Розрахунково-графічна робота «Елементи математичного аналізу».

Завдання 2.1 Знайти границі функцій.

1. $\lim\limits_{x\to\infty}\dfrac{3x^3+3x^2-4}{\sqrt{9x^6+4x}}$, $\lim\limits_{x\to2}\left(\dfrac{1}{x-2}-\dfrac{4}{4-x^2}\right)$, $\lim\limits_{x\to1}\dfrac{x^3-x^2-x}{x^3+x^2-x-1}$, $\lim\limits_{x\to0}\dfrac{\sqrt{x+4}-2}{x}$,

$\lim\limits_{x\to0}\dfrac{1-\cos6x}{5x^2}$, $\lim\limits_{x\to0}\dfrac{e^{3x}-1}{\operatorname{tg}5x}$, $\lim\limits_{x\to0}\dfrac{5^x-5^{2x}}{3x}$, $\lim\limits_{x\to+\infty}x\left[\ln(x+2)-\ln x\right]$, $\lim\limits_{x\to\infty}\left(\dfrac{x+8}{x-2}\right)^x$,

$\lim\limits_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+2x}\right)$.

2. $\lim\limits_{x\to\infty}\left(\dfrac{2x^2-3x}{x+1}-2x\right)$, $\lim\limits_{x\to-1}\dfrac{x^2+3x+2}{x^3+1}$, $\lim\limits_{x\to0}\dfrac{\operatorname{tg}x-\sin x}{x^3}$, $\lim\limits_{x\to2}\dfrac{\sqrt{x-1}-1}{x-2}$,

$\lim\limits_{x\to1}(2-x)^{\frac{x+1}{3x-3}}$, $\lim\limits_{x\to\infty}\left(\dfrac{x+1}{x-3}\right)^{2x}$, $\lim\limits_{x\to0}\dfrac{\sin2x}{\ln(1+3x)}$, $\lim\limits_{x\to0}(1+\sin x)^{\operatorname{cosec}x}$,

$\lim\limits_{x\to+\infty}x\left[\ln x-\ln(x+1)\right]$, $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2}-\sqrt{x^2-1}\right)$.

3. $\lim\limits_{x\to\infty}\left(\dfrac{\sqrt{x^8+3x+4}}{3x^4-2}\right)$, $\lim\limits_{x\to2}\dfrac{x^2-6x+8}{x^2-8x+12}$, $\lim\limits_{x\to1}\dfrac{\sqrt{5-x}-2}{x^2-1}$, $\lim\limits_{x\to0}\dfrac{\sin2x+\sin x}{3\sin2x}$,

$\lim\limits_{x\to0}\dfrac{\ln(1+2x)}{\sin3x}$, $\lim\limits_{x\to+\infty}\left(x\cdot\ln\dfrac{2x+3}{2x+1}\right)$, $\lim\limits_{x\to2}(x-1)^{\frac{3x}{x-2}}$, $\lim\limits_{x\to\infty}\left(\dfrac{x^2+2x}{x+3}-x\right)$,

$\lim\limits_{x\to\infty}\left(\dfrac{3x-2}{3x-4}\right)^{1-x}$, $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-\sqrt{x^2+x-2}\right)$.

4. $\lim\limits_{x\to0}\left(\dfrac{\sqrt{x^2+4}-2}{3x^2}\right)$, $\lim\limits_{x\to\infty}\dfrac{2x+1}{\sqrt{x^2-8}+x}$, $\lim\limits_{x\to0}\dfrac{e^{\sin5x}}{\ln(1+2x)}$, $\lim\limits_{x\to0}\dfrac{\sin4x-\sin2x}{3x}$,

$\lim\limits_{x\to2}\dfrac{x^2-5x+6}{x^3-2x^2}$, $\lim\limits_{x\to+\infty}(2x+1)\ln\dfrac{x}{x+1}$, $\lim\limits_{x\to\infty}\left(\sqrt{x^2+4x-2}-\sqrt{x^2-4}\right)$,

$\lim\limits_{x\to-1}(2+x)^{\frac{x}{3x+3}}$, $\lim\limits_{x\to0}\dfrac{\ln(1-x^2)}{\operatorname{arctg}^2 2x}$, $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x-2}\right)^x$.

5. $\lim\limits_{x\to\infty}\dfrac{2x+1}{\sqrt{x^2-8}+x}$, $\lim\limits_{x\to2}\dfrac{x^2-5x+6}{x^3-2x^2}$, $\lim\limits_{x\to+\infty}(2x+1)\left[\ln 3x-\ln(3x+1)\right]$, $\lim\limits_{x\to0}\dfrac{6^x-1}{4x}$,

$\lim\limits_{x\to0}\dfrac{\sqrt{x^2+4}-2}{3x^2}$, $\lim\limits_{x\to0}\dfrac{\sin 4x-\sin 2x}{3x}$, $\lim\limits_{x\to0}\dfrac{e^{\sin 5x}-1}{\ln(1+2x)}$, $\lim\limits_{x\to-1}(2+x)^{\frac{x}{3x+3}}$,

$\lim\limits_{x\to\infty}\left(\sqrt{x^2+4x}-\sqrt{x^2+3x+1}\right)$, $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x-2}\right)^x$.

6. $\lim\limits_{x\to\infty}\dfrac{3x^2-2x+1}{\sqrt{4x^4+5x^3}}$, $\lim\limits_{x\to2}\dfrac{x^2-x-2}{x^2+2x-8}$, $\lim\limits_{x\to2}(3-x)^{\frac{2x}{x-2}}$, $\lim\limits_{x\to2}\dfrac{\sqrt{2x}-x}{x-2}$, $\lim\limits_{x\to0}\dfrac{\operatorname{tg}^2(x/3)}{2x^2}$,

$\lim\limits_{x\to\infty}\left(\dfrac{2x}{2x+1}\right)^x$, $\lim\limits_{x\to3}\left(\dfrac{1}{x-3}-\dfrac{6}{x^2-9}\right)$, $\lim\limits_{x\to+\infty}x\left[\ln(x-2)-\ln x\right]$, $\lim\limits_{x\to0}(\cos x)^{\operatorname{ctg}^2x}$,

$\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$.

7. $\lim\limits_{x\to\infty}\dfrac{5x^4+x+3}{x^4-12x+1}$, $\lim\limits_{x\to1}\dfrac{x^3-6x^2+11x-6}{x^2-3x+2}$, $\lim\limits_{x\to\infty}\left(\sqrt{2x^2+3x}-\sqrt{2x^2+x+1}\right)$,

$\lim\limits_{x\to0}\dfrac{\sqrt{x^2+16}-4}{2x^2}$, $\lim\limits_{x\to0}\dfrac{1-\cos 4x}{3x^2}$, $\lim\limits_{x\to-2}(3+x)^{\frac{3x}{x+2}}$, $\lim\limits_{x\to0}\dfrac{\sqrt[5]{32x^5-x^8}}{e^{5x}-1}$,

$\lim\limits_{x\to+\infty}(x-1)\left[\ln(2x+1)-\ln x\right]$, $\lim\limits_{x\to0}\dfrac{e^{\sqrt{x}}-1}{3x}$, $\lim\limits_{x\to\infty}\left(\dfrac{3x+1}{3x}\right)^{2x}$.

