1.24

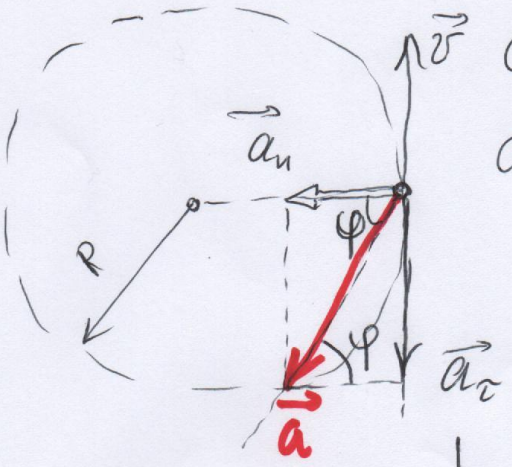
$a_n = 4{,}9 \frac{м}{с^2}$;

$\varphi = (\vec{a}; \vec{a_n})$;

$\varphi = 60^\circ$;

$R = 10$ м;

$\vec{a_\tau}$ - ? $\vec{a}$; $\vec{v}$



$a_n = a \cdot \cos\varphi$

$a = \frac{a_n}{\cos\varphi}$

$a = \frac{4{,}9}{\cos 60^\circ} = \frac{4{,}9}{0{,}5} = 9{,}8 \frac{м}{с^2}$;

$a_\tau = a \cdot \sin\varphi$

$a_\tau = 9{,}8 \cdot \sin 60^\circ = 8{,}487 \frac{м}{с^2}$

$a_n = \frac{v^2}{R} \Rightarrow v = \sqrt{a_n R} = \sqrt{49} = 7 \frac{м}{с}$

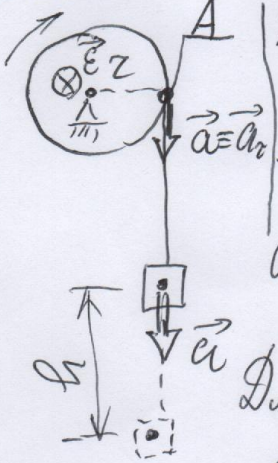
1.32

$z = 4 \cdot 10^{-2}$ м;

$h = 1{,}5$ м;

$t = 3$ с;

$\vec{\varepsilon}$ - ?



$S(t) = S_0 + v_0 t + \frac{at^2}{2}$

$h = \underbrace{h_0}_{0} + \underbrace{v_0}_{0} t + \frac{at^2}{2} \Rightarrow h = \frac{at^2}{2}$

$a = \frac{2h}{t^2} = \frac{3}{9} = \frac{1}{3} = 0{,}333 \frac{м}{с^2}$

Для т. A: $\vec{a} \equiv \vec{a_\tau} \Rightarrow a = \varepsilon z$

$\varepsilon = \frac{a}{z} = \frac{1}{3 \cdot 4 \cdot 10^{-2}} = \frac{100}{12} = 8{,}333 \frac{рад}{с^2}$

1-31

$v_0 = 10\ \frac{м}{с}$;

$\nu = 50\ Гц$;

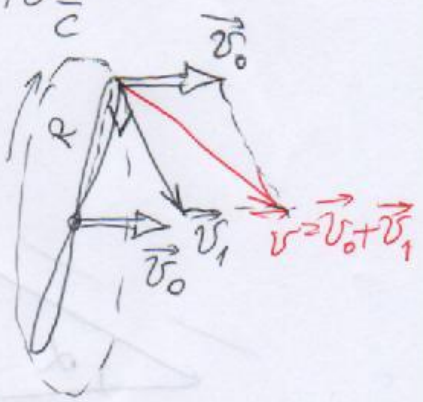
$R = 1\ м$;

$\vec{v}$ - ?

