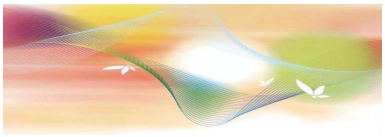
§ 4.2 模糊计算

- 模糊集理论
- 模糊集合的并、交、补运算
- 模糊逻辑
- 模糊控制系统
- 模糊计算局限性



集合的定义

- Definition of a "set"

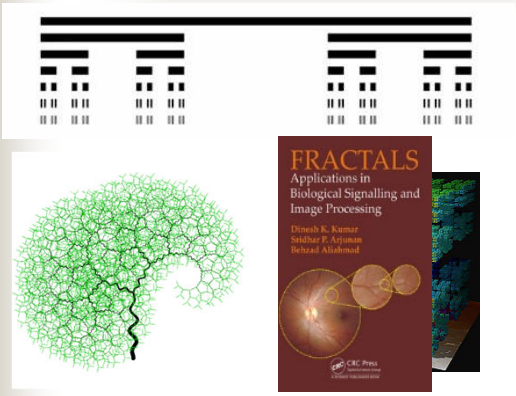
By a set we understand every collection made into a whole of definite, distinct objects of our intuition or of our thought.

For a set in Cantor's sense, the following properties hold:

- $x \neq \{x\}$.
- If $x \in X$ and $X \in Y$, then $x \notin Y$.
- The set of all subsets of X is denoted as 2^x
- $\emptyset$ is the empty set and thus very important.

Georg Cantor (1845-1918)

Cantor Set



集合的定义

Three basic methods to define sets:

- The list method: a set is defined by naming all its members

$$A = \{a_1, a_2, ..., a_n\}$$

- The rule method: a set is defined by a property satisfied by its members

$$A = \{x \mid P(x)\}$$

where '|' denotes the phrase "such that"

$P(x)$: a proposition of the form "x has the property P"

- The characteristic function: A set is defined by a characteristic function

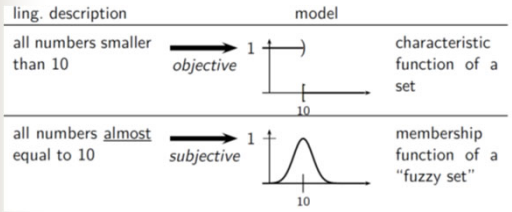
$$\chi_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

the characteristic function

$$\chi_A : X \to \{0,1\}$$

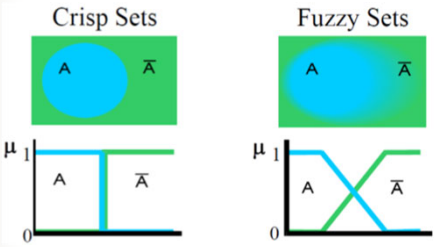
Fuzzy set

- Extension to a fuzzy set



Fuzzy set

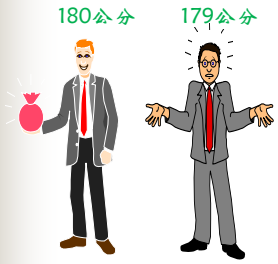
- Crisp set vs. fuzzy set

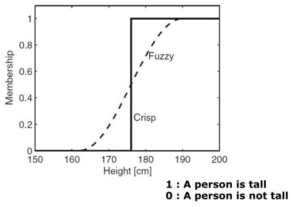


μ-membership degree, possibility distribution, grade of belonging.

Fuzzy set

- 用模糊来调和对立



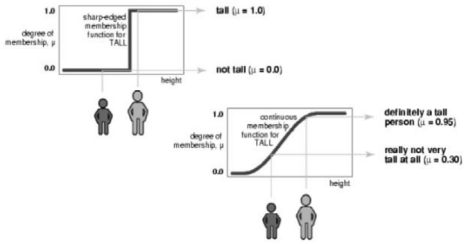


如果高的定义是由这样的*隶属函数*来定义的话，179公分已经相当高了！

Fuzzy set

- Crisp set vs. fuzzy set

What Is Lost.....

