Foreline Heat Conduction

Problem
Need to figure out the length of insulation we need on a foreline to make it touch safe.

Drawing

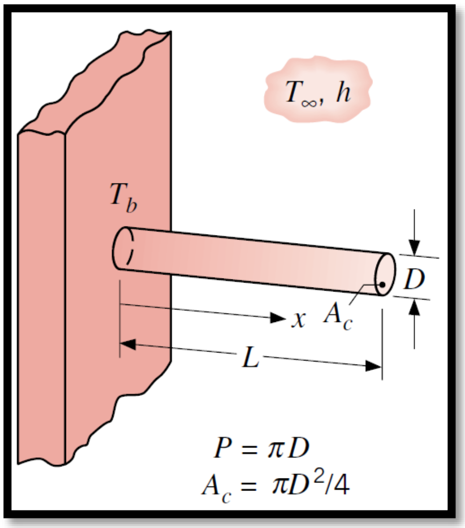

Figure 1: idealized cylindrical fin

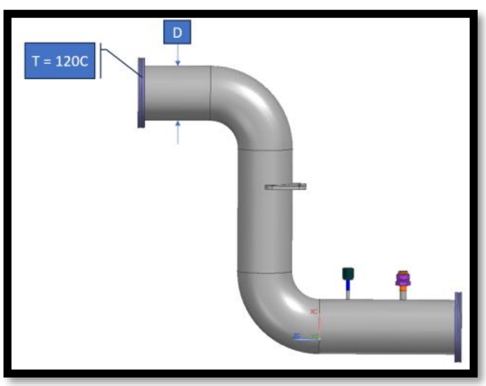

Figure 2: Foreline with base held at 120C

Given
- $k_{SST} = 14\ W/mK$
- $h_{Air} = 100\ W/m^2K$
- $T_b = 120C$
- $T_{Amb} = 30C$
- $D = 4\ in = 0.1016\ m$

Find

Calculate the length at which the fore line becomes touch safe (<60C). That is, when the temperature at a distance of x is equal to 60C.

Assumptions

- No heat transfer internal to pipe
- Free (natural) convection
- Thermal system is at steady state.
- Radiation is neglected.
- Room air is quiescent.
- Constant properties
- Infinite fin boundary condition. $\theta(\infty) = 0$

Relevant Equations

- $A = \frac{\pi D^2}{4}$ **(1.1)** Area of a circle
- $\theta = T(x) - T_{amb}$ **(1.2)** Excess temperature
- $m = \sqrt{\frac{hP}{kA_c}}$ **(1.3)** Fin parameter
    - $h = convection\ coefficient\ of\ air$
    - $k = conduction\ coefficient\ of\ base\ material$
- $\theta(x) = \theta_b e^{-mx}$ **(1.4)** Excess temperature of an infinite fin
- $\theta_b = T_b - T_{amb}$ **(1.5)** Excess temperature at base of fin
- $P = \pi D$ **(1.6)** Perimeter of a circle

Solution

The step-by-step solution.

Step 1: Calculate fin parameter.

Per equation **(1.3)**, the fin parameter is:

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{100\ W/m^2K * \pi * 0.1016\,m}{14\ W/mK * \pi * (0.1016\,m)^2/4}} = \sqrt{\frac{31.92\,W/mK}{0.1135\,Wm/K}} = \mathbf{16.77\ m^{-1}}$$

Step 2: Derive excess temperature equation.

Per equation(s) **(1.2)**,**(1.4)**, and **(1.5)**, the excess temperature is given by the equation:

$$\theta(x) = \theta_b e^{-mx} = (120C - 30C)e^{-16.77m^{-1}x} = \mathbf{90e^{-16.77x}}\ [\mathbf{C}]$$

Step 3: Calculate the x value at which excess temperature reaches 30 degrees C.

Keep in mind that excess temperature is the temperature ***above*** ambient.

$$30 = 90e^{-16.77x} \rightarrow x = 0.0655\,m = \mathbf{2.58\ \mathit{inches}}$$

See figure below for sample values with different materials, taken from (Moran, Munson, Shapiro, & DeWitt, 2003).

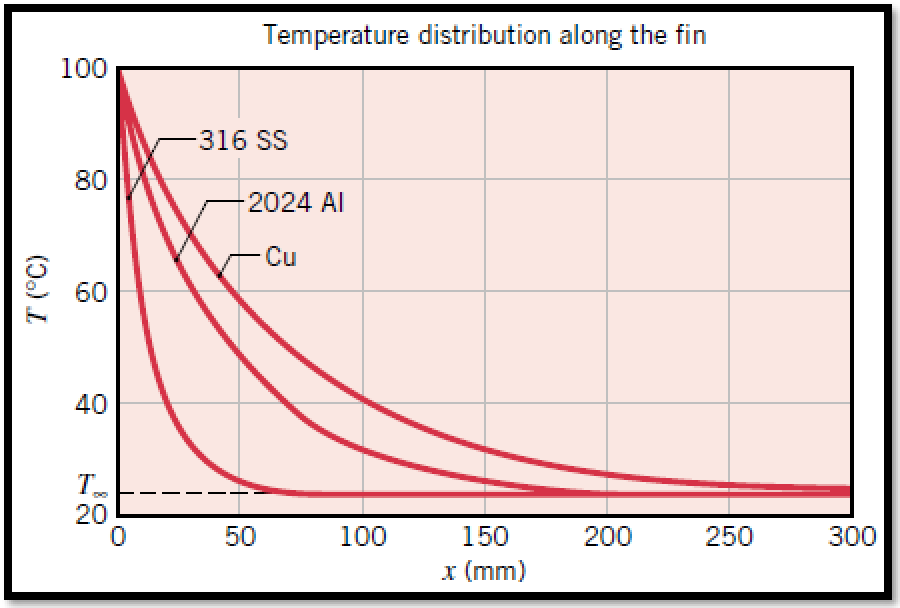

Figure 3: Temperature distribution along fin with base temperature of 100C

Works Cited

Moran, M. J., Munson, B. R., Shapiro, H. N., & DeWitt, D. P. (2003). *Introduction to Thermal Systems Engineering*. John Wiley & Sons, Inc.