Analysis of the Seismicity during the Stimulation of OTN-3 in Helsinki, Finland

Job number 270723-00

Ove Arup & Partners Ltd
13 Fitzroy Street
London
W1T 4BQ
United Kingdom

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Document Verification

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<tr><td>Name</td><td>Thomas Ader</td><td>Michael Chendorain</td><td>Matthew Free</td></tr>
<tr><td>Signature</td><td></td><td></td><td></td></tr>
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1. Earthquake Catalogue Processing

Remove from catalogue the variations of seismicity rates that are not due to injection

1.1 Aftershocks and Clusters

1.2 Periodicities and Magnitude of Completeness

1.3 Magnitude-Frequency Distribution

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Summary

- No need for declustering the catalogue:
  - Aftershocks represent less than 0.5% of events in the catalogue; and
  - Variations of seismicity rate due to variations of detection threshold.
- Catalogue complete for M_L>-1:
  - Daily variations of seismicity rate stabilize at M_L = -1.
- Slight deviation from Gutenberg-Richter at all magnitudes:
  - This might affect the maximum magnitude predictions and should be investigated further.

Goals of Seismic Catalogue Processing

Aim of the study: study the response of seismicity to injection.

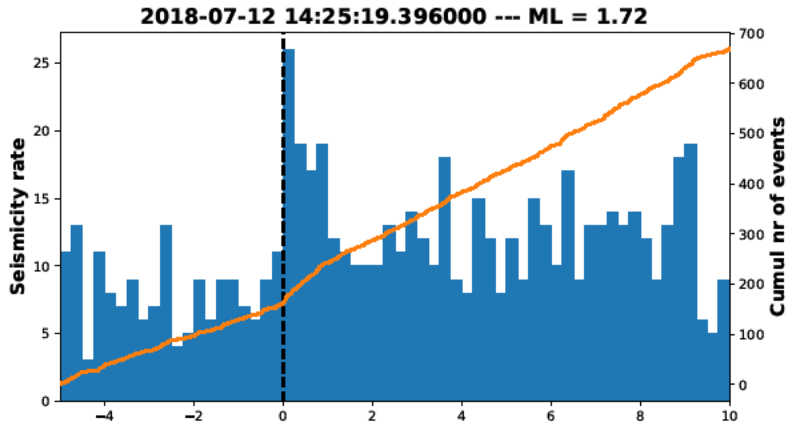
- Remove seismicity not directly triggered by injection
  - Earthquakes related to other earthquakes: aftershocks (and possibly foreshocks) or clusters
  - Variations of seismicity rate due to variations of detection threshold
- Look at the magnitude-frequency distribution of events for maximum magnitude prediction

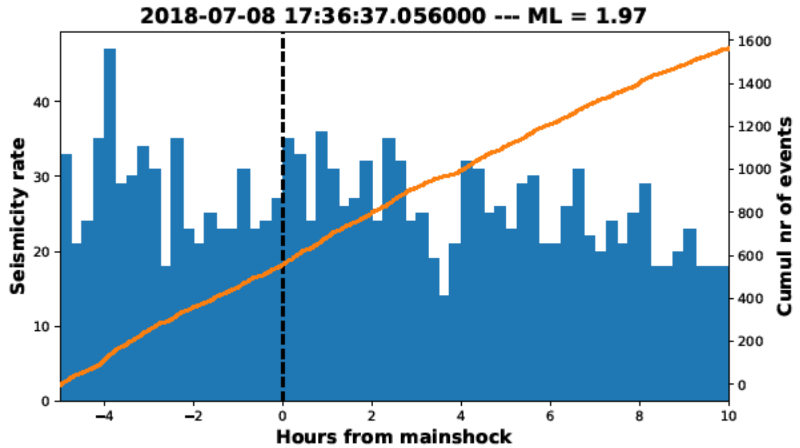
1.1 Aftershocks and Clusters

Are there Aftershocks in the Catalogue?

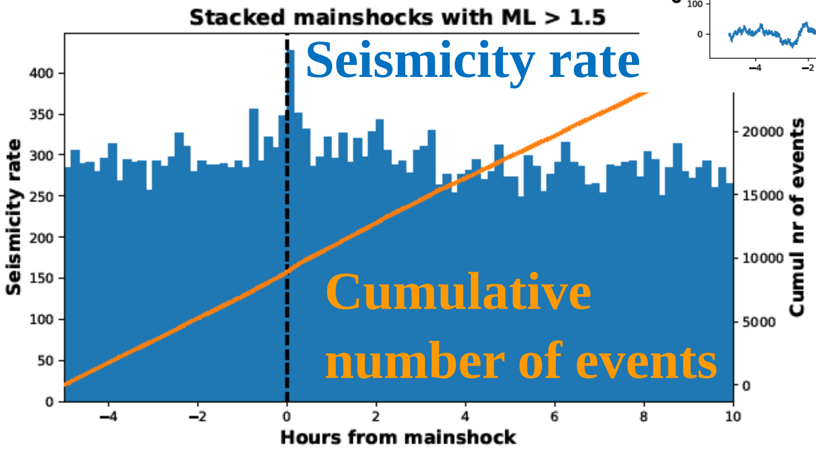
Look at seismicity rate 5h before and 10h after each M_L>1.5 event

- Individual events

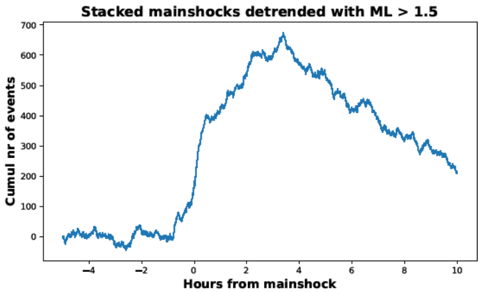




- Stacked events



Detrended cumulative number of events



~300 aftershocks in the ~70,000 event catalogue: less than 0.5% of events

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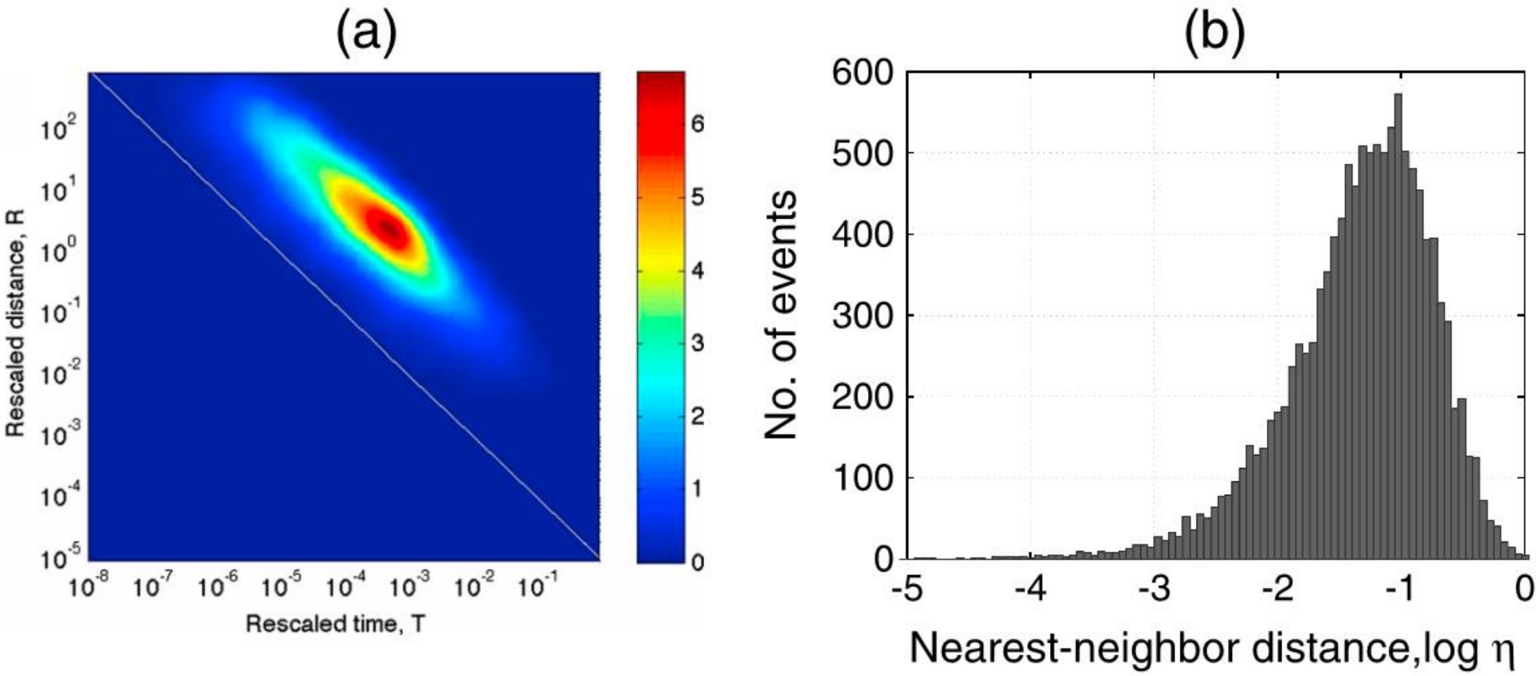
Looking for Clusters, Foreshocks and Aftershocks

Look at the Zaliapin (2013) formalism

- Zaliapin et al (2013) define a magnitude-normalized interevent time T and distance R measure.
- These two measures define an index η = R.T

- For a homogeneous Poisson process:
  - (a) Relative distribution of R vs T.
  - (b) Distribution of η index.



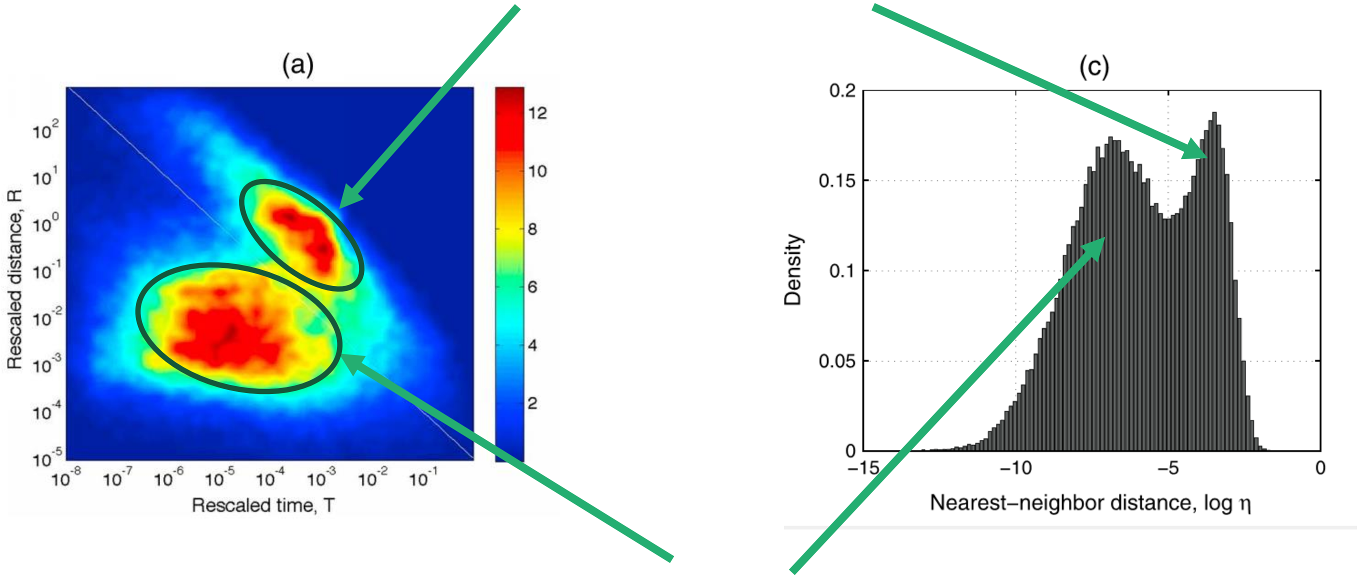
Zaliapin et al (2013), synthetic Poissonian catalogue

Looking for Clusters, Foreshocks and Aftershocks

Look at the Zaliapin (2013) formalism

- When applied to the California earthquake catalogue for $M_L>2$ events, the formalism separates mainshocks (homogeneous Poisson process) and foreshocks/aftershocks

MAINSHOCKS



FORESHOCKS & AFTERSHOCKS

Zaliapin et al (2013), California earthquake catalogue

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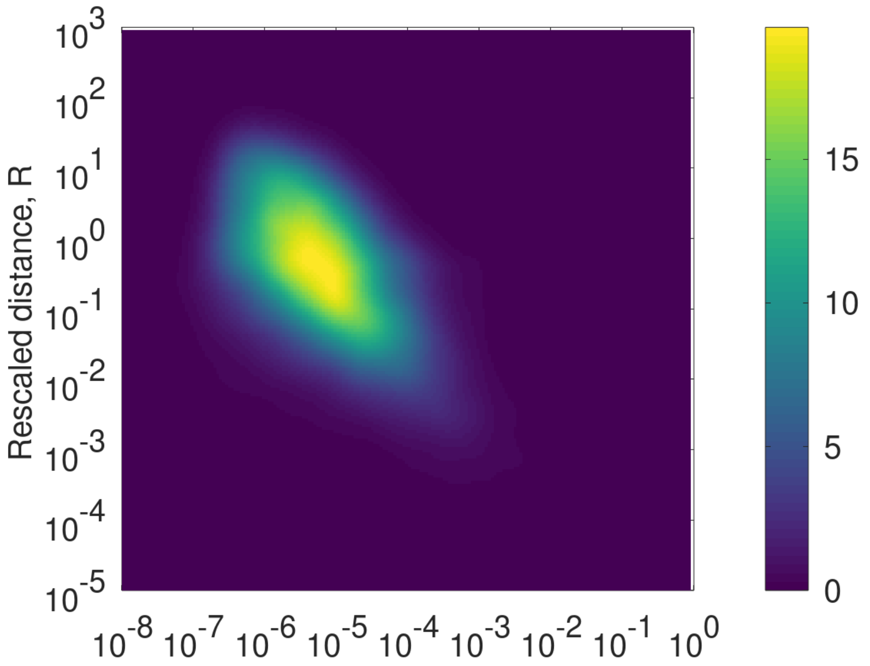
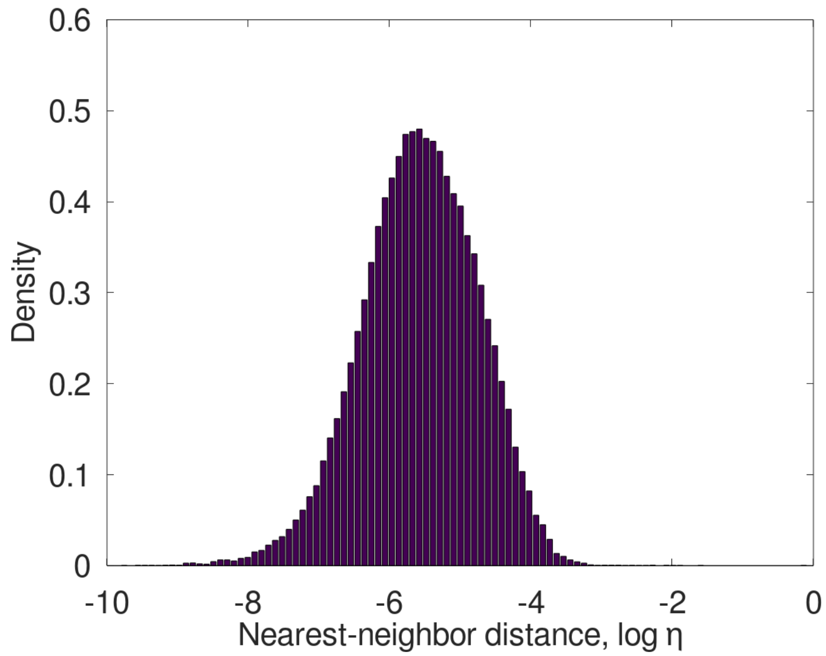
Looking for Clusters, Foreshocks and Aftershocks

Look at the Zaliapin (2013) formalism

- When applied to OTN-3 event catalogue, there seems to be mostly mainshocks:





This study

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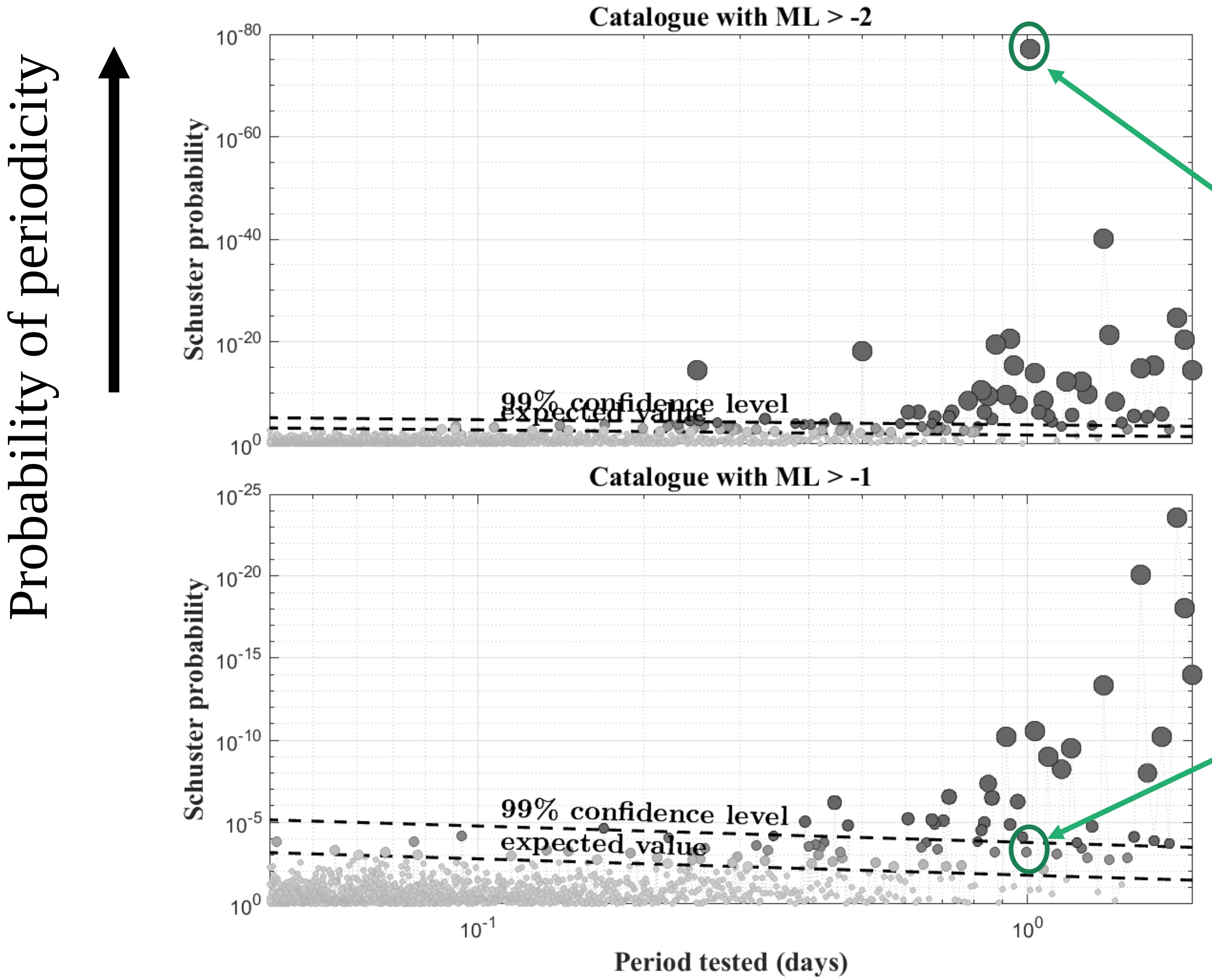
1.2 Periodicities and Magnitude of Completeness

Periodicities in the Catalogue



Full catalogue:
strong periodicity at 1 day

Catalogue for M_L>-1:
1-day periodicity reduced

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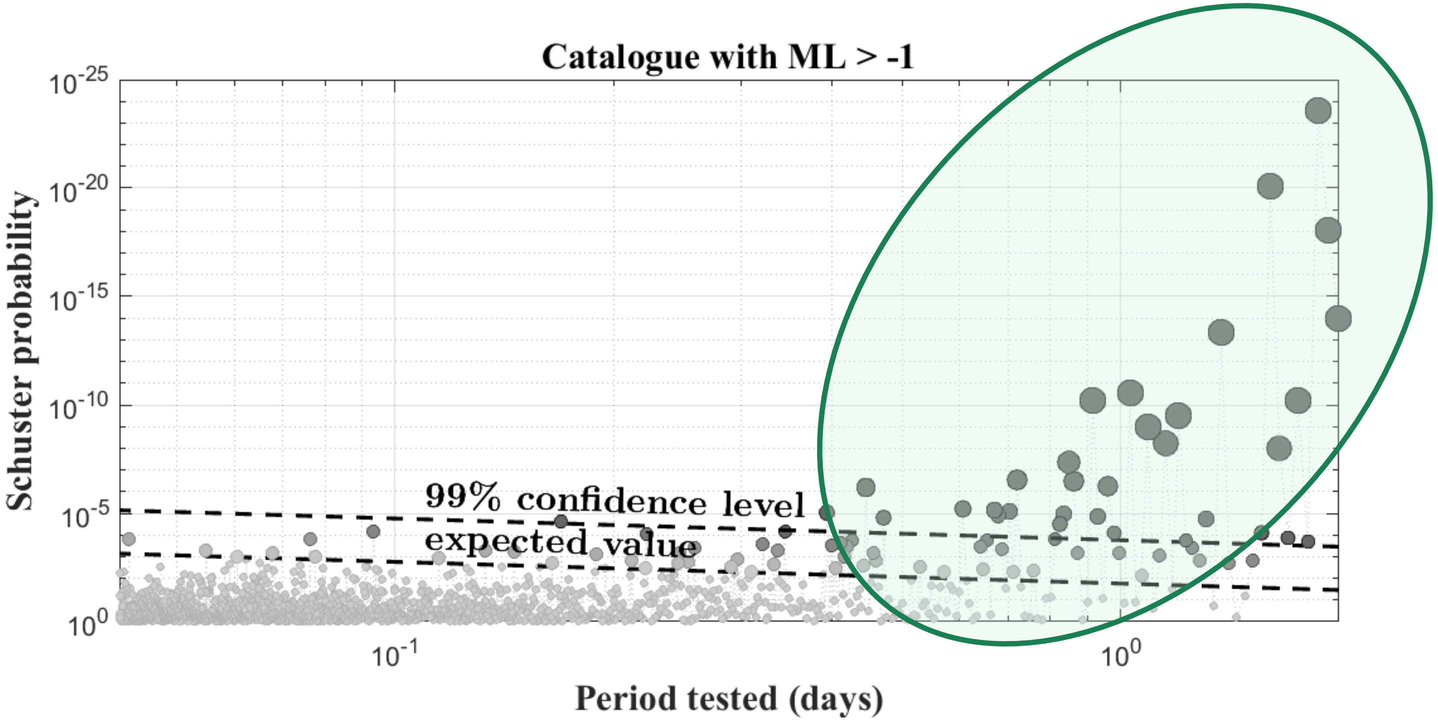
Periodicities in the Catalogue



- Apparent periodicities at large periods suggest clusters of typical duration of days.
- These are the clusters related to the alternating periods with and without injection.
- Non-stationary Poisson process, correlates with injection.

