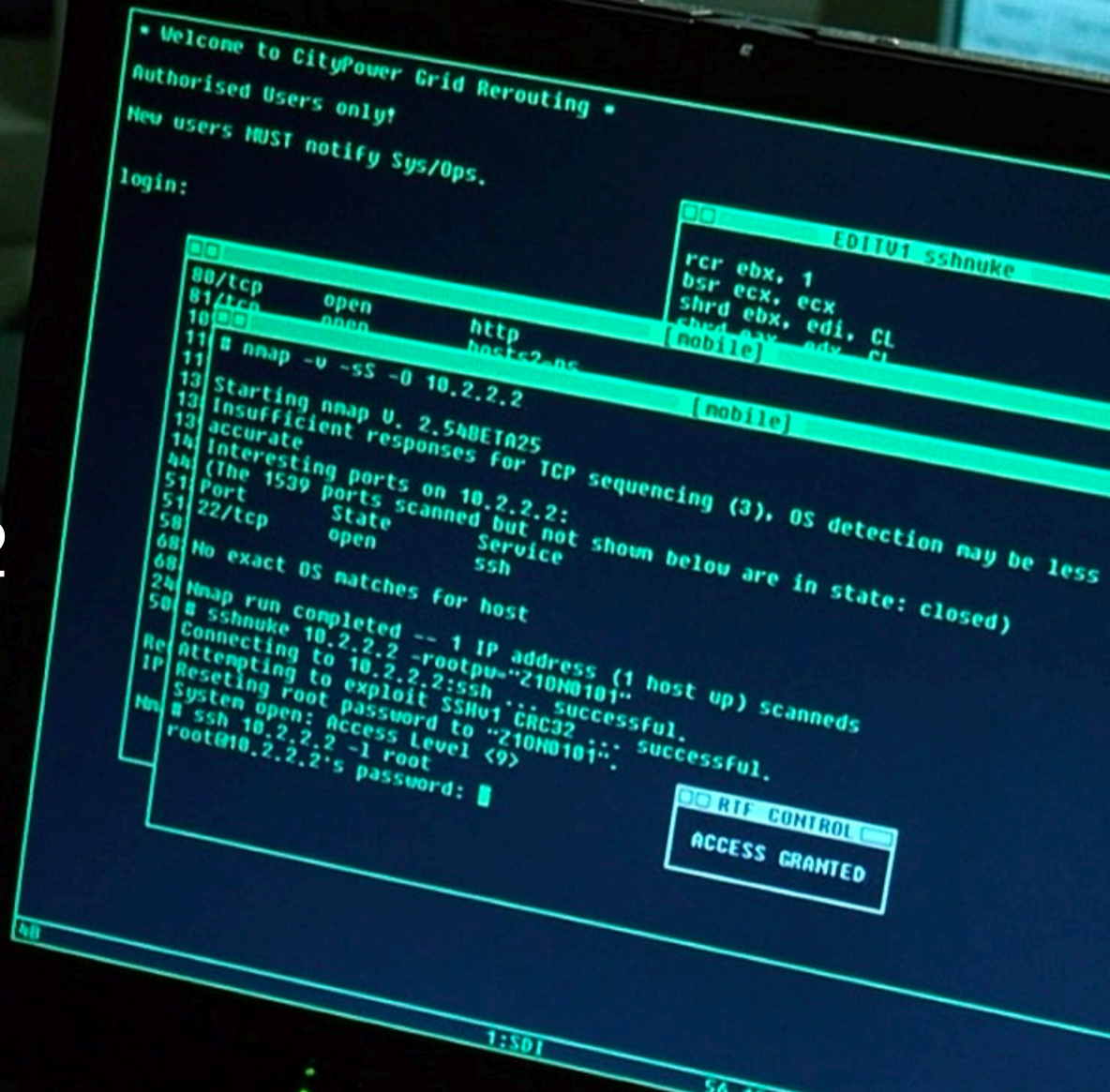


# Computer Network Security

ECE 4112/6612

CS 4262/6262

Prof. Frank Li



\* Slides adopted from Prof. Manos Antonakakis

# Logistics

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- ▶ HW1 due next Monday midnight
- ▶ For those doing the project, start forming teams + brainstorming ideas (feel free to run initial ideas by me in OHs, Piazza, etc.)

