

Assignment - 6

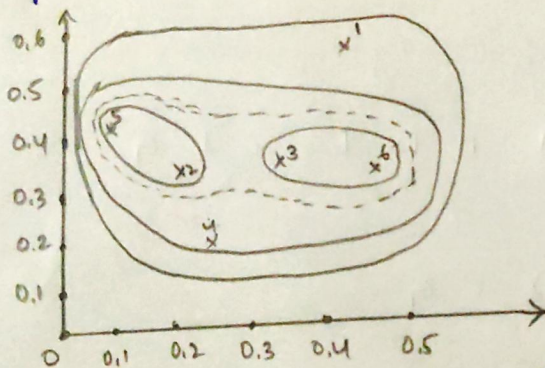
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- 1) Calculate the clustering representation and dendrogram using single, complete and Average link priority function in hierarchical clustering technique.
- a) Single link priority function.

Given below in the table with euclidean distance between each individual points.

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	0					
P_2	0.2357	0				
P_3	0.2218	0.1483	0			
P_4	0.3688	0.2042	0.1513	0		
P_5	0.3421	0.1398	0.2842	0.2932	0	
P_6	0.2347	0.2540	0.1100	0.2216	0.3921	0

Considering lower bound values, since upper bound value are equal to lower bound and finding the cluster according to that.



Graphical representation of the given six points.

after find the euclidean distance b/w each individual points, next step is Merging the two closest members and updating the distance table.

- ① from the above table min value is 0.1100 is between P_3 and P_6
after merging two member we need to find the distance with other member using the formula $\min[\text{dist}(\text{Cluster}, \text{other points})]$

$$\begin{aligned}\Rightarrow \text{Distance with } P_1 &\Rightarrow \min[\text{dist}(P_3 P_6), (P_1, P_3)] \\ &= \min(\text{dist}(P_3 P_1), (P_6 P_1)) \\ &= \min(0.2218, 0.2347) \\ &= 0.2218.\end{aligned}$$

$$\begin{aligned}\text{Distance with } P_2 &= \min[\text{dist}(P_3 P_6), P_2] \\ &= \min(\text{dist}(P_3 P_2), (P_6 P_2)) \\ &= \min(0.1483, 0.2540) \\ &= 0.1483\end{aligned}$$

$$\begin{aligned}\text{Distance with } P_4 &= \min[\text{dist}(P_3 P_6), P_4] \\ &= \min((P_3 P_4), (P_6 P_4)) \\ &= \min(0.1513, 0.2216) \\ &= 0.1513\end{aligned}$$

$$\begin{aligned}\text{Distance with } P_5 &= \min[\text{dist}(P_3 P_6), P_5] \\ &= \min((P_3 P_5), (P_6 P_5)) \\ &= \min(0.2843, 0.3921) \\ &= 0.2843\end{aligned}$$

So updated distance table after merging P_3 and P_6 is

	P_1	P_2	$P_3 P_6$	P_4	P_5
P_1	0				
P_2	0.2357	0			
$P_3 P_6$	0.2218	0.1483	0		
P_4	0.2688	0.2042	0.1513	0	
P_5	0.3421	<u>0.1388</u>	0.2843	0.2932	0

② from the above table which is updated 0.1388 is the min value then join P_2 and P_5

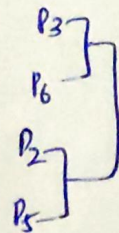
$$\begin{aligned}\text{Distance of } (P_2 P_5) \text{ with } P_1 &= \min(\text{dist}((P_2 P_5) P_1)) \\ &= \min((P_2 P_1), (P_5 P_1)) \\ &= \min(0.2357, 0.3421) \\ &= 0.2357 \Rightarrow P_1\end{aligned}$$

$$\begin{aligned}\text{Distance of } (P_2 P_5) \text{ with } P_3 P_6 &= \min(\text{dist}((P_2 P_5) (P_3 P_6))) = \min((P_2 P_3 P_6), (P_5 P_3 P_6)) \\ &= 0.1483, 0.2843 \\ &= 0.1483 \Rightarrow (P_3 P_6)\end{aligned}$$

$$\begin{aligned}\text{Distance of } (P_2 P_5) \text{ with } P_4 &= \min(\text{dist}((P_2 P_5) P_4)) = \min(\text{dist}((P_2 P_4), (P_5 P_4))) \\ &= (0.2042, 0.2932) = 0.2042 = P_4\end{aligned}$$

updated distance table after merge P_2 with B is

	P_1	$P_2 P_5$	$P_3 P_6$	P_4
P_1	0			
$P_2 P_5$	0.2357	0		
$P_3 P_6$	0.2218	0.1483 min	0	
P_4	0.3688	0.2042	0.1513	0