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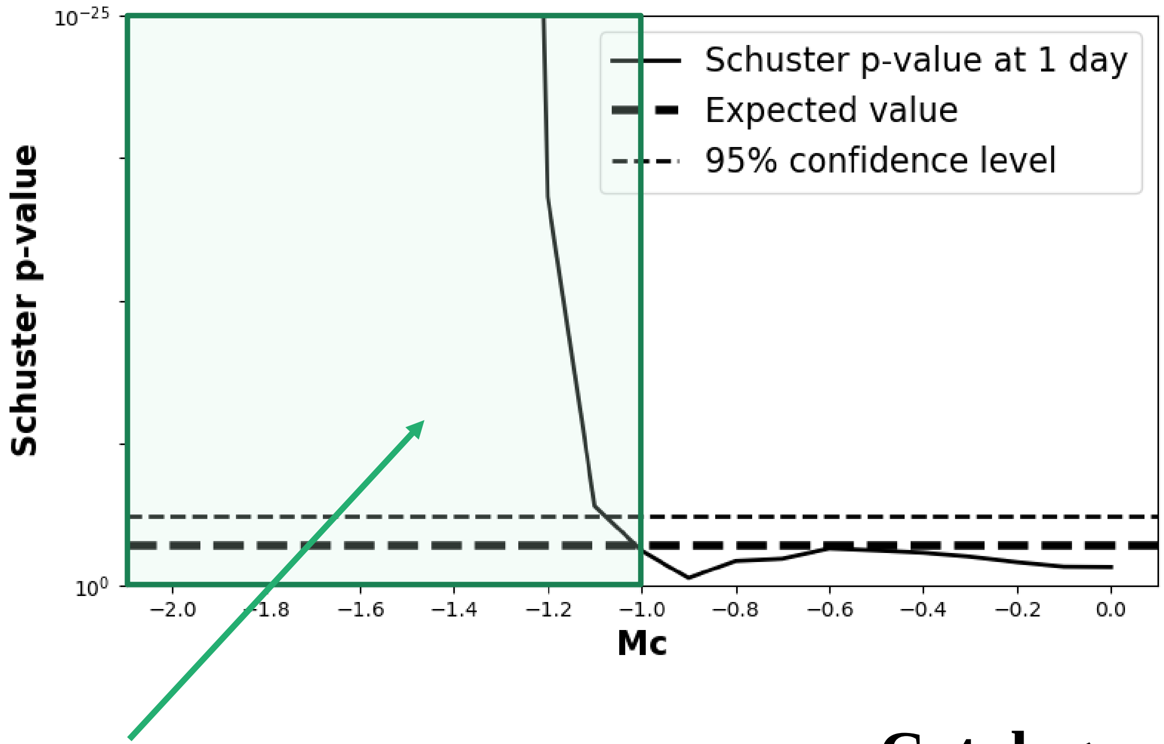
Periodicity at one day in the Catalogue



Large periodicity at one day when including $M_L < -1$ events, due to day-night variations of detection levels.

Catalogue seems to be complete for $M_L > -1$.

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1.3 Magnitude-Frequency Distribution

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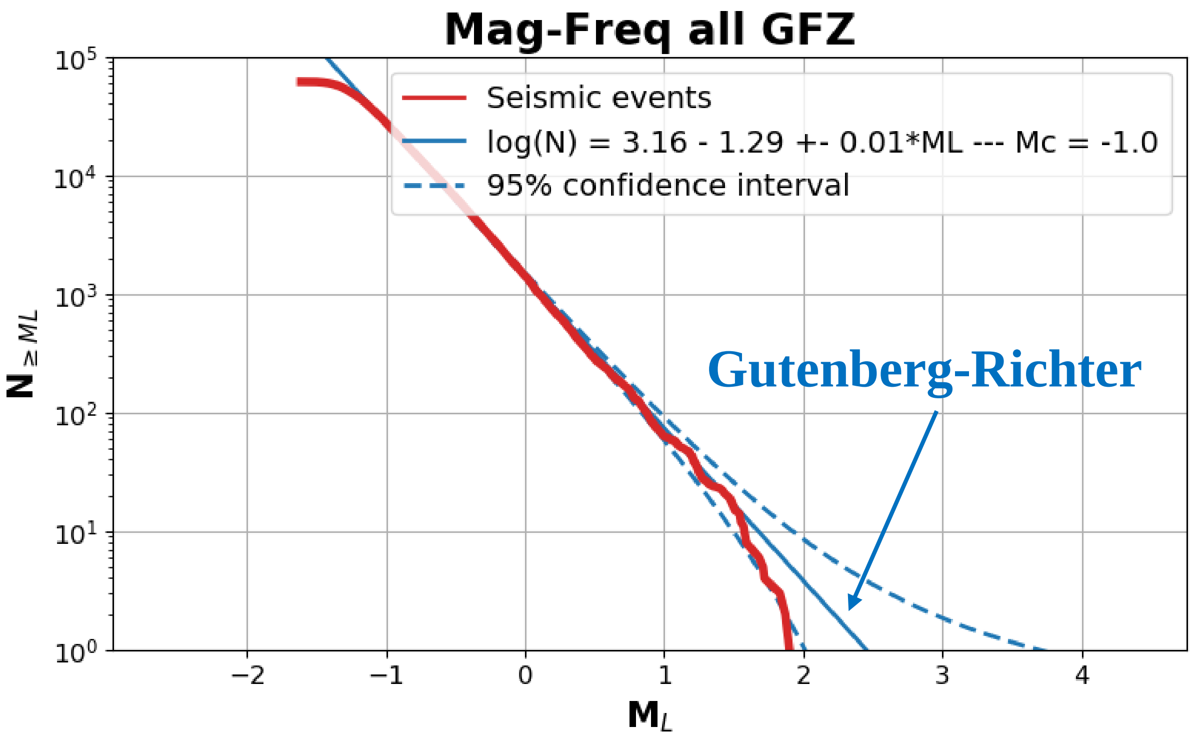
Magnitude-Frequency Distribution



- Roll-off at large magnitudes, just at the limit of the 95% confidence interval

- Implications for maximum magnitude: usual Gutenberg-Richter overestimates maximum magnitude.

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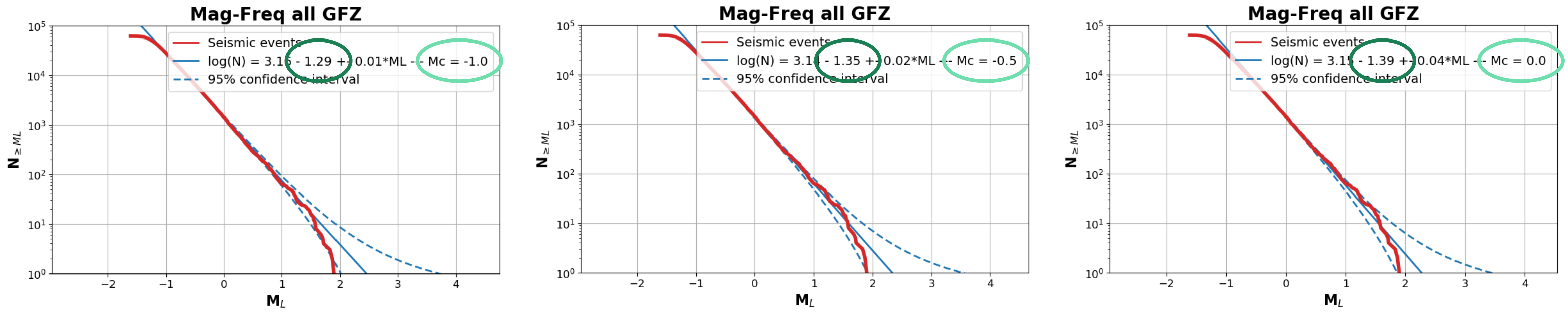
Magnitude-Frequency Distribution



- The roll-off is actually present also at small magnitudes: *b*-value gets larger as Mc is taken larger.

- Fit a Tapered Gutenberg-Richter law: normal Gutenberg-Richter with an additional exponential decay

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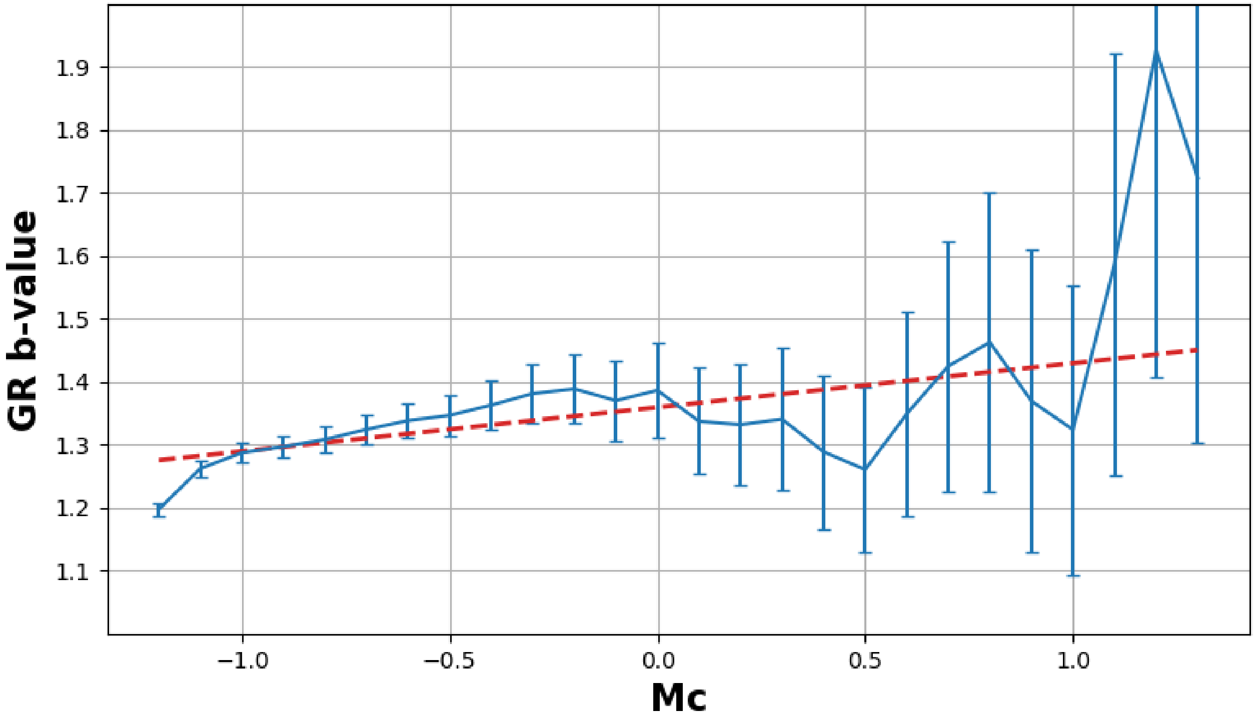
Evolution of GR *b*-value when changing M_c



- Fit a Tapered Gutenberg-Richter law: normal Gutenberg-Richter with an additional exponential decay

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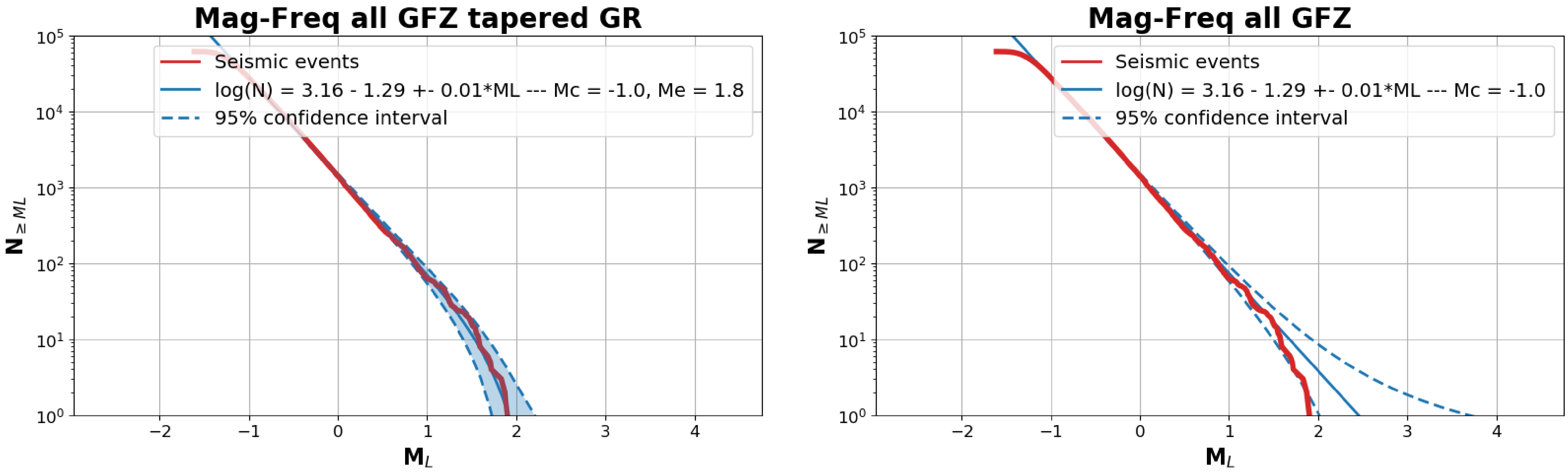
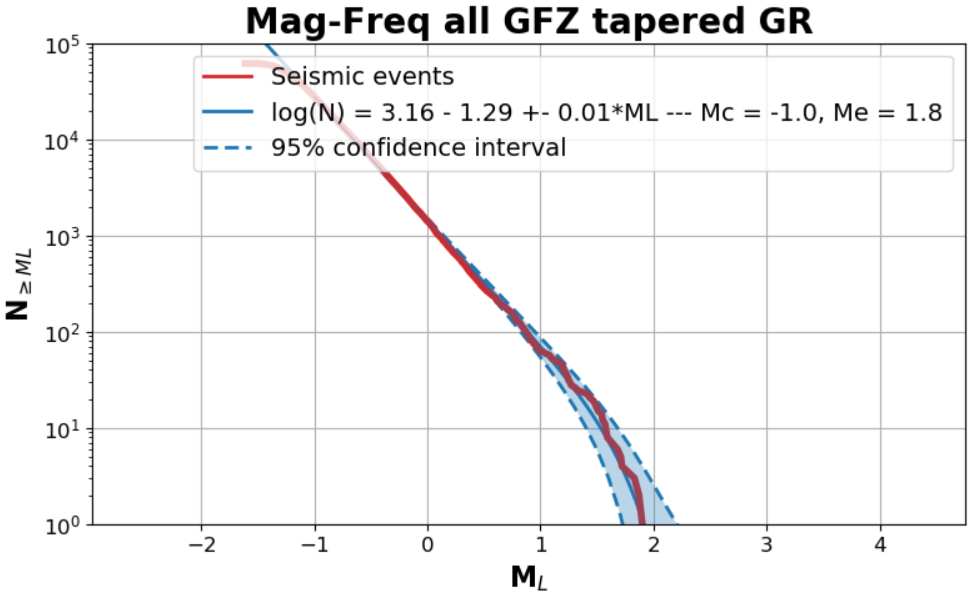
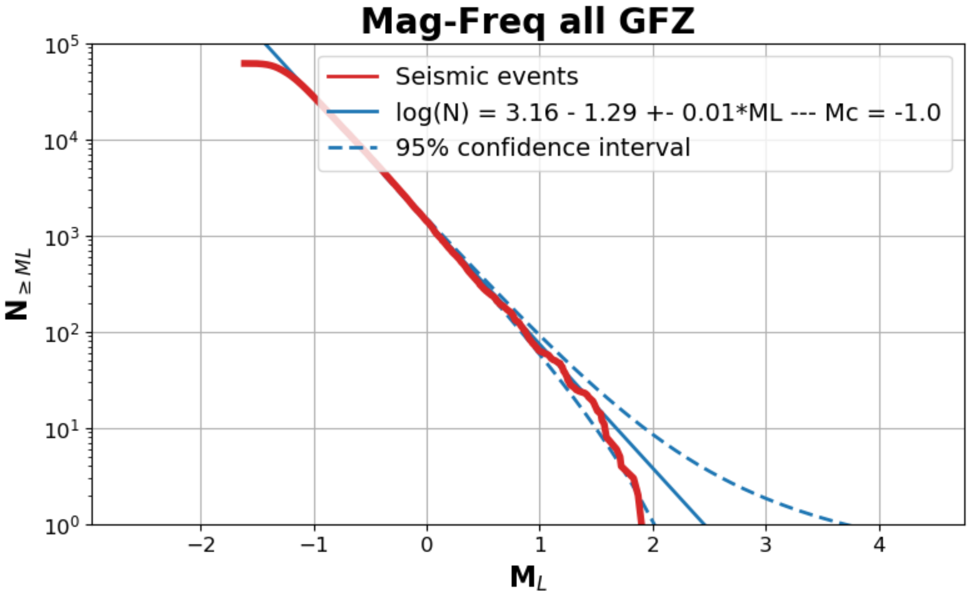
Tapered Gutenberg-Richter

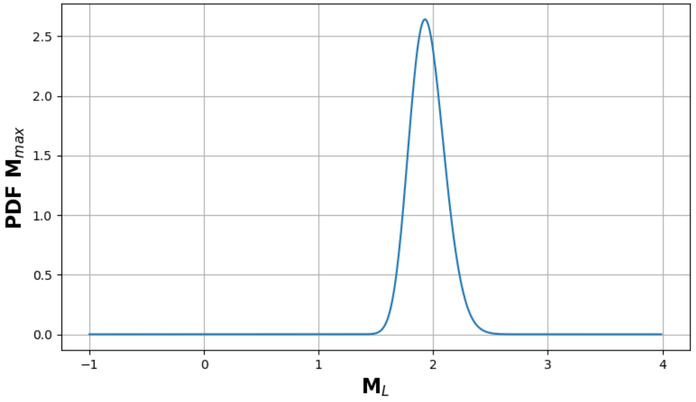


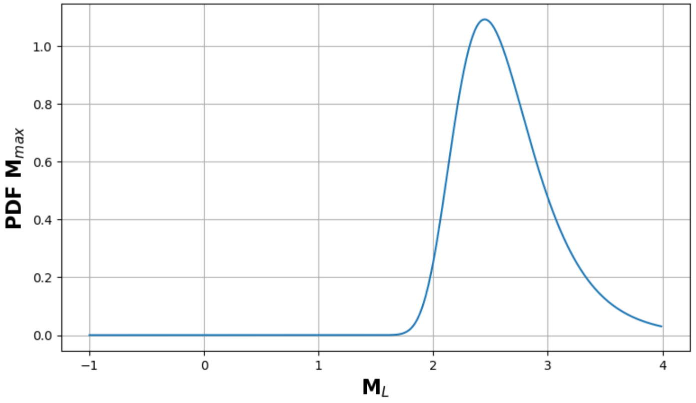
- The Tapered Gutenberg-Richter law suggests a corner magnitude of about 1.77 ± 0.14.