$v_0 = 36\ \frac{км}{ч} = \frac{36 \cdot 10^3\ м}{3{,}6 \cdot 10^3\ с} = 10\ \frac{м}{с}$

$\nu = \frac{3000}{60\ с} = 50\ Гц$

$\begin{cases} v_1 = \omega R = 2\pi\nu R = 314\ \frac{м}{с} \\ \omega = 2\pi\nu \end{cases}$



$v = \sqrt{v_1^2 + v_2^2} = \sqrt{100 + 314^2} = \dots \ \frac{м}{с}$

Динамика поступат. движения

Импульс $\vec{P} = m\vec{v}\ \left[\frac{кг \cdot м}{с}\right]$

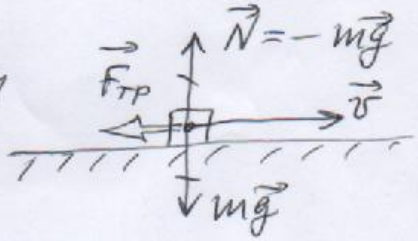
II з-н дин. Н (общ. сл.): $\begin{cases} \vec{F} = \sum_{i=1}^{n} \vec{F}_i = \frac{d\vec{P}}{dt}\ [Н] \\ \text{или} \\ \vec{F} = \frac{d(m\vec{v})}{dt}\ [Н] \end{cases}$

II з-н дин. Н (частн. $m = const$) $\Rightarrow \vec{F} = m\vec{a} \rightarrow \vec{F} \uparrow\uparrow \vec{a}$

Силы

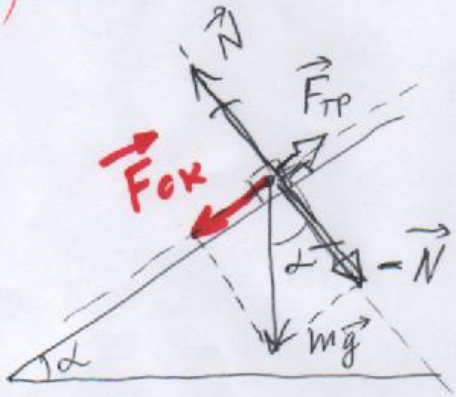
1) Тяжести + Реакция опоры

1а) Горизонталь



$\begin{cases} N = mg \\ F_{тр} = \mu N = \mu mg \end{cases}$ - ускор. торм.

1б) Наклонная пл-ть



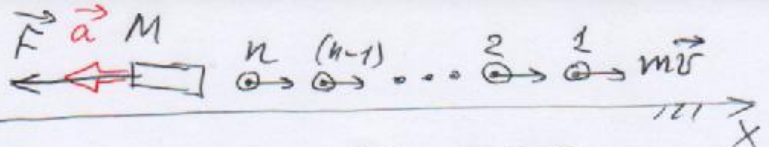
$\begin{cases} N = mg \cdot \cos\alpha - \text{реакц. опоры} \\ F_{ск} = mg \cdot \sin\alpha - \text{скат. сила} \\ F_{тр} = \mu N = \mu mg \cdot \cos\alpha - \text{трен.} \end{cases}$

$\begin{cases} \vec{F}_{ск} + \vec{F}_{тр} = m\vec{a} - \text{II з-н д. Н} \\ F_{ск} - F_{тр} = ma \end{cases}$

1.37

$m = 10^{-2}$ кг;
$v = 5 \cdot 10^2 \frac{м}{с}$;
$n = 6 \cdot 10^2$;
$\Delta t = 60$ с;
$M = 4$ кг;

$\vec{a}$ – ? $\vec{F}$ – ?



II з-н дин. Н: $\begin{cases} F = \frac{\Delta p}{\Delta t} = \frac{n m v}{\Delta t}; \\ \Delta p = n \cdot m v \end{cases}$

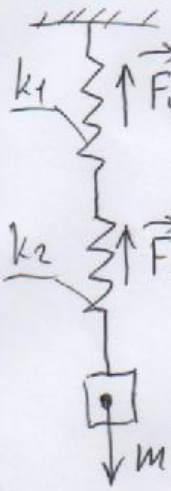
$F = \frac{600 \cdot 10^{-2} \cdot 5 \cdot 10^2}{60} = 50$ Н

II з-н дин. Н.: $F = Ma \rightarrow a = \frac{F}{M} = \frac{50}{4} = 12{,}5 \frac{м}{с^2}$;

$k_1 = 2 \cdot 10^3 \frac{Н}{м}$;
$k_2 = 6 \cdot 10^3 \frac{Н}{м}$;
$m = 10^3$ кг;

1) $(\Delta X)_1$ – ?
2) $(\Delta X)_2$ – ?

