4.1 (1) $1-e^{-at} \xrightarrow{L} \frac{1}{s}-\frac{1}{s+a}$

(3) $te^{-2t} \longrightarrow \frac{1}{(s+2)^2}$

$t \to \frac{1}{s^2}$,

(5) $(1+2t)e^{-t} \longrightarrow \frac{s+3}{(s+1)^2}$

$1+2t \to \frac{1}{s}+\frac{2}{s^2}=\frac{s+2}{s^2}$.

(7) $t^2+2t \longrightarrow \frac{2!}{s^3}+2\frac{1}{s^2}=\frac{2+s}{s^3}$

(9) $e^{-\alpha t}\sinh(\beta t)$

$= e^{-\alpha t}\cdot\frac{e^{\beta t}-e^{-\beta t}}{2}$

$=\frac{1}{2}\left(e^{(\beta-\alpha)t}-e^{-(\beta+\alpha)t}\right) \to \frac{1}{2}\cdot\frac{1}{s-(\beta-\alpha)}-\frac{1}{2}\frac{1}{s+\beta+\alpha}$

(11) $\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t}) \to \frac{1}{\beta-\alpha}\left(\frac{1}{s+\alpha}-\frac{1}{s+\beta}\right)$

(13) $te^{-(t-2)}u(t-1) \longrightarrow \frac{s+2}{(s+1)^2}e^{-s+1}$

$tu(t-1)=(t-1)u(t-1)+u(t-1) \to \frac{1}{s^2}e^{-s}+\frac{1}{s}e^{-s}=\frac{s+1}{s^2}e^{-s}$

4.3 (1) $f(t)=e^{-t}u(t-2)=e^{-2}e^{-(t-2)}u(t-2)$

$e^{-(t-2)}u(t-2) \longrightarrow \frac{e^{-2s}}{s+1}$.

$\therefore F(s)=\frac{e^{-2(s+1)}}{s+1}$. $Re(s)>-1$

(2) $e^{-(t-2)}u(t-2) \to \frac{e^{-2s}}{s+1}$, $Re(s)>-1$

(3) $f(t)=e^2\cdot e^{-t}u(t)=e^2\cdot\frac{1}{s+1}$. $Re(s)>-1$

(4) $f(t)=\sin[2(t-1)+2]u(t-1)$

$=\cos 2\sin 2(t-1)\,u(t-1)+\sin 2\cos 2(t-1)\,u(t-1)$

$\to \cos 2\cdot\frac{2}{s^2+4}e^{-s}+\sin 2\frac{s}{s^2+4}e^{-s}$. $Re(s)>0$

(5) $f(t)=(t-1)[u(t-1)-u(t-2)]$

$=(t-1)u(t-1)-(t-2)u(t-2)-u(t-2)$

$\to \frac{1}{s^2}e^{-s}-\frac{1}{s^2}e^{-2s}-\frac{1}{s}e^{-2s}$

$=\frac{e^{-s}-e^{-2s}-se^{-2s}}{s^2}$ $Re(s)>-\infty$

4-5 (1) $f(0_+) = \lim\limits_{s\to+\infty} sF(s) = \lim\limits_{s\to\infty} s\dfrac{s+6}{(s+2)(s+5)} = 1$

极点在左半平面:

$f(+\infty) = \lim\limits_{s\to 0} sF(s) = 0$

(2) $f(0_+) = \lim\limits_{s\to+\infty} s\cdot\dfrac{s+3}{(s+1)^2(s+2)} = 0$ 极点在左半平面

$f(+\infty) = \lim\limits_{s\to 0} sF(s) = 0$

4-20 (b)

$f_1(t)$

$$f_1(t) = \sin\frac{2\pi}{T}t\left[u(t) - u\left(t-\frac{T}{2}\right)\right] = \sin\frac{2\pi}{T}t\,u(t) + \sin\left[\frac{2\pi}{T}\left(t-\frac{T}{2}\right)\right]u\left(t-\frac{T}{2}\right)$$

$$F_1(s) = \frac{\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2}\left(1+e^{-\frac{sT}{2}}\right)$$

$$F_b(s) = \frac{F_1(s)}{1-e^{-s\frac{T}{2}}} = \frac{\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2}\cdot\frac{1+e^{-\frac{sT}{2}}}{1-e^{-\frac{sT}{2}}} \quad Re(s)>0$$