Tapered Gutenberg-Richter









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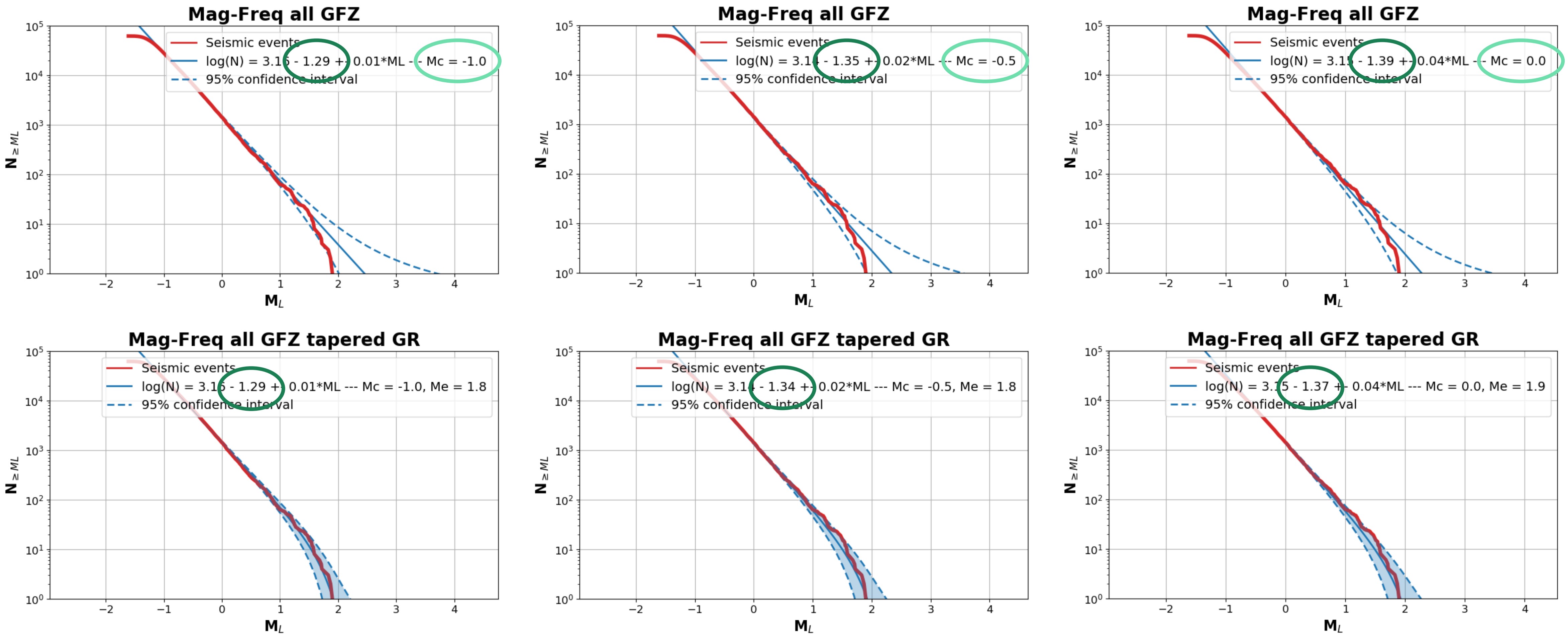
Tapered Gutenberg-Richter



- The Tapered Gutenberg-Richter law does not completely solve the problem of the dependence of the *b*-value with the magnitude of completeness.

Tapered Gutenberg-Richter

- Given that the b-value still depends on the magnitude of completeness M_c, the choice of M_c is important.

- For prediction purpose, we will rely on a value of $M_c = 0$, higher than the actual M_c of the available catalogue ($M_c = -1$), for two reason:
  - It only uses the MF distribution closer to M_{max} to compute the b-value; and
  - During the stimulation, the available catalogue is more likely to have a higher M_c, so this ensures that the algorithm developed will be appropriate for such a catalogue.

2. Analysis of the Earthquake Catalogue

Time, space and magnitude-frequency variations of the seismicity during injection

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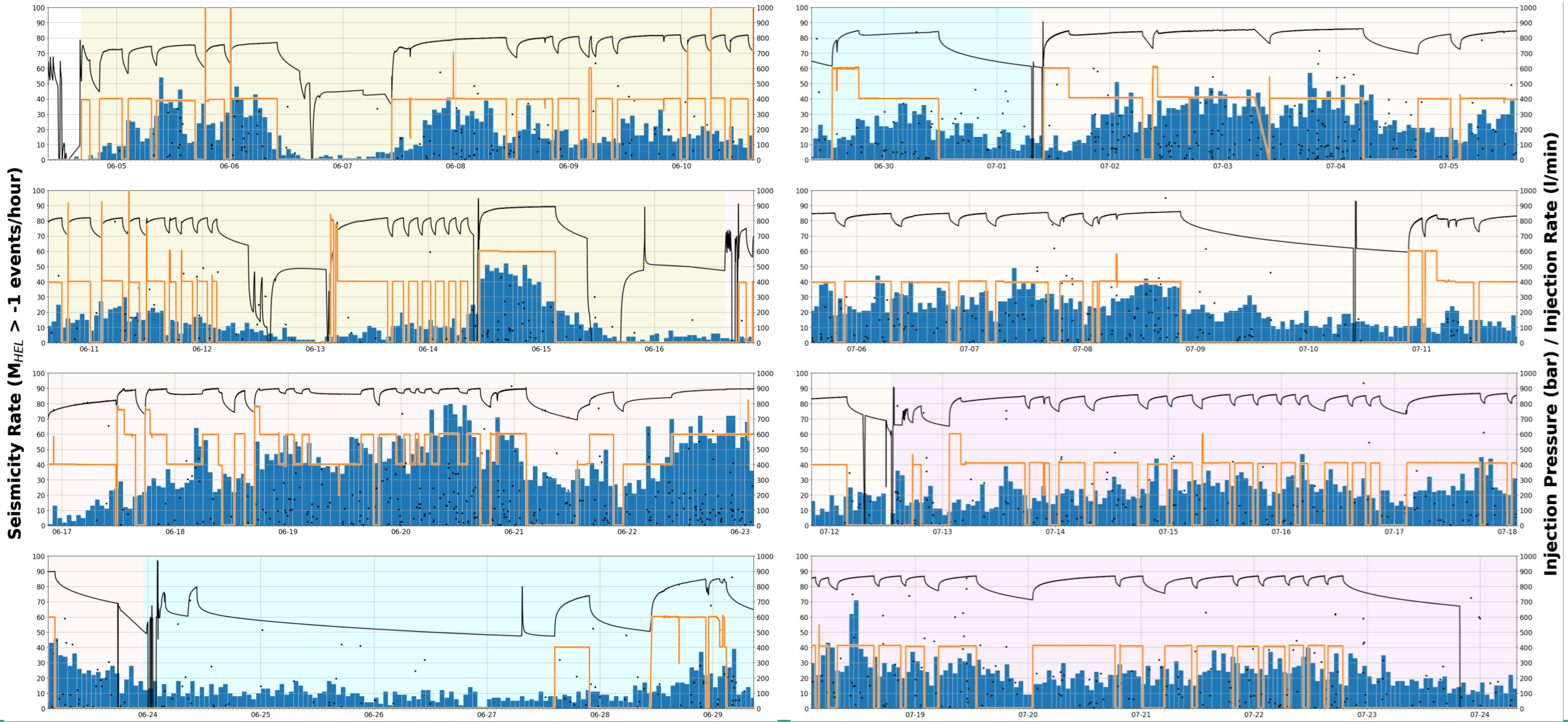
Seismicity Rate Injection Rate Injection Pressure



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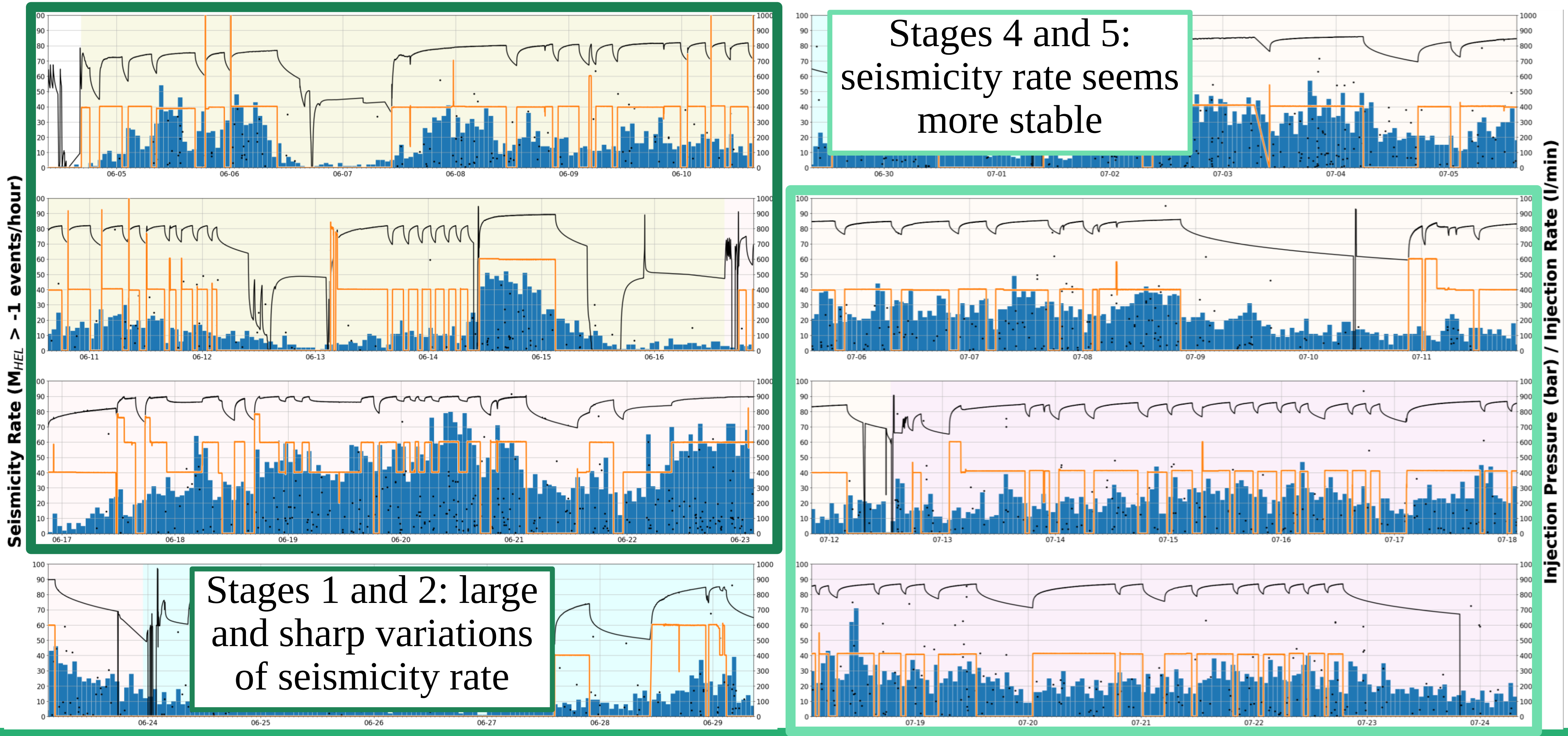
2.1 Visual Inspection and Initial Observations

Seismicity rate varies more during initial injection stages



Stages 4 and 5: seismicity rate seems more stable

Stages 1 and 2: large and sharp variations of seismicity rate

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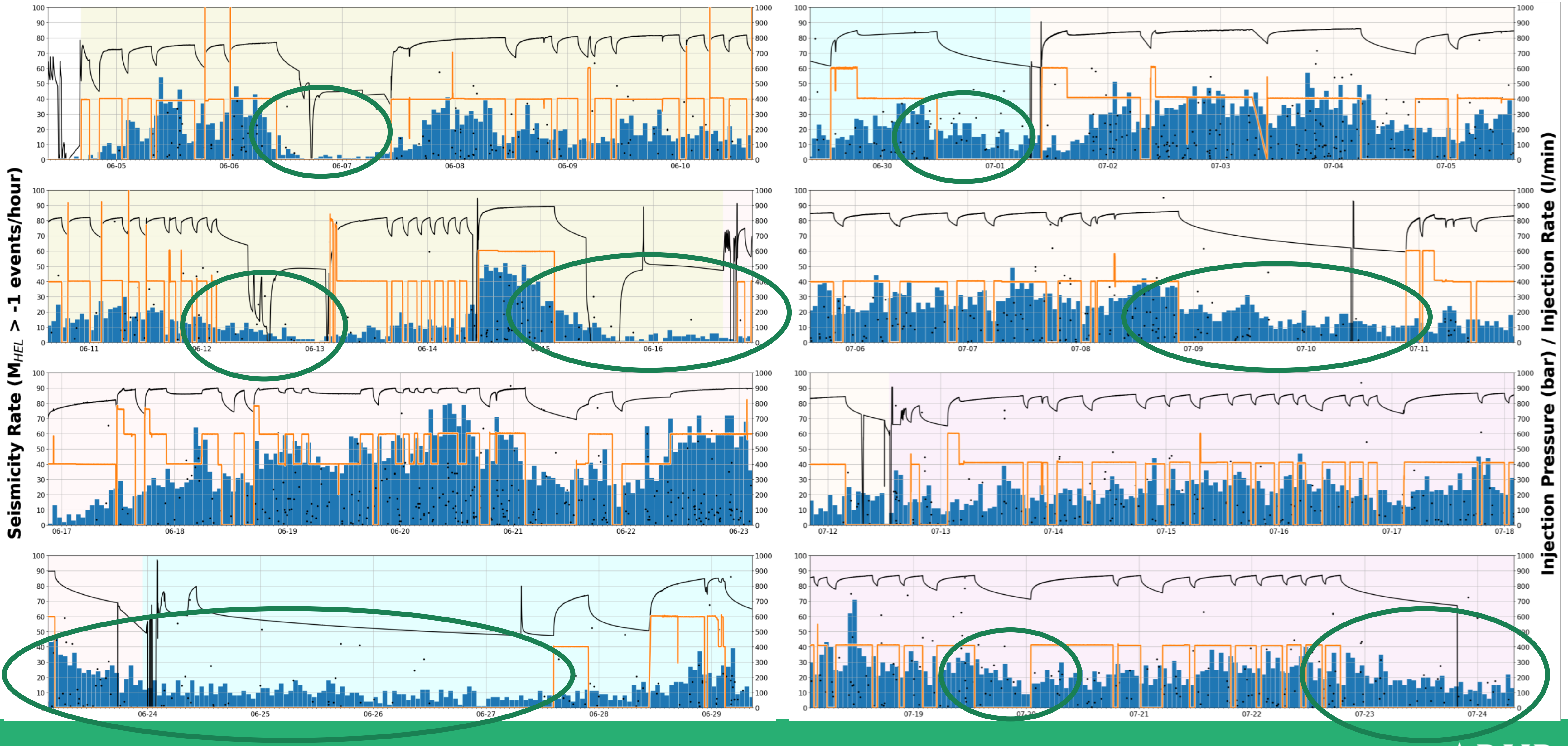
Seismicity rate decays gradually when injection stops



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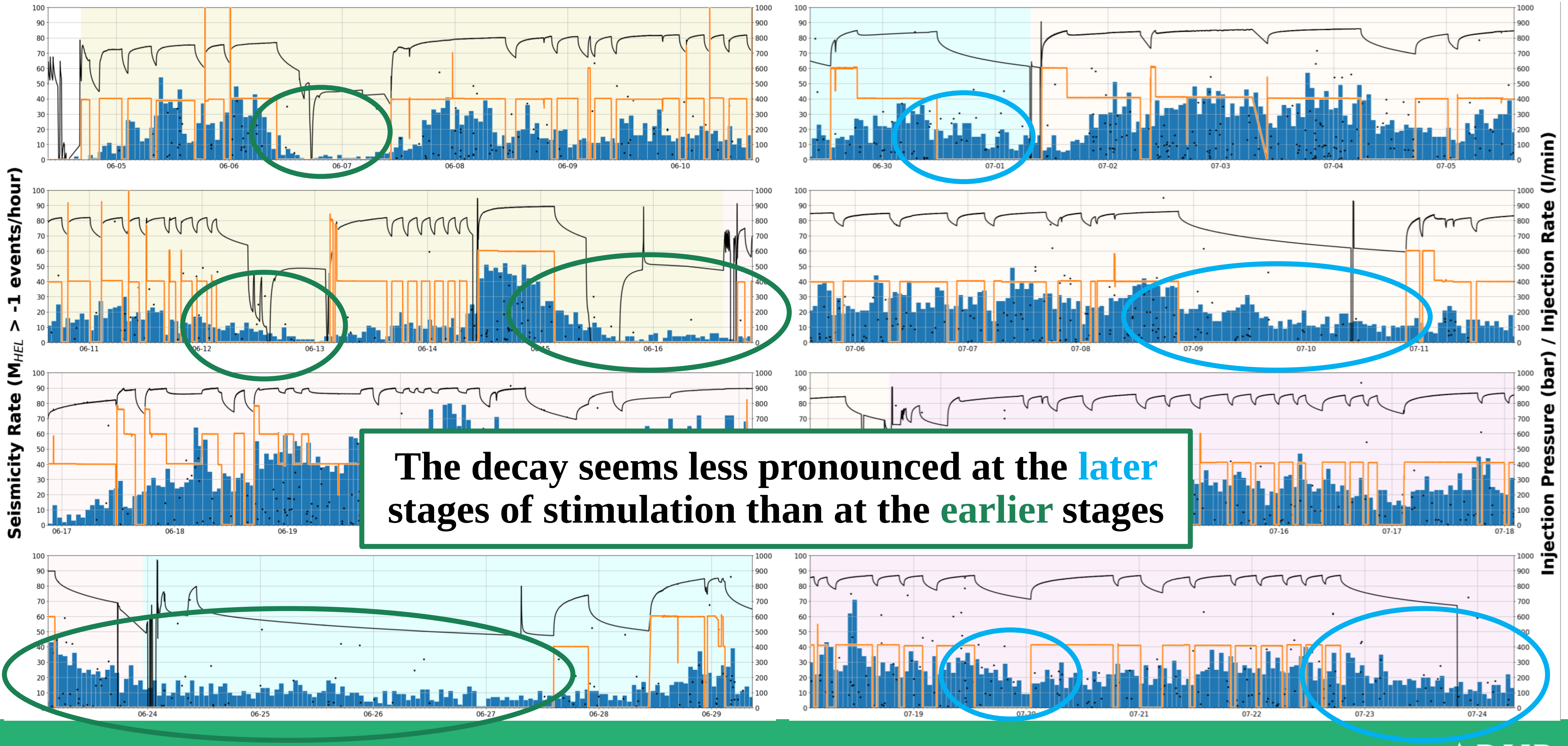
Seismicity rate decays gradually when injection stops



The decay seems less pronounced at the later stages of stimulation than at the earlier stages

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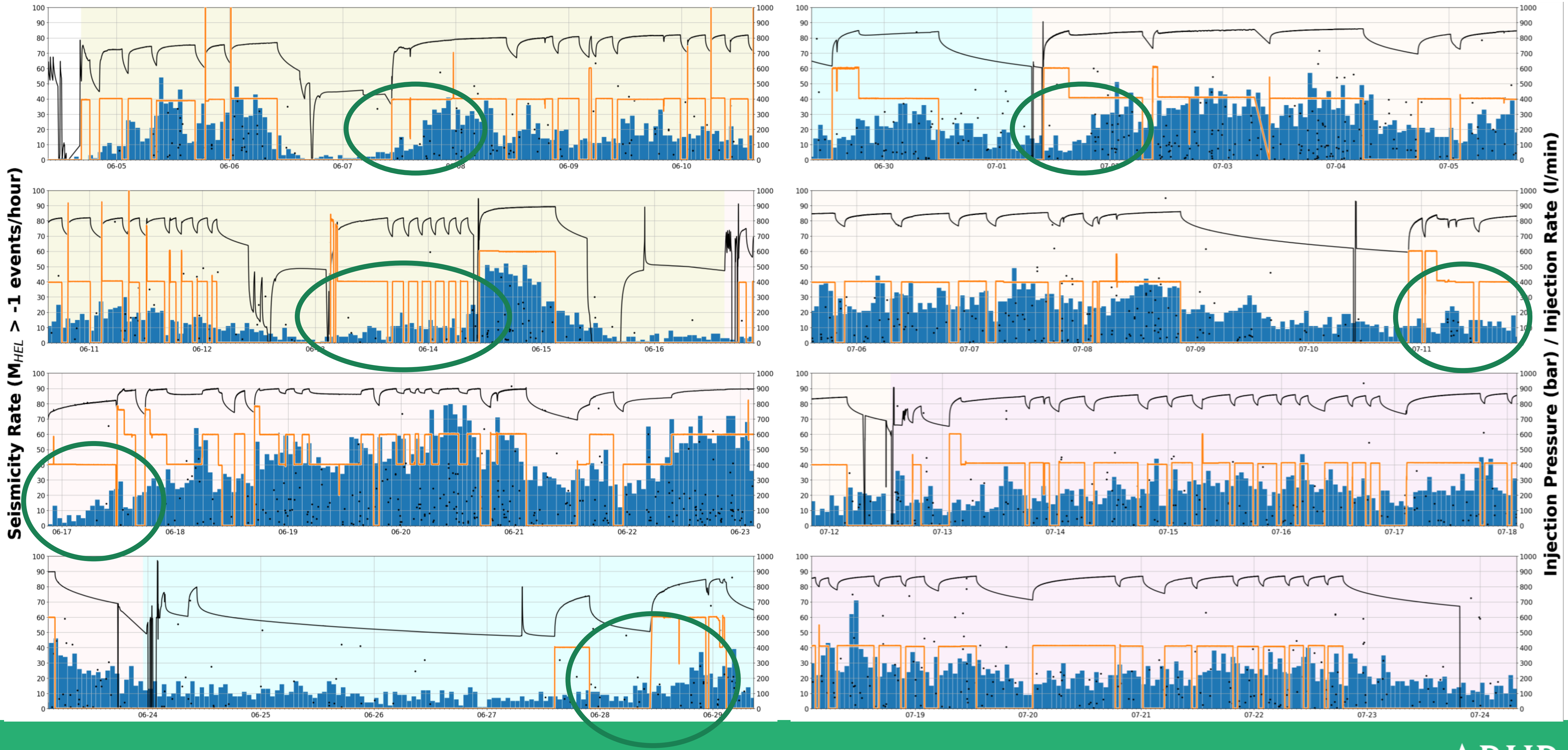
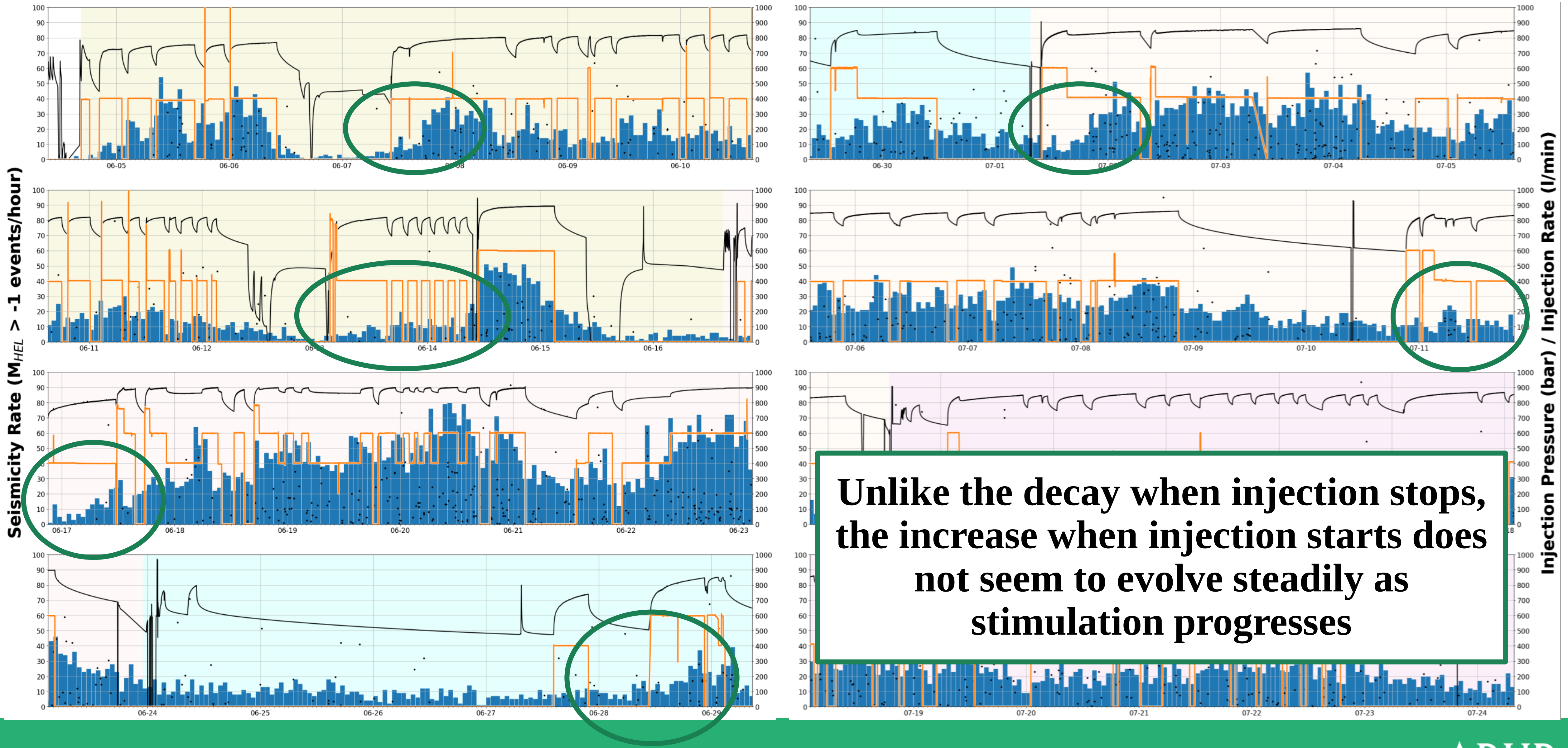
Seismicity rate increases gradually when injection starts