8. $\lim\limits_{x\to\infty}\dfrac{2x+1}{\sqrt{9x^2-x+3}}$, $\lim\limits_{x\to0}\dfrac{4^x-1}{5x}$, $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+1}\right)^{x+1}$, $\lim\limits_{x\to0}\dfrac{\sqrt{x^2+4}-2}{6x^2}$, $\lim\limits_{x\to0}\dfrac{1-\cos 5x}{1-\cos 3x}$,

$\lim\limits_{x\to-1}(2+x)^{\frac{x-1}{x+1}}$, $\lim\limits_{x\to\infty}\left(\sqrt{3x^2+2x-1}-\sqrt{3x^2+x+2}\right)$, $\lim\limits_{x\to+\infty}x\left[\ln x-\ln(4x-3)\right]$,

$\lim\limits_{x\to0}\dfrac{2\operatorname{arctg}3x}{\ln(1-2x)}$, $\lim\limits_{x\to2}\dfrac{x^2+2x-8}{x^3-8}$.

9. $\lim\limits_{x\to\infty}\dfrac{2x^4-3x^2}{3x^4+1}$, $\lim\limits_{x\to0}\dfrac{e^x-1}{5x}$, $\lim\limits_{x\to\infty}\left(\sqrt{5x^2-x+2}-\sqrt{5x^2+2x-1}\right)$, $\lim\limits_{x\to0}\dfrac{\sqrt{x^2+4}-2}{5x^2}$,

$\lim\limits_{x\to\infty}\left(\dfrac{x-1}{x+1}\right)^{3x}$, $\lim\limits_{x\to1}(2-x)^{\frac{3x+2}{x-1}}$, $\lim\limits_{x\to0}\dfrac{e^{\sin 3x}-1}{2x}$, $\lim\limits_{x\to3}\dfrac{x^2-5x+6}{x^2-9}$,

$\lim\limits_{x\to+\infty}(x+2)\left[\ln(x+1)-\ln x\right]$, $\lim\limits_{x\to0}\dfrac{\sin 3x+\sin 5x}{x}$.

10. $\lim\limits_{x \to 0} \dfrac{7^x - 1}{4x}$, $\lim\limits_{x \to 1} \dfrac{x^2 + 5x - 6}{x^3 - x}$, $\lim\limits_{x \to \infty}\left(\sqrt{x^2 + 5x - 2} - \sqrt{x^2 + 3x}\right)$, $\lim\limits_{x \to 3} \dfrac{\sqrt{x-2} - 1}{x - 3}$,

$\lim\limits_{x \to 0} \dfrac{1 - \cos 3x}{\operatorname{tg}^2 2x}$, $\lim\limits_{x \to -3}(4 + x)^{\frac{x-2}{x+3}}$, $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^2 + 3x - 2}}{x + 5}$, $\lim\limits_{x \to \infty}\left(\dfrac{x-1}{x+3}\right)^{2x+1}$, $\lim\limits_{x \to 0} \dfrac{\arcsin 2x}{e^{3x} - 1}$,

$\lim\limits_{x \to +\infty}(3x + 1)\left[\ln(x-1) - \ln(x+2)\right]$.

11. $\lim\limits_{x \to \infty}\left(\dfrac{x^3 + 3x^2}{x^2 + 1} - x\right)$, $\lim\limits_{x \to 2} \dfrac{x^2 - 7x + 10}{x^2 - 8x + 12}$, $\lim\limits_{x \to \infty}\left(\sqrt{x^2 + 4} - \sqrt{x^2 + x}\right)$, $\lim\limits_{x \to 0} \dfrac{8^x - 1}{5x}$,

$\lim\limits_{x \to 0} \dfrac{\cos 4x - \cos 2x}{x^2}$, $\lim\limits_{x \to 1} \dfrac{2x - 2}{\sqrt{2 - x} - 1}$, $\lim\limits_{x \to 0}(1 + x)^{\frac{3x+2}{x}}$, $\lim\limits_{x \to 0} \dfrac{e^{4x} - 1}{\ln(1 + 3x)}$,

$\lim\limits_{x \to +\infty}(2x - 1)\left[\ln(x-1) - \ln(x+1)\right]$, $\lim\limits_{x \to \infty}\left(\dfrac{3x + 1}{3x - 2}\right)^{-4x}$.

12. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2 + 1}}{2x + 5}$, $\lim\limits_{x \to 0} \dfrac{6^x - 1}{4x}$, $\lim\limits_{x \to \infty}\left(\sqrt{4x^2 - x} - \sqrt{4x^2 + 2x + 5}\right)$, $\lim\limits_{x \to 1} \dfrac{\sqrt{5 - x} - 2}{3x - 3}$,

$\lim\limits_{x \to 0} \dfrac{\cos 2x - 1}{x \sin x}$, $\lim\limits_{x \to 1}\left(\dfrac{1}{1 - x^3} - \dfrac{1}{x - 1}\right)$, $\lim\limits_{x \to 1}(2 - x)^{\frac{3x+4}{x-1}}$, $\lim\limits_{x \to 0} \dfrac{\operatorname{arctg} 3x}{e^{7x} - 1}$,

$\lim\limits_{x \to +\infty}(3x + 2)\left[\ln(2x) - \ln(2x - 1)\right]$, $\lim\limits_{x \to \infty}\left(\dfrac{2x - 4}{2x - 1}\right)^{3x}$.

13. $\lim\limits_{x \to \infty} \dfrac{x^3 + 2x^2 + 4}{4x^3 + 3x^2 + 1}$, $\lim\limits_{x \to 0} \dfrac{9^x - 1}{5x}$, $\lim\limits_{x \to \infty}\left(\sqrt{x^2 + 5} - \sqrt{x^2 - 1}\right)$, $\lim\limits_{x \to 0} \dfrac{\sqrt[4]{16x^4 - x^8}}{e^{8x} - 1}$,

$\lim\limits_{x \to 2} \dfrac{\sqrt{6 - x} - 2}{2x - 4}$, $\lim\limits_{x \to 0}(x + 1)^{\frac{2x-3}{x}}$, $\lim\limits_{x \to -5} \dfrac{x^2 + 4x - 5}{x^2 + 5x}$, $\lim\limits_{x \to +\infty}(3x + 1)\left[\ln(4x) - \ln(4x + 1)\right]$,

$\lim\limits_{x \to 0} \dfrac{\sin 7x - \sin 2x}{\sin x}$, $\lim\limits_{x \to \infty}\left(\dfrac{x^2 + 2}{x^2 + 1}\right)^{x^2}$.

14. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2 + 3} + 2x}{4x + 3}$, $\lim\limits_{x \to 0} \dfrac{6x}{2^x - 1}$, $\lim\limits_{x \to \infty}\left(\sqrt{x^2 + 5x + 3} - \sqrt{x^2 - 3x - 1}\right)$,

$\lim\limits_{x \to 0} \dfrac{1 - \cos 4x}{\operatorname{tg}^2 2x}$, $\lim\limits_{x \to \infty}\left(\dfrac{2x + 3}{2x + 1}\right)^{4x}$, $\lim\limits_{x \to 2}(3 - x)^{\frac{5x-3}{x-2}}$, $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x^2 + 4x + 5}$,

$\lim\limits_{x \to -1} \dfrac{\sqrt{4 + x + x^2} - 2}{x + 1}$, $\lim\limits_{x \to 0} \dfrac{\arcsin 3x}{e^{\sin x} - 1}$, $\lim\limits_{x \to +\infty} x\left[\ln(2x - 3) - \ln(x + 1)\right]$.