1)



$\begin{cases} F_{упр1} = mg \rightarrow k_1 X_1 = mg \rightarrow X_1 = \frac{mg}{k_1} \\ F_{упр2} = mg \rightarrow k_2 X_2 = mg \rightarrow X_2 = \frac{mg}{k_2} \end{cases}$

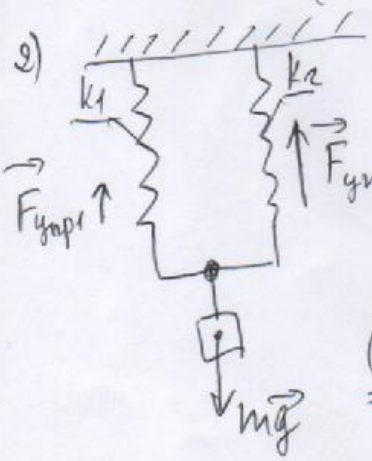
$X_1; X_2$ – удл. пружин

$(\Delta X)_1 = X_1 + X_2 = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

$(\Delta X)_1 = 10^3 \cdot 9{,}81\left(\frac{1}{2 \cdot 10^3} + \frac{1}{6 \cdot 10^3}\right) =$

$= 9{,}81\left(0{,}5 + \frac{1}{6}\right) = 6{,}540$ м;

2)



$(\Delta X)_2 = X_1 = X_2 \Rightarrow \begin{cases} F_{упр1} = k_1 (\Delta X)_2 \\ F_{упр2} = k_2 (\Delta X)_2 \end{cases}$

$F_{упр1} + F_{упр2} = mg$

$(\Delta X)_2 \cdot (k_1 + k_2) = mg$

$(\Delta X)_2 = \frac{mg}{k_1 + k_2} = \frac{9{,}81 \cdot 10^3}{8 \cdot 10^3} = 1{,}226$ м

1.34

$m_1 = 10$ кг;

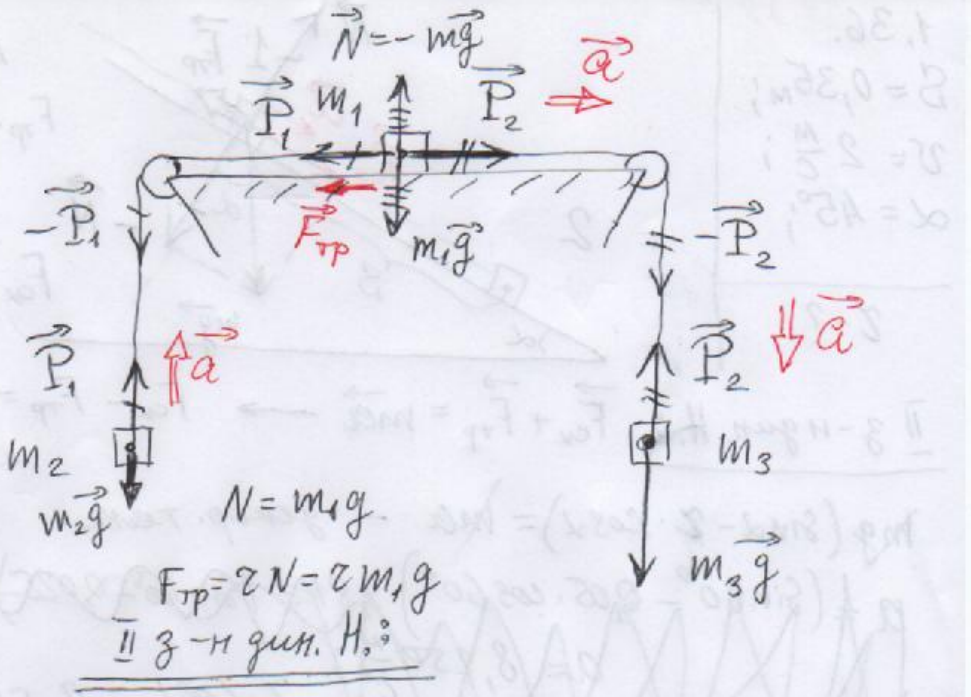
$m_2 = 5$ кг;

$m_3 = 8$ кг;

$\tau = 0{,}05$;