Seismicity rate increases gradually when injection starts



Unlike the decay when injection stops, the increase when injection starts does not seem to evolve steadily as stimulation progresses

2.2 Hypotheses

Hypotheses

INJECTION PAUSES

- Seismicity diffuses away from injection point
  - Pressure front takes longer to reach seismogenic areas
  - Seismicity takes longer to respond to injection changes

STABLE REGIMES

- If injecting at lower pressure and lower rates, pressure has time to diffuse away from injection point, without high-pressure front building up
  - Lower seismicity & smaller events

    i.e., lower number of events per volume unit injected

    i.e., larger b-value

2.3 Hypotheses Verification

2.3.1 Diffusion of Seismicity From Injection Point

Injection Pauses

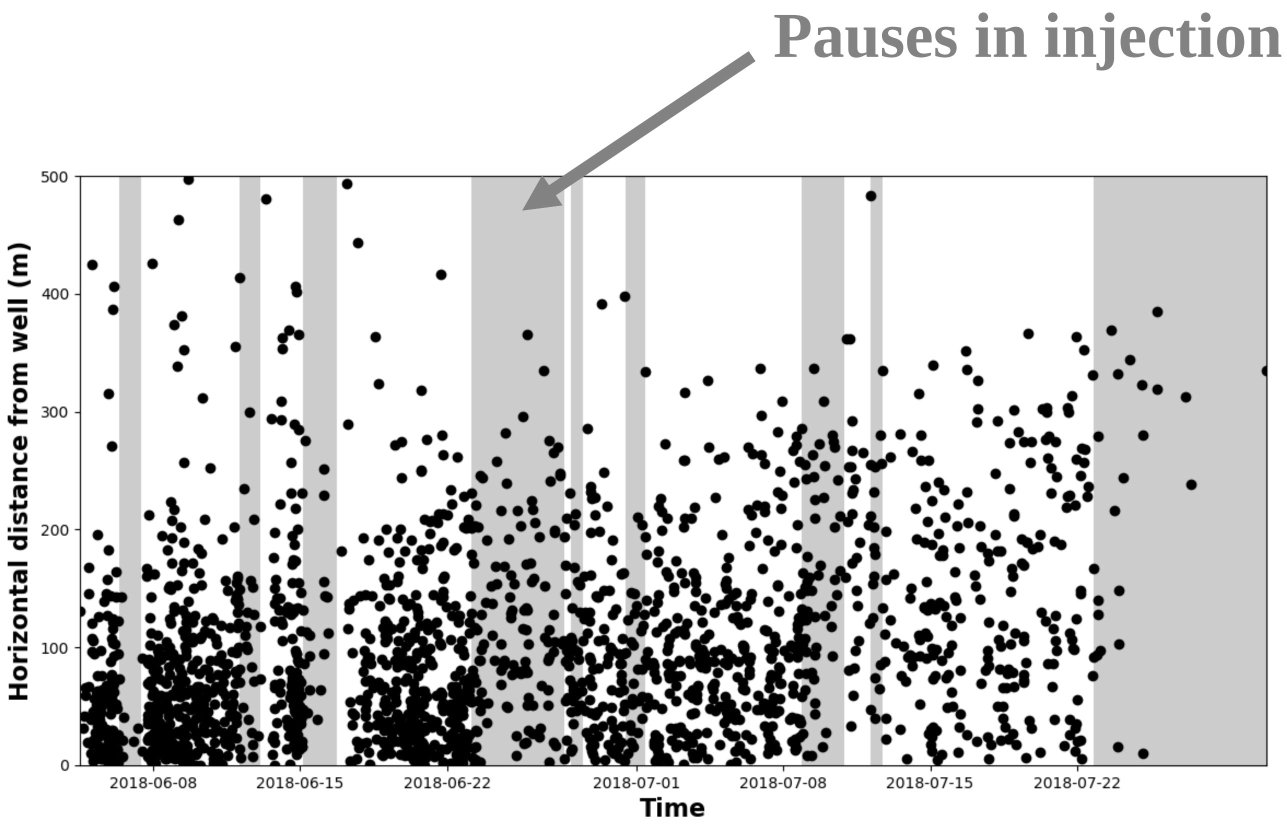
Hypothesis

- Seismicity diffuses away from injection point
  - Pressure front takes longer to reach seismogenic areas
  - Seismicity takes longer to respond to injection changes

Verification

1. Look at spatial evolution of seismicity throughout stimulation
2. Look at evolution of response time of seismicity when injection stops throughout stimulation

Distance of Seismicity from Injection Well



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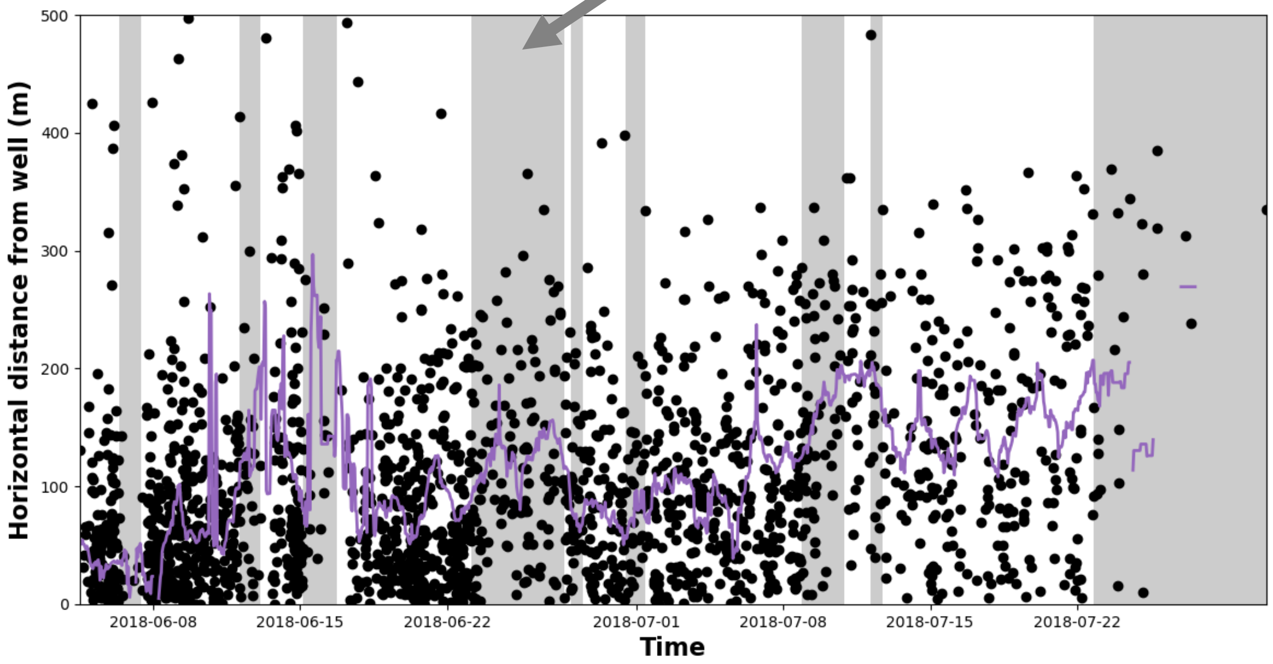
Distance of Seismicity from Injection Well

Pauses in injection



Standard deviation of distance: sliding windows of 24h. The standard deviation is corrected for the expected spread due to uncertainties in earthquake locations.

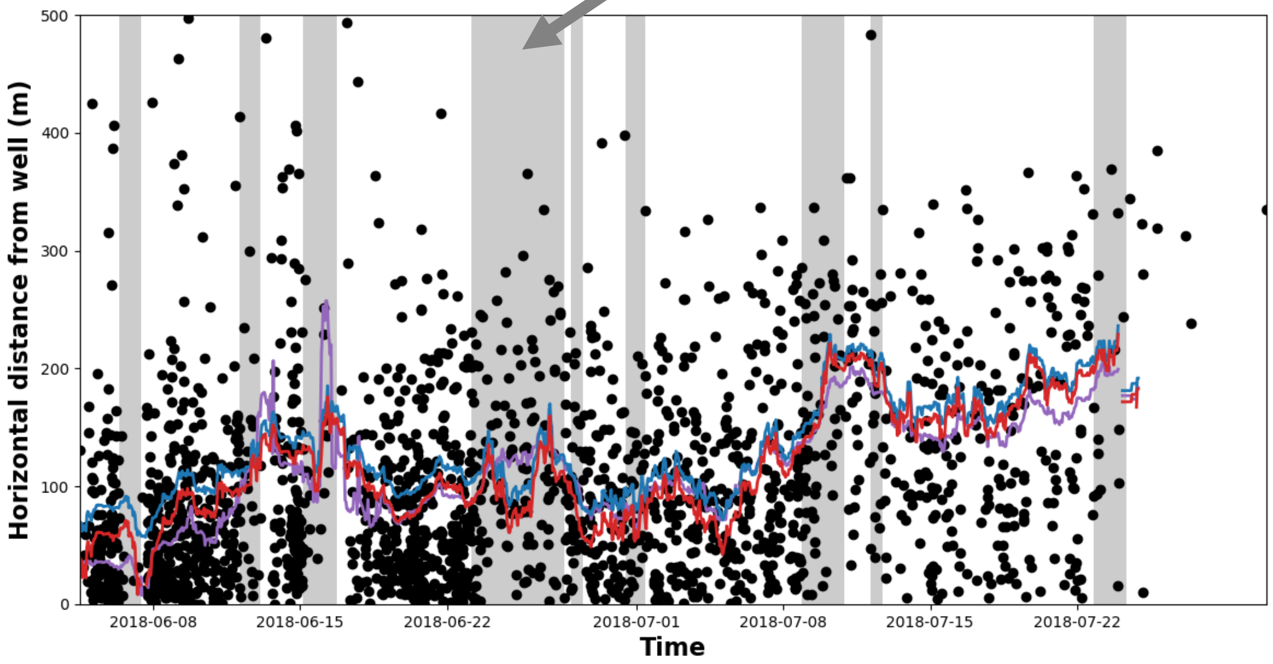
Distance of Seismicity from Injection Well

Standard deviation

68% of events

68% of events corrected with uncertainty on event location

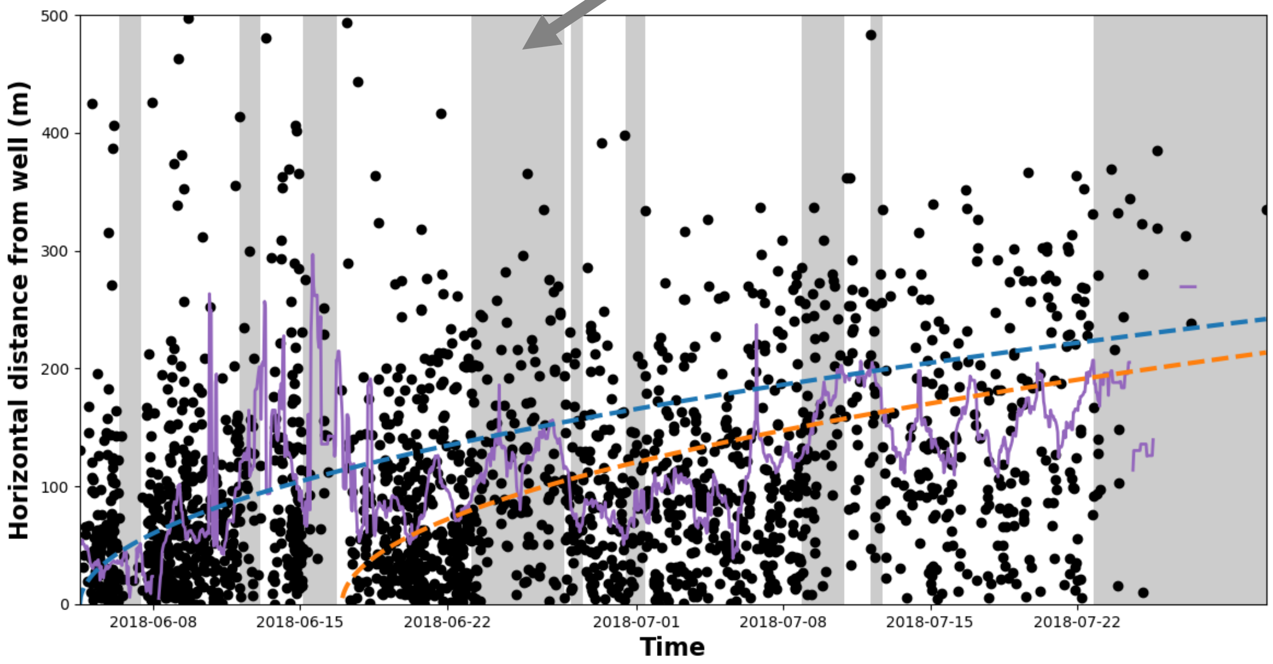
Pauses in injection



Standard deviation of distance: sliding windows of 24h. The standard deviation is corrected for the expected spread due to uncertainties in earthquake locations.

Distance of Seismicity from Injection Well

Pauses in injection



Diffusion curves $r = 2\sqrt{Dt}$ from the start of phases 1 and 2, where the diffusivity is D = 0.003 m^2/s. **The seismicity seems to spread according to diffusion laws.**

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Injection Pauses

- Two models tested for evolution of seismicity with time when injection stops
  - Exponential decay:

$$R(t) = R_0 e^{-t\ln 2/t_a},$$

where t_a is the time such that $R(t_a) = R_0/2$.

  - Omori decay (used to describe aftershock sequences):

$$R(t) = \frac{R_0}{1+t/t_a},$$

where t_a is the time such that $R(t_a) = R_0/2$.

- Look at the magnitude-frequency distribution before and after injection stops.
- The computation of t_a and confidence intervals is detailed in Appendices A1 and A3.

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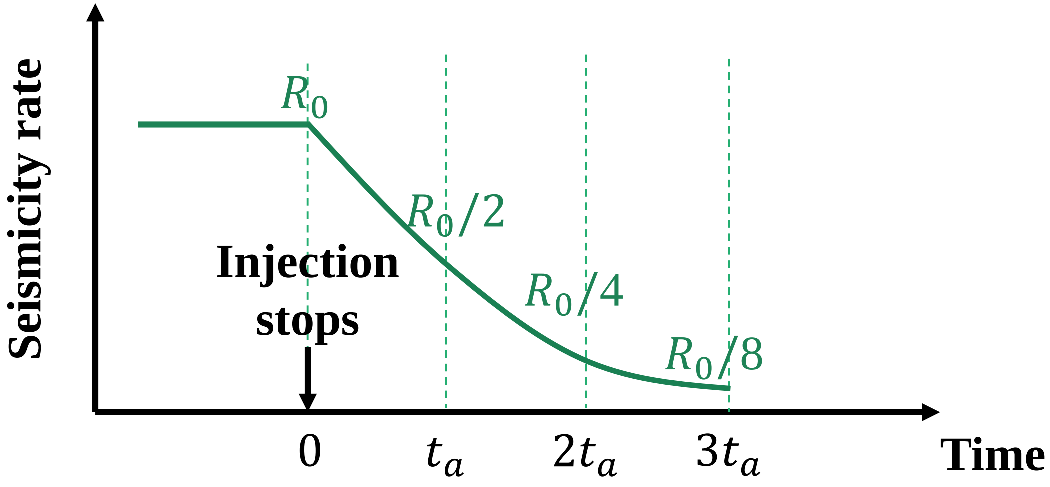
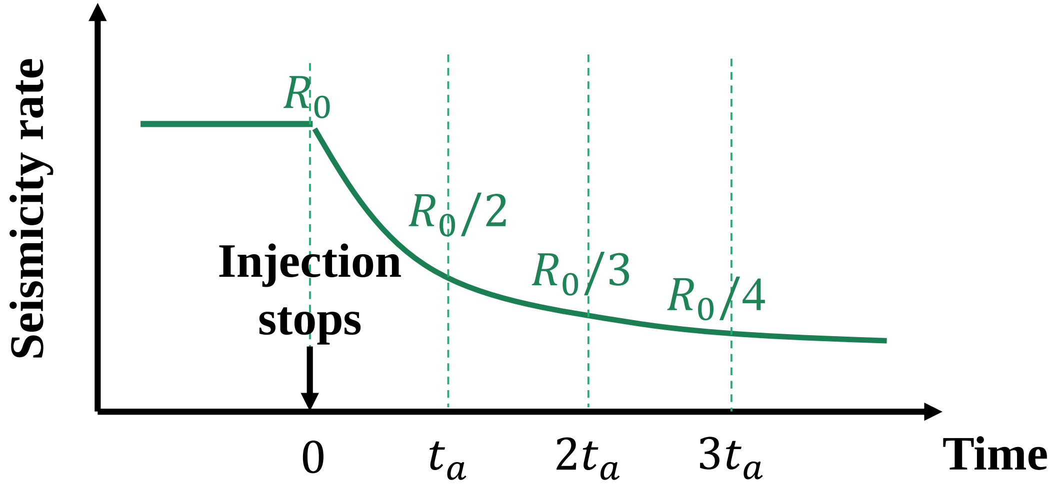
Exponential vs Omori decay





- **Exponential decay:**

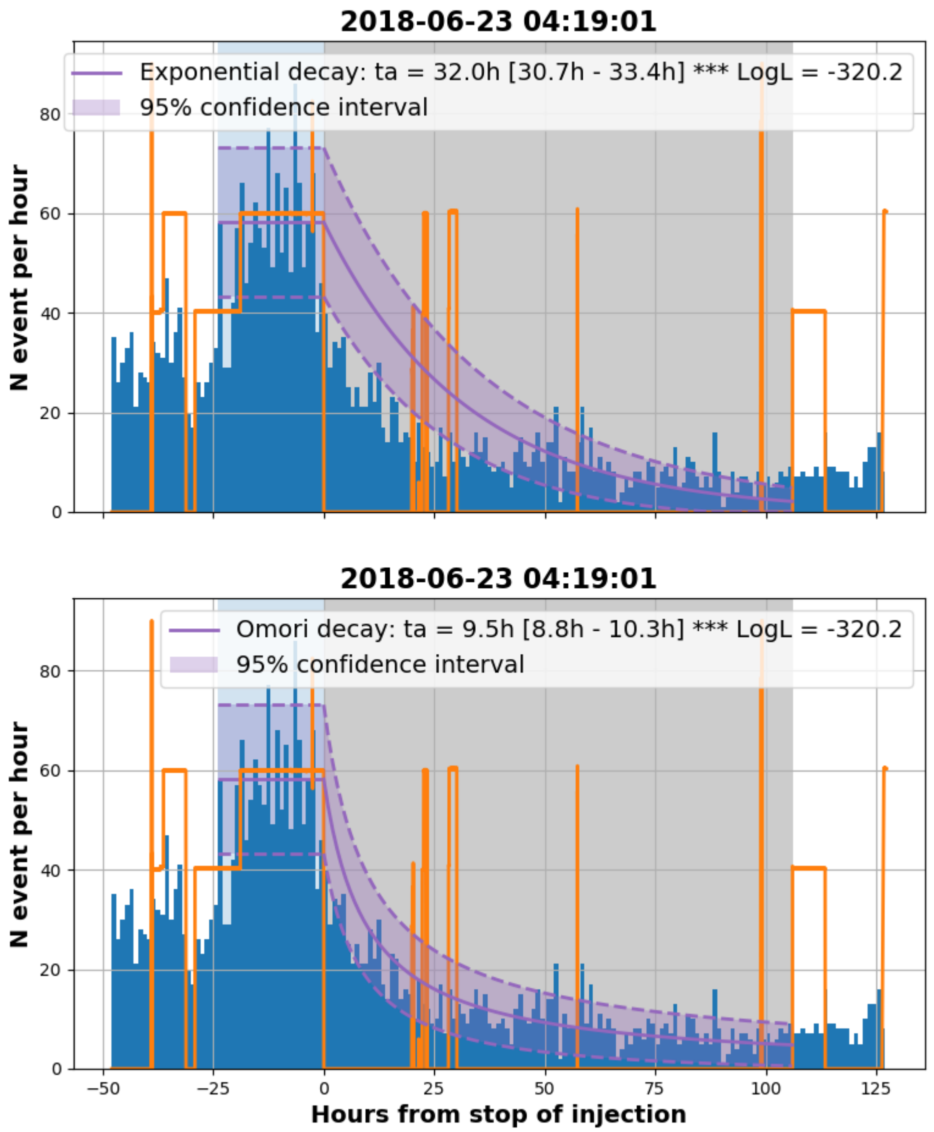
The seismicity rate decreases by a factor of 2 at each time interval t_a.