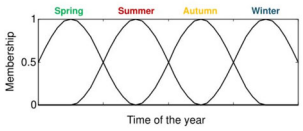


模糊集理论

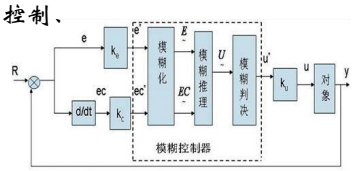
1.模糊集理论

- 扎德1965年提出

Fuzzy sets



- 是模糊计算的数学基础
- 广泛应用在推理、控制、决策等领域



Example

Example

所有“大苹果”看作是一个集合，那么“大苹果”就是一个*模糊集合*，因为没有确切的定义什么样的苹果叫做大苹果。如果认为3两以上的苹果算是绝对的大苹果，也就是说3两以上的苹果属于“大苹果”的程度为1，那么2.9两的苹果属于“大苹果”的程度可以是0.9左右，2.8两的苹果是0.8。这种属于程度就称为*隶属度函数*，其值在0～1之间连续变化。

Example

若用A来表示模糊集合“大苹果”，用μ来表示*隶属度函数*，A中的元素用x来表示，则$\mu_A(x)$便表示x属于A的隶属度，对于上面的例子就可以写成

$$\mu_A(3)=1 \quad \mu_A(2.9)=0.9 \quad \mu_A(2.8)=0.8$$

模糊集定义

- 模糊集定义：

给定论域U（问题所限定的范围），μ_A是把任意$x\in U$映射为$[0,1]$上某个实数，即

$$\mu_A: U \to [0,1] \qquad x \to \mu_A(x)$$

则称μ_A为定义在U上的一个隶属函数，由$\mu_A(x)$（对所有$x\in U$）所构成的集合A称为U上的一个模糊集，$\mu_A(x)$称为x对A的隶属度

论域U指所讨论的事物全体，如在“大苹果”的例子中，U指得是所有篮子里的苹果。

1.模糊集理论

- 什么是模糊集?
  - $A=\{x \mid x$是指年龄不超过30岁的人$\}$
  - $B=\{x \mid x$是个年轻人$\}$

$\mu_A : U \rightarrow \{0,1\}$　　$\mu_B : U \rightarrow [0,1]$

$$\mu_A(x)=\begin{cases}1 & x \in A\\ 0 & x \notin A\end{cases}$$

Crisp Set　　Fuzzy Set

定义论域U的模糊子集A

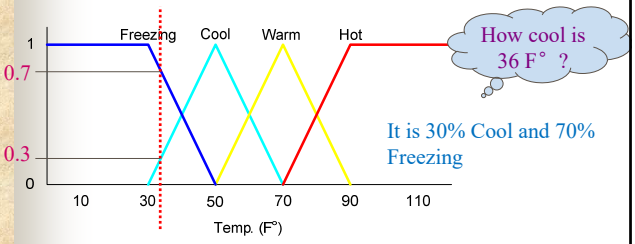
$$A=\{(x,\mu_A(x)) \mid x \in U\},$$

- $\mu_A(x)$　　*Membership functions*（隶属函数）
  - x对A的隶属度，取值范围—[0,1]
- 例：设有论域：U={ 1, 2, 3, 4, 5 }，用模糊集表示出模糊概念“大数”
  - 设A表示“大数”的模糊集，μ_A为其隶属函数

则有：A={ 0/1, 0.1/2, 0.5/3, 0.8/4, 1/5 }

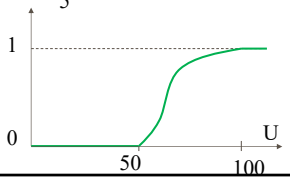
1.模糊集理论：隶属函数

- Temp: {Freezing, Cool, Warm, Hot}
- Degree of Truth or "Membership"



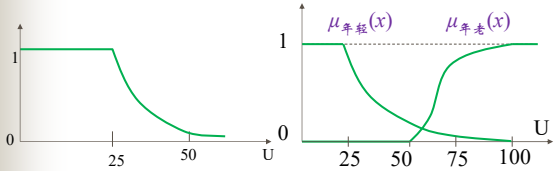
- 论域U是连续的情况
  - 模糊集可用实函数表示
  - 例：考虑年龄集U=[0, 100]，A=“年老”，扎德给出的“年老”集函数刻画

$$\mu_A(x)=\begin{cases}0 & 0 \le x \le 50\\ (1+(\frac{x-50}{5})^{-2})^{-1} & 50 \le x \le 100\end{cases}$$



- B=“年轻”，扎德给出它的隶属函数：

$$\mu_B(x)=\begin{cases}1 & 0 \le x \le 25\\ (1+(\frac{x-25}{5})^{2})^{-1} & 25 \le x \le 100\end{cases}$$



- 模糊集的表示方式

若以年龄为论域，并设$U=[0, 200]$。设O表示模糊集合“年老”，Y表示模糊集合“年轻”。已知“年老”和“年轻”的隶属度函数分别表示为

$$\mu_o(x)=\begin{cases}0 & 0 \le x \le 50\\ \dfrac{1}{1+(\frac{5}{x-50})^2} & 50 < x \le 200\end{cases}$$

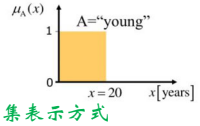
$$\mu_Y(x)=\begin{cases}1 & 0 \le x \le 25\\ \dfrac{1}{1+(\frac{x-25}{5})^2} & 25 < x \le 200\end{cases}$$

