C) or G–L quadrature rules are used to compute the impedance matrices, which induce errors in solving the dual SIEs [1]. Physically, for numerical solutions of surface scattering, the near-field interactions play a dominant role in computational accuracy. In this work, based on the MoM–SMCG, the impedance matrices are decoupled into near-fields and far-fields. Thus, to accurately compute the impedance matrix elements in near-fields, we propose an adaptive quadrature method using high-order Legendre polynomials. For the intuitive illustration, we consider the computation of Z_D which corresponds to the Dirichlet boundary condition, and transform the Kernel integral into local coordinate, which writes as

$$Z_D = \int_{\sigma_p} \mathrm{d}t' K_D(\mathbf{t}_m;\mathbf{t}') = \int_{-1}^{1} \mathrm{d}t'_{\mathrm{local}} \frac{\Delta t}{2} K_D(\mathbf{t}_m;\mathbf{t}') \tag{3}$$

with the coordinate transformation defined as

$$t' = t'_p + \frac{\Delta t}{2} t'_{\mathrm{local}} t'_{\mathrm{local}} \in (-1, 1) \tag{4}$$

where t'_p and Δt represent the centre and length of pth interval σ_p, respectively.

For the non-self-patch interactions, i.e. $m \neq p$, define

$$\kappa_{mk} = \int_{-1}^{1} \mathrm{d}t'_{\mathrm{local}} \frac{\Delta t}{2} K_D(\mathbf{t}_m;\mathbf{t}')\cdot F_k(t'_{\mathrm{local}}) = \sum_{j=1}^{N_j} \widetilde{\omega}_j^{m,p} F_k(t_{j,\mathrm{local}}) \tag{5}$$

where F_k represents the kth-order Legendre polynomial. $t_{j,\mathrm{local}}$ is the jth sampling points in σ_p in local coordinate. In this work, to be in accordance with the far-field calculations which use G–L quadrature for the inner integrations, the sampling points are selected as the zeros of N_jth Legendre polynomial. All terms in the integral in (5) are known so that κ_{mk} can be calculated using normal quadrature. Note that the summation $\sum_{j=1}^{N_j} \widetilde{\omega}_j^{m,p}$ is the numerical result of Z_D in (3). To solve the N_j numbers of $\widetilde{\omega}^{m,p}$, we construct N_j equations like (5) using 0th to $(N_j - 1)$th

Legendre polynomials. Then, the set of equations can be expressed in matrix equation form as

$$\overline{\kappa}_m = \overline{\overline{L}} \cdot \overline{\overline{\omega}}^{m,p} \tag{6}$$

According to the property of orthogonal polynomials, $\overline{\overline{L}}$ is a non-singular matrix with $N_j \times N_j$ size in which the elements are $L_{kj} = F_k(t_{j,\text{local}})$. Thus the unknown vector $\overline{\overline{\omega}}^{m,p}$ can be solved directly as

$$\overline{\overline{\omega}}^{m,p} = \overline{\overline{L}}^{-1} \cdot \overline{\kappa}_m \tag{7}$$

For the self-patch interactions, i.e. $m = p$, the integral in (5) has a logarithmic singularity at $t'_{\text{local}} = 0$ and is to be calculated accurately. Here, we employ the generalised Gaussian quadrature (GGQ) which does not need to do singularity extraction [10]. Considering that the GGQ typically performs in the region of (0, 1) and t_m equals to t'_p in this case, (5) could be split into two integrals with coordinate transformation, which writes as

$$\begin{aligned}\kappa_{mk} = &\int_0^1 \mathrm{d}t''_{\text{local}} \frac{\Delta t}{2} K_D\left(\mathbf{t}_m; \mathbf{t}_m - \frac{\Delta t}{2}\mathbf{t}''_{\text{local}}\right) \cdot F_k\left(-t''_{\text{local}}\right) \\ &+ \int_0^1 \mathrm{d}t''_{\text{local}} \frac{\Delta t}{2} K_D\left(\mathbf{t}_m; \mathbf{t}_m + \frac{\Delta t}{2}\mathbf{t}''_{\text{local}}\right) \cdot F_k\left(t''_{\text{local}}\right) \quad t''_{\text{local}} \in (0, 1)\end{aligned} \tag{8}$$

Both integrals in (8) can be calculated by the GGQ rules. Then the impedance matrix elements for self-patches are computed following (6) and (7). Similarly, integrations of other impedance matrix elements in (2) can also be calculated using the adaptive quadrature.