- **Omori decay:**

The seismicity rate decreases quickly at first and then more slowly than the exponential model.

If it takes a time t_a for the seismicity rate to decrease by half, if will only decrease to a third after $2t_a$, by a quarter after $3t_a$, etc.

Exponential vs Omori decay



- **Exponential decay**: Poor fit to the observed seismicity rate.

- **Omori decay** mimics better two features:
  - Sharp decay after injection stops
  - Long term flat seismicity rate

➢ **Used to fit decay of seismicity and estimate characteristic time.**

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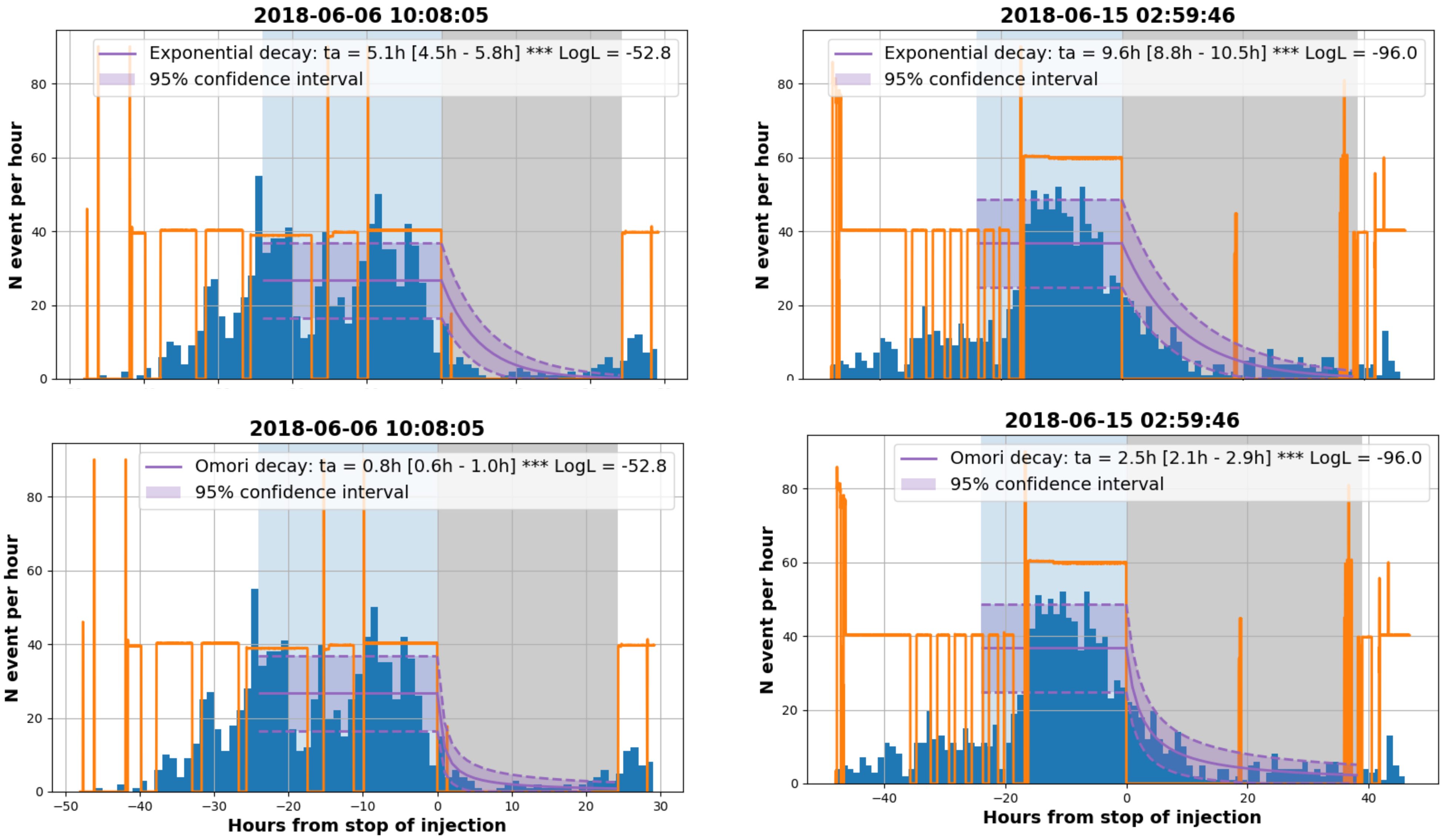
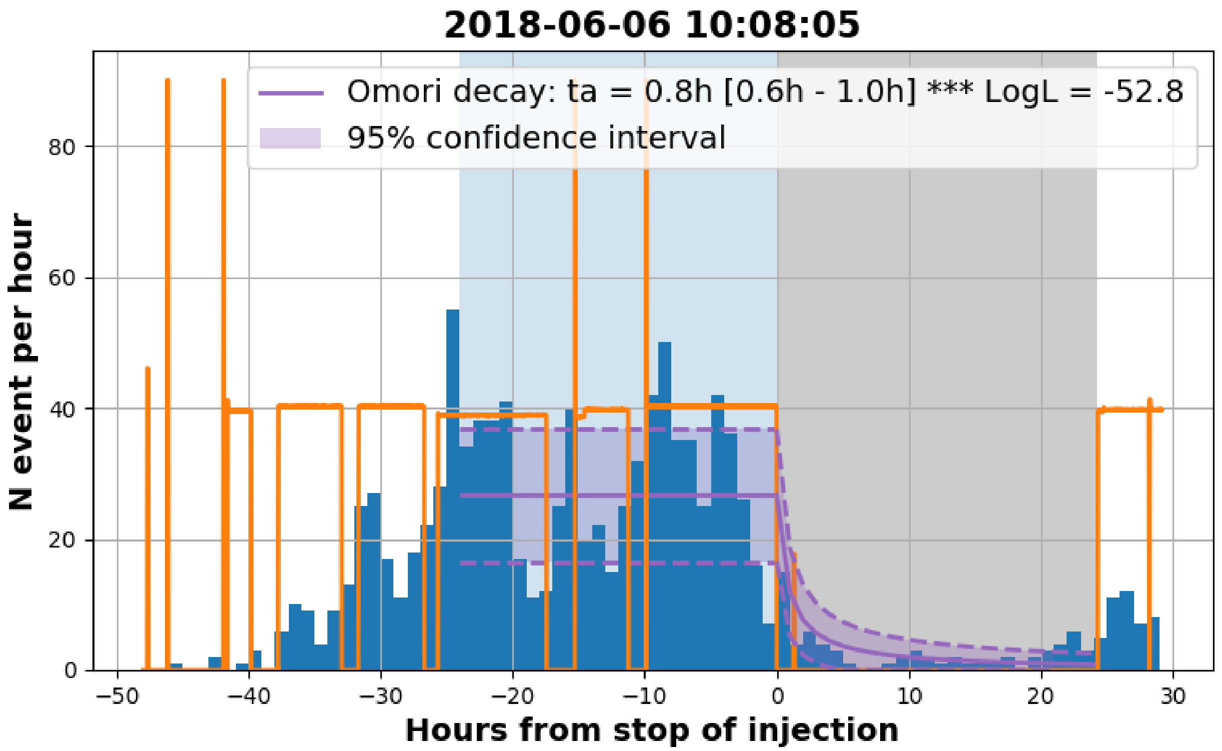
Exponential vs Omori decay

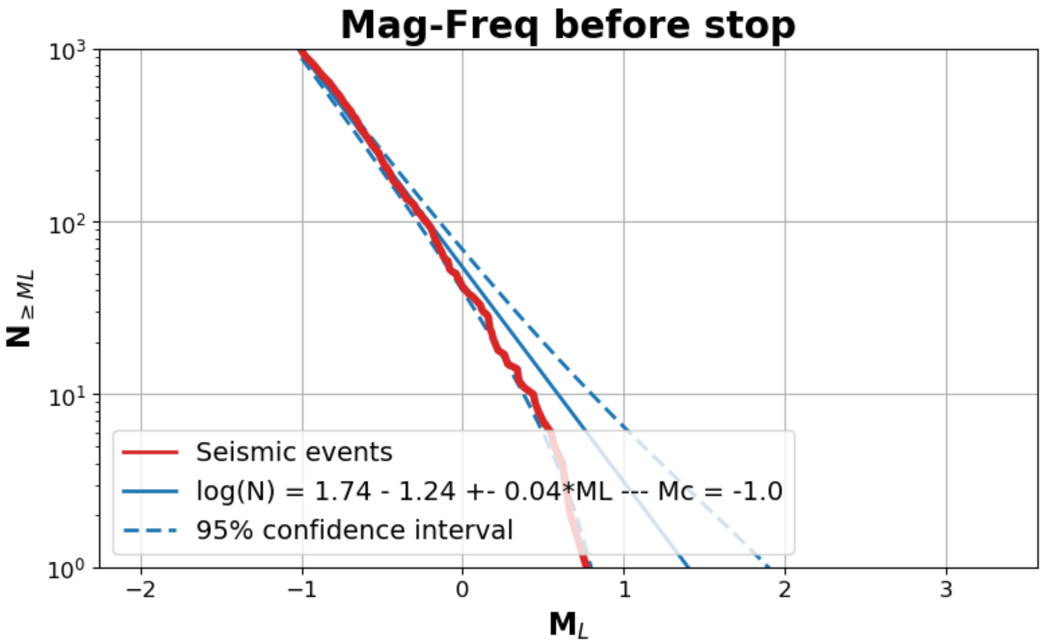


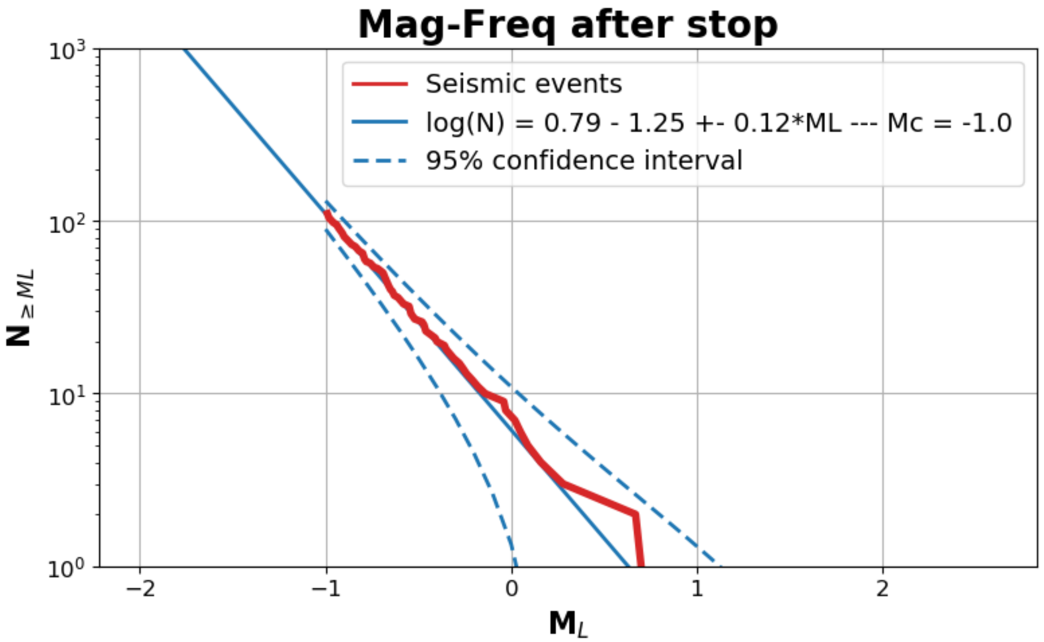
EXPONENTIAL

OMORI

Injection Pauses







- Omori law fits well the decay of seismicity rate
- *b*-value similar before and after injection stops

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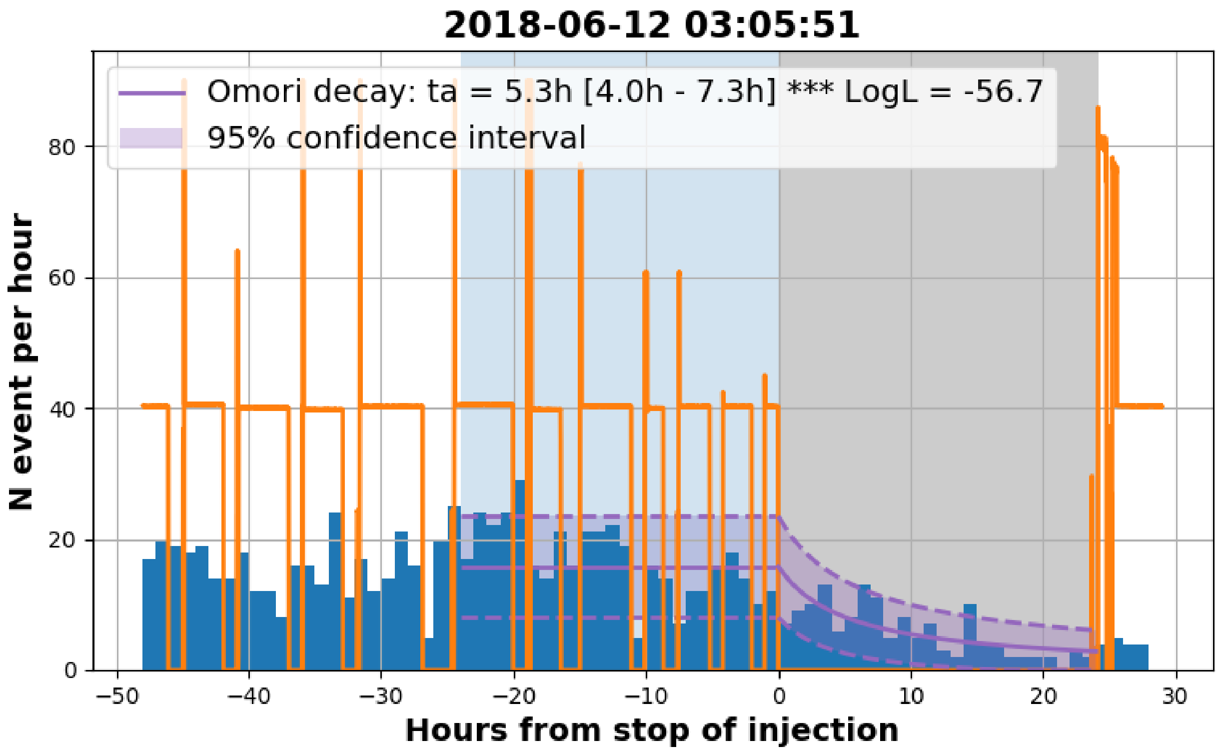
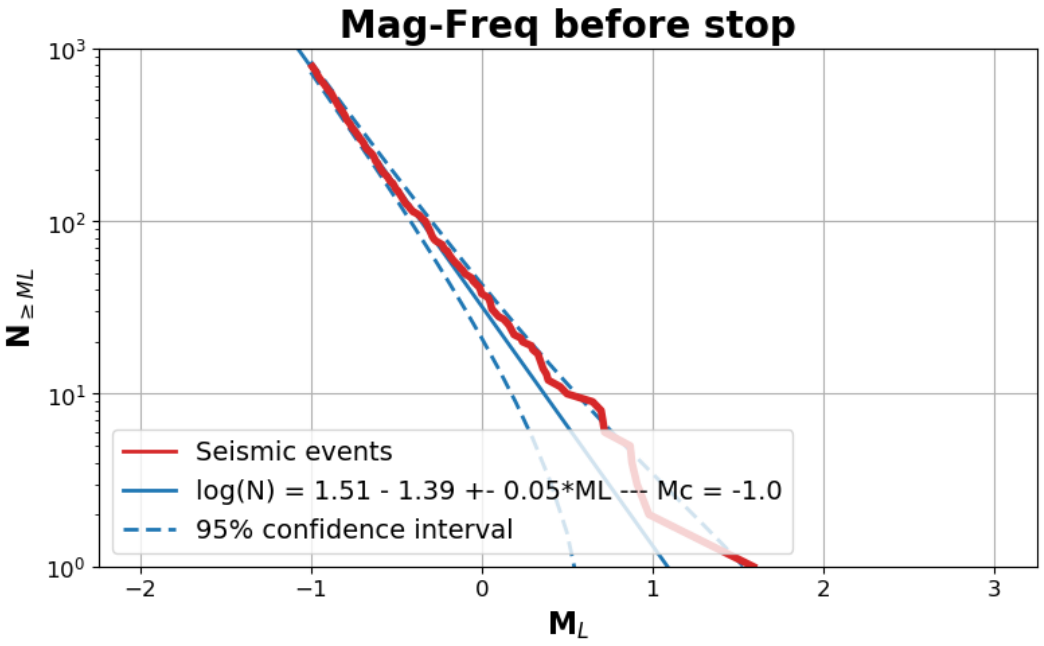
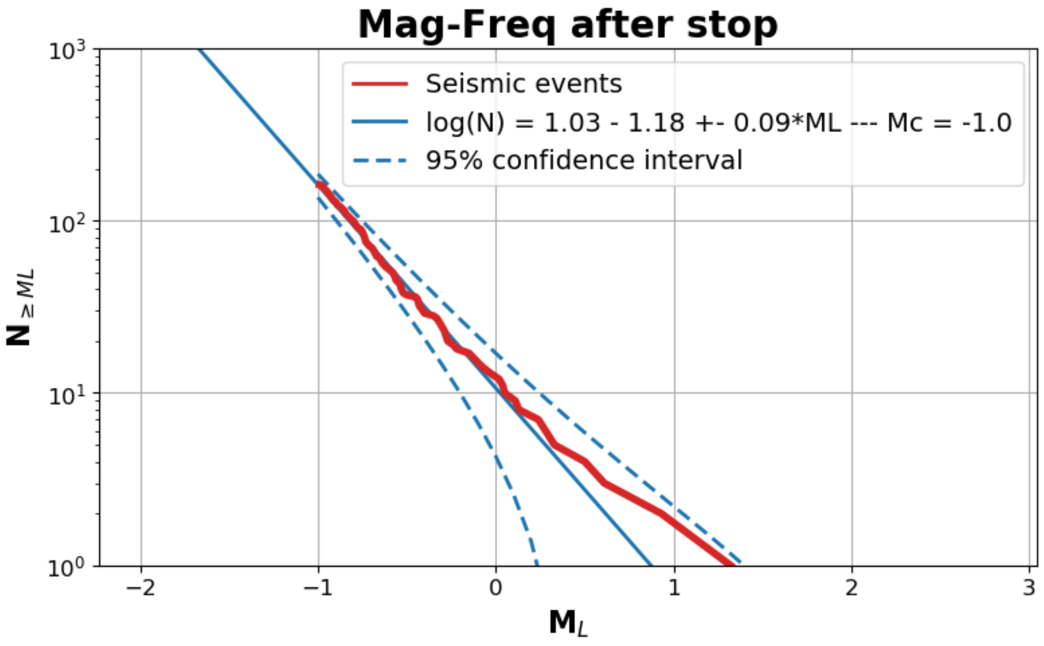
Injection Pauses







- Omori law fits well the decay of seismicity rate
- *b*-value larger before than after injection stops