15. $\displaystyle\lim_{x\to\infty}\frac{x^4+5x^3+7}{2x^5+3x^4+1}$, $\displaystyle\lim_{x\to0}\frac{\sin3x+\sin5x}{2x}$, $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+7x-1}-\sqrt{x^2+3x-2}\right)$,

$\displaystyle\lim_{x\to3}\frac{2x-6}{\sqrt{x-1}-1}$, $\displaystyle\lim_{x\to\infty}\frac{3^x-1}{8x}$, $\displaystyle\lim_{x\to0}\frac{e^{5x}-1}{4x}$, $\displaystyle\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{2x-1}$, $\displaystyle\lim_{x\to+\infty}x\left[\ln(3x+5)-\ln3x\right]$,

$\displaystyle\lim_{x\to\frac{\pi}{2}}\left(\operatorname{tg}x-\frac{1}{\cos x}\right)$, $\displaystyle\lim_{x\to2}\frac{x^3-3x-2}{x^2-x-2}$.

16. $\displaystyle\lim_{x\to\infty}\frac{6x^3-3x+1}{2x^5+3x^4+1}$, $\displaystyle\lim_{x\to-1}\frac{x^3-3x+4}{x^3+1}$, $\displaystyle\lim_{x\to+\infty}x^2\left[\ln(x-3)-\ln x\right]$, $\displaystyle\lim_{x\to1}\frac{x^2-1}{\sqrt{5-x}-2}$,

$\displaystyle\lim_{x\to0}\frac{1-\cos x}{3\sin^2 2x}$;, $\displaystyle\lim_{x\to\infty}\left(\frac{3x-1}{3x+2}\right)^{x-3}$, $\displaystyle\lim_{x\to0}\frac{e^{x^2}-1}{3x^2}$, $\displaystyle\lim_{x\to0}\frac{\ln(1-3x)}{5x}$, $\displaystyle\lim_{x\to-2}\frac{\operatorname{arctg}(x+2)}{4-x^2}$,

$\displaystyle\lim_{x\to\infty}\left(x-\sqrt{x^2-2x}\right)$.

17. $\displaystyle\lim_{x\to\infty}\frac{5x+1}{2x+\sqrt[3]{x}}$, $\displaystyle\lim_{x\to-2}\frac{x^2-3x-10}{x^3+x^2-2x}$, $\displaystyle\lim_{x\to+\infty}(x-3)\left[\ln5x-\ln(4x+3)\right]$, $\displaystyle\lim_{x\to0}\frac{\operatorname{tg}10x}{\sin^2 2x}$,

$\displaystyle\lim_{x\to1}\frac{\sqrt{10-x}-3}{2x-2}$, $\displaystyle\lim_{x\to\frac{\pi}{2}}(1+\cos x)^{-\frac{1}{\cos x}}$, $\displaystyle\lim_{x\to0}\frac{e^{4x}-1}{\sin3x}$, $\displaystyle\lim_{x\to0}\frac{\arcsin\sqrt[3]{x^4}}{x-\sqrt[3]{x}}$,

$\displaystyle\lim_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^{x+3}$, $\displaystyle\lim_{x\to\infty}\left(\sqrt{x+5}-\sqrt{x}\right)$.

18. $\displaystyle\lim_{x\to\infty}\frac{(x+1)^2(3-7x)^2}{(2x-1)^4}$, $\displaystyle\lim_{x\to1}\frac{x^2+x-2}{x^2+5x-6}$, $\displaystyle\lim_{x\to+\infty}(x+1)\left[\ln(2x-1)-\ln x\right]$,

$\displaystyle\lim_{x\to0}\frac{6x}{\sqrt{x+4}-2}$, $\displaystyle\lim_{x\to0}\frac{7^x-1}{3x}$, $\displaystyle\lim_{x\to0}\frac{1-\cos2x}{3\sin^2 x}$, $\displaystyle\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^{5x}$, $\displaystyle\lim_{x\to0}(1+\operatorname{tg}x)^{\frac{2}{\sin x}}$,

$\displaystyle\lim_{x\to0}\frac{e^{\sin2x}-1}{\operatorname{arctg}3x}$, $\displaystyle\lim_{x\to\infty}\left(x-\sqrt{x^2+3}\right)$.

19. $\displaystyle\lim_{x\to\infty}\frac{\sqrt{x^2+14}+x}{3x+5}$, $\displaystyle\lim_{x\to5}\frac{x^2-4x-5}{x^3-3x^2-50}$, $\displaystyle\lim_{x\to+\infty}x\left[\ln(3x+2)-\ln x\right]$,

$\displaystyle\lim_{x\to0}\frac{\sin x}{\sin6x-\sin7x}$, $\displaystyle\lim_{x\to64}\frac{\sqrt{x}-8}{4-\sqrt[3]{x}}$, $\displaystyle\lim_{x\to0}\frac{\ln(1-2x^2)}{5x}$, $\displaystyle\lim_{x\to0}\frac{x^3+2x^2}{\sin^2(x/4)}$, $\displaystyle\lim_{x\to0}\frac{e^{2x^2}-1}{1-\cos x}$,

$\displaystyle\lim_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{x-1}$, $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+5}-\sqrt{x^2-1}\right)$.

20. $\lim\limits_{x\to\infty}\dfrac{(x+5)^2 x}{5x^3+2}$, $\lim\limits_{x\to2}\left(\dfrac{1}{x+2}-\dfrac{4}{4-x^2}\right)$, $\lim\limits_{x\to+\infty}x\left[\ln x^2-\ln(x-1)\right]$, $\lim\limits_{x\to1}\dfrac{\sqrt{x}-1}{x\sqrt{x}-1}$,

$\lim\limits_{x\to0}\dfrac{1-\sqrt{1-x}}{\sin4x}$, $\lim\limits_{x\to0}\dfrac{\ln(1+6x)}{\sin4x}$, $\lim\limits_{x\to0}\dfrac{9^x-1}{3x}$, $\lim\limits_{x\to0}\dfrac{e^{\sin3x}-1}{\arcsin2x}$, $\lim\limits_{x\to\infty}\left(\dfrac{x-5}{x+3}\right)^{2x-1}$,

$\lim\limits_{x\to\infty}\left(x-\sqrt{x^2+4x}\right)$.

21. $\qquad\lim\limits_{x\to\infty}\dfrac{2x^3+x^2-5}{x^3+x-2}$, $\qquad\lim\limits_{x\to2}\dfrac{x^3-3x-2}{x^3-8}$, $\qquad\lim\limits_{x\to+\infty}(x-1)\left[\ln(4x-1)-\ln x\right]$,

$\lim\limits_{x\to3}\dfrac{x-\sqrt{3x}}{x-3}$, $\quad\lim\limits_{x\to0}\dfrac{1-\cos5x}{\sin3x}$, $\quad\lim\limits_{x\to0}\dfrac{e^{\sin6x}-1}{2x}$, $\quad\lim\limits_{x\to0}\dfrac{\ln(x+1)}{\operatorname{tg}x}$, $\quad\lim\limits_{x\to1}(2-x)^{\frac{3}{x-1}}$,

$\lim\limits_{x\to\infty}\left(\dfrac{2x-3}{2x+1}\right)^{3x}$, $\lim\limits_{x\to\infty}\left(\sqrt{x^2+8}-x\right)$.

22. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{8x^3+2}}{4x+1}$, $\lim\limits_{x\to-4}\dfrac{x^2+x-12}{x^2+4x}$, $\lim\limits_{x\to+\infty}(6x-1)\left[\ln x-\ln(x-2)\right]$, $\lim\limits_{x\to0}\dfrac{e^{3x}-e^x}{\sin2x}$,

$\lim\limits_{x\to16}\dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}$, $\qquad\lim\limits_{x\to0}\dfrac{\sin5x}{\operatorname{tg}2x}$, $\qquad\lim\limits_{x\to0}(1+x)^{\frac{1-3x}{4x}}$, $\qquad\lim\limits_{x\to0}\dfrac{6x}{2^x-1}$, $\qquad\lim\limits_{x\to\infty}\left(\dfrac{x-3}{x+1}\right)^{x-1}$,

$\lim\limits_{x\to\infty}\left(\sqrt{x+6\sqrt{x}}-\sqrt{x}\right)$.

23. $\lim\limits_{x\to\infty}\dfrac{x-2x^2+5x^4}{2+3x^2+x^4}$, $\lim\limits_{x\to-3}\dfrac{x^2-2x-15}{x^2+3x}$, $\lim\limits_{x\to0}\dfrac{a^x-1}{3x},a>0,a\neq1$, $\lim\limits_{x\to0}\dfrac{\ln(1-5x)}{6x}$,

$\lim\limits_{x\to0}\dfrac{1-\cos4x}{3\sin^22x}$, $\lim\limits_{x\to-1}\dfrac{x^3+1}{\arcsin(x+1)}$, $\lim\limits_{x\to0}\dfrac{3-\sqrt{x+9}}{2x^2+x}$, $\lim\limits_{x\to+\infty}(x+4)\left[\ln x-\ln(2x-3)\right]$,

$\lim\limits_{x\to\infty}\left(\dfrac{x+4}{x+1}\right)^{2x}$, $\lim\limits_{x\to\infty}\left(\sqrt{x^2+8x}-x\right)$.

24. $\qquad\lim\limits_{x\to\infty}\dfrac{x^4+5x^3+2}{\sqrt{4x^8+2x^2+4}}$, $\quad\lim\limits_{x\to4}\dfrac{x^2-2x-8}{x^3-16x}$, $\quad\lim\limits_{x\to2}\dfrac{\sqrt[3]{x-6}+2}{x^3+8}$, $\quad\lim\limits_{x\to\infty}\dfrac{\ln(1+3x)}{\sin2x}$,

$\lim\limits_{x\to0}\dfrac{1-\cos6x}{3\operatorname{tg}^2x}$, $\qquad\lim\limits_{x\to1}\dfrac{\sin\pi x}{e^{x-1}-1}$, $\qquad\lim\limits_{x\to3}(4-x)^{\frac{2}{x-3}}$, $\qquad\lim\limits_{x\to+\infty}x\left[\ln(5x-3)-\ln x\right]$,

$\lim\limits_{x\to\infty}\left(\dfrac{5x-1}{5x}\right)^{2x+1}$, $\lim\limits_{x\to\infty}\left(\sqrt{x^2+3}-\sqrt{x^2-3}\right)$.

25. $\lim\limits_{x\to\infty}\dfrac{2x^3+4x+1}{3x^3+2x-1}$, $\quad\lim\limits_{x\to0}\dfrac{4x^2-3x}{2x^2-9x}$, $\quad\lim\limits_{x\to+\infty}(x+2)\big[\ln x-\ln(6x-1)\big]$, $\quad\lim\limits_{x\to0}\dfrac{\sin^2 2x}{e^{x^2}-1}$,

$\lim\limits_{x\to0}\dfrac{\cos 5x-\cos 3x}{1-\cos x}$, $\quad\lim\limits_{x\to\infty}\left(\sqrt{x^2+5}-\sqrt{x^2-5}\right)$, $\quad\lim\limits_{x\to+\infty}\left(\dfrac{x}{x+3}\right)^{2x}$, $\quad\lim\limits_{x\to0}\dfrac{\ln(1+2x)}{3^x-1}$,

$\lim\limits_{x\to2}\dfrac{\sqrt{x+1}-\sqrt{3}}{2-x}$, $\lim\limits_{x\to-1}(2+x)^{\frac{3}{x+1}}$.

26. $\lim\limits_{x\to\infty}\dfrac{4x^2+x+1}{\sqrt{x^4-x+2}}$, $\quad\lim\limits_{x\to4}\dfrac{\sqrt{1+2x}-3}{\sqrt{x}-2}$, $\quad\lim\limits_{x\to+\infty}\left(\sqrt{x^2+3}-\sqrt{x^2-5}\right)$, $\quad\lim\limits_{x\to-1}\dfrac{x^2-2x-3}{x^3+1}$,

$\lim\limits_{x\to-2}(3+x)^{\frac{5}{x+2}}$, $\quad\lim\limits_{x\to0}\dfrac{\arcsin 2x}{\sin 3x+\sin 7x}$, $\quad\lim\limits_{x\to1}\dfrac{e^{x-1}-1}{2x-2}$, $\quad\lim\limits_{x\to+\infty}x\big[\ln(x+1)-\ln(3x+2)\big]$,

$\lim\limits_{x\to0}\dfrac{\ln(1-3x)}{\operatorname{tg}4x}$, $\lim\limits_{x\to\infty}\left(\dfrac{2x+3}{2x}\right)^{x-1}$.

Завдання 2.2 Дослідити функцію на неперервність, визначити характер її точок розриву.

1. $\quad y=\dfrac{-6}{(x+3)^2}$; $\qquad y=e^{\frac{x}{x+2}}$; $\qquad y=\operatorname{arcctg}\dfrac{1}{x-2}$; $\qquad y=\begin{cases} x+4, & x<-1, \\ x^2+2, & -1\le x\le1, \\ 2x, & x>1.\end{cases}$

2. $\quad y=\dfrac{3}{(x+1)^3}$; $\qquad y=2^{\frac{2x}{3-x}}$; $\qquad y=\dfrac{x^2-9}{x-3}$; $\qquad y=\begin{cases} x+2, & x\le-1, \\ x^2+1, & -1<x\le1, \\ 3-x, & x>1.\end{cases}$

3. $\quad y=\dfrac{5}{(x-2)^4}$; $\qquad y=3^{\frac{1}{2x+1}}$; $\qquad y=\operatorname{arctg}\dfrac{2}{x}+3$; $\qquad y=\begin{cases} -x, & x\le0, \\ -(x-1)^2, & 0\le x\le2, \\ x-3, & x>2.\end{cases}$

4. $\quad y=\dfrac{2}{x-1}$; $\qquad y=e^{\frac{x+1}{x-2}}$; $\qquad y=\dfrac{x^2-4}{x+2}$; $\qquad y=\begin{cases} \cos x, & x\le0, \\ x^2+1, & 0<x\le1, \\ x, & x>1.\end{cases}$