In the MoM–SMCG for rough surface problems, the far-field interactions of source and field points in integration kernels are decoupled into several SMCG orders based on Taylor series expansions of Green's functions [1]. The fast Fourier transform (FFT) and iterative solver are utilised to solve the dual SIEs efficiently by exploiting the translational invariance of impedance matrices. As stated above, the inner integrals in each SMCG order are computed by the G–L quadrature instead of the simpler N–C quadrature [3, 8] (see (31) in [8]). In addition, the recently developed NIBC technique is also incorporated to accelerate the iterative solver by reducing the conditional number of matrix equation [3, 8].

Numerical Results and Discussions: To validate the performance of computational accuracy and efficiency of the proposed method, we consider the EM scattering and emission from rough ocean surfaces which have multi-scale roughness and large permittivity (~74+67*i* in this work). Generated by Elfouhaily spectrum at the wind speed of 5 m/s [11], the ocean surface profiles have same surface length of 100 λ (incidence wavelength) and various discretising densities of 10, 15, 20, 25 and 30 points/λ. The root-mean-square heights are about 0.63 λ (~0.135 m). Several existing MoM-based numerical methods are also involved for comparison including the original MoM [1], the standard MoM–SMCG [1], the Nystrom [5] and our recently proposed NIBC/Nystrom/SMCG [8] and MLSD–SMCG [7]. Notably, the later three methods incorporate high-order Nystrom method and the orders of their basis functions (selected as Legendre polynomials following [3, 8]) are selected as five in this work. Thus, the corresponding numbers of unknowns to be solved are five times those of the left methods and the proposed method using pulse basis (see Figure 3). For a fair comparison, we also use 5th-order adaptive quadrature ($N_j = 5$ in (5)) in the proposed method. The bandwidths of relevant methods are chosen as 4 λ. The 4th order SMCG is used as in [7, 8]. The iterative solver used here is the generalised minimal residual method (GMRES). Numerical results are illustrated at L-band (1.4 GHz) and the incidence angle is 40°. Energy conservations of all simulations are obeyed to within 0.5%.

Figure 1 compares the bistatic scattering coefficients (BSCs, in linear scale) calculated by the original MoM and the proposed method from the ocean surface profile with discretising density of 10 points/λ. Considering the high accuracy of the Nystrom, we take it as a benchmark. It is seen that the BSCs given by the proposed method in this paper agree well with those of Nystrom. Comparing to the original MoM, the proposed method presents a significant improve of computation