$\vec{a} - ?$

$\vec{P}_1 - ?$ $\vec{P}_2 - ?$



$N = m_1 g$

$F_{тр} = \tau N = \tau m_1 g$

II з-н дин. Н.:

Тело 1: $\vec{P}_1 + \vec{P}_2 + \vec{F}_{тр} = m_1\vec{a} \longrightarrow P_2 - P_1 - F_{тр} = m_1 a$

$P_2 - P_1 - \tau m_1 g = m_1 a;$ (1)

Тело 2: $\vec{P}_1 + m_2\vec{g} = m_2\vec{a} \longrightarrow P_1 - m_2 g = m_2 a;$

$P_1 = m_2 g + m_2 a;$ (2)

Тело 3: $\vec{P}_2 + m_3\vec{g} = m_3\vec{a} \longrightarrow -P_2 + m_3 g = m_3 a;$

$P_2 = m_3 g - m_3 a;$ (3)

(2); (3) → (1): $m_3 g - m_3 a - m_2 g - \underbrace{m_2 a}_{-\tau m_1 g} = m_1 a;$

$a(m_1 + m_2 + m_3) = g(m_3 - m_2 - \tau m_1);$

$$a = \frac{(m_3 - m_2 - \tau m_1)g}{m_1 + m_2 + m_3} = \frac{(8 - 5 - 0{,}5)\,9{,}81}{23} = \frac{2{,}5 \cdot 9{,}81}{23} = 1{,}066 \ \frac{м}{с^2}$$ (4)

(4) → (2): $P_1 = m_2(g - a) = 5(9{,}81 - 1{,}066) = 43{,}720$ Н

(4) → (3): $P_2 = m_3(g - a) = 8(9{,}81 - 1{,}066) = 69{,}952$ Н

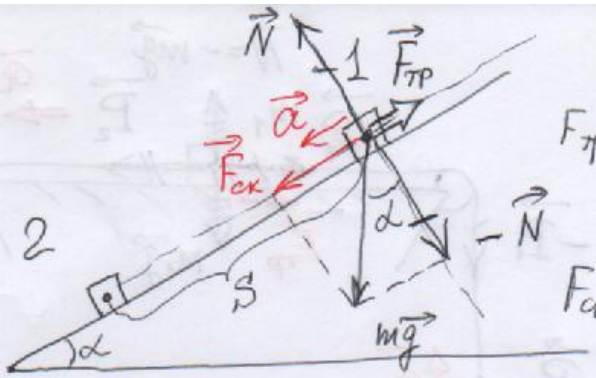
1.36.

$S = 0{,}35_м$;

$v = 2\frac{м}{с}$;

$\alpha = 45°$;

$\tau - ?$



$N = mg \cdot \cos\alpha$

$F_{тр} = \tau N = \tau mg \cdot \cos\alpha$

$F_{ск} = mg \cdot \sin\alpha$

II з-н дин. Н.: $\vec{F}_{ск} + \vec{F}_{тр} = m\vec{a} \longrightarrow F_{ск} - F_{тр} = ma$;

$mg(\sin\alpha - \tau \cdot \cos\alpha) = ma$ – ускор. тела

$a = (\sin 60° - 0{,}05 \cdot \cos 60°) \cdot 9{,}81 = 8{,}250 \frac{м}{с^2}$;

$S = S_0 + v_0 t + \frac{at^2}{2}$

$\tau \cdot \cos\alpha = g \cdot \sin\alpha - a$

$\tau = \frac{g \cdot \sin\alpha - a}{\cos\alpha}$; (1)

Путь: $S = \underset{0}{S_0} + \underset{0}{v_0} t + \frac{at^2}{2} \rightarrow S = \frac{at^2}{2}$ (2)

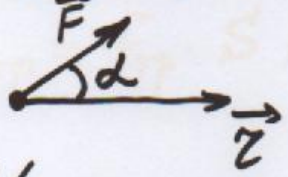
Скорость: $v = at \rightarrow t = \frac{v}{a} \rightarrow t^2 = \frac{v^2}{a^2}$; (3)

(3) → (2): $S = \frac{a v^2}{2a^2} \rightarrow a = \frac{v^2}{2S} = \frac{4}{0{,}75} = 5{,}300 \frac{м}{с^2}$; (4)

(4) → (1): $\tau = \frac{9{,}81 \cdot \sin 45° - 5{,}3}{\cos 45°} = 2{,}315$;

Энергия. Работа. Законы сохранения.