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Continuing with hashes/message digests

# Message Authentication Code (MAC)

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- ▶ Designed to provide both *authentication* and *integrity*.
- ▶ How can we use hash functions to compute a MAC?
  - ▶  $H$  is a hash function
  - ▶  $m$  is a message
  - ▶  $K$  is a secret key.
- ▶ Let's talk about different constructions
  - ▶ Secret Prefix Construction  $\Rightarrow H(K \parallel m)$
  - ▶ Secret Suffix Construction  $\Rightarrow H(m \parallel K)$
  - ▶ HMAC  $\Rightarrow H((K \oplus \text{opad}) \parallel H((K \oplus \text{ipad}) \parallel m))$

# Secret Prefix Construction

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$$\text{MAC} = H(K \parallel m)$$

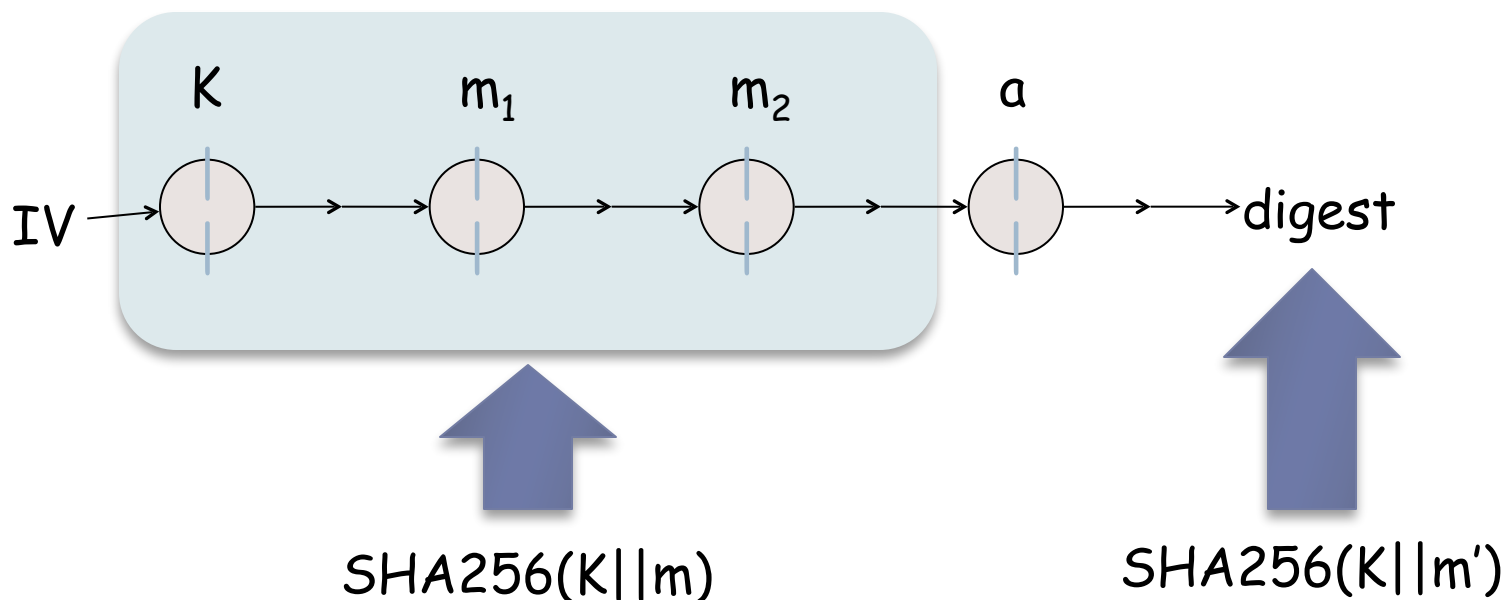
H = cryptographic hash function

K = secret key

m = message to send

# Uh oh! Length Extension Attack!

- ▶ Because most hash functions are **iterated hash functions**
  - ▶ Attacker knows the message  $m$  and  $H(K \parallel m)$
  - ▶ They could append something to  $m$  to get  $m' = m \parallel a$ , and use  $H(K \parallel m)$  to initialize the computation of  $H(K \parallel m')$



# Secret Suffix Construction

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$$\text{MAC} = H(m \parallel K)$$