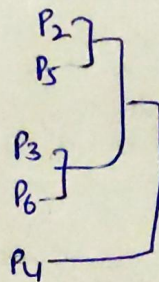
Since the min value is 0.1483 merge $(P_2 P_5)$ and $(P_3 P_6)$ and update distance matrix

$$\begin{aligned} \text{Distance of } (P_2 P_5)(P_3 P_6) \text{ with } P_1 &= \min(\text{dist}((P_2 P_5)(P_3 P_6)P_1)) \\ &= \min(\text{dist}((P_2 P_5)P_1), \text{dist}((P_3 P_6)P_1)) \\ &= \min(0.2357, 0.2218) \\ &= 0.2218 \rightarrow P_1 \end{aligned}$$

$$\begin{aligned} \text{Distance of } (P_2 P_5)(P_3 P_6) \text{ with } P_4 &= \min(\text{dist}((P_2 P_5)(P_3 P_6)P_4)) \\ &= \min(\text{dist}((P_2 P_5)P_4), \text{dist}((P_3 P_6)P_4)) \\ &= \min(0.2042, 0.1513) \\ &= 0.1513 \rightarrow P_4 \end{aligned}$$

So, the updated distance table is as shown below

	P_1	$P_2 P_5 P_3 P_6$	P_4
P_1	0		
$P_2 P_5 P_3 P_6$	0.2218	0	
P_4	0.3688	0.1513 min	0



Merge P_4 and P_2, P_5, P_3, P_6 and update the distance table

$$\begin{aligned} \text{Distance with } P_1 &= \min[\text{dist}((P_2 P_5 P_3 P_6)P_1)] \\ &= \min[\text{dist}((P_2 P_5 P_3 P_6)P_1), \text{dist}(P_4 P_1)] \\ &= (0.2218, 0.3688) = 0.2218 \end{aligned}$$

	P_1	$P_2 P_5 P_3 P_6 P_4$
P_1	0	
$P_2 P_5 P_3 P_6 P_4$	0.2218 min	0

$$\begin{aligned} \text{Now min } &(\text{dist}((P_2 P_5 P_3 P_6 P_4)P_1)) \\ &= 0.2218 \end{aligned}$$

So final cluster is shown in figure 1 and below is the dendrogram

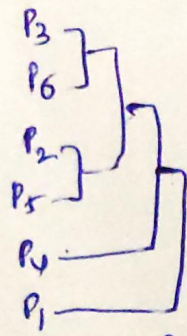


fig 2

② complete link proximity function.

For complete link proximity we need merge the closest point and update the distance table with below formula accordingly to below formula.

$$\text{Distance} = \max(\text{dist}(\text{clustered point}, \text{points}))$$

Graphical representation of given point

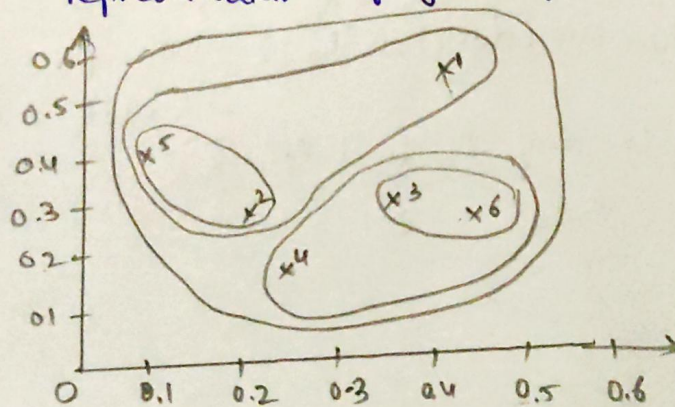


fig 1

from the table 1 min value is 0.1100, i.e. merge P_3 and P_6 and update the distance table.