1) Работа: $\vec{F}$ [Н] – сила; $\vec{r}$ [м] – перемещен.



$$A\,[Дж] = F r \cdot \cos\alpha;$$

2) Кинетическая энергия:

$$E_к = \frac{m v^2}{2}\,[Дж];$$ m [кг]; v [$\frac{м}{с}$];

3) Потенциальная энергия:

а) в поле тяжести: $E_п = m g h\,[Дж]$

б) упругой деформации:

$$E_п = \frac{k x^2}{2}\,[Дж];$$

k [Н/м] – коэф. упр.
x [м] – деформация сжатия (растяж.)

З-н сохр. энерг.

а) Замкнутая мех. сист. ($F_{тр} = 0$):

$$\left.\begin{array}{l} E_к + E_п = const; \\ \text{или} \\ \Delta E_к + \Delta E_п = 0; \end{array}\right\} \Rightarrow \left\{\begin{array}{l} E_{к1} + E_{п1} = E_{к2} + E_{п2}; \\ \text{или} \\ \Delta E_к = \Delta E_п; \end{array}\right.$$

б) Незамкн. мех. сист. ($F_{тр} \neq 0$)

$$\begin{cases} \Delta E = A_{тр}; \\ A_{тр} = F_{тр} \cdot S; \end{cases} \text{ или } \boxed{E_1 = E_2 + A_{тр};}$$

Мощность – энергия (или р-та) за 1 сек. (Power)

$$\begin{cases} P = \dfrac{dA}{dt} = \dfrac{d(F \cdot S)}{dt} = F \cdot \underbrace{\dfrac{dS}{dt}}_{v} = F \cdot v; \\ \vec{F} = const; \end{cases}$$

$$P = F \cdot v \left[Вт \equiv \frac{Дж}{с} \right];$$

КПД – коэф. полезного действия:

$$\boxed{\eta = \frac{A}{E} [-];}$$

A – полезн. р-та
E – затрач. энергия

1-62

$k = 1{,}25 \cdot 10^3 \frac{Н}{м}$;

$\Delta l = 8 \cdot 10^{-2}$ м;

$m = 5 \cdot 10^{-3}$ кг;

$F_{тр} = 0$;

v – ?

$F_{тр} = 0 \Rightarrow$ $\Delta E_п = \Delta E_к$ — з-н сохр. энерг.

$\frac{k(\Delta l)^2}{2} = \frac{m v^2}{2} \Rightarrow k(\Delta l)^2 = m v^2$

$v = \Delta l \cdot \sqrt{\frac{k}{m}}$

$v = 8 \cdot 10^{-2} \sqrt{\frac{1250}{5 \cdot 10^{-3}}} = 8 \cdot 10^{-2} \sqrt{25 \cdot 10^4} =$

$= 40 \frac{м}{с}$;

1-63

$m = 10^3$ кг;

$F_{тр} = 0{,}1 mg$;

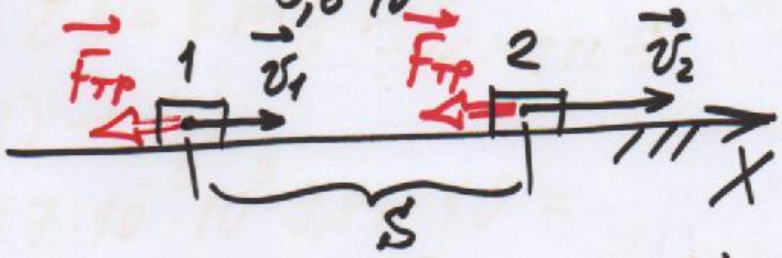
$v_1 = 2{,}778$ м/с;

$v_2 = 11{,}111$ м/с;

$S = 5 \cdot 10^2$ м;

A – ?