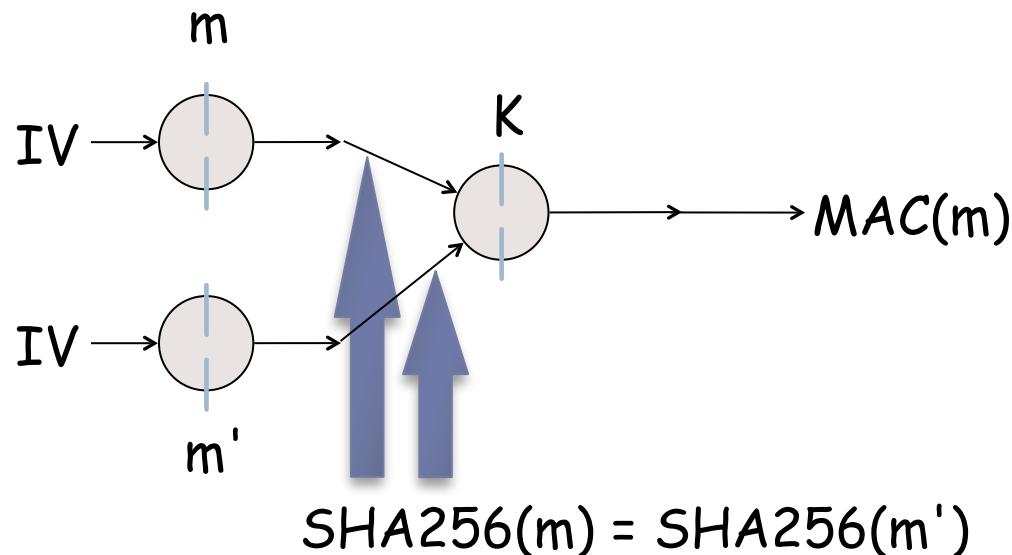
H = cryptographic hash function

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m = message to send

# Secret Suffix Construction

- ▶ Better! But subject to weakness in the hash function.
  - ▶ Say the attacker can find a hash collision with the original message  $m$ , so an  $m'$  where  $H(m') = H(m)$ .
  - ▶ If  $H$  is an iterated hash function,  $\text{MAC}(m') = \text{MAC}(m)$
  - ▶ Even w/o the secret key, attacker can find MAC collisions.





# HMACs

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$$\text{HMAC} = H(K \parallel H(K \parallel m))$$

H = cryptographic hash function

K = secret key

m = message to send

# HMAC

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$$\text{HMAC} = H(K \parallel H(K \parallel m))$$

- ▶ Inner + outer layers of hashing is much stronger!
- ▶ Resistant to length extension attack
- ▶ Technically can use same key, but in practice, slightly alters the keys

$$\text{HMAC} = H((K \oplus \textit{opad}) \parallel H((K \oplus \textit{ipad}) \parallel m))$$

where *ipad* and *opad* are standardized values

# Public Key Cryptography

# Public Key Cryptography

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## Symmetric Key Crypto

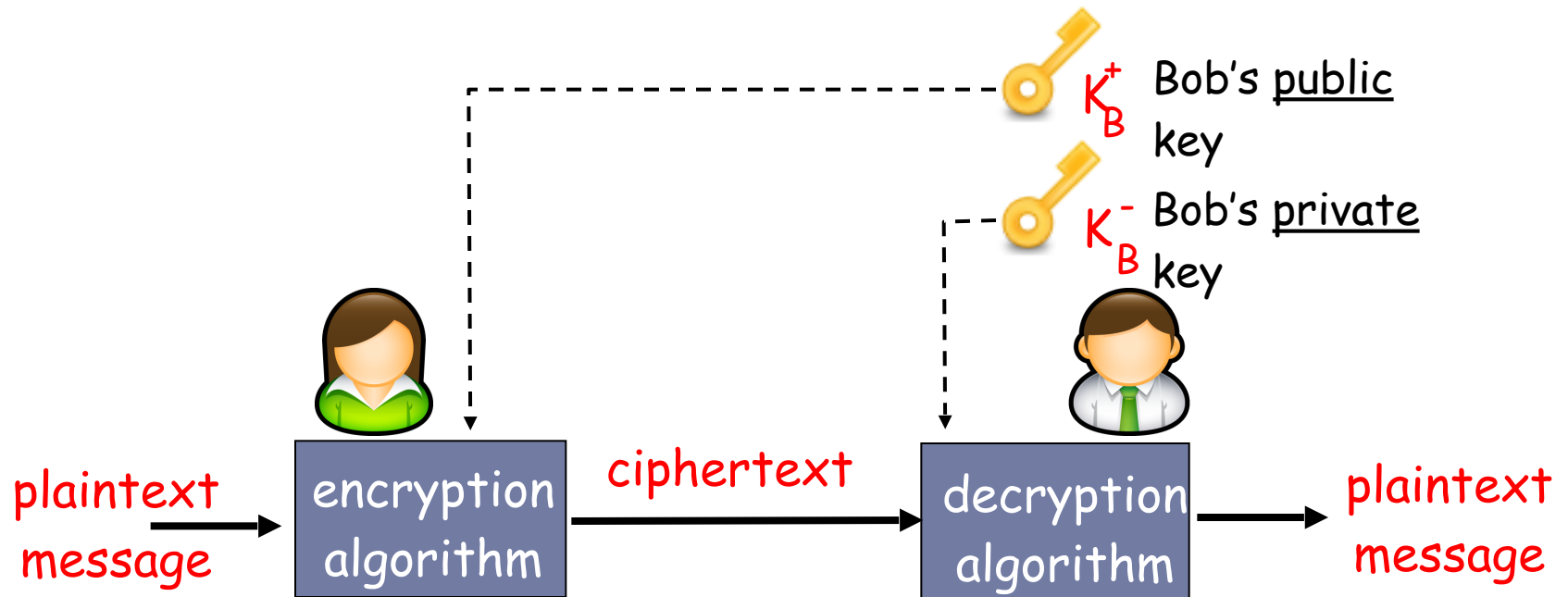
- ▶ Requires sender and receiver know shared secret key
- ▶ What's the challenge?
  - ▶ Key distribution is hard and fraught with peril!