模糊集表示方式

young = { $x \in P$ | age(x) ≤ 20 }

Characteristic function:

$$\mu_{young}(x)=\begin{cases}1, & age(x) \le 20\\ 0, & age(x) > 20\end{cases}$$



Crisp集表示方式

模糊集表示

- 论域U是离散的情况

在整数1，2，…，10组成的论域中，即论域X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10}，设A表示“几个”这样一个模糊集合。并设各元素属于A的隶属度函数依次为 = {0, 0, 0.3, 0.7, 1, 1, 0.7, 0.3, 0, 0}，则A可表示为

$A=\{(x,\mu_A(x))|x\in X\}=\{(1,0),(2,0),(3,0.3),(4,0.7),(5,1),(6,1),(7,0.7),(8,0.3),(9,0),(10,0)\}$

或者$A=\sum_{i=1}^{10}\frac{\mu_A(x_i)}{x_i}=\frac{0}{1}+\frac{0}{2}+\frac{0.3}{3}+\frac{0.7}{4}+\frac{1}{5}+\frac{1}{6}+\frac{0.7}{7}+\frac{0.3}{8}+\frac{0}{9}+\frac{0}{10}$

模糊集合运算

- 设A、B为论域U上的两个模糊集合。则A∪B、A∩B、A的补集也是模糊集合
- 并集：将对应的论域元素的隶属度两两取大
- 交集：将对应的论域元素的隶属度两两取小
- 补集：将集合的每一个元素的隶属度取反

两个集合之间的运算是两个集合的隶属函数之间的运算

模糊集合运算

模糊集运算的数学表达

定义：设A，B是论域U的两个模糊子集，隶属函数

包含：$A\subseteq B\Leftrightarrow\mu_A(u)\le\mu_B(u)$

并集：$\mu_{A\cup B}(u)=\mu_A(u)\vee\mu_B(u)$

交集：$\mu_{A\cap B}(u)=\mu_A(u)\wedge\mu_B(u)$

余集：$\mu_{A^c}(u)=1-\mu_A(u)$

∨表示取大；

∧表示取小。

模糊集合运算

模糊集合运算的基本性质

- 分配律

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

- 结合律

$(A\cap B)\cap C=A\cap(B\cap C)$

$(A\cup B)\cup C=A\cup(B\cup C)$

模糊集合运算

模糊集合运算的基本性质

- 交换律

$A\cup B=B\cup A$　　$A\cap B=B\cap A$

- 吸收律

$(A\cap B)\cup A=A$　　$(A\cup B)\cap A=A$

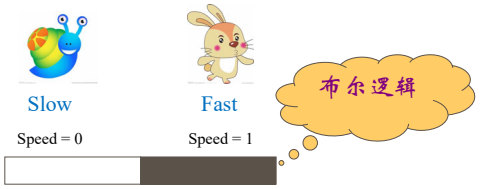
- 幂等律

$A\cup A=A\cap A=A$

模糊逻辑

3.模糊逻辑

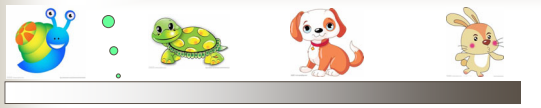
- 逻辑的传统表示




```
bool speed;
get the speed
if ( speed == 0) {// speed is slow}
else {// speed is fast}
```

Multi-valued Logic

多值模糊逻辑



Slowest Slow Fast Fastest

```
float speed;
get the speed
if speed is slowest
{……}
else if speed is slow
{……}
else if speed is fast
{……}
else speed is fastest
{……}
```

模糊语言变量
(*Fuzzy Linguistic Variables*)

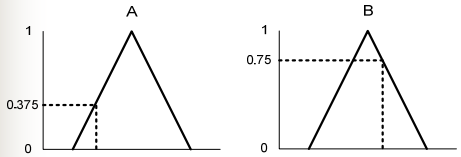
连接词

- How do we use fuzzy membership functions in predicate logic?
- Fuzzy logic Connectives:
  - Fuzzy Conjunction, ∧
  - Fuzzy Disjunction, ∨
- Operate on degrees of membership in fuzzy sets

Cont.