5. $y = \dfrac{-2}{(x-3)^2}$; $y = 4^{\frac{2x}{x-1}}$; $y = \dfrac{x^2 - 3x + 2}{x - 1}$; $y = \begin{cases} -x, & x \le 0, \\ x^2, & 0 < x \le 2, \\ x+1, & x > 2. \end{cases}$

6. $y = \dfrac{3}{(x-3)^3}$; $y = 5^{\frac{2x+1}{x-3}}$; $y = \operatorname{arcctg}\dfrac{2}{x-1}$; $y = \begin{cases} -x, & x \le 0, \\ \sin x, & 0 < x \le \pi, \\ x-2, & x > \pi. \end{cases}$

7. $y = \dfrac{4}{(x+1)^5}$; $y = 2^{\frac{1}{2x+2}}$; $y = \dfrac{x^2 + x - 6}{x + 2}$; $y = \begin{cases} -(x+1), & x \le 0, \\ (x-1)^2, & 0 < x \le 1, \\ x-1, & x > 1. \end{cases}$

8. $y = \dfrac{-3}{(x+5)^2}$; $y = 3^{\frac{1}{2-2x}}$; $y = \operatorname{arctg}\dfrac{3}{x+1}$; $y = \begin{cases} -x^2, & x \le 0, \\ \operatorname{tg} x, & 0 < x \le \pi/4, \\ 2, & x > \pi/4. \end{cases}$

9. $y = \dfrac{-2}{x-4}$; $y = e^{\frac{1}{6-3x}}$; $y = \dfrac{x^2 + 3x - 4}{x + 4}$; $y = \begin{cases} -2x, & x \le 0, \\ \sqrt{x}, & 0 < x \le 4, \\ 1, & x > 4. \end{cases}$

10. $y = \dfrac{-3}{(x+3)^3}$; $y = 6^{\frac{4+x}{3-x}}$; $y = \dfrac{x^3 - 1}{x - 1} + 2$; $y = \begin{cases} -2x, & x \le 0, \\ \sqrt{x}, & 0 < x \le 4, \\ 1, & x > 4. \end{cases}$

11. $y = \dfrac{1}{(x-5)^4}$; $y = 2^{\frac{1}{4-2x}}$; $y = \operatorname{arcctg}\dfrac{2}{x-2}$; $y = \begin{cases} \sin x, & x \le 0, \\ 0.5x, & 0 < x \le 2, \\ 3, & x > 2. \end{cases}$

12. $y = \dfrac{2}{(x-4)^5}$; $y = 5^{\frac{1+2x}{1-x}}$; $y = \dfrac{x^2 - 2x - 8}{x+2}$; $y = \begin{cases} x^2, & x \le 1, \\ -2x+3, & 1 < x \le 2, \\ 1, & x > 2. \end{cases}$

13. $y = \dfrac{4}{x+2}$; $y = 4^{\frac{1}{3-3x}}$; $y = \dfrac{x^3 - 1}{x-1}$; $y = \begin{cases} x^2 + 1, & x \le 1, \\ 2x, & 1 < x \le 3, \\ x+2, & x > 3. \end{cases}$

14. $y = \dfrac{-1}{(x-4)^2}$; $y = 3^{\frac{3x}{2-x}}$; $y = \operatorname{arctg}\dfrac{3}{x+2}$; $y = \begin{cases} -x, & x \le -1, \\ 2 - x^2, & -1 < x \le 1, \\ x-1, & x > 1. \end{cases}$

15. $y = \dfrac{3}{(x+4)^3}$; $y = e^{\frac{1+2x}{3-x}}$; $y = \dfrac{x^2 + 2x - 3}{x+3}$; $y = \begin{cases} x-3, & x \le 0, \\ x+1, & 0 < x \le 4, \\ 3 + \sqrt{x}, & x > 4. \end{cases}$

16. $y=\dfrac{-6}{(x+1)^4}$; $\quad y=2^{\frac{1+x}{x-3}}$; $\quad y=\dfrac{x^3+8}{x+2}$; $\quad y=\begin{cases}\sqrt{1-x}, & x\le 0,\\ 0, & 0<x\le 2,\\ x-2, & x>2.\end{cases}$

17. $y=\dfrac{5}{(x-1)^5}$; $\quad y=7^{\frac{1-x}{x-2}}$; $\quad y=\dfrac{x^2+3x-4}{x+4}$; $\quad y=\begin{cases}2x^2, & x\le 0,\\ x, & 0<x\le 1,\\ 2, & x>1.\end{cases}$

18. $y=\dfrac{2+x}{(x-6)^2}$; $\quad y=9^{\frac{1+2x}{1-x}}$; $\quad y=\dfrac{x^3+1}{x+1}$; $\quad y=\begin{cases}\sin x, & x<0,\\ x, & 0\le x\le 2,\\ 0, & x>2.\end{cases}$

19. $y=\dfrac{6+x}{(x-3)^2}$; $\quad y=e^{\frac{1}{8-4x}}$; $\quad y=\dfrac{x^2+3x-10}{x-2}$; $\quad y=\begin{cases}x-1, & x\le 0,\\ x^2, & 0<x\le 2,\\ 2x, & x>2.\end{cases}$

20. $y=\dfrac{7}{(x+1)^3}$; $\quad y=4^{\frac{1}{2x+2}}$; $\quad y=\operatorname{arcctg}\dfrac{1}{x}+\dfrac{\pi}{2}$; $\quad y=\begin{cases}\cos x, & x\le \pi/2,\\ 0, & \pi/2<x\le \pi,\\ x, & x>\pi.\end{cases}$

21. $y=\dfrac{4-x}{(x+5)^2}$; $\quad y=7^{\frac{4x}{x+1}}$; $\quad y=\dfrac{x^3-8}{x-2}$; $\quad y=\begin{cases}0, & x\le 0,\\ \operatorname{tg}x, & 0<x<\pi/2,\\ x, & x\ge \pi/2.\end{cases}$

22. $y=\dfrac{3}{(x+6)^3}$; $\quad y=10^{\frac{4-x}{x+2}}$; $\quad y=\dfrac{x^2-x-12}{x+3}$; $\quad y=\begin{cases}3x+1, & x<0,\\ x^2+1, & 0\le x<1,\\ 0, & x\ge 1.\end{cases}$

23. $y=\dfrac{-7+x}{(x-2)^4}$; $\quad y=9^{\frac{1+3x}{x-3}}$; $\quad y=\operatorname{arctg}\dfrac{2}{1-x}$; $\quad y=\begin{cases}-x, & x\le 0,\\ x^2, & 0<x\le 2,\\ x+1, & x>2.\end{cases}$

24. $y=\dfrac{6}{(x-4)^5}$; $\quad y=3^{\frac{5-x}{1-x}}$; $\quad y=\dfrac{x^3-x}{x-1}$; $\quad y=\begin{cases}2, & x\le -\pi/4,\\ \operatorname{tg}x, & -\pi/4<x<0,\\ 3x, & x\ge 0.\end{cases}$

25. $y=\dfrac{-2-x}{(x+1)^2}$; $y=6^{\frac{1}{2x+2}}$; $y=\dfrac{x^2-3x+2}{x-2}$; $y=\begin{cases} 0, & x<-2, \\ \sqrt{x+2}, & -2\le x<2, \\ x+1, & x\ge 2. \end{cases}$

26. $y=\dfrac{3x+3}{x-2}$; $y=2^{\frac{2x}{3-3x}}$; $y=\operatorname{arcctg}\dfrac{2}{x+3}$; $y=\begin{cases} -0.5x, & x<-2, \\ x^2+1, & 0\le x<1, \\ 2, & x\ge 1. \end{cases}$

Завдання 2.3 Знайти похідні функцій.