4-21 将连续信号 $f(t)$ 以时间间隔 T 进行冲激抽样得到 $f_s(t)=f(t)\delta_T(t)$, $\delta_T(t)=\sum\limits_{n=0}^{\infty}\delta(t-nT)$,求:

4-21

(1) 抽样信号的拉氏变换 $\mathscr{L}[f_s(t)]$;

(2) 若 $f(t)=e^{-at}u(t)$ 求 $\mathscr{L}[f_s(t)]$。

(1) $f_s(t) = f(t)\sum\limits_{n=0}^{+\infty}\delta(t-nT) = \sum\limits_{n=0}^{+\infty} f(nT)\,\delta(t-nT)$

$F_s(s) = \sum\limits_{n=0}^{+\infty} f(nT)e^{-nsT}$

(2) $f_s(t) \xrightarrow{\mathscr{L}} \sum\limits_{n=0}^{+\infty} e^{-anT}\cdot e^{-nsT} = \sum\limits_{n=0}^{+\infty}\left(e^{-(a+s)T}\right)^n = \dfrac{1}{1-e^{-(a+s)T}}$, $Re(a+s)>0$, $Re\, s>-a$

4-4 (1) $\frac{1}{s+1} \xrightarrow{\mathcal{L}^{-1}} e^{-t}u(t)$

(5) $\frac{3}{(s+4)(s+2)} = \left(\frac{1}{s+2} - \frac{1}{s+4}\right)\frac{3}{2}$

$\xrightarrow{\mathcal{L}^{-1}} \frac{3}{2}(e^{-2t} - e^{-4t})u(t)$

(9) $\frac{1}{s(RCs+1)} = \frac{A}{s} + \frac{B}{RCs+1} = \frac{1}{s} - \frac{\frac{1}{RC}}{s+\frac{1}{RC}} = \left(e^{-t} - \frac{1}{(RC)^2}e^{-\frac{1}{RC}}\right)u(t)$

$\begin{cases} ARCs + A + Bs = 1. \\ ARC + B = 0. \\ A = 1. \end{cases} \quad \begin{cases} A = 1 \\ B = -\frac{1}{RC} \end{cases}$

(13) $= \frac{100(s+50)}{(s+1)(s+200)} = \frac{A}{s+1} + \frac{B}{s+200}$

$As + 200A + Bs + B = 100s + 5000$

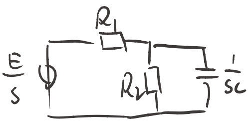
$\begin{cases} A + B = 100 \\ 200A + B = 5000 \end{cases} \quad \begin{cases} A = \\ B = \end{cases}$

4-7 $t<0$ 时, $i(t) = 0$. $V_c(t) = 0$.

$t>0$ 时, $i(t) = C\frac{dV_c(t)}{dt}$

$I(s) = C(sV_c(s) - V_c(0_-))$

$V_c(s) = \frac{1}{sC}I(s) + \frac{V_c(0-)}{s}$



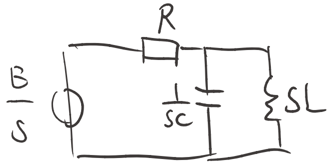
$V_c(s) = \frac{R_2}{R_2 + R_1(sR_2C+1)} \cdot \frac{E}{s}$

$= \frac{\frac{E}{R_1C}}{\left(s + \frac{R_1+R_2}{R_1R_2C}\right)s} = \frac{\frac{ER_2}{R_1+R_2}}{s} + \frac{-\frac{ER_2}{R_1+R_2}}{s + \frac{R_1+R_2}{R_1R_2C}}$