## Public Key Cryptography

- ▶ Radically different approach [Diffie-Hellman76, RSA78]
- ▶ Sender and receiver do not share secret key
  - ▶ Receiver has a public encryption key known to all
  - ▶ Receiver has a private decryption it keeps secret

# Public Key Cryptography

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# Computational Hardness

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- ▶ **Why do we care about computational hardness?**
  - ▶ Hard computational problems are cornerstone of modern cryptography.
  - ▶ Computational hardness will be the basis of the security for the two public key algorithms we discuss.

# The Discrete Logarithm Problem (DLP)

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Given  $x$  where  $x \equiv g^y \pmod{p}$ , find  $y$ .  
 $g$  is a number within group  $Z_p^*$ , for prime  $p$ .

- ▶ What is  $Z_p^*$ ?
  - ▶  $Z_p^*$  is the multiplicative group of integers modulo prime  $p$ .
    - ▶ It contains the set of all integers from 1 to  $p-1$ , so  $\{1, 2, \dots, p-1\}$ .
    - ▶ Only operation permitted is multiplying group members mod  $p$ .
  - ▶ *Group axioms* (required properties) include closure, associativity, identity existence, and inverse existence
- ▶ Is DLP NP-Complete?
  - ▶ This problem is NP and seems difficult but probably not NP-complete.

# Diffie-Hellman (DH) Key Exchange

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- ▶ First developed public key cryptosystem
  - ▶ Useful to perform key exchange when communication channel is not private
- ▶ How does it work?
  1. Two prime numbers **g** and **p** are publicly known (e.g., standard values).
  2. Alice picks a secret number **a**, computes  **$g^a \bmod p = A$** , and sends **A** to Bob.
  3. Bob picks a secret number **b**, computes  **$g^b \bmod p = B$** , and sends **B** to Alice.
  4. Alice takes her secret number **a**, and the **B** received from Bob, and computes the shared secret key as:  **$B^a \bmod p = g^{ab} \bmod p$** .
  5. Bob takes his secret number **b**, and the **A** received from Alice, and computes the shared secret key as:  **$A^b \bmod p = g^{ab} \bmod p$** .

$$\begin{aligned}(g^a \bmod p)^b \bmod p &= g^{ab} \bmod p \\ (g^b \bmod p)^a \bmod p &= g^{ba} \bmod p\end{aligned}$$



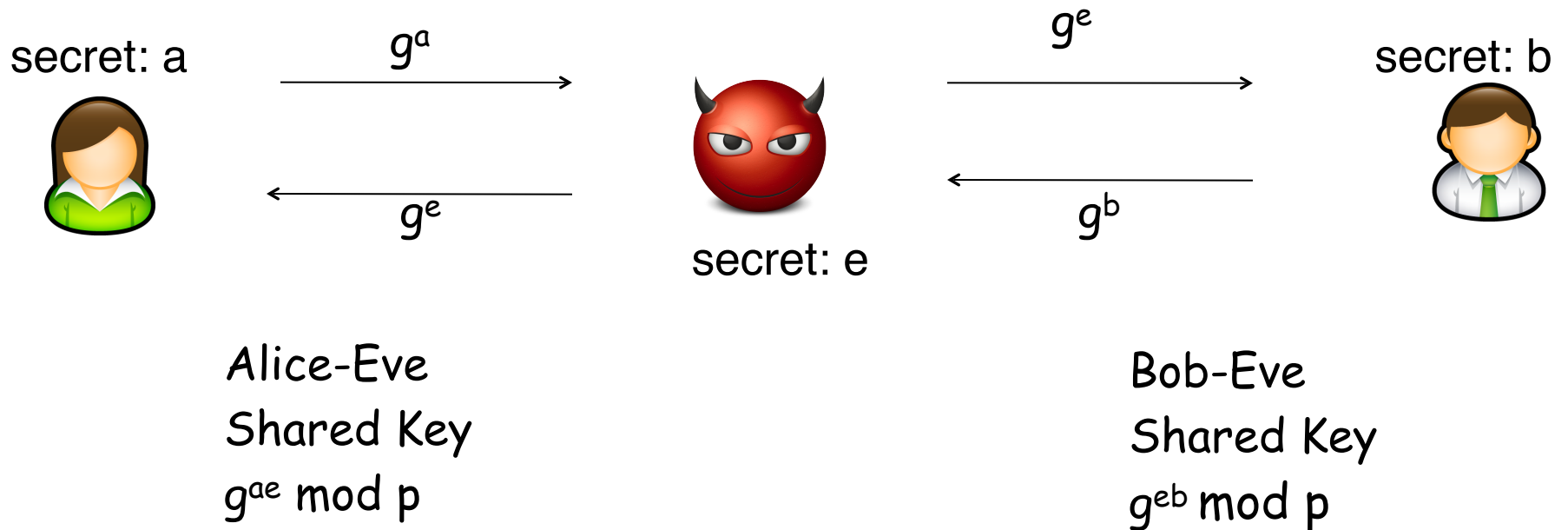
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  4. Alice takes her secret number  $a$ , and the  $B$  received from Bob, and computes the shared secret key as:  $B^a \bmod p = g^{ab} \bmod p$ .
  5. Bob takes his secret number  $b$ , and the  $A$  received from Alice, and computes the shared secret key as:  $A^b \bmod p = g^{ab} \bmod p$ .
- ▶ Both parties now have the same secret key, and note that the secret key itself was not directly sent (i.e.,  $g^{ab} \bmod p$  was never directly sent)
- ▶ Figuring out  $a$  from  $A = g^a \bmod p$  would require solving DLP (same for  $b$ ).