1. $y=3xe^{-3x^2}+2$; $y=\dfrac{5x}{\sin 3x+2}$; $y=\dfrac{\sqrt{x^2+1}}{x+1}$; $y=\ln(x+\sqrt{x^2+1})$; $y=2xe^{\sin 5x}$;
$y=\sin(\ln(1+e^{\sqrt{x}}))$; $y=\operatorname{arctg}^3 5x$; $y=\cos^4(1+\sqrt{x})$; $y=x^{\operatorname{tg}x}$; $x\sin 2y+y^2=4$.

2. $y=(4-x^2)e^{\sqrt{x}}+\pi$; $y=\dfrac{1-3x^2}{\cos 5x+4}$; $y=\dfrac{\sqrt[3]{1-3x^2}}{5x+4}$; $y=\ln(x+\dfrac{1}{x+4})$;
$y=(3+x^5)\cdot e^{\sin x}$; $y=\operatorname{arctg}\left(\ln(\dfrac{1}{\sqrt{x}}+x)\right)$; $y=\arcsin^4(1-x)$; $y=\sin^3(e^x+3)$;
$y=(\operatorname{tg}x)^{\sqrt{x}}$; $\cos(x+y)=\sqrt[3]{y}$.

3. $y=x^2 2^{-x}+5$; $y=\dfrac{7x}{\sin 5x+2}$; $y=\dfrac{\sqrt{1-x}}{x^2+3}$; $y=\ln(3x-\sqrt{1-x})$; $y=3x\cdot e^{\cos 4x}$;
$y=\sin\ln(1-3^{x^2})$; $y=\arcsin^5 3x$; $y=\sin^8(1+\sqrt[3]{x})$; $y=x^{\sin x}$; $x\sin 2y=y^3$.

4. $y=5x^3\cdot 3^{2-x}+4$; $y=\dfrac{3-x}{\sin 2x-4}$; $y=\dfrac{\sqrt{1-2x^3}}{4-x}$; $y=\ln(\sqrt[3]{1-x^2}+3)$;
$y=x^2e^{-\cos 3x}$; $y=\ln\sin(4+e^{-x})$; $y=\operatorname{arctg}^5(1-3x)$; $y=x^{\cos(1-x)}$; $y=\cos^3(1-e^{3x})$;
$y=x\operatorname{tg}(xy)-e^{-y}=0$.

5. $y=(1-4x)e^{-x^3}+\ln 2$; $y=\dfrac{\arcsin(1-5x^2)}{1-x}$; $y=(1-x^2)\cdot 2^{\cos(1+x)}$;
$y=\sin\ln(\sqrt{x}+\dfrac{1}{\sqrt{x}})$; $y=\dfrac{x^3+2}{\sqrt{x-x^2}}$; $y=\ln\dfrac{1}{x^2+\sqrt{x+3}}$; $y=\arccos^3 7x$;
$y=\sin^5(e^x+x)$; $y=(x+7)^{\cos x}$; $\cos(x+y)+\dfrac{x}{y}=3$.

6. $y = (x+4)e^{1-x^2} + \sqrt{2}$; $y = \dfrac{3x+1}{\arcsin(1-x)}$; $y = \dfrac{\sqrt{1-x^2}}{3-x^3}$; $y = \ln\dfrac{1}{x+\sqrt{1-x}}$;

$y = (3+x^2)\cdot e^{\arccos 2x}$; $y = \operatorname{arctg}\left(\ln(e^{\frac{1}{x}}+5)\right)$; $y = \sin^3(1-e^{2x})$; $y = (x+\sin x)^x$;

$y = \cos(1-y) + \dfrac{x^3}{y} = 10$.

7. $y = (x-1)\arccos x + \ln 2$; $y = \dfrac{\cos(1-x)+5}{x^4}$; $y = \dfrac{\sqrt{x^5+4}}{3x+2}$; $y = x^3 e^{\operatorname{arctg}3x}$;

$y = \ln(x^2 - \sqrt[3]{1-x})$; $y = \ln\operatorname{arctg}\sqrt[5]{x}$; $y = \sin^8 3x$; $y = \ln^7(9x) - e^{-x}$; $y = x^{\ln(1-e^x)}$;

$y^3 e^{xy} + \cos x = 3$.

8. $y = x^2 4^{1-x^2} + \pi$; $y = \dfrac{1+3x^2}{1+3\cos 5x}$; $y = \dfrac{5-x}{\sqrt{1-3x^3}}$; $y = x^3 e^{\operatorname{arctg}3x}$;

$y = \ln(\sqrt[3]{5-x} + x^2)$; $y = \cos\operatorname{tg}(\sqrt{x} - x^2)$; $y = \arcsin^5(1+\dfrac{1}{\sqrt{x}})$; $y = \operatorname{arctg}^3(1-x)$;

$y = (1-x)^{\cos 3x}$; $x\arccos y + y^2 = 3$.

9. $y = x^4 e^{1-\sqrt{x}} + 4$; $y = \dfrac{\sqrt[3]{x^2}}{(1+x)^3}$; $y = \dfrac{\cos(1-5x)}{4+x^2}$; $y = \ln(\sqrt[3]{4+x^2}+1)$; $y = x^5 2^{\operatorname{tg}x}$;

$y = \arccos\left(\ln(e^{-x}+x)\right)$; $y = \operatorname{arctg}^7(1-x)$; $y = (\arcsin x)^{-x^2}$; $y = \sin^8(1+2^{\sqrt{x}})$;

$y\sqrt{x} + \cos(3x+y) = 4$.

10. $y = 3x\cdot e^{-3+2x^3} + 2$; $y = \dfrac{5x}{\sin 3x + 2}$; $y = \dfrac{\sqrt{x^2+1}}{x+1}$; $y = \ln(x+\sqrt{x^2+1})$;

$y = 2x e^{\sin 5x}$; $y = \sin\ln(1+e^{\sqrt{x}})$; $y = \operatorname{arctg}^3 5x$; $y = \cos^4(1+\sqrt{x})$; $y = x^{\operatorname{tg}3x}$;

$x\sin 2y + y^2 = 4$.

11. $y = 3x^5 4^{-7x}$; $y = \dfrac{\operatorname{arctg}(5-x)}{1+x^4}$; $y = \dfrac{1-3x}{\sqrt[5]{(2x+4)^3}}$; $y = \ln(x+e^{-x^3})$;

$y = 2^{\ln 3x}(1-3x^2)$; $y = \operatorname{arctg}\left(\ln(e^{2x}-4)\right)$; $y = \arcsin^3(1-\dfrac{1}{x})$; $y = \cos^6(1-x)$;

$(3\sin x)^{\sqrt{x}} = y$; $y\ln x - x\ln y = x+y$.