$\mathcal{L}^{-1}\downarrow$

$V_c(t) = \frac{ER_2}{R_1+R_2}\left(1 - e^{-\frac{R_1+R_2}{R_1R_2C}t}\right)u(t)$

4-9



$$I(s)=\frac{\frac{E}{S}}{R+\frac{1}{SC}\parallel SL}=\frac{E(S^2LC+1)}{S(S^2RLC+SL+R)}$$

$$=\frac{\frac{E}{R}}{S}+\frac{-\frac{E}{R^2C}}{S^2+\frac{1}{RC}S+\frac{1}{LC}}$$

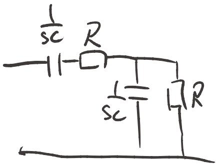
当 $\frac{1}{2RC}<\frac{1}{\sqrt{LC}}$ 时.

记 $\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2=w_d^2$ 则

$$I(s)=\frac{\frac{E}{R}}{s}+\frac{\left(-\frac{E}{R^2C}\frac{1}{w_d}\right)w_d}{\left(s+\frac{1}{2RC}\right)^2+w_d^2}$$

$$i(t)=\left[\frac{E}{R}-\frac{E}{R^2Cw_d}e^{-\frac{1}{2RC}t}\sin w_d t\right]u(t)$$

4-13 (a)



$$H(s)=\frac{V_2(s)}{V_1(s)}=\frac{\frac{1}{SC}\parallel R}{\frac{1}{SC}\parallel R+\frac{1}{SC}+R}=\frac{SRC}{S^2R^2C^2+3SRC+1}$$

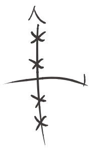
4-26 (1)

$$H(s)=\frac{\frac{1}{2s}\parallel\left(2s+\frac{1}{2s}\right)}{\frac{1}{2s}\parallel\left(2s+\frac{1}{2s}\right)+2s}\cdot\frac{\frac{1}{2s}}{\frac{1}{2s}+2s}=\frac{\frac{s^2+\frac{1}{4}}{2s^3+s}}{\frac{s^2+\frac{1}{4}}{2s^3+s}+2s}\cdot\frac{1}{4s^2+1}$$

$$=\frac{1}{16s^4+12s^2+1}$$

$s^2=-\frac{3}{8}\pm\frac{\sqrt{5}}{8}<0$.

$s=-\sqrt{\frac{3+\sqrt{5}}{8}}j.\ \sqrt{\frac{3+\sqrt{5}}{8}}j.\ -\sqrt{\frac{3-\sqrt{5}}{8}}j\ \sqrt{\frac{3-\sqrt{5}}{8}}j.$



4-28

$g(t)=(1-e^{-2t})u(t)$

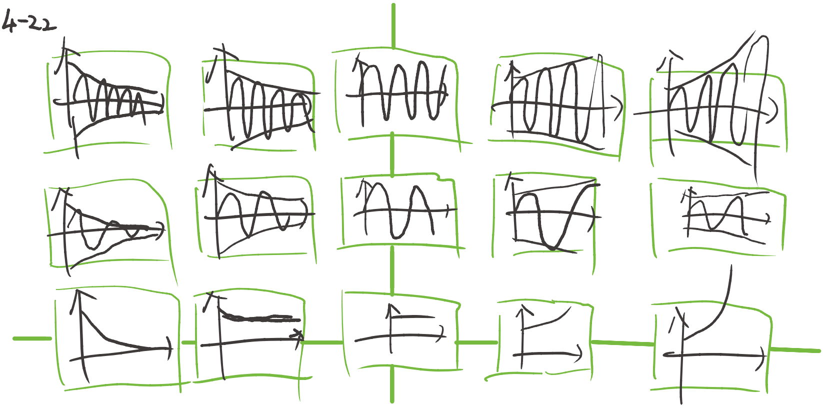
$h(t)=g'(t)=2e^{-2t}u(t)$

$H(s)=\frac{2}{s+2}$, $Re(s)>-2$.

