Statistical Inference Course Project - Part 1

Alex McBride

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Contents

Synopsis

Investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. Investigate the distribution of averages of 40 exponentials running a thousand simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should 1. Show the sample mean and compare it to the theoretical mean of the distribution. 2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution. 3. Show that the distribution is approximately normal.

Create the Distribution

```
# set seed for reproducability
set.seed(10)

# set lambda to 0.2
lambda <- 0.2

# 40 samples
n <- 40

# 1000 simulations
sim <- 1000

# simulate
sim_exp <- replicate(sim, rexp(n, lambda))

# calculate mean of exponentials
means_exp <- apply(sim_exp, 2, mean)
```

Show and compare the distribution mean and variance

Theoretical mean is `1/lambda` = 5

Our analytical mean is `mean(means_exp)` = 5.0450596

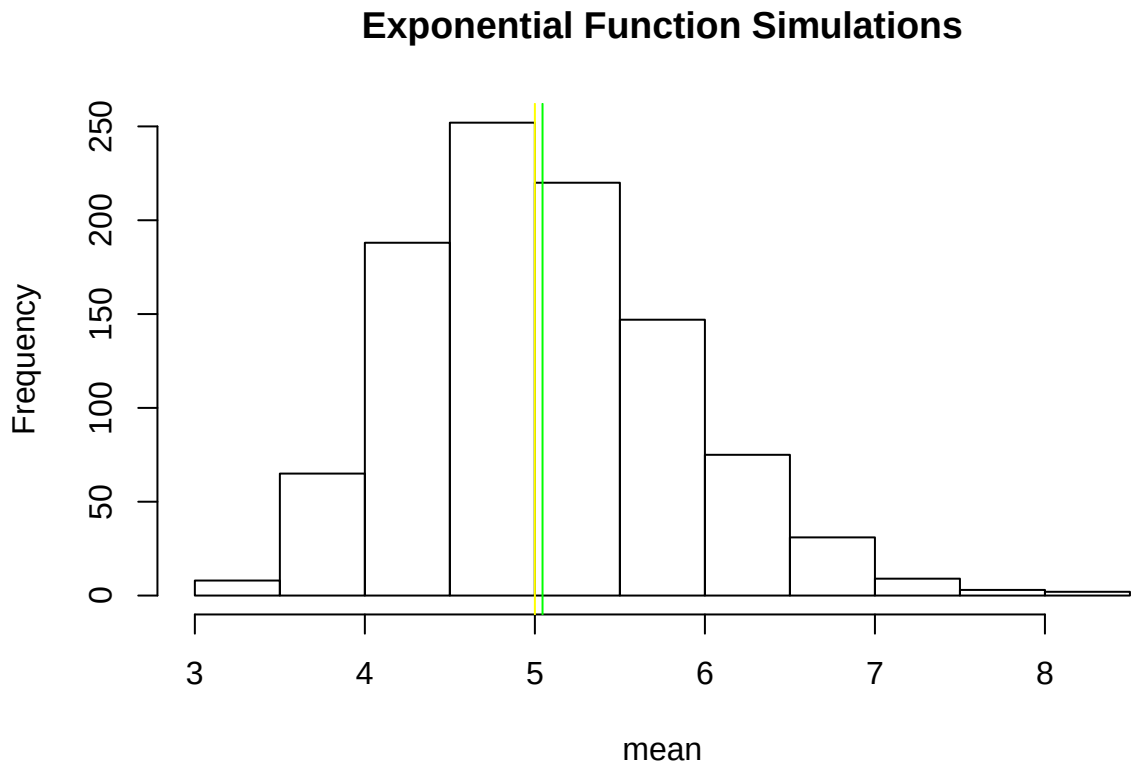
The theoretical standard deviation is `1/lambda/sqrt(n)` = 0.7905694

Our analytical standard deviation is `sd(means_exp)` = 0.7982821

The theoretical variance is `((1/lambda)*(1/sqrt(n)))^2` = 0.625

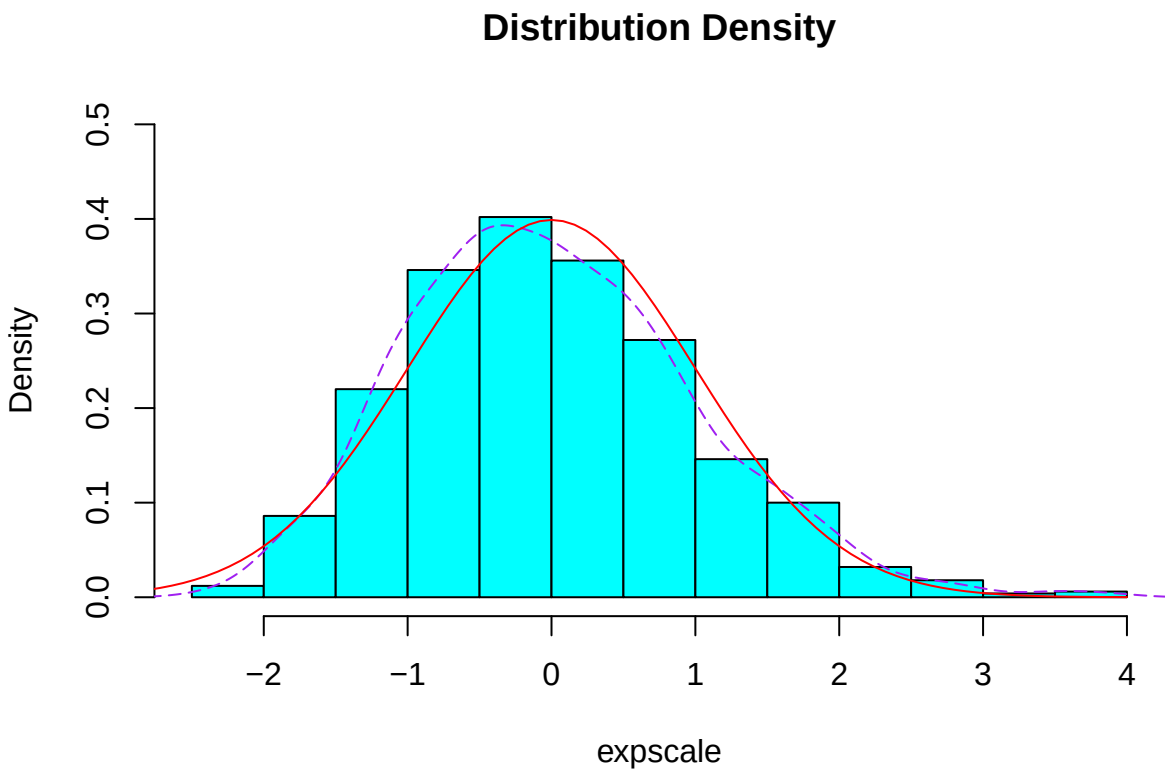
Our analytical variance is `var(means_exp)` = 0.6372544

```
hist(means_exp, xlab = "mean", main = "Exponential Function Simulations")
abline(v = mean(means_exp), col = "green")
abline(v = 1/lambda, col = "yellow")
```



Show that the distribution is approximately normal

```
expscale <- scale(means_exp)
hist(expscale,probability=T, main="Distribution Density", col = "cyan", ylim=c(0, 0.5))
lines(density(expscale), col = "purple", pch = 22, lty=5)
# Compare with the standard normal distribution
curve(dnorm(x,0,1), -4, 4, col="red", add=T)
```


Distribution Density
Density
0.0
0.1
0.2
0.3
0.4
0.5
−2
−1
0
1
2
3
4
expscale