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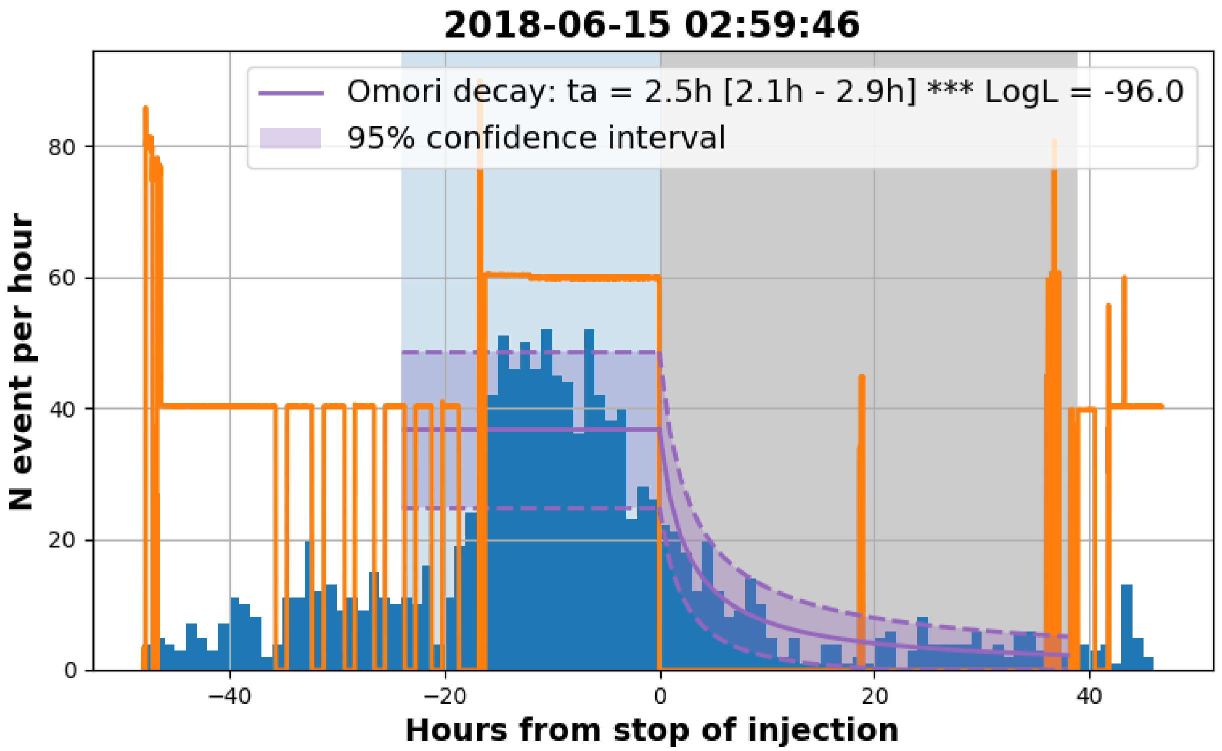
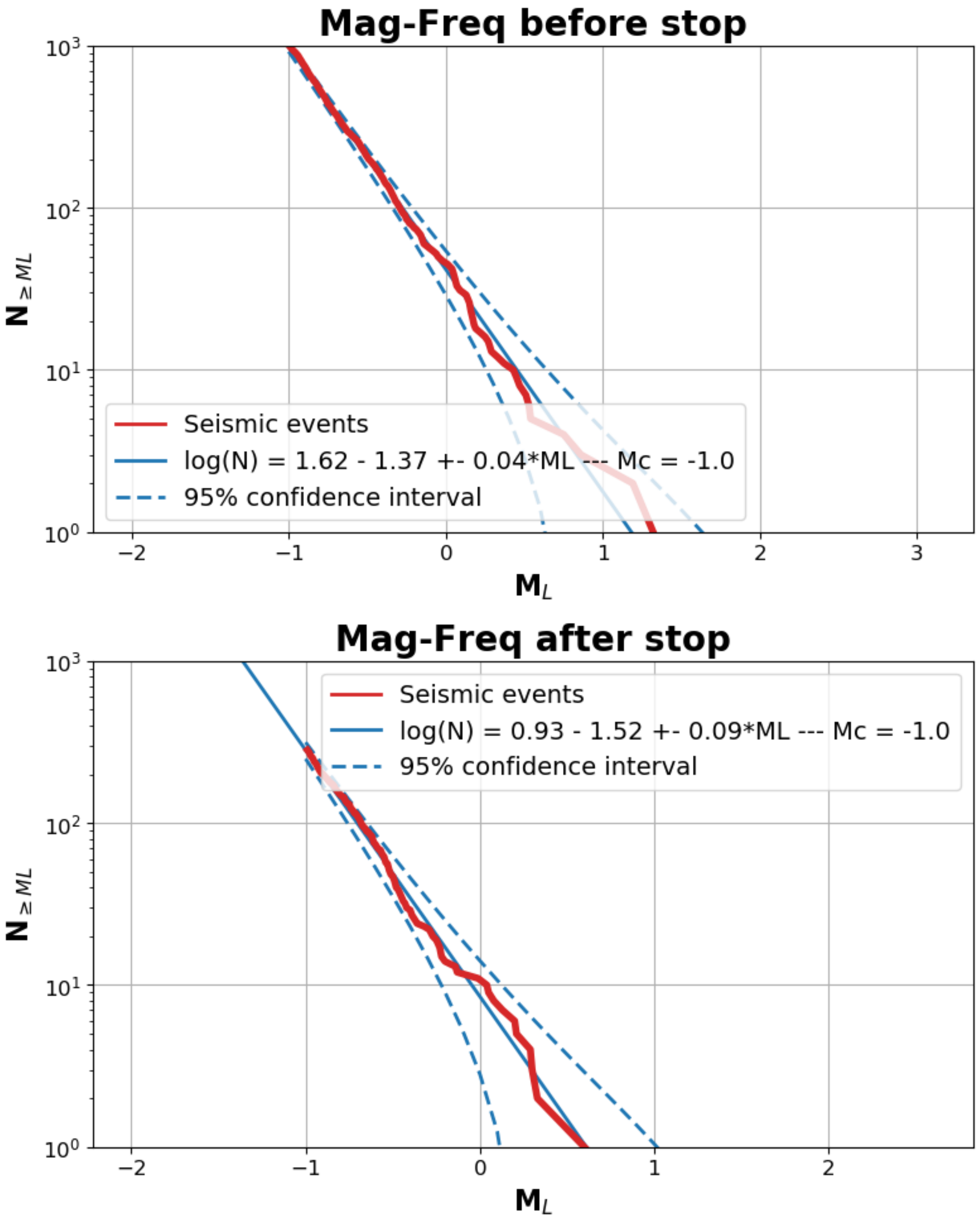
Injection Pauses





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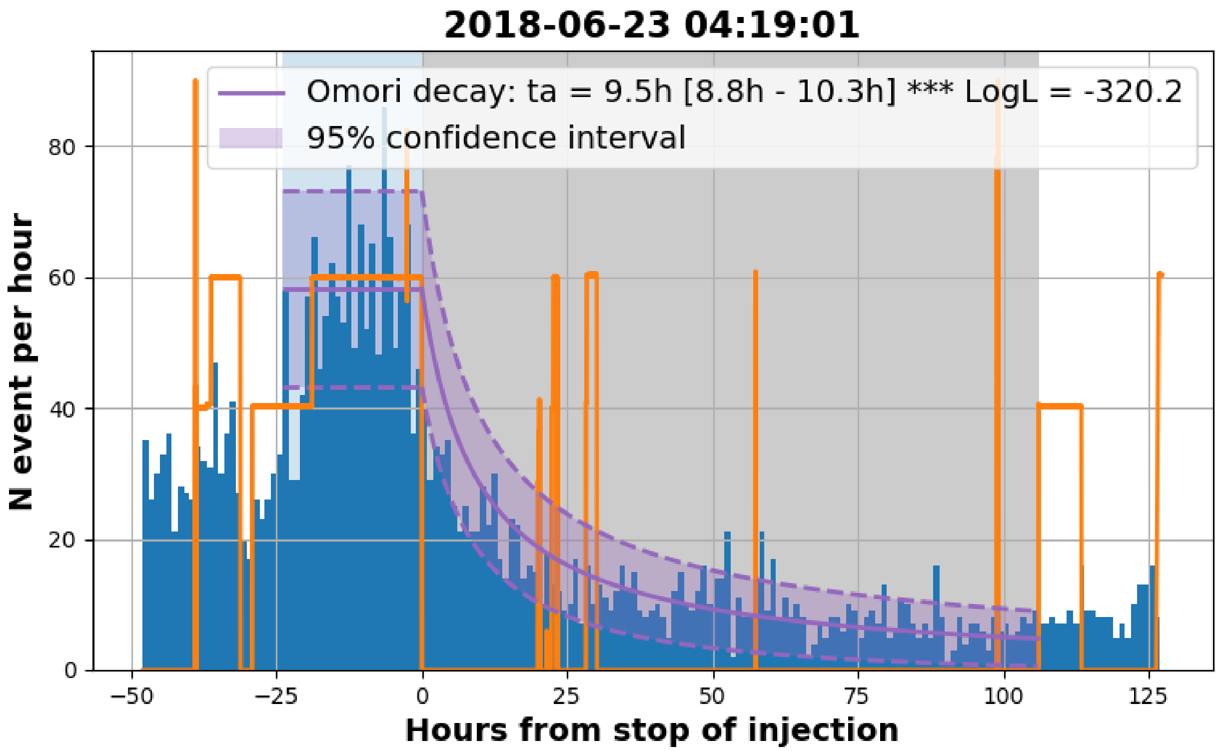
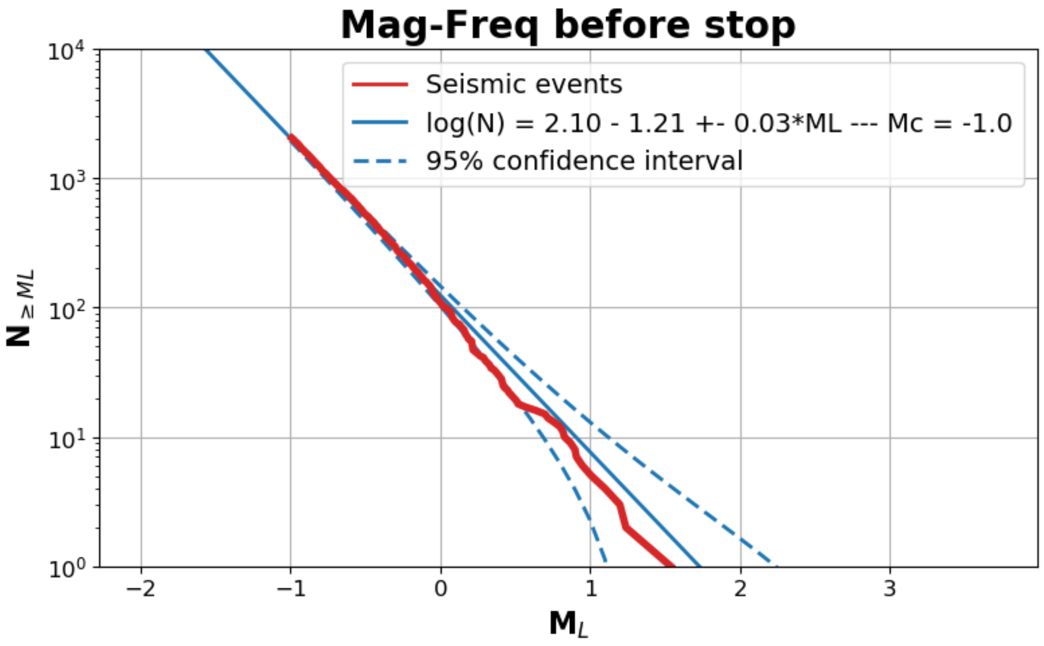
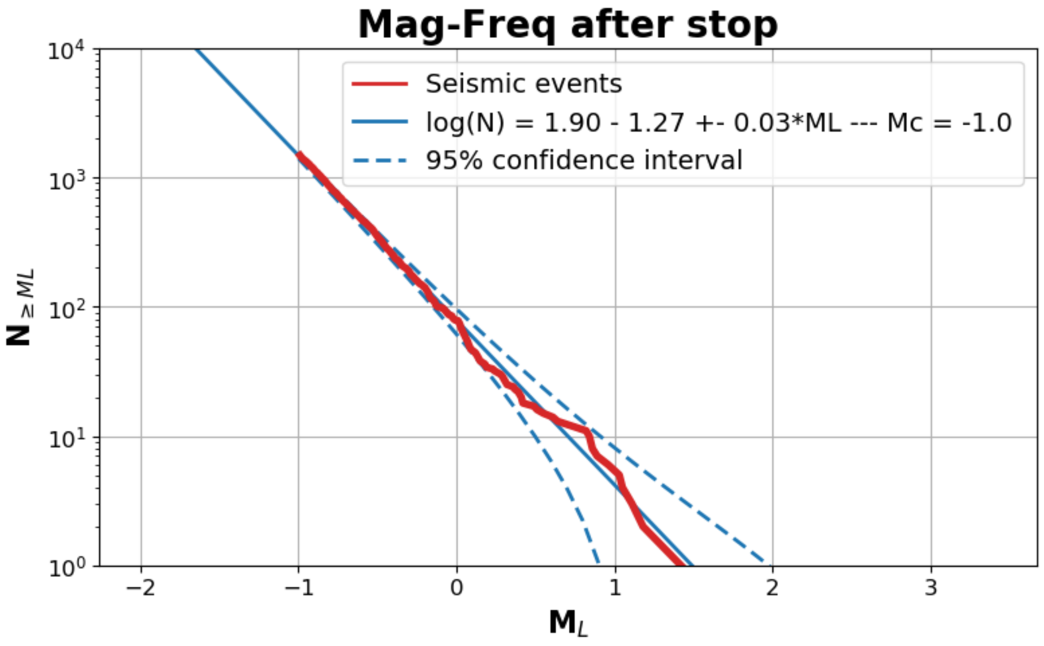
Injection Pauses







- Omori law fits well the decay of seismicity rate
- *b*-value similar before and after injection stops

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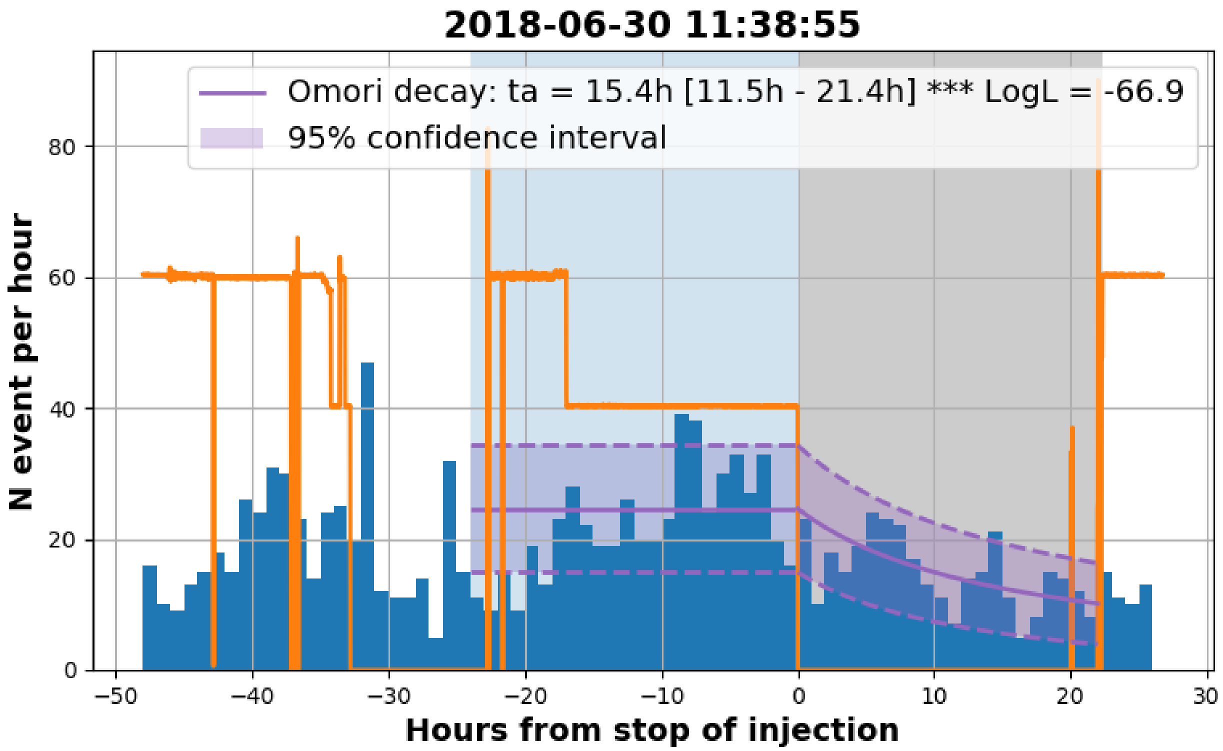
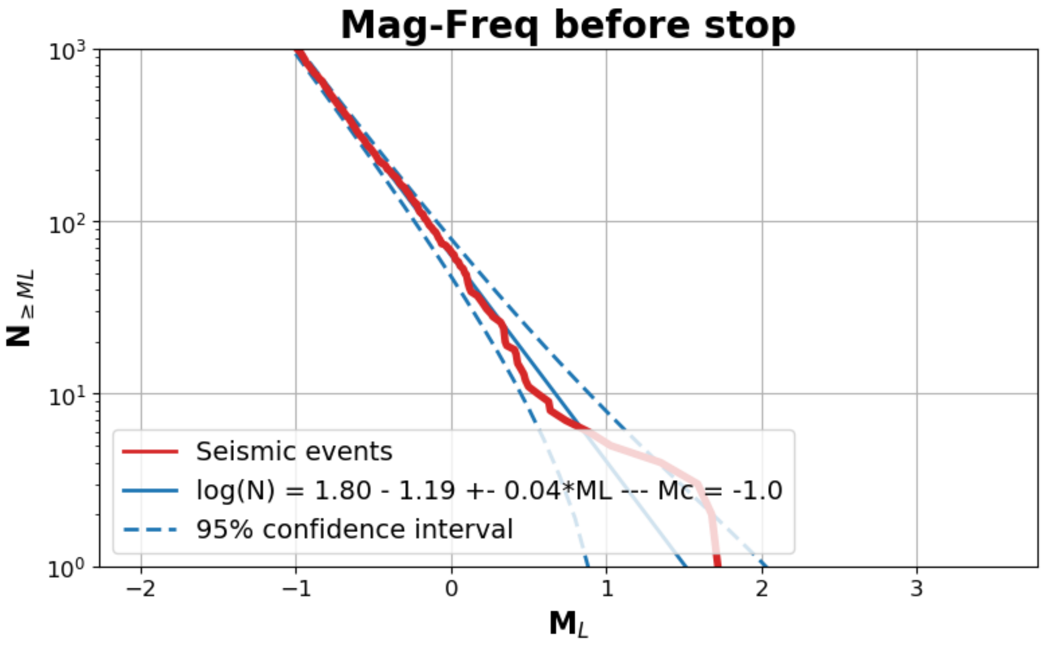
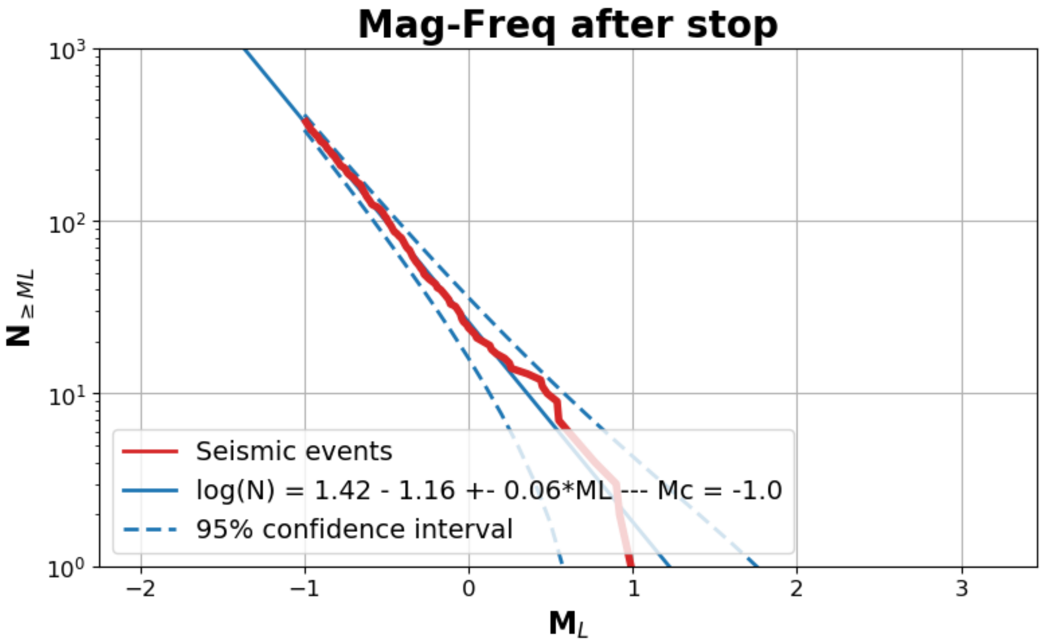
Injection Pauses