12. $y = 2x^5 e^{1-7x}$; $\quad y = \dfrac{1-7x^2}{\arccos 3x + 4}$; $\quad y = \dfrac{\sqrt{3-4x^2}}{1-5x}$; $\quad y = \ln \dfrac{1}{4+\sqrt[3]{4+2x}}$;

$y = 5^{\cos 4x}(1+4x^3)$; $\quad y = \cos^3(4+e^{\sqrt{x}})$; $\quad y = x^6 \cdot 2^{\operatorname{tg} x}$; $\quad y = \operatorname{arcctg}\left(\ln(1+4e^{4x-5})\right)$;

$y = (\sqrt{x})^{\sin(1-x)}$; $\quad y = \ln(x+y) + x^2$.

13. $y = 4^{3-x}(1-7x^3) + e^2$; $\quad y = \dfrac{2x+7}{\cos(5-x^2)}$; $\quad y = \dfrac{\sqrt[7]{2x-x^3}}{x-7}$; $\quad y = \ln(x^3 + \dfrac{1}{\sqrt{x}})$;

$y = (4x-7)e^{\sin(1-x)}$; $\quad y = \arcsin \ln(2^{\sqrt{1-x}} + 4)$; $\quad y = \operatorname{tg}^7(1-3x^2)$; $\quad y = \sin^5(1-e^{-x})$;

$y = x^{\operatorname{arctg} 5x}$; $\quad y = \sqrt{x^2+y^2} + \ln \dfrac{x}{y} = e^2$.

14. $y = (1-3x) \cdot 4^{2x} + 7$; $\quad y = \dfrac{3+2x}{\sin(1-x)}$; $\quad y = \dfrac{\sqrt[3]{4-5x}}{x^3+4}$; $\quad y = x^2 e^{\sin(1-2x)}$;

$y = \ln(\sqrt[4]{1-x} + 4x)$; $\quad y = \arccos^2(1+3x)$; $\quad y = \operatorname{tg}\ln\left(1-e^{x^2}\right)$; $\quad y = \operatorname{arctg}^3(3^{\sqrt{x}} + x)$;

$y = (4-7x^3)^{\ln x}$; $\quad \cos(x+y^2) + xy = 3$.

15. $y = (x-1)^2 e^{x^3} + 7$; $\quad y = \dfrac{\arcsin 5x + 3}{1-7x^3}$; $\quad y = \dfrac{1+\sqrt[3]{x-x^2}}{1+4x^3}$; $\quad y = \ln(\dfrac{1}{x+4} + \sqrt{x})$;

$y = 3^{\sin 8x}(4+7x^4)$; $\quad y = \cos^5(x^2 - 7x)$; $\quad y = \ln^7(e^{\sqrt[3]{x}} + x)$; $\quad y = \ln\operatorname{arctg}(\sqrt[3]{x} + x)$;

$y = (\sqrt{x} + 1)^{2x^2+3}$; $\quad \arcsin(x+y) = \dfrac{y}{x}$.

16. $y = (\sqrt{x} + 3)e^{x^2} + 7$; $\quad y = \dfrac{(3x-4)^2}{\sqrt{x}+7}$; $\quad y = \dfrac{1+\sqrt[3]{x-x^2}}{1+4x^3}$; $\quad y = \ln(\sqrt[5]{1-x^3+2})$;

$y = x^3 3^{\operatorname{arctg} 5x}$; $\quad y = \arcsin \ln(e^{\sqrt{x}} + 3x)$; $\quad y = \operatorname{arcctg}^6(5-3x)$; $\quad y = \sin^6(e^{\sqrt[3]{x}} + x)$;

$y = (\arcsin x)^{\sqrt{x}}$; $\quad xy + \ln(x+5y) = 3$.

17. $y = 2x^3 4^{1-x^8}$; $\quad y = \dfrac{\operatorname{arctg}\left(5x-3x^3\right)}{\sqrt{x}+4}$; $\quad y = \dfrac{5-3x^2}{\sqrt{3x^3+4}}$; $\quad y = x^5 2^{\ln(1-9x)}$;

$y = \ln\left(x+3^{-\sqrt{x+4}}\right)$; $\qquad y = \arccos\left(\ln\left(e^{-\sqrt{x^2+4}}\right)\right)$; $\qquad y = \sin^7(3-8x)$;

$y = \arcsin^4(1+x^2)$; $\quad y = (5\operatorname{tg} x)^{1-\sqrt{x}}$; $\quad \cos(x^2 - y) + \dfrac{3x+1}{y} = 4$.

18. $y = \sqrt[3]{x^2} e^{1-3x^3}$; $\quad y = \dfrac{x-7x^2}{\sqrt{x^2-4x}}$; $\quad y = \dfrac{\sin(3x-4x^2)}{5+x^3}$; $\quad y = \operatorname{tg}^7(3-7x)$;

$y = \ln\left(\sqrt[3]{1-2x^2}+1\right)$; $\ y = x^4 2^{\arcsin\sqrt{x}}$; $\ y = \operatorname{arctg}\ln\left(e^{\sqrt[3]{x^2-x}}+3\right)$; $\ y = \sin^4(e^{-x^3}+x^2)$;

$y = (\arccos x)^{\sqrt[3]{x}}$; $\ \ln(x^2+y) + \dfrac{y}{x^2} = 3$.

19. $y = (2-7x^2)\cdot 3^{-3x} + \sqrt{2}$; $\quad y = \dfrac{1-5x}{\cos(3-2x^3)}$; $\quad y = \dfrac{1-4x^4}{\sqrt[3]{5-x}}$; $\quad y = (1-4x^2)^{\sqrt{x}}$;

$y = \ln(3x - \sqrt[3]{x^2})$; $\quad y = (3x-4)e^{\sin 4x}$; $\quad y = \arccos^5(1-4x^2)$; $\quad y = \operatorname{arctg}^4(e^{5x}+3)$;

$y = \operatorname{tg}\ln(5^{\sqrt{x}}+4)$; $\ \cos(x^3-y) + \dfrac{y}{x} = 1$.

20. $y = (1+7x)\cdot e^{3x+x^2} + 7$; $\ y = \dfrac{\arcsin(1-7x)}{\sqrt{1-x^2}}$; $\ y = \dfrac{\sqrt{x+4}}{x^5+4x^3}$; $\ 7 = \ln\left(x+\sqrt[3]{1-x}\right)$;

$y = (4-x^3)\cdot 3^{\sin(1-3x)}$; $\ y = \ln\left(\sqrt[3]{x} - \dfrac{4}{x}\right)$; $\ y = \operatorname{arctg}^2 3x$; $\ y = \sin^3\left(e^{-\sqrt{x}} - \dfrac{1}{\sqrt{x}}\right)$;

$y = (\cos x)^{1-\sqrt{x}}$; $\ \ln(x^3-y^3) = x$.