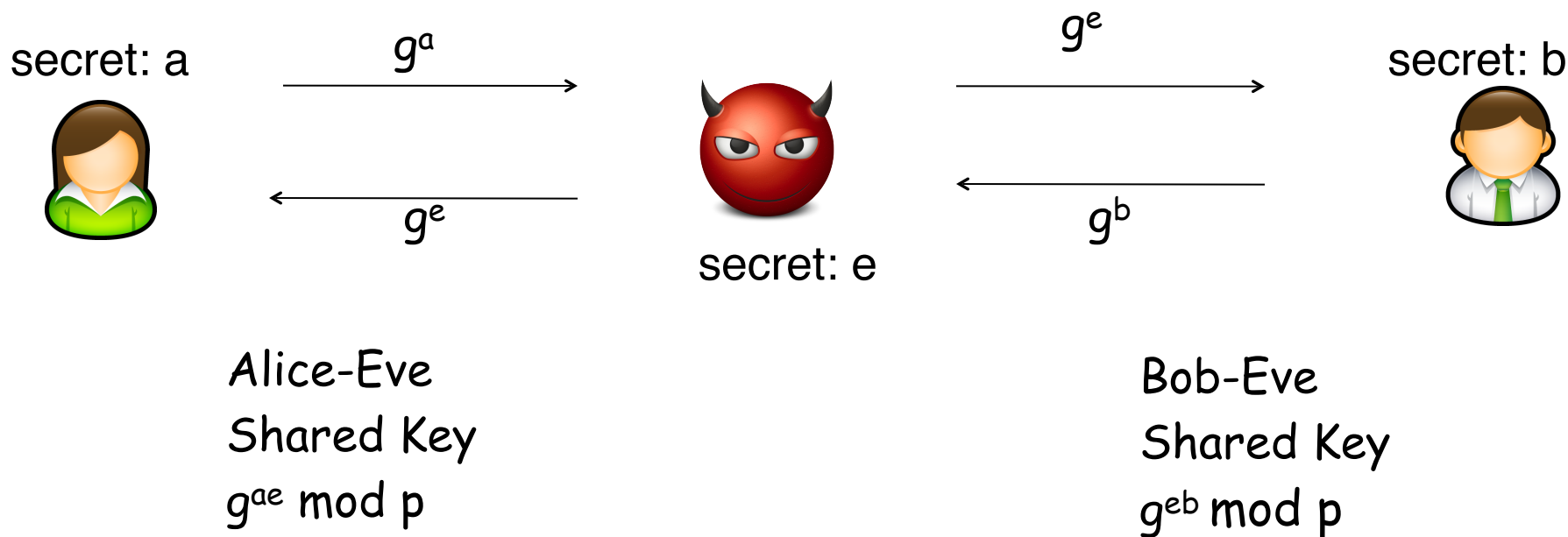
# DH: Man-in-the-Middle Attack

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# DH: Man-in-the-Middle Attack

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Need some way to verify that the public key Bob receives is truly from Alice (and vice versa). How to solve? (End of today's lecture)

# The Factoring Problem

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Given a large number  $N = p \times q$ , find the prime factors  $p$  and  $q$

- ▶ **Factoring Large Numbers in Practice**
  - ▶ The fastest factoring algorithm for large numbers is the general number field sieve (GNFS).
  - ▶ GNFS complexity is (roughly)  $= \exp(1.92x(\ln n)^{1/3}(\ln \ln n)^{2/3})$
- ▶ **Is Factoring NP-Complete?**
  - ▶ Factoring is NP and "seems hard to solve" (i.e., we don't know how to do so efficiently).
  - ▶ However, probably not NP-complete, but no proof for that.

# RSA Overview

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- ▶ A message is a bit pattern.
- ▶ A bit pattern can be uniquely represented by an integer number.
- ▶ Thus encrypting a message is equivalent to encrypting a number.

## Example

- ▶  $m = 10010001 = 145$ 
  - ▶ This message is uniquely represented by the decimal number 145.
  - ▶ To encrypt  $m$ , we encrypt the corresponding number, which gives a new number (the ciphertext).

# RSA: Creating Public/Private Key Pair

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1. Choose two large prime numbers  $p, q$ .  
(e.g., 1024 bits each)
2. Compute  $n = pq$ ,  $z = (p-1)(q-1)$
3. Choose  $e$  (with  $e < n$ ) that has no common factors with  $z$ . ( $e, z$  are "relatively prime").
4. Choose  $d$  such that  $ed-1$  is exactly divisible by  $z$ .  
(in other words:  $ed \bmod z = 1$  ).
5. Public key is  $(n, e)$ . Private key is  $(n, d)$ .