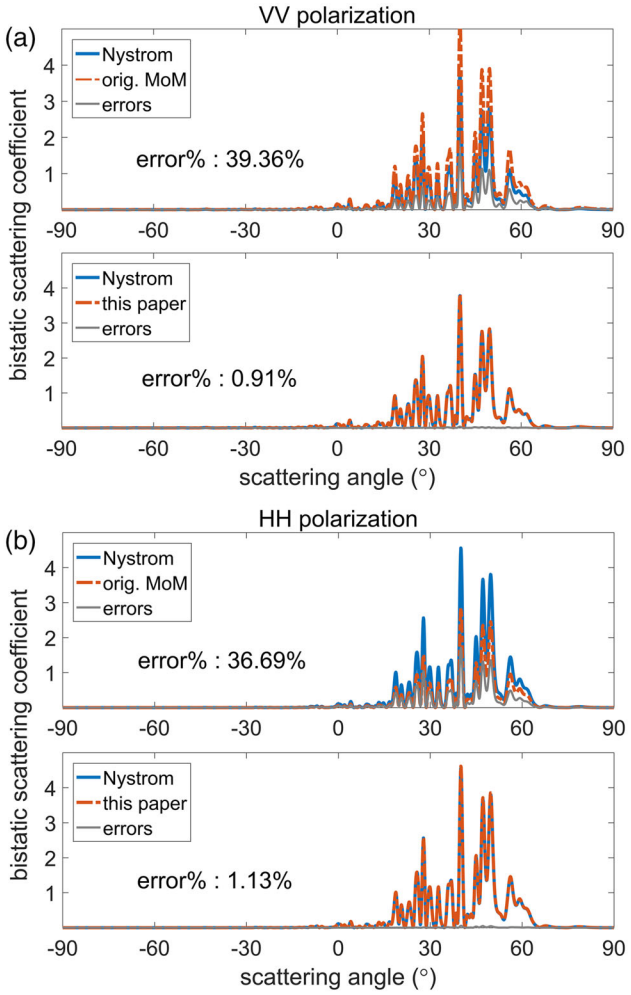

Fig. 1 *Comparison of BSCs calculated by the original MoM and the proposed method from the ocean surface profile with discretising density of 10 points/λ. Nystrom is taken as the benchmark. (a) VV polarisation, (b) HH polarisation*

accuracy with the percent errors decreasing to about 1% from close to 40%. The error percentage is defined as [7]

$$\text{error\%} = \frac{\sqrt{\sum_{i=1}^{N}\left(\sigma^i_{\text{cal.}} - \sigma^i_{b.m.}\right)^2}}{\sqrt{\sum_{i=1}^{N}\left(\sigma^i_{b.m.}\right)^2}} \times 100 \tag{9}$$

where $\sigma^i_{\text{cal.}}$ and $\sigma^i_{b.m.}$ represent the model results and the benchmark of the BSCs, respectively. Figure 2 plots the ocean surface emissivities calculated by various methods versus discretising density for vertical and horizontal polarisations. The proposed method gives close results to the high-order methods, particularly for surface profile with sparse discretisation. These indicate the high accuracy of the proposed method. In addition, the accuracy performance of proposed method has also been examined with the (Gaussian and exponential) rough soil surfaces and shows robustness on solving the rough surface problems.

Figures 3 and 4 present the consumptions of CPU time and memory of various methods in terms of discretising density. In this work, computations are carried out on the device equipped with the Intel Core i7-8700 CPU at 3.2 GHz × 12 and 32 GB of memory capacity. One could see that the benchmark method manifests the largest costs of both CPU power and memory. Although the our previous NIBC/Nystrom/SMCG and MLSD-SMCG methods significantly reduce numerical consumptions, large number of unknowns induced by high-order basis functions still yields larger computational complexity compared with those using pulse basis. As for the proposed method, by implementing the pulse basis function as well as the acceleration algorithms, it basically has best computational efficiency and memory saving among the comparing methods.

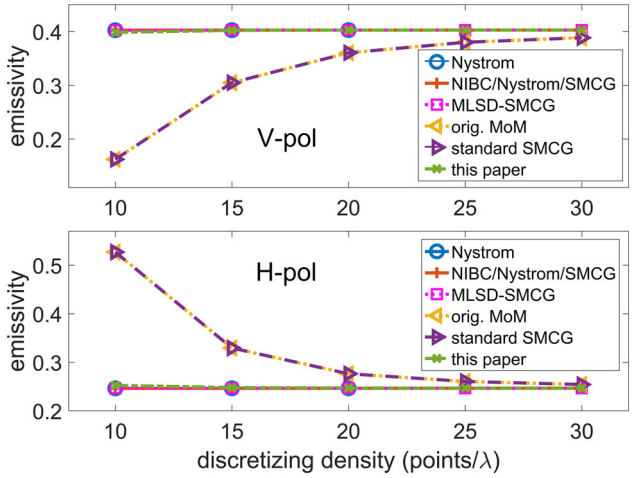

Fig. 2 *Ocean surface emissivities calculated by various methods versus discretising density for vertical and horizontal polarisations*