21. $y = (1-x^2)\cdot 2^{1-\sqrt{x}} + 4$; $\quad y = \dfrac{1-7x}{1-2\sin x}$; $\quad y = \dfrac{\sqrt[3]{1-x^2}}{3-4x}$; $\quad y = x^4 e^{\cos 5x}$;

$y = \ln\left(\sqrt[3]{1-4x}+x\right)$; $\ y = \sin\operatorname{arctg}\left(\sqrt{x}+4\right)$; $\ y = \operatorname{arctg}^5(1-x)$; $\ y = \ln^7\left(1 - \dfrac{1}{\sqrt[3]{x}}\right)$;

$y = (1-4x^2)^{\sin 3x}$; $\ x^3\arccos y - y^4 = \sqrt{e}$.

22. $y = (5-3x)3^{x^2} + 7$; $\quad y = \dfrac{\sin(5x-x^2)}{1-x}$; $\quad y = \dfrac{5x+4}{\sqrt[3]{x}+4x}$; $\quad y = \cos^3(e^{\sqrt{x}}+4)$;

$y = \ln\left(5x - \sqrt[3]{x^2-7x}\right)$; $\ y = \ln\sin(5-e^{-\sqrt{x}})$; $\ y = x^5 e^{\operatorname{arctg} x}$; $\ y = \arccos^7(1-5x)$;

$y = \left(\sqrt{x}\right)^{\ln x}$; $\ \dfrac{x}{y^2} + \operatorname{arctg} xy = 1$.

23. $y = 5xe^{x^3+4} + \pi$; $\quad y = \dfrac{\operatorname{arctg}(5-x)}{x^2+4}$; $\quad y = \dfrac{15+x^3}{\sqrt{x-7x^2}}$; $\quad y = \ln\dfrac{x}{1-\sqrt[3]{x}}$;

$y = \arccos\left(\ln\left(7-\sqrt[3]{x}\right)\right)$; $\quad y = x^4 2^{\cos\sqrt{x}}$; $\quad y = \operatorname{arcctg}^4 5x$; $\quad y = \sin^9(3-e^{\sqrt{x}})$;

$y = (x+\cos 3x)^{x^2}$; $\ \sin(x-y) + x^3 y = 3$.

24. $y = (1-x^3)e^{5x} + 7$; $y = \dfrac{\text{tg}(1-3x)}{x^3+4}$; $y = \dfrac{\cos\sqrt{x}+4}{\sqrt[3]{x}+x}$; $y = \sin^7(1-x^2)$;

$y = x^7 7^{\arcsin 5x}$; $\quad y = \ln\dfrac{x}{\sqrt[6]{1+5x}-x}$; $\quad y = \ln\cos(x - \dfrac{1}{\sqrt[3]{x}})$; $\quad y = \ln^4(1+e^{5x})$;

$y = \left(\text{arctg}\left(1-\sqrt{x}\right)\right)^x$; $x^3 e^{x+y} - y = 4$.

25. $\quad y = (5x-4)e^{\frac{1}{\sqrt{x}}+3}$; $\quad y = \dfrac{(2x+4)^5}{\sqrt[4]{x^3}}$; $\quad y = \dfrac{5+4x^3}{\text{tg}(1-7x)}$; $\quad y = x^3 5^{\cos(1-x)}$;

$y = \ln(\sqrt[3]{1-x^2}-x)$; $\quad y = \arcsin\ln\left(e^{1-5x} - \dfrac{1}{x}\right)$; $\quad y = \cos^7(e^{\sqrt{x}}+3x)$;

$y = \sin^9(1+3^{\sqrt{x}})$; $y = [\cos(1-x)]^x$; $\sqrt{yx} - \sin(5y-x) = 0$.

26. $y = (3+2x^2)\cdot e^{\sqrt{3-x}} + 4$; $y = \dfrac{4-5x}{\cos 3x - 7}$; $y = \dfrac{\sqrt[3]{4-7x}}{3x^2+7}$; $y = (4-x^3)e^{\sin(2+4x)}$;

$y = \ln\left(5x - \dfrac{1}{1-x}\right)$; $\quad y = \sin\ln(1-2^{5x})$; $\quad y = \arcsin^3(2+3x)$; $\quad y = \sin^4(e^{3x}-1)$;

$y = (\text{tg}\,x)^{3-\sqrt{x}}$; $\cos(x+y) + \sqrt[4]{y} = 0$.

Завдання 2.4 Провести повне дослідження функцій і накреслити їх графіки.

1. $\quad y = \dfrac{x+1}{x^3}$; $\qquad y = xe^{-x} + 1$.

2. $\quad y = \dfrac{x^2}{x-1}$; $\qquad y = (2+x)e^x - 1$.

3. $\quad y = \dfrac{x^3}{x^3-2}$; $\qquad y = (x-1)e^{-2x} + 2$.

4. $\quad y = \dfrac{-x}{x^3-1}$; $\qquad y = (3x-1)e^{2x} - 3$.

5. $y = \left(\dfrac{2x+3}{x-1}\right)^2;$ $y = (2x+1)e^{2x} + 1.$

6. $y = \dfrac{3x^3}{x^3+6};$ $y = 2xe^x - 1.$

7. $y = \dfrac{x^2+1}{2x+3};$ $y = (3-x)e^{-x} + 3.$

8. $y = \dfrac{x^3}{x^2+2x+3};$ $y = (3x+1)e^{-x} + 2.$

9. $y = \dfrac{16}{x^2(x-4)};$ $y = (1-x)e^{-x} + 1.$

10. $y = \dfrac{x^2+1}{x-1};$ $y = (2x+1)e^x - 1.$

11. $y = \dfrac{x^4}{x^3-1};$ $y = (1-3x)e^{2x} + 2.$

12. $y = \dfrac{x^3}{x^2+1};$ $y = 3xe^{-x} + 1.$

13. $y = \dfrac{-x^2}{(x-2)^2}$; $\qquad$ $y = (5x - 2)e^{-x} + 3.$

14. $y = \dfrac{x}{x^3 - 2}$; $\qquad$ $y = (2x - 1)e^{2x} + 3.$

15. $y = \dfrac{4x^3}{x^3 - 1}$; $\qquad$ $y = (4 - 2x)e^x - 2.$

16. $y = \dfrac{x^2 - 4x + 3}{x^2 - 2x}$; $\qquad$ $y = (x + 3)e^{-2x} + 1.$

17. $y = \dfrac{x + 3}{x^3}$; $\qquad$ $y = -xe^x + 2.$

18. $y = \dfrac{4x^3 + 5}{x}$; $\qquad$ $y = (2x + 5)e^{2x} + 1.$

19. $y = \dfrac{x^2}{x^3 + 1}$; $\qquad$ $y = (1 - x)e^x + 1.$

20. $y = \dfrac{x^3}{2(x-1)^2}$; $y = (x+1)e^{-x} + 3.$

21. $y = \dfrac{x}{2-x^3}$; $y = (2x+3)e^{x} + 2.$

22. $y = \dfrac{x}{x^2-4}$; $y = (x-3)e^{-2x} + 4.$

23. $y = \dfrac{4x^3}{x^3-1}$; $y = 2xe^{-x} - 1.$

24. $y = \dfrac{2-4x^2}{1-4x^2}$; $y = (2x-1)e^{-x} - 2.$

25. $y = \dfrac{x^2-2x+2}{x-1}$; $y = (x-2)e^{2x} - 3.$

26. $y = \dfrac{-x}{x^3+3}$; $y = (2x-3)e^{-3x} + 1.$