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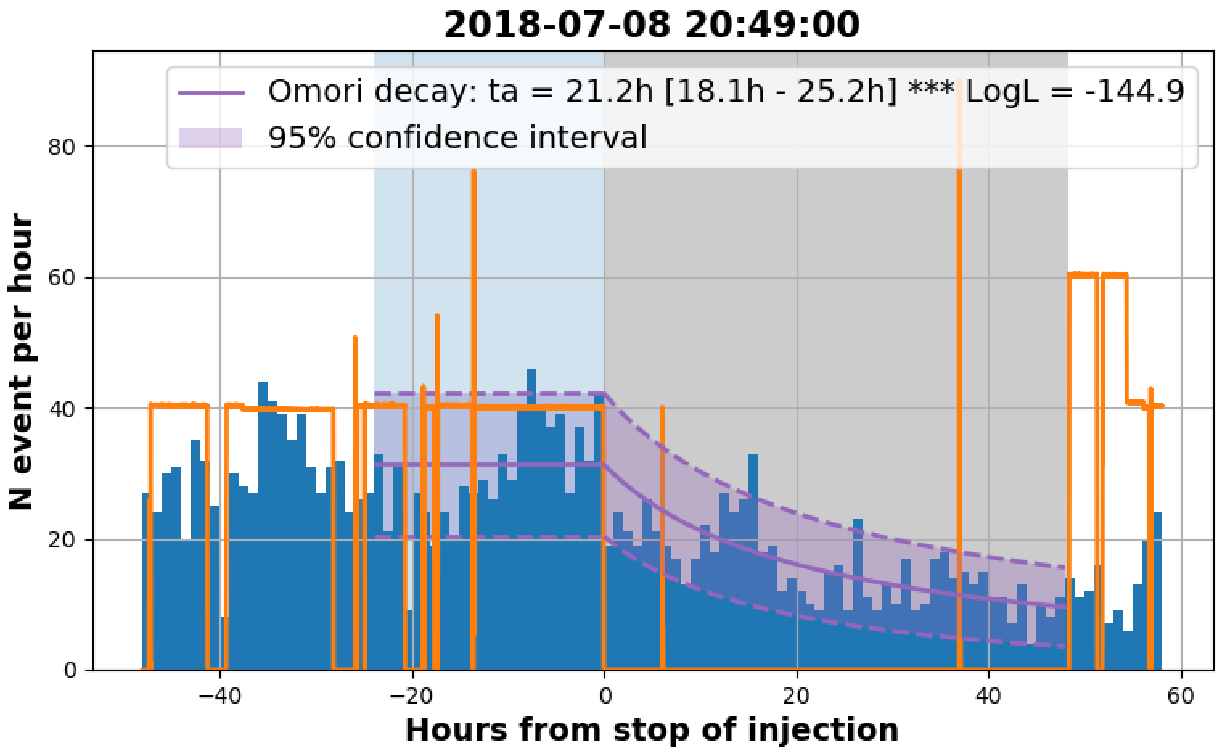
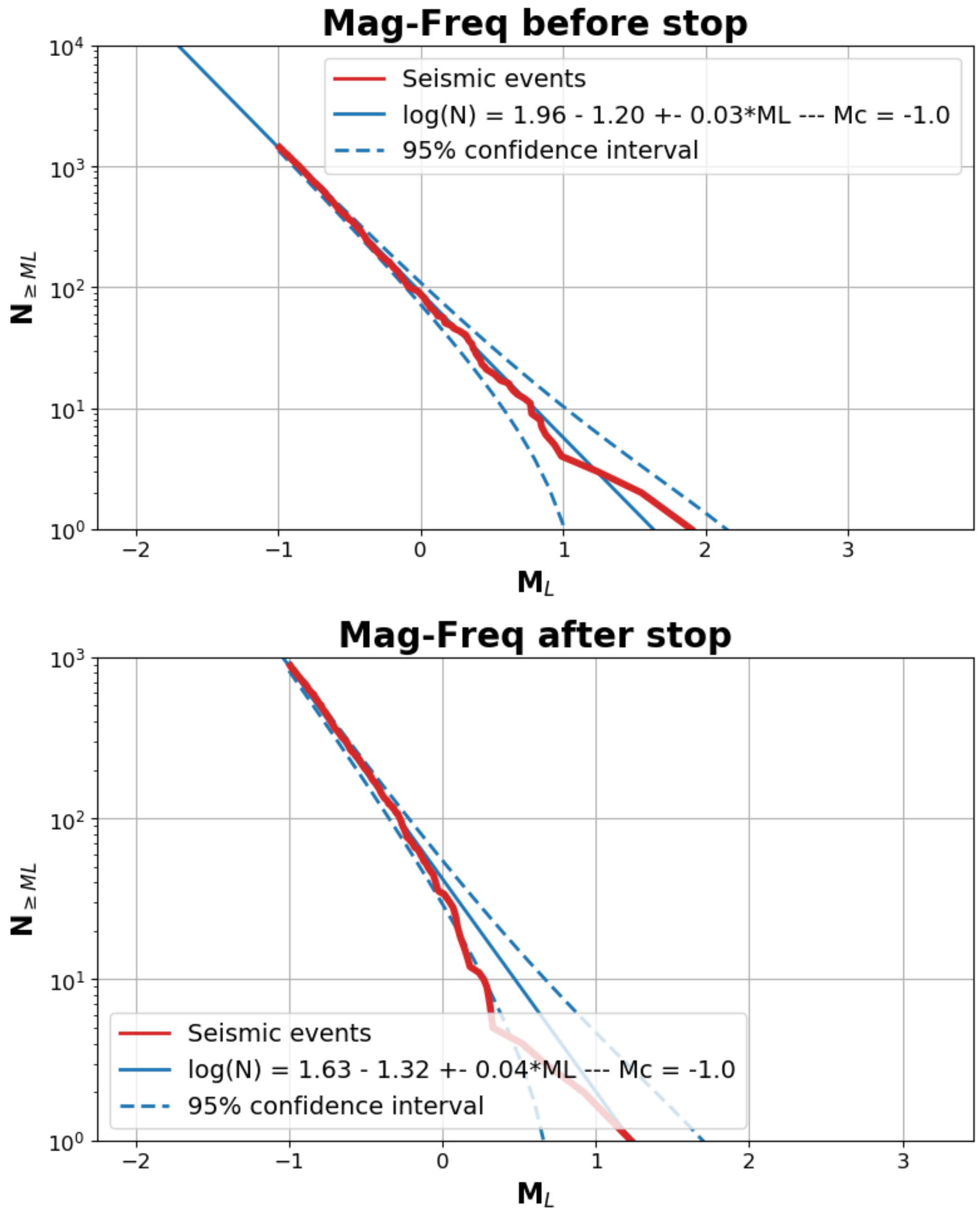
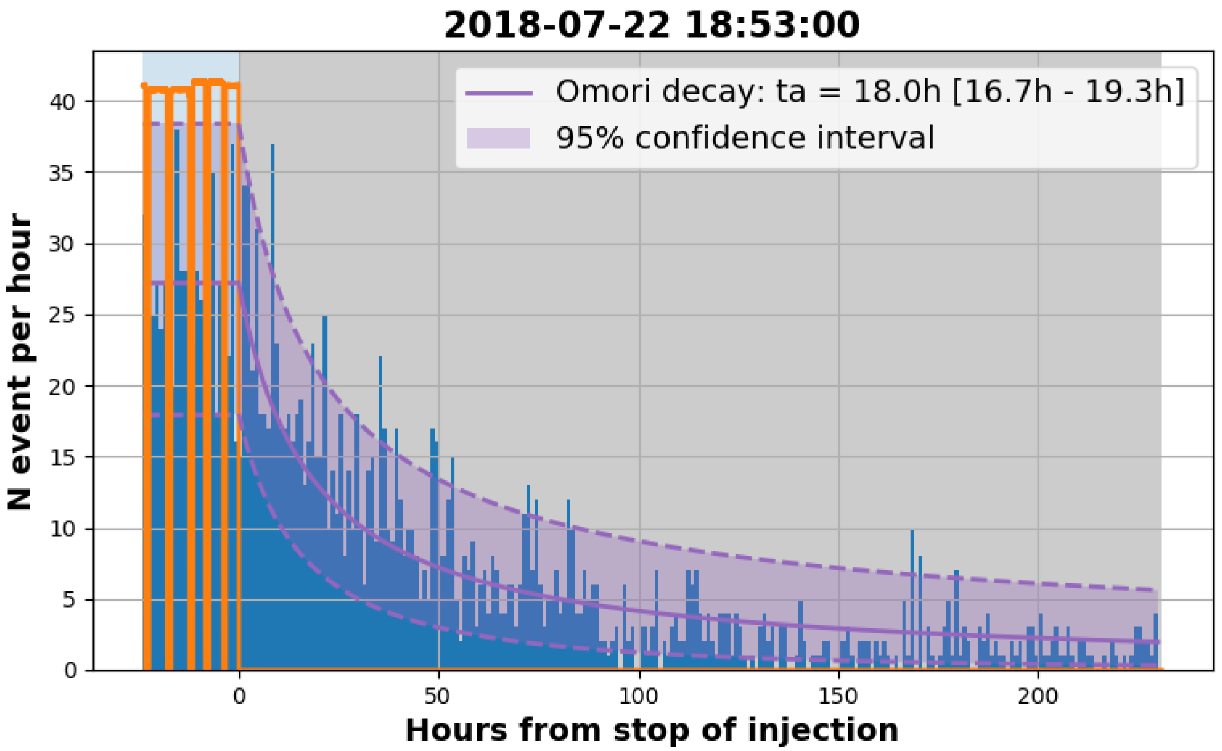
Injection Pauses

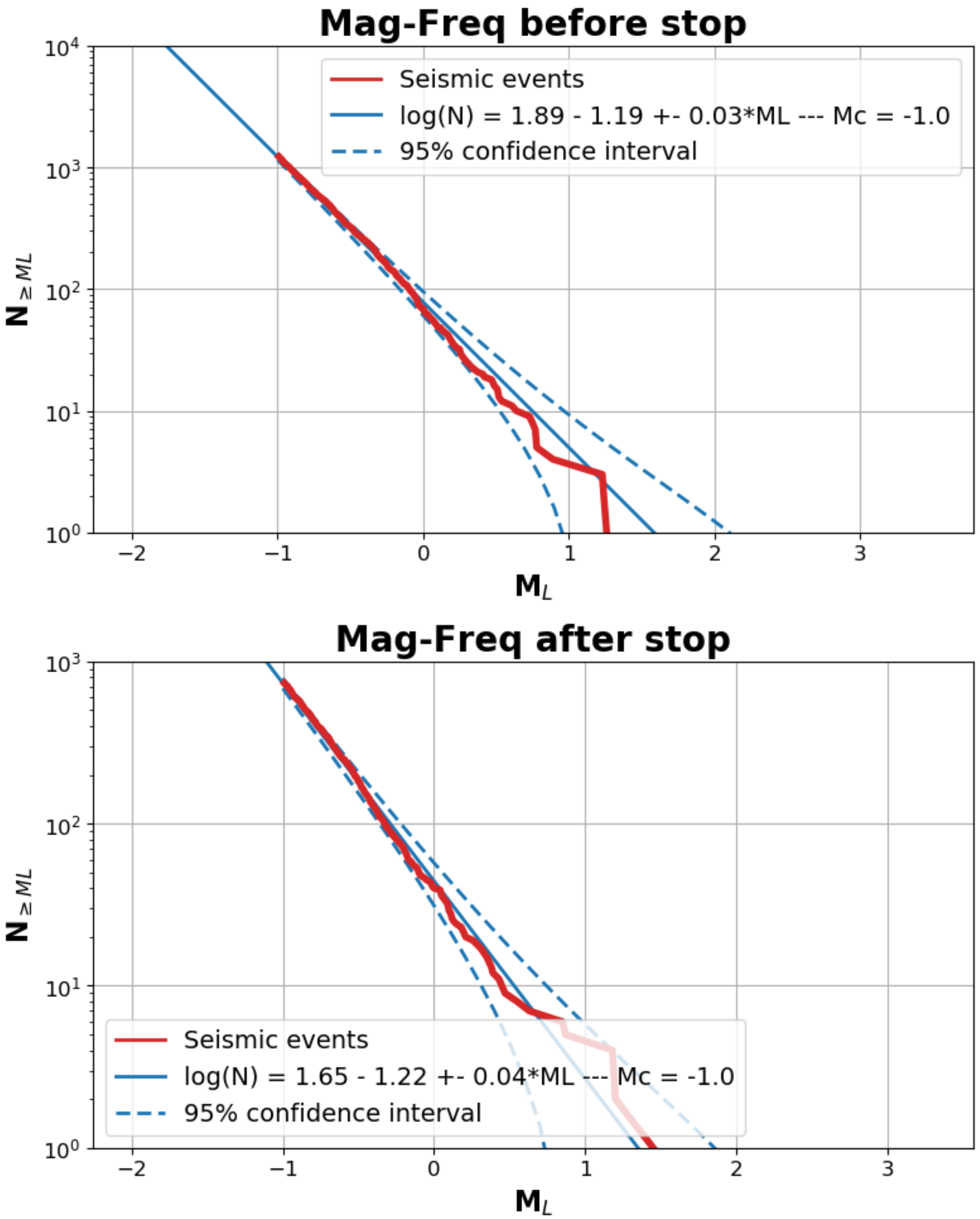




- Omori law fits well the decay of seismicity rate
- *b*-value smaller before and after injection stops

Injection Pauses (End of Stimulation)





- Omori law fits well the decay of seismicity rate
- *b*-value similar before and after injection stops

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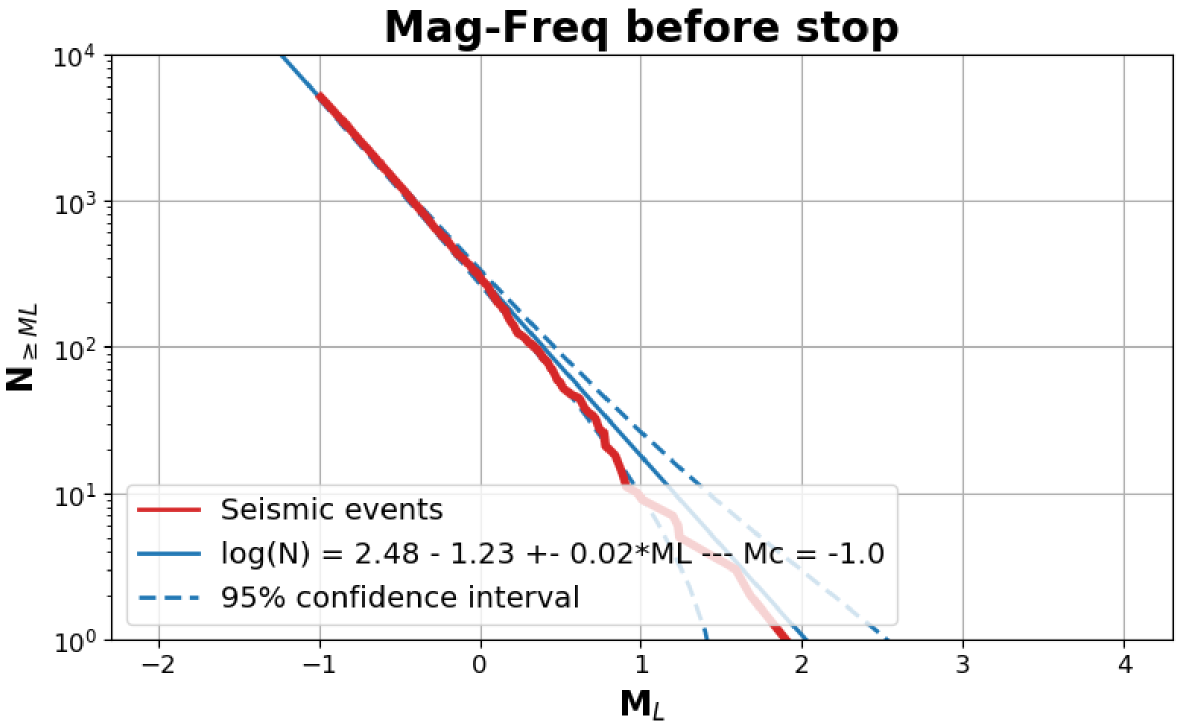
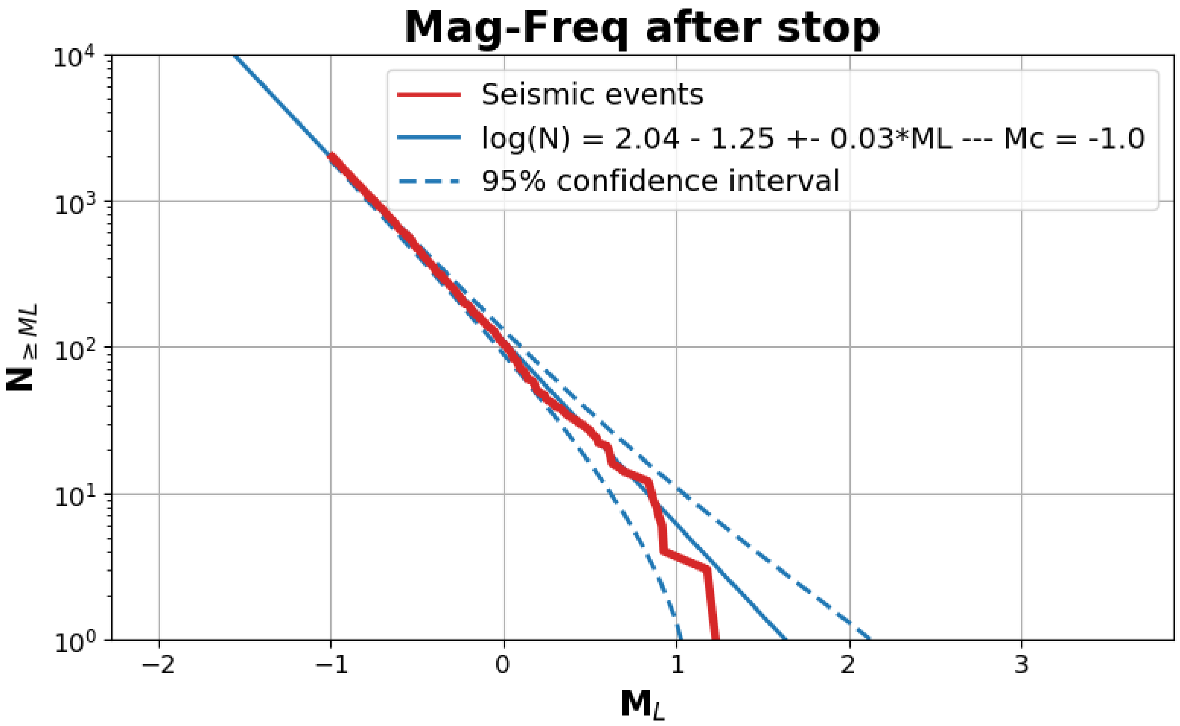
Global Evolution of Magnitude-Frequency when Injection Stops





- The b-value does not change when stopping injection

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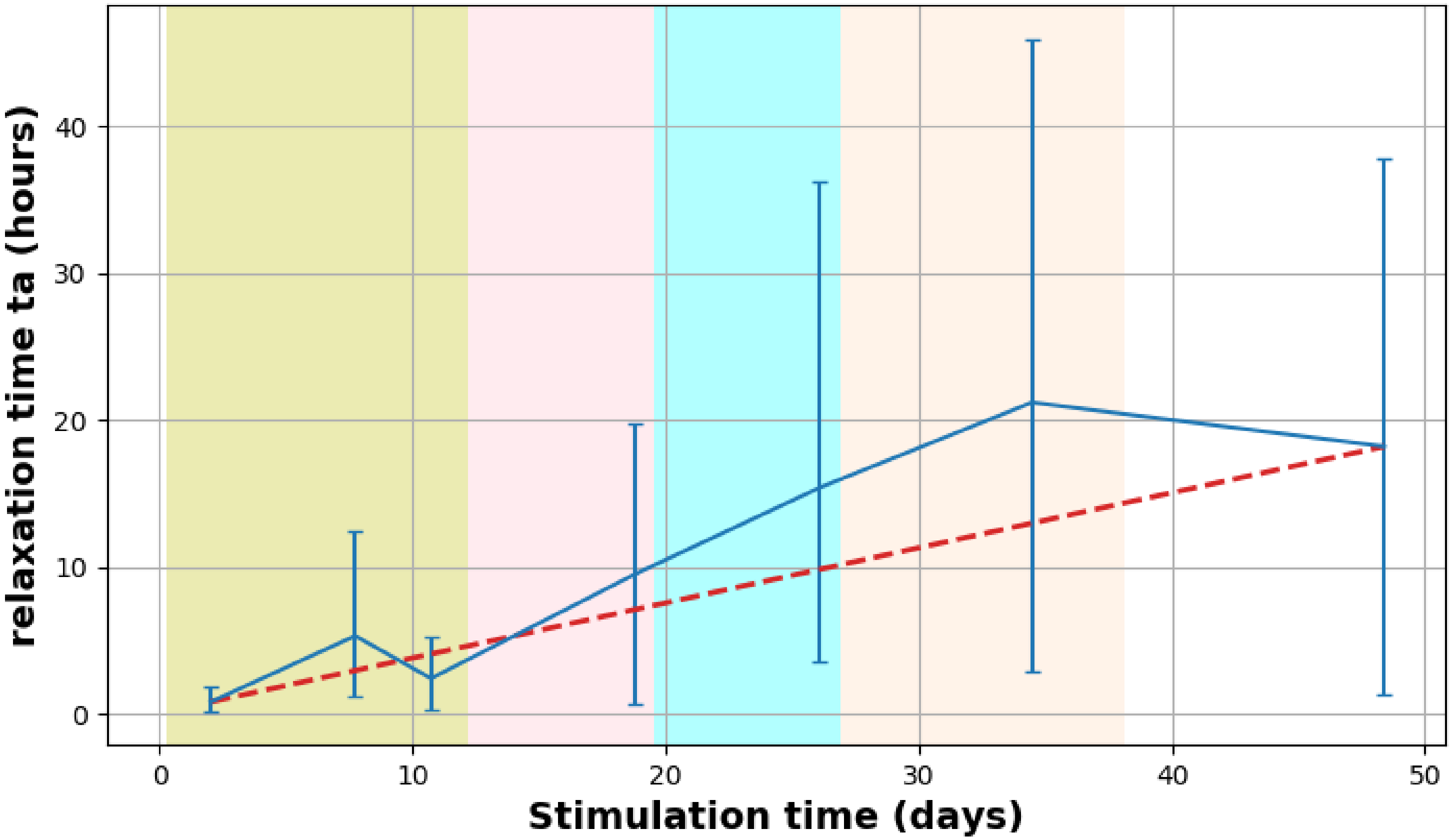
Global Evolution of t_a as Stimulation Proceeds through Time