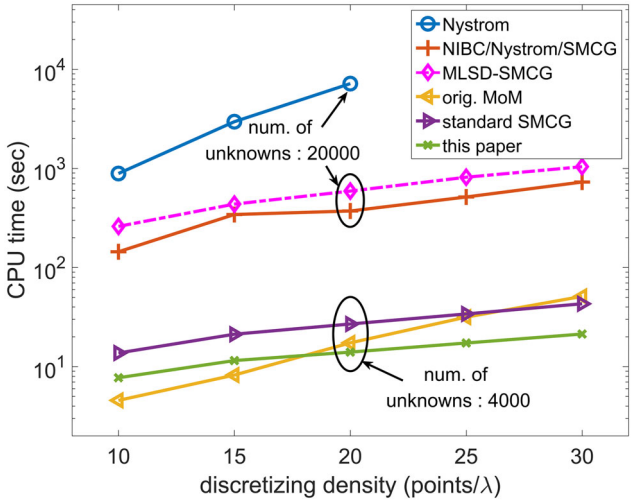

Fig. 3 *CPU time consumptions of various methods versus discretising density*

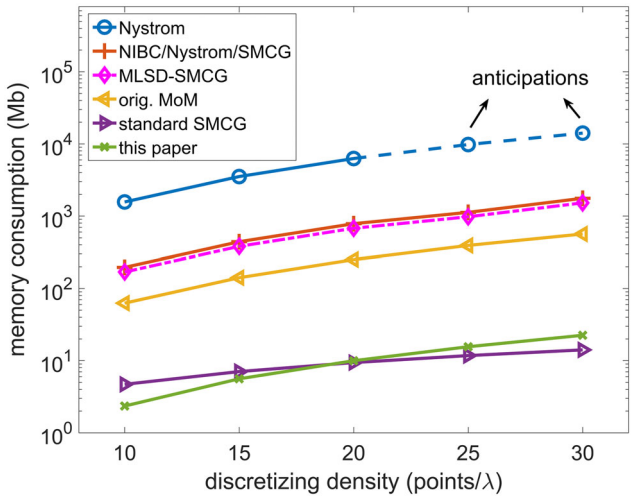

Fig. 4 *Memory consumptions of various methods versus discretising density*

For a brief summary, we qualitatively and fairly compare the MoM-based mentioned above in terms of computational accuracy and consumptions in Table 1. Here, "≈" denotes that the compared methods are basically identical. ">" denotes that the method on the left side is better or slightly better than that on the right side. " >> " denotes that the specific performance of the method on the left side far outweighs that on the right side. One could note that the proposed method basically achieves a good balance among the computational accuracy and consumptions.

Table 1. Qualitative comparison of computational accuracy and consumptions of various MoM-based methods

Accuracy	Nystrom ≈ NIBC/Nystrom/SMCG ≈ MLSD-SMCG > this paper > > standard MoM ≈ standard SMCG
Efficiency	This paper > standard SMCG > standard MoM > > NIBC/Nystrom/SMCG > MLSD-SMCG > > Nystrom
Memory consumption	This paper ≈ standard SMCG > standard MoM > > NIBC/Nystrom/SMCG ≈ MLSD-SMCG > > Nystrom

Conclusion: This Letter presents an adaptive quadrature method implemented with the MoM–SMCG to accurately and efficiently solve the rough surface scattering problems. By exploiting the orthogonal Legendre polynomials, high accuracy of computing the near-field impedance matrices is archived. Also, the usages of pulse basis function and the fast algorithms, e.g. the FFT, the NIBC and the iterative solver, lead to low computational complexity and memory requirement in solving the dual SIEs. The proposed method has been assessed by comparing to existing methods for 2D ocean scattering problems. Numerical simulations demonstrate its reasonable computational accuracy and efficiency. This method will also be promising to 3D problems.

Acknowledgments: This work was supported by the China Postdoctoral Science Foundation (grant 2020M680554) and the National Natural Science Foundation of China (grant 61771043).

Received: *17 April 2021* Accepted: *7 May 2021*
doi: 10.1049/ell2.12229

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