# RSA: Encryption and Decryption

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public      private

0. Given  $(n,e)$  and  $(n,d)$  as computed above
1. To encrypt message  $m (<n)$ , compute
$$c = m^e \bmod n$$
2. To decrypt received bit pattern,  $c$ , compute
$$m = c^d \bmod n$$

Magic  
happens!

$$m = \underbrace{(m^e \bmod n)}_c^d \bmod n$$

# RSA: Example

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Bob chooses  $p=5$ ,  $q=7$ . Then  $n=35$ ,  $z=24$ .

$e=5$  (so  $e$ ,  $z$  relatively prime).

$d=29$  (so  $ed-1$  exactly divisible by  $z$ )

$ed = 145 \equiv 1 \pmod{z}$  (b/c  $144/24=6$ )

Encrypting 8-bit messages.

	<u>bit pattern</u>	<u><math>m</math></u>	<u><math>m^e</math></u>	<u><math>c = m^e \pmod{n}</math></u>
encrypt:	00001000	12	24832	17

	<u><math>c</math></u>	<u><math>c^d</math></u>	<u><math>m = c^d \pmod{n}</math></u>
decrypt:	17	481968572106750915091411825223071697	12



# RSA

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- ▶ **Why is this secure?**
  - ▶ Suppose you know Bob's public key  $(n,e)$ . It's really hard to determine  $d$  without knowing the factors of  $n$  (factoring is hard!)
- ▶ **How to handle large messages?**
  - ▶ Combine RSA with symmetric crypto.
  - ▶ Use RSA to securely transmit a symmetric key, then use symmetric cipher to encrypt message.
  - ▶ RSA is slow anyways, AES is fast

# Digital Signature

## Simple digital signature for message $m$ :

- ▶ Bob signs  $m$  by encrypting with his private key  $K_B^-$ , creating “signed” message,  $K_B^-(m)$ . (Could use RSA)

Bob's message,  $m$

Dear Alice

Oh, how I have missed you. I think of you all the time! ... (blah blah blah)