$$E(s)=\frac{R(s)}{H(s)}=\frac{\frac{1}{s}-\frac{1}{s+2}-\frac{1}{(s+2)^2}}{\frac{2}{s+2}}=\frac{1}{s}-\frac{1}{2(s+2)}$$

$$e(t)=\left(1-\frac{1}{2}e^{-2t}\right)u(t)$$

4-22



4-35

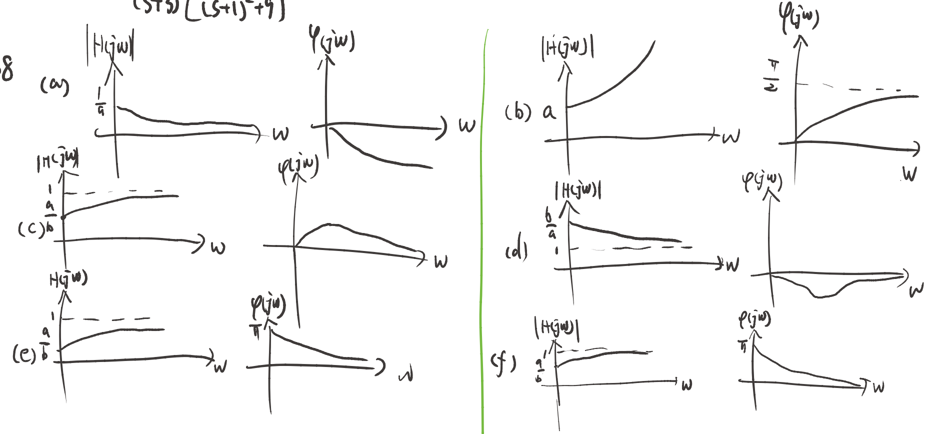
$$H(s)=\frac{k[s-(2+j1)]\cdot[s-(-2-j1)](s-0)}{(s+3)[s-(-1+j3)][s-(-1-j3)]}$$

$$=\frac{ks\cdot[(s+2)^2+1]}{(s+3)[(s+1)^2+9]}$$

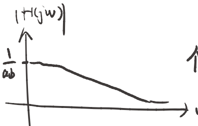
$$H(\infty)=k=5.$$

$$H(s)=\frac{5s(s^2+4s+5)}{(s+3)(s^2+2s+10)}$$

4-38



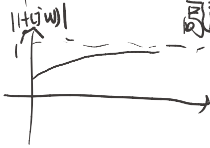
4-39

(a) 

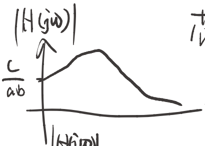
低通

(b) 

带通

(c) 

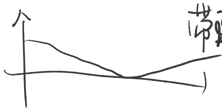
高通

(d) 

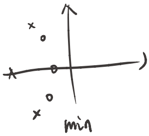
带通

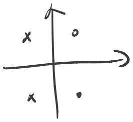
(e) 

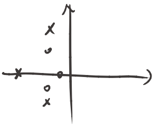
带通

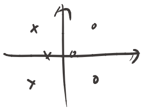
(f) 
带阻

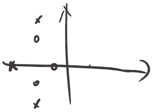
4-42 (a) 是 (b)(c)(d) 不是

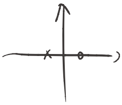
(b) 
min


Ap

(c) 
min


Ap

(d) 
min


Ap

4-45

(1) $\begin{cases} V_1(s)+V_2(s)=E(s) \\ E(s)\dfrac{s}{s^2+4s+4}\cdot k = V_2(s) \end{cases} \Rightarrow V_1(s)=\dfrac{s^2+(4-k)s+4}{ks}V_2(s)$

$H(s)=\dfrac{V_2(s)}{V_1(s)}=\dfrac{ks}{s^2+(4-k)s+4}$

(2) $s^2+(4-k)s+4$ 根在左半平面 $\Leftrightarrow \begin{cases} 4-k>0 \\ 4>0 \end{cases} \Rightarrow k<4.$

(3) $k=4.$

$H(s)=\dfrac{4s}{s^2+4}.$

$h(t)=4\cos 2t\, u(t).$