$$\begin{aligned} \text{Distance with } P_1 &\Rightarrow \max(\text{dist}(P_3 P_6) P_1) = \max(\text{dist}(P_3 P_1) (P_6 P_1)) \\ &= \max(0.2218, 0.2347) = 0.2347 \end{aligned}$$

$$\begin{aligned} \text{Distance with } P_2 &\Rightarrow \max(\text{dist}(P_3 P_6) P_2) = \max(\text{dist}(P_3 P_2) (P_6 P_2)) \\ &= \max(0.1483, 0.2540) = 0.2540 \end{aligned}$$

$$\text{Distance with } P_4 = \max(\text{dist}(P_3 P_6) P_4) = \max(0.1513, 0.2216) = 0.2216$$

$$\text{Distance with } P_5 = \max(\text{dist}(P_3 P_6) P_5) = \max(0.2843, 0.3921) = 0.3921$$

So, the updated distance table after merging P_3 and P_6 is.

	P_1	P_2	$P_3 P_6$	P_4	P_5	
P_1	0					P_2 P_6
P_2	0.2357	0				P_3 P_5
$P_3 P_6$	0.2347	0.2500	0			
P_4	0.3688	0.2042	0.2216	0		
P_5	0.3421	<u>0.1358</u> min	0.3921	0.2932	0	

min value is 0.1358 merge P_2, P_5 and update the distance table.

$$\text{Distance of } (P_2 P_5) \text{ with } P_1 = \max(\text{dist}((P_2 P_5) P_1)) = \max(0.2357, 0.3421) = 0.3421 \rightarrow P_1$$

$$\text{Distance of } (P_2 P_5) \text{ with } P_3 P_6 \Rightarrow \max(\text{dist}((P_2 P_5) (P_3 P_6))) = \max(0.2500, 0.3921) = 0.3921 \rightarrow P_3 P_6$$

$$\text{Distance of } (P_2 P_5) \text{ with } P_4 = \max(\text{dist}((P_2 P_5) P_4)) = \max(0.2042, 0.2932) = 0.2932 \rightarrow P_4$$

updated distance table after merging P_2 and P_5 is

	P_1	$P_2 P_5$	$P_3 P_6$	P_4	
P_1	0				
$P_2 P_5$	0.3421	0			
$P_3 P_6$	0.2347	0.3921	0		
P_4	0.3688	0.2932	<u>0.2216</u> min	0	

$$\text{Distance of } P_4 (P_3 P_6) \text{ with } P_1 = \max(\text{dist}((P_3 P_6) P_1)) = \max(0.2347, 0.3688) = 0.3688 \Rightarrow P_1$$

$$\text{Distance with } P_2 P_5 = \max((P_4 (P_3 P_6)) (P_2 P_5)) = \max(0.2932, 0.3921) = 0.3921 \Rightarrow P_2 P_5$$

	P_1	$P_2 P_5$	$P_3 P_6 P_4$
P_1	0		
$P_2 P_5$	0.3421	0	
$P_3 P_6 P_4$	<u>0.3688</u> min	0.3921	0

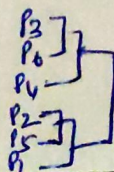
merge $P_2 P_5$ and P_1

$$\text{Distance with } P_3 P_6 P_4 = \max((P_1 (P_2 P_5)) (P_3 P_6 P_4)) = \max(0.3688, 0.3921) = 0.3921$$

Final table is

	$P_2 P_5 P_1$	$P_3 P_6 P_4$
$P_2 P_5 P_1$	0	
$P_3 P_6 P_4$	0.3921	0

Final clustering is shown in fig ① & ② and dendrogram is



The minimum value in lower bound 0.1100 b/w P_3 & P_6
 first cluster $\boxed{3 \text{ } 6}$

The distance matrix is $\text{AVG}[\text{dist}(P_3, P_6), P_1] = \text{dis}((P_3, P_6), P_1)$
 $= \frac{1}{2} (\text{dis}(P_3, P_1) + \text{dis}(P_6, P_1))$
 $= \frac{1}{2} (0.2218 + 0.2247)$
 $= 0.2232$

The distance matrix is, $\text{AVG}[\text{dist}(P_3, P_6), P_2] = \text{dis}((P_3, P_6), P_2)$
 $= \frac{1}{2} [\text{dis}(P_3, P_2) + \text{dis}(P_6, P_2)]$
 $= \frac{1}{2} (0.1482 + 0.2540)$
 $= 0.2011$