Bob



$K_B^-$

Bob's private key

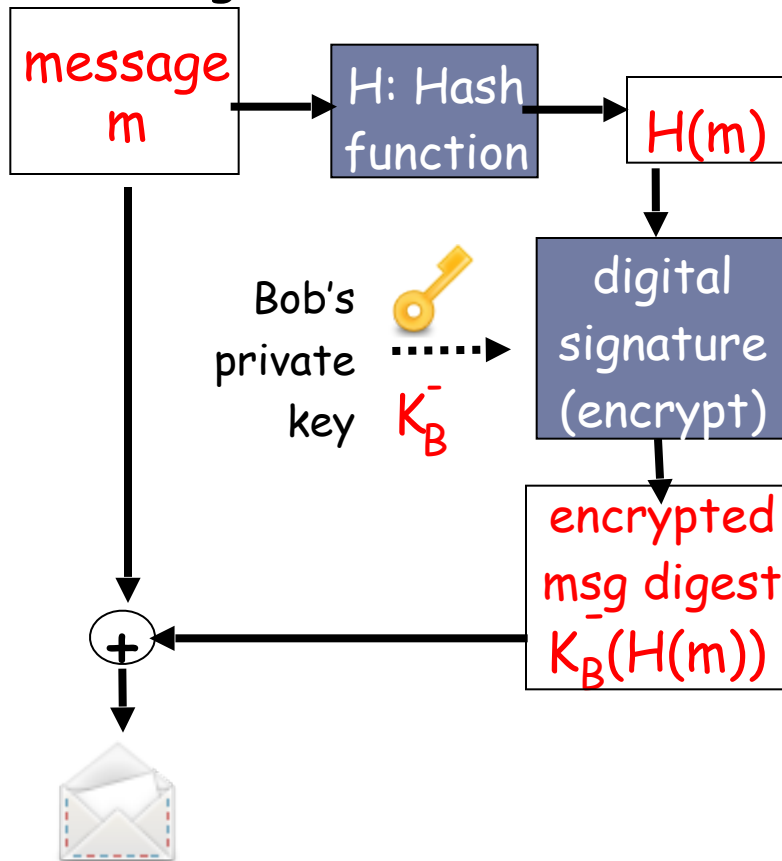
Public key encryption algorithm

$K_B^-(m)$

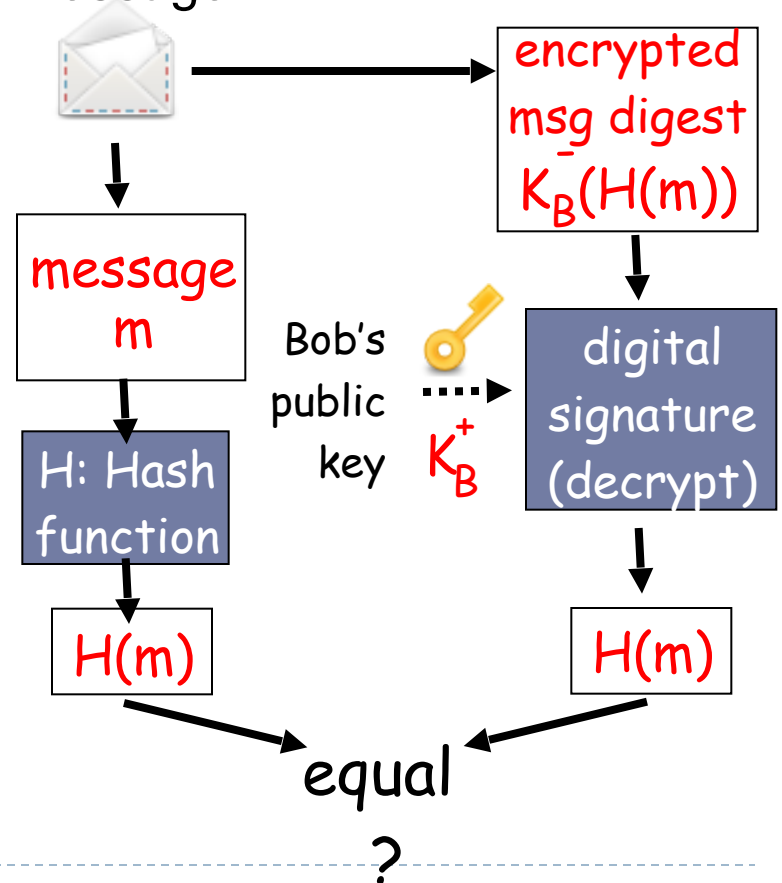
Bob's message,  $m$ , signed (encrypted) with his private key

# Digital Signature with Hash

Bob sends digitally signed message:



Alice verifies signature and integrity of digitally signed message:



# Digital Signatures Summary

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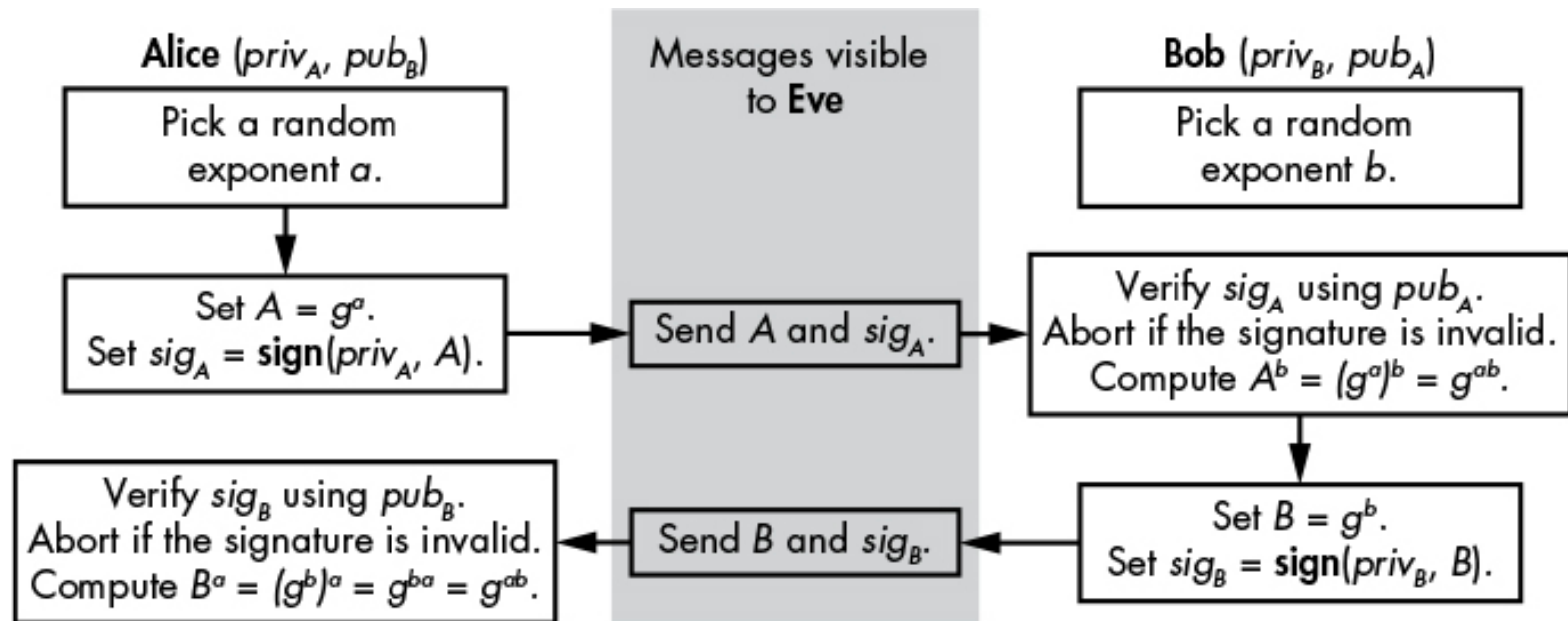
What are the general steps?