$v_1 = 10 \frac{км}{ч} = \frac{10 \cdot 10^3}{3{,}6 \cdot 10^3}$ –



$A = \Delta E_к + A_{тр}$; (1)

$\Delta E_к = \frac{m(v_2^2 - v_1^2)}{2}$; (2)

$A_{тр} = F_{тр} \cdot S = 0{,}1 mgS$; (3)

(2); (3) → (1):

$$A = m\left[\frac{v_2^2 - v_1^2}{2} + g \cdot S\right];$$

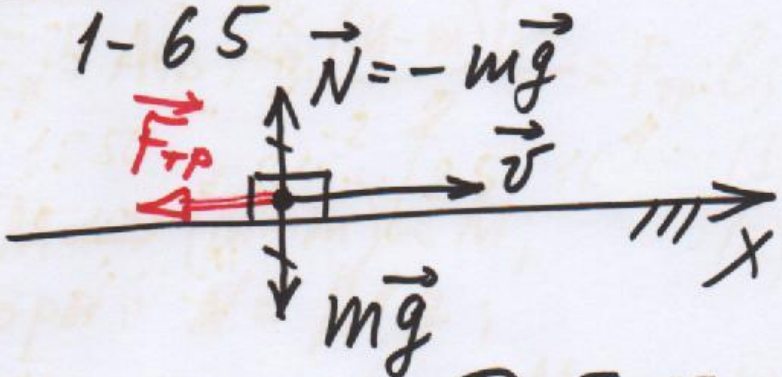
$$\underline{\underline{A}} = 10^3\left[\frac{11{,}111^2 - 2{,}778^2}{2} + 9{,}81 \cdot 500\right] =$$

$$= \ldots\ldots \text{ Дж};$$

1-65

$m = 10^3$ кг;
$\vec{v} = const$;
$v = 10$ м/с;
$z = 0{,}07$;

P - ?



$A_{тр} = F_{тр} \cdot S \Rightarrow P = F_{тр} \cdot v$; (1)

$N = mg$ – реакция опоры

$F_{тр} = zN = zmg$; (2)

(2) → (1):

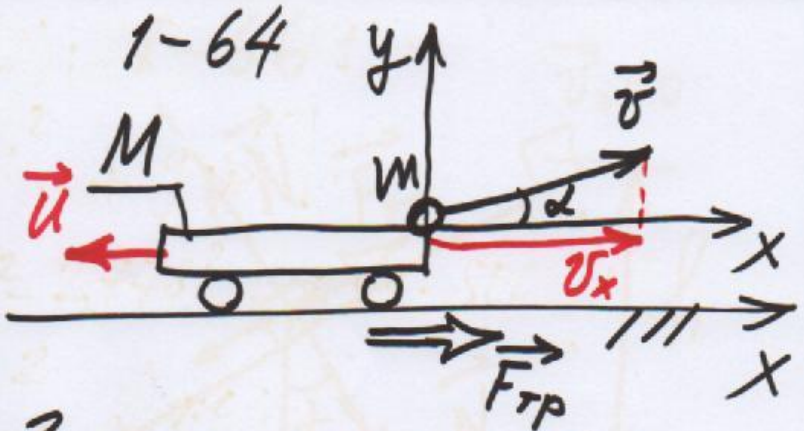
$$\underline{\underline{P}} = zmgv = 7 \cdot 10^{-2} \cdot 10^3 \cdot 9{,}81 \cdot 10 =$$

$$= 7 \cdot 9{,}81 \cdot 10^2 = \underline{\underline{6{,}867 \cdot 10^3 \text{ Вт} = 6{,}867 \text{ кВт}}};$$

1-64

$M = 2 \cdot 10^4$ кг;
$m = 10$ кг;
$v = 10^3$ м/с;
$\alpha = 30°$;
$\tau = 2 \cdot 10^{-3}$;

l - ?



З-н сохр. энерг.:

$$E_к = A_{тр} \rightarrow \frac{(M-m)u^2}{2} = F_{тр} \cdot l; \quad (1)$$

$$m \ll M \Rightarrow (M-m) \approx M; \quad (2)$$

Реакция опоры: $N = Mg$;

Сила трения: $F_{тр} = \tau N = \tau Mg$; (3)

(2); (3); → (1): $\frac{Mu^2}{2} = \tau Mgl \rightarrow l = \frac{u^2}{2\tau g}$; (4)

З-н сохр импульса:

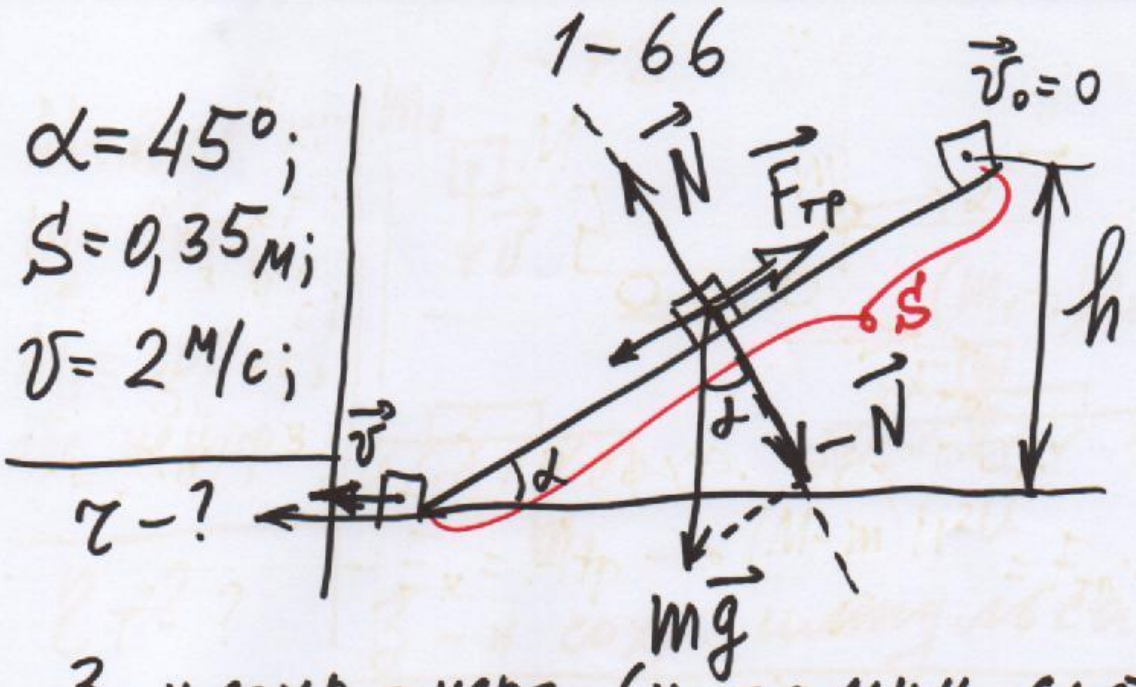
$$\begin{cases} Mu = mv_x; \\ v_x = v \cdot \cos\alpha; \end{cases} \Rightarrow Mu = mv \cdot \cos\alpha$$

$$u = \frac{mv \cdot \cos\alpha}{M} = \frac{10 \cdot 10^3 \cdot \cos 30°}{2 \cdot 10^4} =$$

$= 0{,}865 \frac{м}{с}$ (5) → (4):

$$l = \frac{0{,}865^2}{4 \cdot 9{,}81 \cdot 10^{-3}} = 1{,}907 \cdot 10 = 19{,}07 \text{ м};$$

1-66



$\alpha = 45^\circ$;
$S = 0{,}35$ м;
$v = 2$ м/с;

З-н сохр. энерг. (не замкн. сист.):

$$E_п = E_к + A_{тр} \longrightarrow mgh = \frac{mv^2}{2} + F_{тр} \cdot S; \quad (1)$$

$$N = mg \cdot \cos\alpha \longrightarrow F_{тр} = \tau N = \tau mg \cdot \cos\alpha;$$
$$h = S \cdot \sin\alpha; \quad (2)$$

