

$\sum_{k=0}^n B(n, p) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} = 1$
 $\text{泊松 } \pi(\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$
 $\text{二项 } G(p) = (1-p)^{n-1} p$
 $\text{超几何 } H = \frac{\binom{M}{k} \binom{N-M}{n-k}}{\binom{N}{n}}$
 $\text{正态 } U(a, b) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}$
 $\text{指数 } E(\lambda) = \lambda e^{-\lambda}$
 $\text{Gamma } \Gamma(n, \lambda) = \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}$
 $\text{Beta } B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} x^{a-1} (1-x)^{b-1}$
 $\text{F } F(n_1, n_2) = \frac{f_1(x)/g_1}{f_2(x)/g_2}$

$f_{X|Y}(x|y) = f(x, y) / f_Y(y)$ 独立性: $P_{ij} = \sum P_i \sum P_j$ and $f(x, y) = f_X(x) f_Y(y)$
 $Z = X + Y, f_Z(z) = \int_{-\infty}^{+\infty} f(x-y, y) dy$
 $ax + by, \int \frac{1}{|b|} f(x, \frac{z-ax}{b}) dx$
 $XY, \int \frac{1}{|x|} f(x, \frac{z}{x}) dx$
 $X/Y, \int |x| f(x, xz) dx$
 $E(Z) = \pi F_{X_0}(z)$
 $E(Z) = 1 - \pi[1 - F_{X_0}(z)]$
 $\lim_{n \rightarrow \infty} P(X - p < \epsilon) = 1$

$F(X) = P(Y \leq X) \quad Y = g(X) \text{ 则 } f_Y(y) = f_X(g^{-1}(y)) \left| \frac{g^{-1}(y)}{dy} \right|$
 $E(X^2) = \int x^2 f(x) dx$
 $D(X) = E[(X - EX)^2] = EX^2 - E^2 X$
 $D(CX) = C^2 DX$
 $D(X+Y) = DX + DY + 2 \text{cov}(X, Y)$
 $\text{cov} = E[(X - EX)(Y - EY)] = E(XY) - EXEY$
 $\text{相关系数 } \rho_{XY} = \frac{\text{cov}(X, Y)}{\sqrt{DXDY}} \leq 1$

$\text{Markov } P(X \geq a) \leq EX/a, a > 0, X \geq 0$
 $\text{Chebyshev } P\{|X - EX| \geq \epsilon\} \leq DX/\epsilon^2, \epsilon > 0$
 $\text{Bernoulli } \text{fre } \rightarrow \text{pro}$
 $\text{Chebyshev } \text{avg } \rightarrow \text{avg of exp}$
 $\text{Khinchin } \text{avg } \rightarrow \text{exp}$
 $\text{CLT } \frac{\bar{X} - EX}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$
 $\text{Lyapunov } S_n^2 = \sum_{i=1}^n \sigma_i^2, \frac{1}{S_n^3} \sum (x_i - EX_i)^3 \xrightarrow{d} N(0, 1)$
 $\text{De Moivre-Laplace } \frac{X - np}{\sqrt{np(1-p)}} \xrightarrow{d} N(0, 1), X \sim B(n, p)$

$E(\lambda): P\{X > t\} = P\{X > s + t | X > s\}$
 $\pi(n), \chi^2(n): \text{独立可加性 } X+Y \sim \pi(n+m)$
 $n \rightarrow \infty, t \sim N(0, 1)$

样本 $E\bar{X} = \mu$, $D\bar{X} = \sigma^2/n$. $ES_{n-1} = \sigma^2$, $DS_{n-1}^2 = 2\sigma^4/(n-1)$

无偏: $E\hat{\theta} = \theta \rightarrow$ 有效: σ^2_{min}
相合: $n \rightarrow \infty, \hat{\theta} \rightarrow C$

对总体 $X \sim N(\mu, \sigma^2)$ 有 $\bar{X} \sim N(\mu, \sigma^2/n)$, $\frac{(n-1)S^2}{\sigma^2} = \chi^2_{(n-1)}$ $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{(n-1)}$

$\frac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2} \sim F(n_1-1, n_2-1)$, 当 $\sigma_1^2 = \sigma_2^2$ 时, $\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{S_W \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}$, $S_W = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$

$\bar{f}_x = \frac{1}{f_{max}}$

矩估计: 用 k 阶矩代替矩数量 λ 最大似然: $L(p) = \prod_{i=1}^n p(x_i = X_i)$, $\hat{\theta} = \frac{d \ln L(\theta)}{d\theta} = 0$ 量级

置信 $1-\alpha$ 区间: 估计 μ $\bar{X} \pm \frac{\sigma}{\sqrt{n}} Z_{\alpha/2}$ ~~$\bar{X} \pm \frac{S}{\sqrt{n}} t_{\alpha/2}(n-1)$~~

估计 σ^2 μ $\left(\frac{\sum (X_i - \mu)^2}{\chi^2_{\alpha/2}(n)}, \frac{\sum (X_i - \mu)^2}{\chi^2_{1-\alpha/2}(n)} \right)$ ~~$\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2}(n-1)}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}(n-1)} \right)$~~

双正态, σ_1^2, σ_2^2 $\bar{X} - \bar{Y} \pm Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ ~~或 $n_1, n_2 \gg 0, S$ 估计~~ $\hat{\sigma}_1^2 = \hat{\sigma}_2^2$ $\left(\bar{X} - \bar{Y} \pm t_{\alpha/2}(n_1+n_2-2) \cdot S_W \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$

估计 σ_1^2/σ_2^2 μ_1/μ_2 $\left(\frac{S_1^2}{S_2^2} \cdot \frac{1}{F_{\alpha/2}}, \frac{S_1^2}{S_2^2} \cdot \frac{1}{F_{1-\alpha/2}} \right) (F(n_1-1, n_2-1))$ $\begin{cases} a = n + Z_{\alpha/2}^2 \\ b = -(2n\bar{X} + Z_{\alpha/2}^2) \\ c = n\bar{X}^2 \end{cases}$

0-1 估计 P , (p_1, p_2) , $p_{1,2} = \frac{1}{2a}(-b \pm \sqrt{b^2 - 4ac})$

正态检验 H_0, H_1 , 拒绝域: $\hat{\sigma}^2 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$, 双边 $|Z| \geq Z_{\alpha/2}$ 右边 $Z \geq Z_{\alpha}$

检验 μ : $\hat{\sigma}^2 T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1)$ 双边 $|T| \geq t_{\alpha/2}$ 右边 $T \geq t_{\alpha}$

$\chi^2 = \frac{\sum (X_i - \mu)^2}{\sigma^2}$ 检验 σ^2 : M 双 $(0, \chi^2_{1-\alpha/2}(n)) \cup (\chi^2_{\alpha/2}(n), +\infty)$ 右 $(\chi^2_{\alpha/2}(n), +\infty)$ 左 $(0, \chi^2_{1-\alpha}(n))$

$\chi^2 = \frac{(n-1)S^2}{\sigma^2} \rightarrow$ ~~χ^2~~ 上面自由度 $\rightarrow n-1$