Insulated window heat transfer.

Problem

Most modern windows use double pane glass to reduce the amount of heat transferred into or out of buildings. Consider a double pane window with 1 m^2 area and a 1 mm air gap between 2 window panes when the inner pane is held at 20 C, and the outer pane is held at 0 C. Please list your assumptions and ignore the effects of sunlight.

Find

Estimate how much heat is transferred from one pane to the other via conduction. Estimate how much heat is transferred from one pane to the other via convection. Estimate how much heat is transferred from one pane to the other via radiation.

Drawing

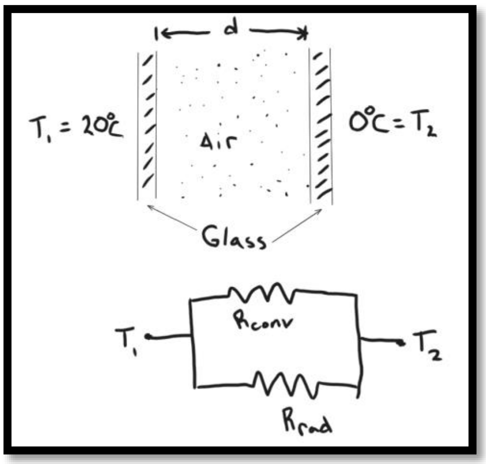


Figure 1: Drawing of thermal system

Assumptions

Natural (free) convection between glass planes

- No heat transfer through edges of window/air volume.
- Neglect effect of sunlight.
- Only radiation from one glass pane to the other is considered.
- Transmissivity of glass equal to that of low-iron glass at room temperature between the wavelengths of 0.3-2.5 micron. (Introduction to Thermal Systems Engineering, p. 511)
- Thermal properties (Introduction to Thermal Systems Engineering, pp. 516-517):
  - Glass
    - Conductivity - $k_{glass} = 1.4 \frac{W}{m*K}$
    - Emissivity - $\varepsilon_{glass} = 0.92$
    - Transmissivity - $\tau_{glass} = 0.9$

    - Reflectivity - $\rho_{glass} = 0.02$
  - Air @ 300K
    - Density - $\rho_{air} = 1.1614 \frac{kg}{m^3}$
    - Conductivity - $k_{air} = 0.0263 \frac{W}{m*K}$
    - Diffusivity - $\alpha_{air} = 22.5 * 10^{-6} \frac{m^2}{s}$
    - Dynamic viscosity - $\nu_{air} = 15.89 * 10^{-6} \frac{m^2}{s}$
- Air is quiescent.
- Properties are constant.
- System is in steady state.
- $g = 9.8 \frac{m}{s^2}$
- Stefan-Boltzmann constant - $\sigma = 5.670 * 10^{-8} \frac{W}{m^2 * K^4}$
- View factor is 1.

Relevant Equations

See (Introduction to Thermal Systems Engineering) for appropriate equations.

- Newton's law of cooling - $q = \bar{h} A_s (T_s - T_\infty)$
- Volumetric thermal expansion coefficient for ideal gases **(17.69)** - $\beta = \frac{1}{T}$
- Nusselt - Raleigh empirical correlation **(17.70)** - $\overline{Nu_L} = \frac{\bar{h}L}{k} = CRa_L^n$
- Raleigh number **(17.71)** - $Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$
- Churchill-Chu correlation **(17.74)** - $\overline{Nu_L} = \left\{0.825 + \frac{0.387 Ra_L^{\frac{1}{6}}}{\left[1+\left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right\}^2$
- Stefan-Boltzmann law **(18.9)** - $E_b = \sigma T^4$
- Gray body radiation - $q"_{rad} = \varepsilon \sigma T^4$
- Surface radiation balance of semi-transparent medium **(18.26)** - $\rho + a + \tau = 1$
- Prandtl number - $Pr = \frac{\nu}{\alpha}$
- Radiative heat transfer between diffuse gray infinite planes **(18.58)** - $q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$

Solution

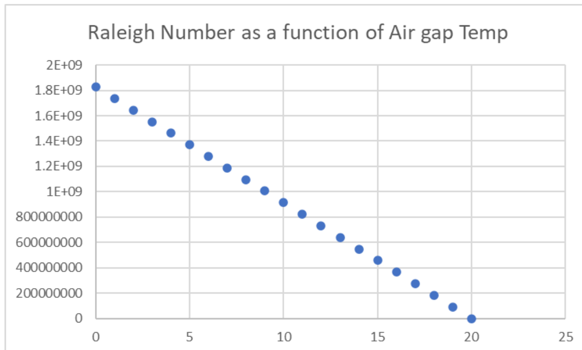
First, a comment on conduction since the prompt asks for the heat transfer due to conduction. It is 0 based on the assumptions, there is no solid with which to transfer heat through in this scenario.