The distance matrix is $\text{AVG}[\text{dis}(P_3, P_6), P_4] = \text{dis}((P_3, P_6), P_4)$
 $= \frac{1}{2} [\text{dis}(P_3, P_4) + \text{dis}(P_6, P_4)]$
 $= \frac{1}{2} (0.1513 + 0.2216)$
 $= 0.1864$

The distance matrix is, $\text{AVG}[\text{dist}(P_3, P_6), P_5] = \text{dis}((P_3, P_6), P_5)$
 $= \frac{1}{2} (\text{dis}(P_3, P_5) + \text{dis}(P_6, P_5))$
 $= \frac{1}{2} (0.2843 + 0.2921)$
 $= 0.3382$

The updated distance matrix for cluster (P_3, P_6)

	P_1	P_2	P_3, P_6	P_4	P_5
P_1	0.0000				
P_2	0.2357	0			
P_3, P_6	0.2282	0.2011	0		
P_4	0.3688	0.2042	0.1864	0	
P_5	0.3421	0.1388	0.3382	0.2932	0

The min value is 0.1388 b/w P_2 & P_5

Second cluster $\boxed{2 \text{ } 5}$

To update the distance matrix, $\text{AVG}[\text{dist}(P_2, P_5), P_1]$
 $\text{dis}((P_2, P_5), P_1) = \frac{1}{2} (\text{dis}(P_2, P_1) + \text{dis}(P_5, P_1)) = \frac{1}{2} (0.2357 + 0.3421)$
 $= 0.2889$

To update the distance matrix, $\text{AVG}[\text{dist}((P_2, P_5), (P_3, P_6))]$
 $\text{dis}((P_2, P_5), (P_3, P_6)) = \frac{1}{2} (\text{dis}(P_2, (P_3, P_6)) + \text{dis}(P_5, (P_3, P_6)))$
 $= \frac{1}{2} (0.2011 + 0.3382) = 0.2696$

To update the distance matrix, $\text{AVG}[\text{dist}(P_2, P_5), P_4]$
 $\text{dis}((P_2, P_5), P_4) = \frac{1}{2} (\text{dis}(P_2, P_4) + \text{dis}(P_5, P_4)) = \frac{1}{2} (0.2042 + 0.2932)$
 $= 0.2487$

The updated distance matrix for cluster (P_2, P_5)

	P_1	P_2, P_5	P_3, P_6	P_4
P_1	0			
P_2, P_5	0.2889	0		
P_3, P_6	0.2282	0.2696	0	
P_4	0.3688	0.2487	0.1864	0

The min value in lower bound is 0.1864 b/w P_4 & P_3, P_6
third cluster



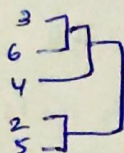
To update the distance matrix, $AVG (dist(P_2, P_6, P_4), P_1) = dist((P_2, P_6, P_4), P_1)$
 $= \frac{1}{2} [dist((P_2, P_6), P_1) + (P_4, P_1)]$
 $= \frac{1}{2} [(0.2282) + (0.3688)]$
 $= 0.2985$

To update the distance matrix, $AVG (dist(P_2, P_6, P_4), (P_2, P_5))$
 $dist((P_2, P_6, P_4), (P_2, P_5)) = \frac{1}{2} [dist((P_3, P_6), (P_2, P_5)) + (P_4, (P_2, P_5))]$
 $= \frac{1}{2} (0.2696 + 0.2487) = 0.2591$

updated matrix for cluster $(P_4, (P_3, P_6))$

	P_1	P_2, P_5	P_3, P_6, P_4
P_1	0		
P_2, P_5	0.2889	0	
P_3, P_6, P_4	0.2985	0.2591	0

The min value is 0.2593 b/w P_2, P_5 & P_3, P_6, P_4
fourth cluster.



To update the distance matrix, $AVG (dist((P_3, P_6, P_4), (P_2, P_5)), P_1)$
 $dist((P_3, P_6, P_4), (P_2, P_5)) \text{ with } P_1 = \frac{1}{2} [dist((P_3, P_6, P_4), P_1) + ((P_2, P_5), P_1)]$
 $= \frac{1}{2} (0.2985 + 0.2889)$
 $= 0.2937$

	P_1	P_2, P_5, P_3, P_6, P_4
P_1	0	
P_2, P_5, P_3, P_6, P_4	0.2937	0

The last cluster P_2, P_5, P_3, P_6, P_4 with P_1