连接词

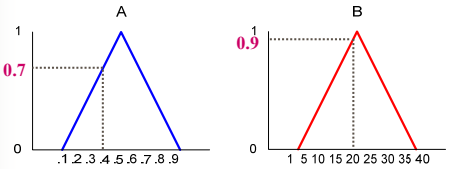
- A∨B ≙ max(A, B)
- A∨B = C "Quality C is the disjunction of Quality A and B"



- (A∨B = C) ⇒ (C = 0.75)

Example

Example: Calculate A∧B given that A is 0.4 and B is 20

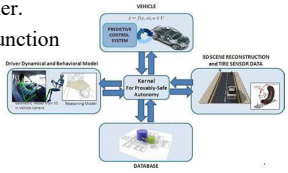


- Determine degrees of membership:
  - A = 0.7, B = 0.9
- Apply fuzzy AND
  - A∧B = min(A, B) = 0.7

Example

4. Fuzzy Control System

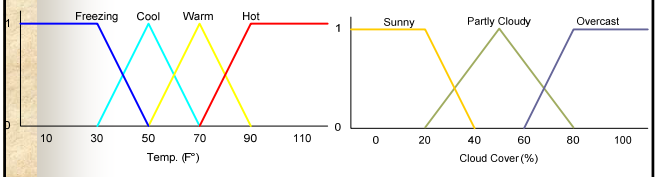
- Fuzzy Logic provides a more efficient and resourceful way to solve *Control Systems Problem*
- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Examples
  - Speed Control
    - How fast am I going to drive today?
    - Depends on the weather.
    - Disjunction and Conjunction



Example

4. 模糊控制系统— 速度控制

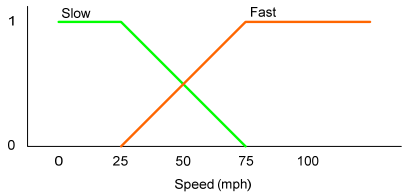
- Inputs
  - Temperature: {Freezing, Cool, Warm, Hot}
  - Cover: {Sunny, Cloudy, Overcast}



Example

4.模糊控制系统— 速度控制

- Output
  - Speed: {Slow, Fast}



Example

4.模糊控制系统— 速度控制

- Rules
  - If it's Sunny and Warm, drive Fast

    Sunny(*Cover*)∧*Warm*(*Temp*)⇒ *Fast*(*Speed*)
  - If it's Cloudy and Cool, drive Slow

    Cloudy(*Cover*)∧*Cool*(*Temp*)⇒ *Slow*(*Speed*)
  - Driving Speed is the combination of output of these rules...

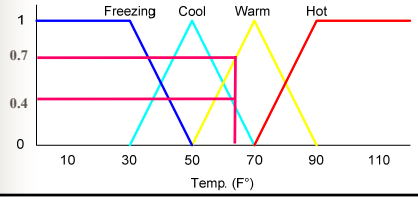
Example

- Speed Calculation: How fast will I go if it is
  - 65 F°， 25 % Cloudy ?
- Step

  Fuzzification：

  Calculate Input Membership Levels

65 F° ⇒ Cool = 0.4,
Warm= 0.7

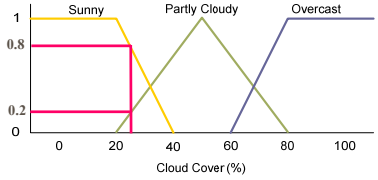


Example

- Step

  Fuzzification (模糊化)：*Calculate Input Membership Levels*
  - Temperature
  - Cover

25% Cover ⇒Sunny = 0.8,
Cloudy = 0.2



Example

- Step
  - Fuzzification
  - Calculating

    If it's Sunny and Warm, drive Fast

    Sunny(*Cover*)∧*Warm*(*Temp*)⇒*Fast*(*Speed*)

    0.8 ∧ 0.7 = 0.7 ⇒ *Fast* = 0.7

    If it's Cloudy and Cool, drive Slow

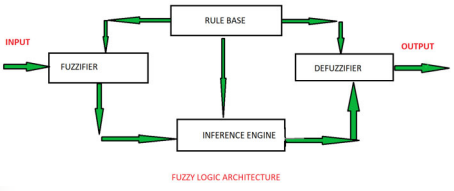
    Cloudy(*Cover*)∧*Cool*(*Temp*)⇒*Slow*(*Speed*)

    0.2 ∧ 0.4 = 0.2 ⇒ *Slow* = 0.2

Example

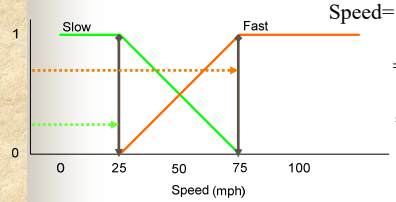
- Step
  - Fuzzification
  - Calculating
  - Defuzzification： Constructing the Output

How fast can I drive?



Example

- Defuzzification
  - Speed is 20% Slow and 70% Fast
  - Find centroids: Location where membership is 100%



Speed= weighted mean

= (0.2*25+ 0.7*75) /0.9

= 63.9 mph

基本结构

5.模糊计算—基本结构