Convection

From equation 17.71

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

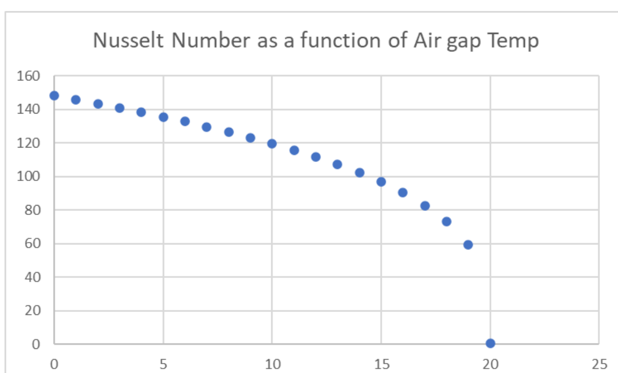
One of the challenges here is the bulk fluid temperature (Temperature between the plates) is not known, plotting Raleigh number as a function of air gap temperature yields the following plot:



Using the Churchill-Chu correlation (eq. 17.74)

$$\overline{Nu_L} = \left\{ 0.825 + \frac{0.387 Ra_L^{\frac{1}{6}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}} \right\}^2$$

We may generate a plot of Nusselt number as a function of air gap temperature.



Taking the function average, we find that:

$$\overline{Nu_L} = 110.61$$

Back solving (drawing a line on the plot) for air gap temperature, we find that:

$$T_\infty = 12\ C$$

The average convection coefficient is

$$\overline{Nu_L} = \frac{\bar{h}L}{k} \rightarrow \bar{h} = \frac{\overline{Nu_L} * k}{L}$$

$$\bar{h} = \frac{110.61 * 0.0263 \frac{W}{m * K}}{1\ m} = 2.91 \frac{w}{m^2 * K}$$

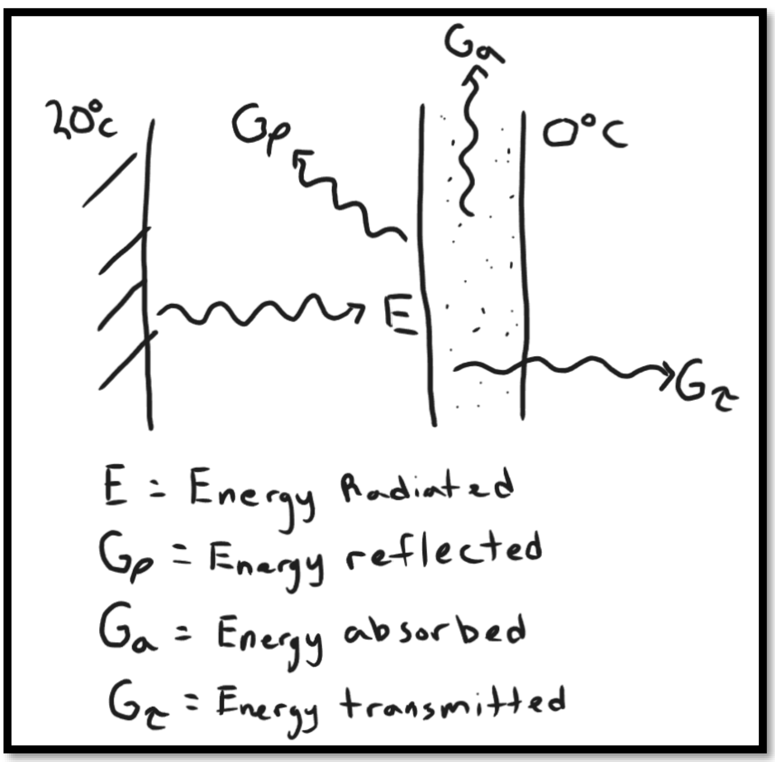
The heat transfer from the 20 C pane to the 12 C air gap via convection is

$$\boxed{q_{conv} = 2.91 \frac{w}{m^2 * K} * 1\ m^2 * (20 - 12)C = \mathbf{23.28\ W}}$$

Since our assumptions necessitates only heat transfer through convection and radiation with no heat loss through the sides, we may assume that the heat transferred from the hot (20C) pane to the air gap (12C) is then transferred to the cold pane (0C).

Radiation

A diagram below shows the thermal system for radiative heat transfer between these two panes of glass.



Our quantity of interest is G_a, or Energy absorbed into the 0C plate.

To find the portion of this energy absorbed by the 0C glass pane, we may use the surface radiation balance for a semi-transparent medium.

$$\rho + a + \tau = 1$$

Solving for a, we have:

$$a = 1 - 0.02 - 0.9 = 0.08$$

This suggests that only 8% of the incident energy is absorbed into the 0C pane. From here on out, we will treat both panes as large, infinite diffuse gray surfaces to simplify the calculations. The radiative heat transfer between two such planes is as follows.

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$q_{12} = \frac{1\ m^2 * 5.670 * 10^{-8} \frac{W}{m^2 * K^4} * ((293.15\ K)^4 - (273.15\ K)^4)}{\frac{1}{0.92} + \frac{1}{0.92} - 1} = 87.83\ W$$

Using our previously calculated absorption coefficient, we may calculate the heat absorbed by the 0C pane:

$$\boxed{q_{rad} = 87.83\ W * 0.08 = \mathbf{7.03\ W}}$$

Works Cited

Moran, M. J., Munson, B. R., Shapiro, H. N., & DeWitt, D. P. (2003). *Introduction to Thermal Systems Engineering*. John Wiley & Sons, Inc.