(2) → (1): $mgS \cdot \sin\alpha = \frac{mv^2}{2} + \tau mgS \cdot \cos\alpha$

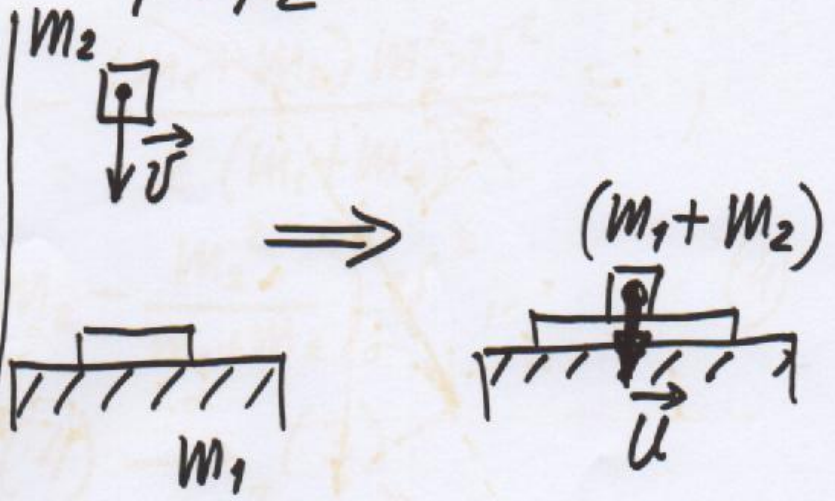
$\tau gS \cdot \cos\alpha = gS \cdot \sin\alpha - 0{,}5v^2$;

$$\underline{\tau} = \frac{gS \cdot \sin\alpha - 0{,}5v^2}{gS} = \ldots\ldots$$

1-72

Молот:
$m_2 = 8$ кг;
$m_1 = 300$ кг;
абс. неупр.

$\eta - ?$



З-н сохр. импульса:

$\underbrace{P_1}_{0} + P_2 = P \longrightarrow m_2 v = (m_1 + m_2) u$

$u = \frac{m_2 v}{m_1 + m_2}$ и $u^2 = \frac{m_2^2 v^2}{(m_1+m_2)^2}$; (1)

Полезная работа:

$A = \Delta E_к = \frac{m_2 v^2}{2} - \frac{(m_1+m_2)u^2}{2}$; (2)

КПД: $\eta = \frac{A}{E_{к1}} = \frac{A}{0{,}5 m_2 v^2} = \frac{2A}{m_2 v^2}$; (3)

(1) → (2):

$$A = \frac{m_2 v^2}{2} - \frac{(m_1 + m_2) m_2^2 v^2}{2 (m_1 + m_2)^2} =$$

$$= 0{,}5 \left(m_2 - \frac{m_2^2}{m_1 + m_2} \right) v^2 ; \qquad (4)$$

(4) → (3):

$$\eta = \frac{0{,}5 v^2 \left(m_2 - \frac{m_2^2}{m_1 + m_2} \right)}{0{,}5 m_2 v^2} = 1 - \frac{m_2}{m_1 + m_2} ;$$

$$\underline{\eta} = 1 - \frac{8}{308} = \underline{0{,}974} = 97{,}4\% ;$$

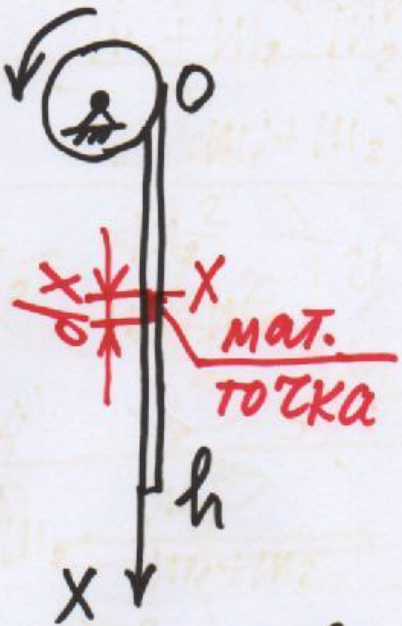
1-73

$h = 10^3$ м;

$\tilde{c} = 2 \frac{кг}{м}$;

$F_{тр} = 0$;

A - ?



На расст. $x = [0; h]$ выделим мат. точку ($dx \to 0$) массой $dm = \tilde{c} \cdot dx$ [кг]

Р-та подъёма мат. точки:

~~$dA = dm \cdot g \cdot h$~~ $dA = dm \cdot g \cdot x = \tilde{c} g x \cdot dx$

Полная р-та:

$$A = \int dA = \tilde{c} g \underbrace{\int_0^h x \cdot dx}_{x^2/2} = \frac{\tilde{c} g x^2}{2} \Big|_0^h =$$

$$= \frac{\tilde{c} g h^2}{2} - 0$$

$$\underline{\underline{A = \frac{2 \cdot 9{,}81 \cdot 10^6}{2} = 9{,}81 \cdot 10^6 Дж = 9{,}81 МДж}}$$

(9)