1. Sender signs a message with private key.
2. Receiver verifies message with public key.

What does this get us?

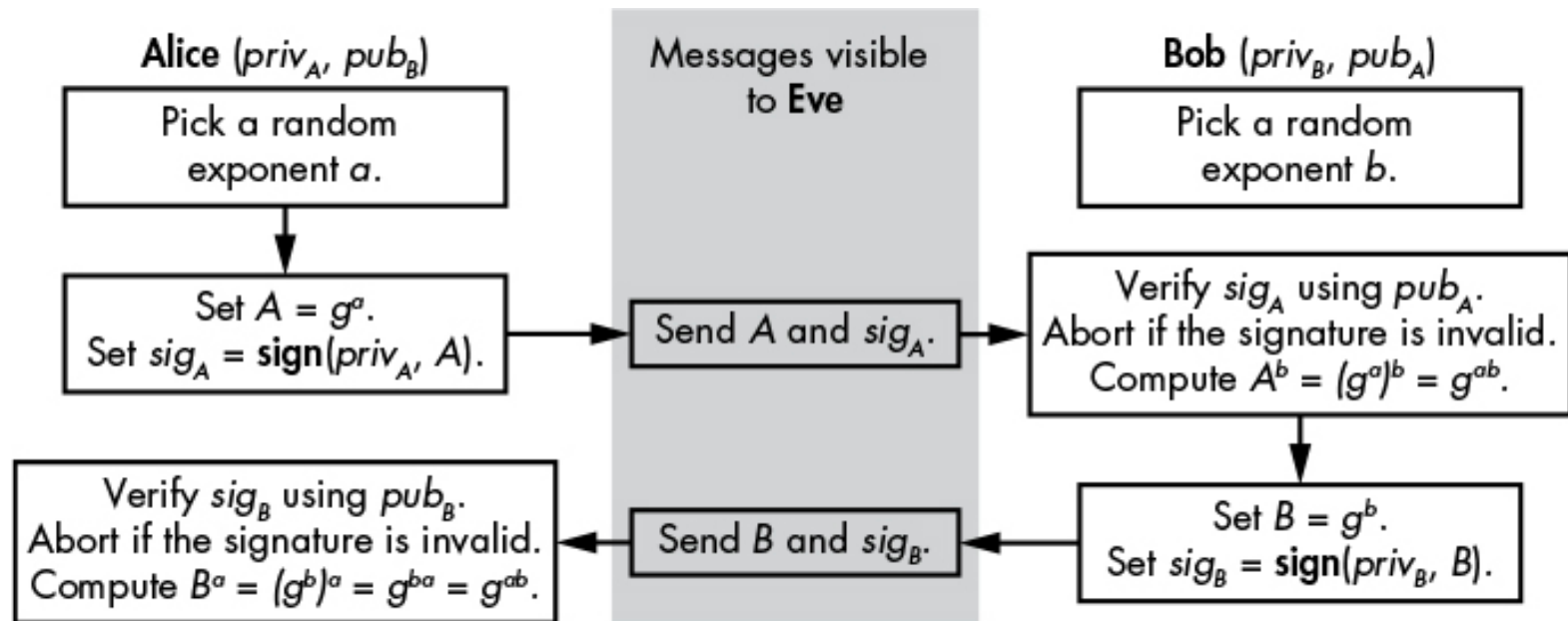
1. **Verifiability**
  - ➡ Receiver can verify sender signed message (m).
  - ➡ No one else could have signed message (m).
2. **Non-repudiation:**
  - ➡ Sender cannot claim to have signed  $m'$  and not  $m$ .

# Back to Diffie-Hellman: Man-in-the-Middle Defense



The authenticated Diffie-Hellman protocol

# Back to Diffie-Hellman: Man-in-the-Middle Defense



The authenticated Diffie-Hellman protocol

Let's say Eve recorded all traffic b/w Alice and Bob. After Alice and Bob finish communicating, they "forget"  $a$  and  $b$  (they are ephemeral/short-lived keys). **Perfect Forward Secrecy:** Even if Alice or Bob's private key is leaked, Eve still can't decrypt the recorded traffic (w/o solving the DLP).

# Putting it all together

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Use public-key crypto to establish symmetric keys:

- RSA encryption to secretly agree on symmetric key
- Diffie-Hellman key-exchange with RSA signatures to prevent MITM attack, provides perfect forward secrecy

With the symmetric key:

- Encrypt/decrypt messages using AES in some secure block cipher mode (e.g., CBC, CTR)
- Use HMAC (which relies on a secret key) to achieve integrity + authentication

General recommendation: use different keys for different algorithms

- different RSA key pair for signing and for encrypting
- different secret keys for AES and HMACs