- ta seems to evolve steadily in time throughout the stimulation

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2.3.2 Stable Regimes

Stable Regimes

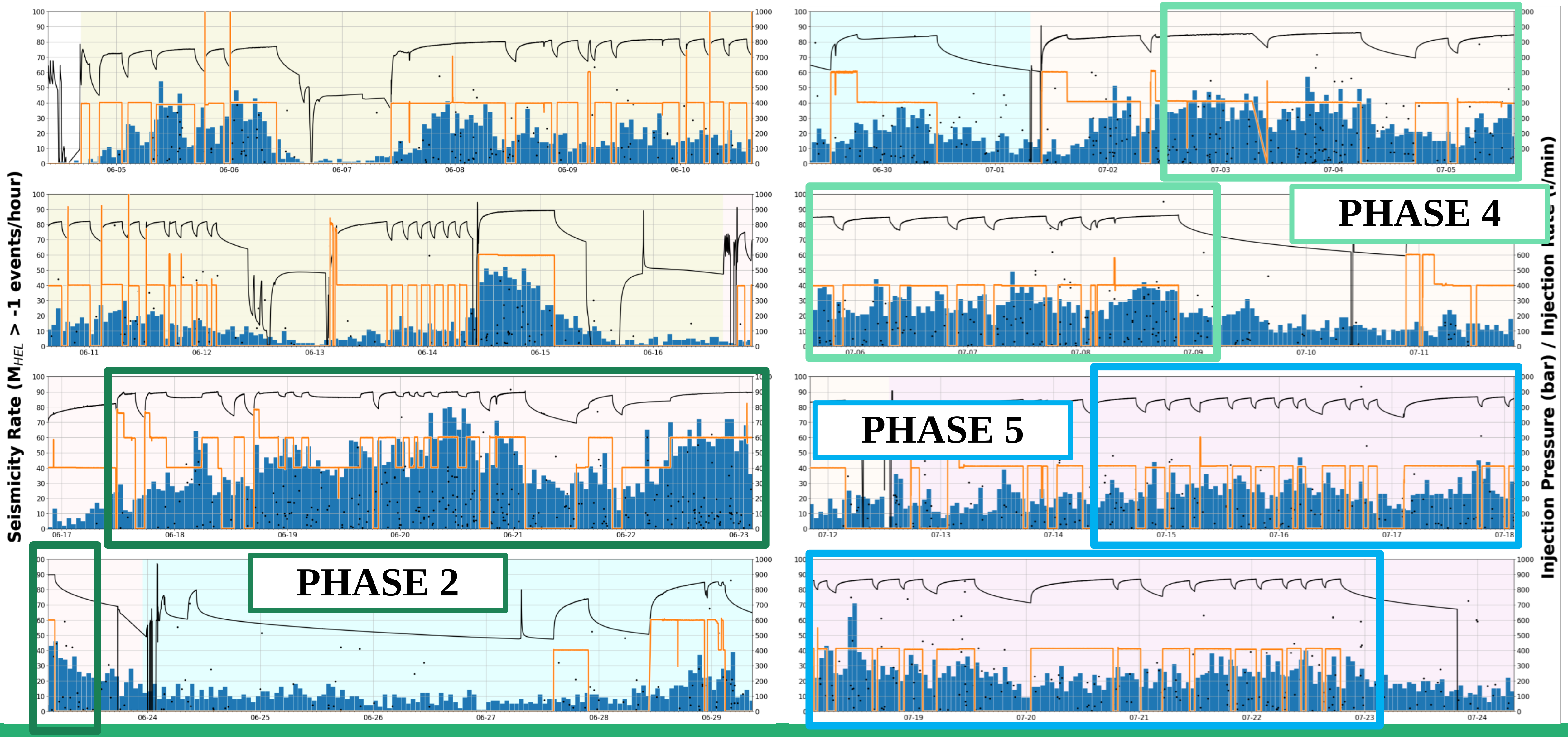
- Hypothesis: if injecting at lower pressure and lower rates
  - Pressure has time to diffuse
  - No high pressure front has time to build up
  - Lower seismicity & smaller events

    i.e., lower number of events per volume unit injected

    i.e., larger b-value

- Check this by comparing stable regimes of phases 2, 4 and 5

Seismicity Rate / Injection Rate / Injection Pressure



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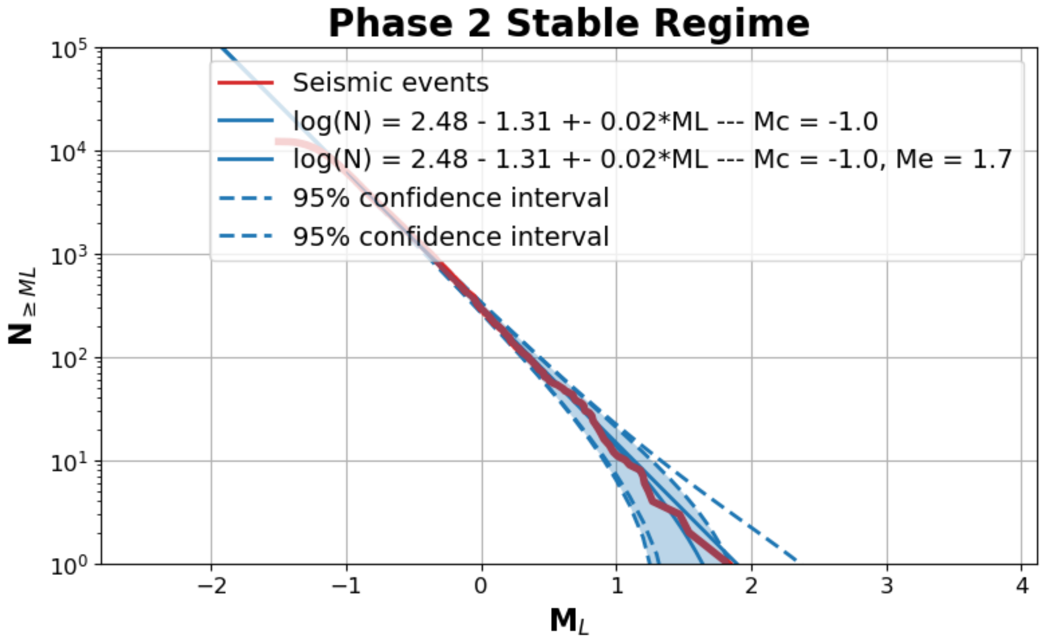
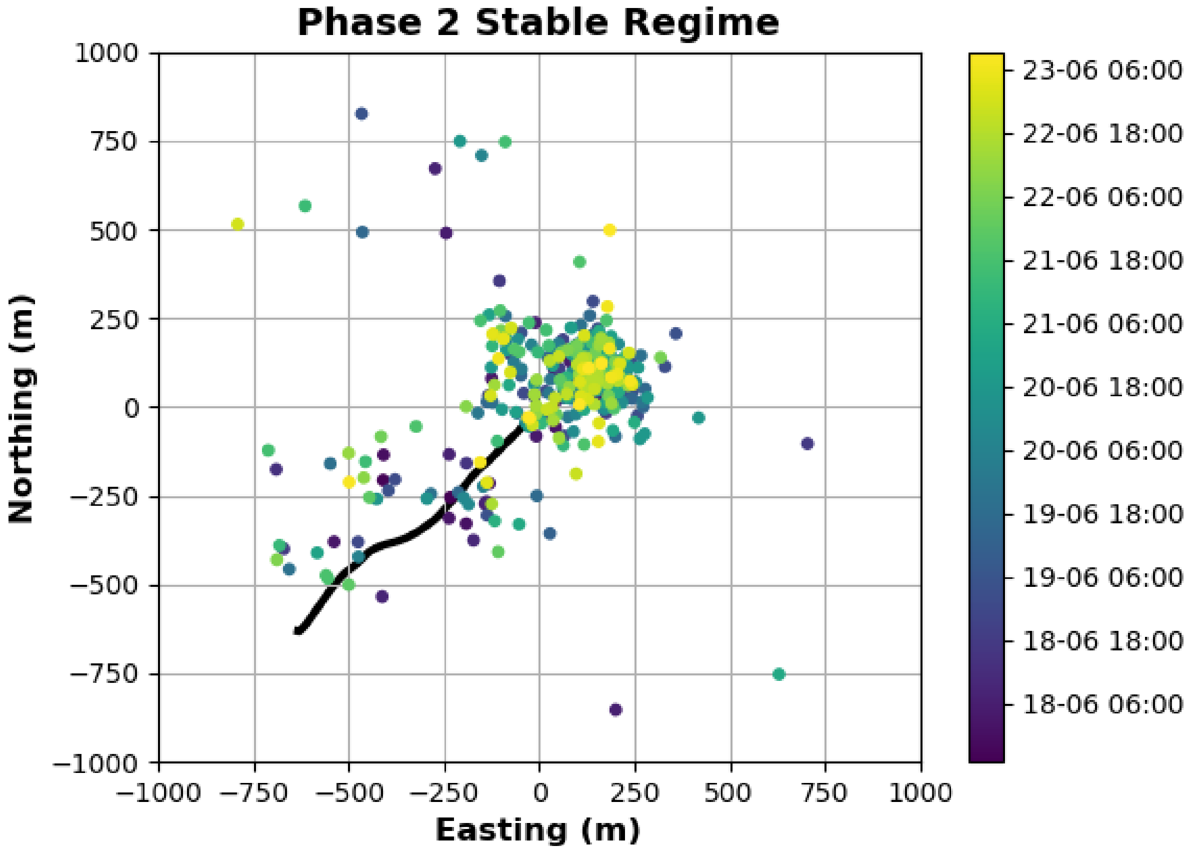
Phase 2 Stable Regime



PARAMETER	VALUE
Dates	2018-06-17 18h to 2018-06-23 10h
Extent of seismicity	99 m
Volume injected	3448 m^3
Eq productivity	8.64 ± 0.49 M_L>0 / 100m^3
GR b-value	1.31 ± 0.02



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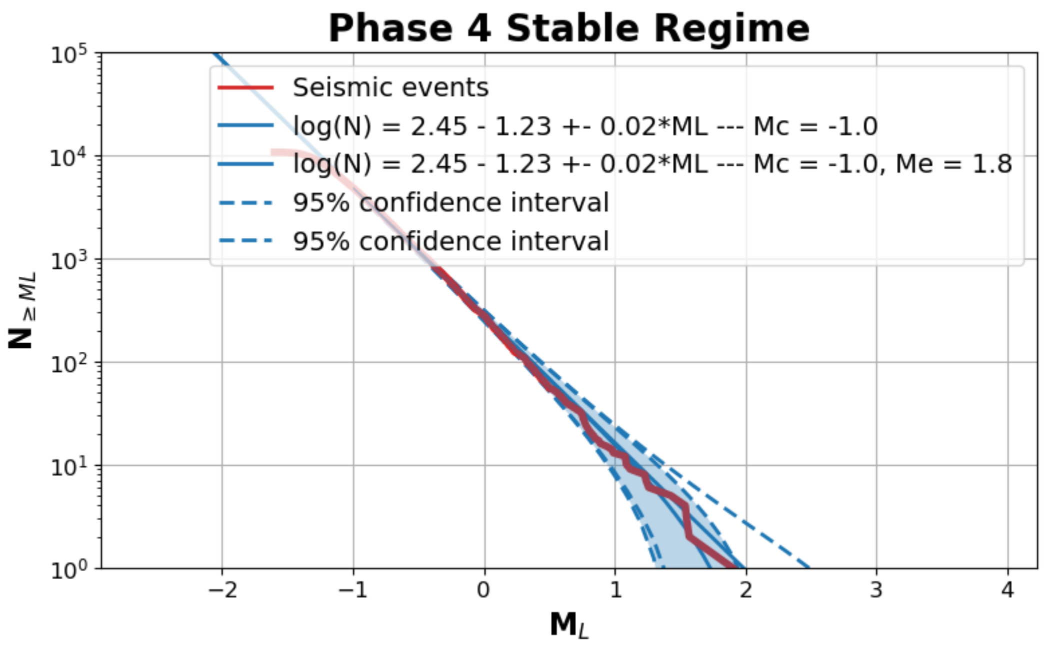
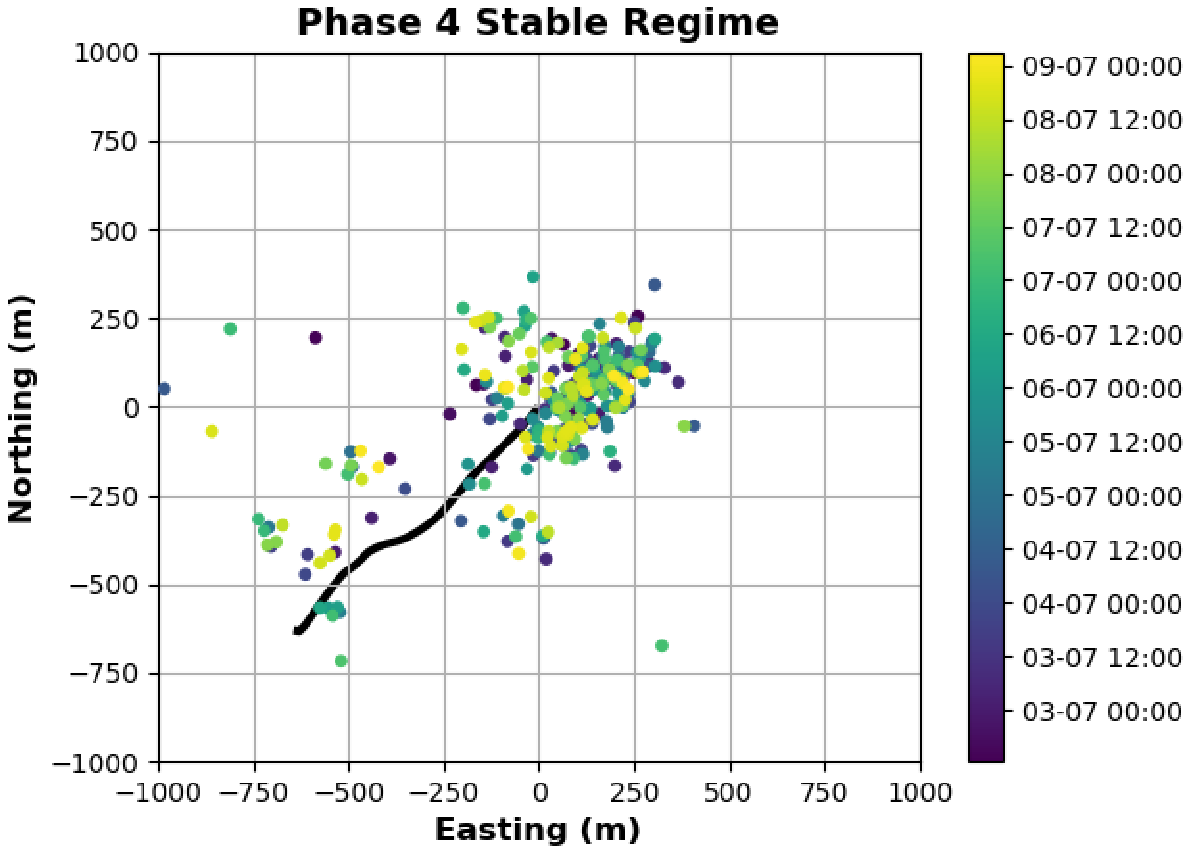
Phase 4 Stable Regime



PARAMETER	VALUE
Dates	2018-07-02 12h to 2018-07-09 3h
Extent of seismicity	118 m
Volume injected	3045 m^3
Eq productivity	8.71 ± 0.52 M_L>0 / 100m^3
GR b-value	1.23 ± 0.02



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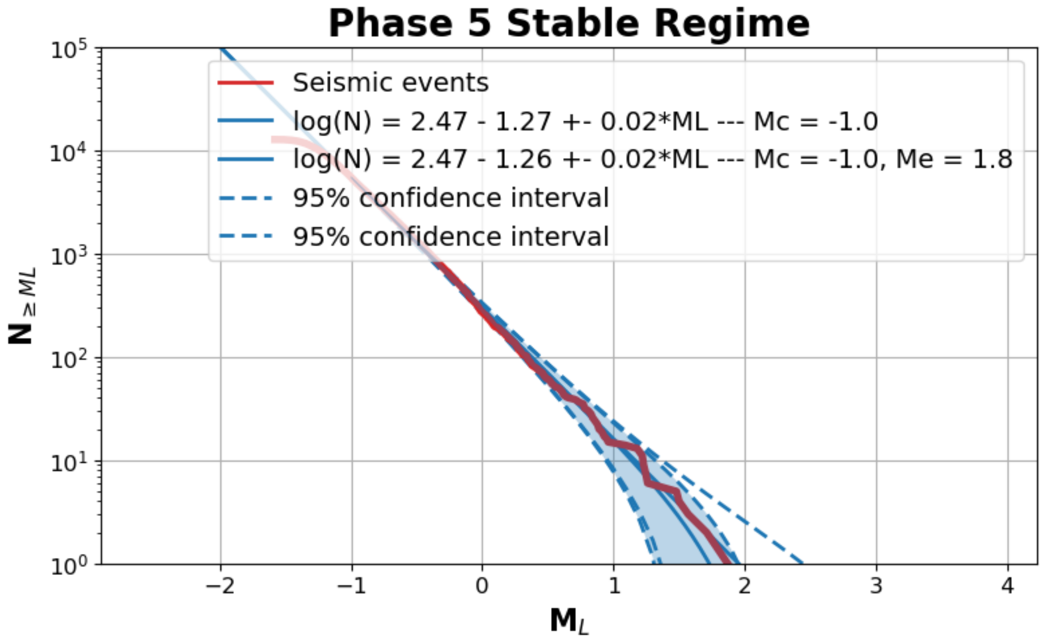
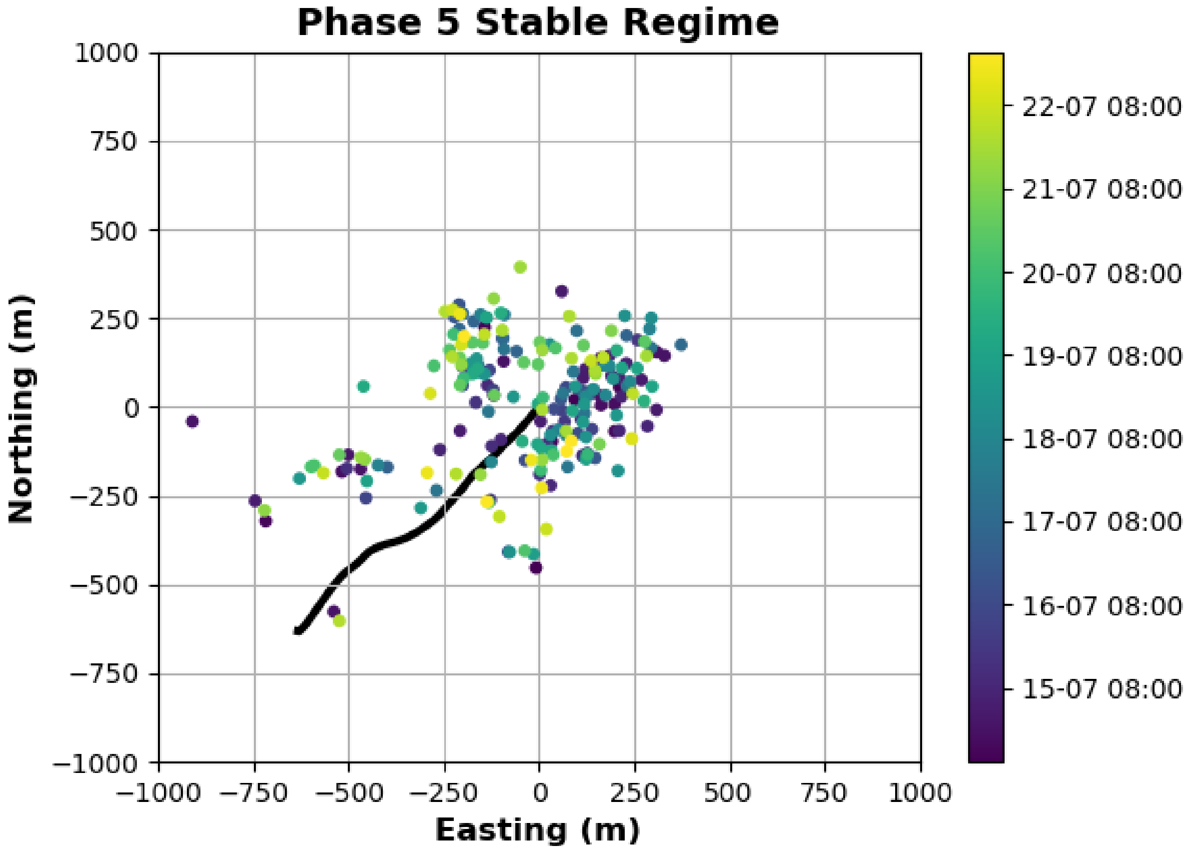
Phase 5 Stable Regime



PARAMETER	VALUE
Dates	2018-07-14 8h to 2018-07-23 1h
Extent of seismicity	159 m
Volume injected	3691 m^3
Eq productivity	7.71 ± 0.45 M_L>0 / 100m^3
GR b-value	1.27 ± 0.02



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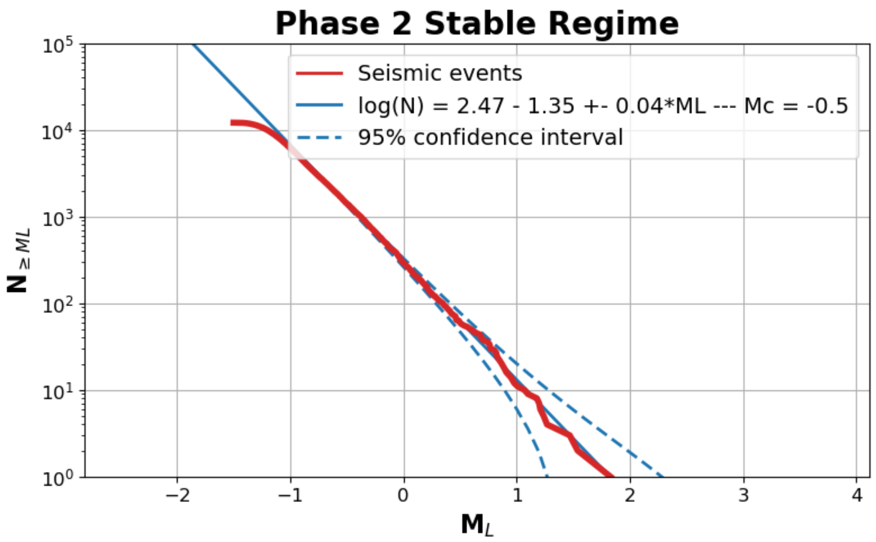
Stable Regimes Phases 2, 4 and 5

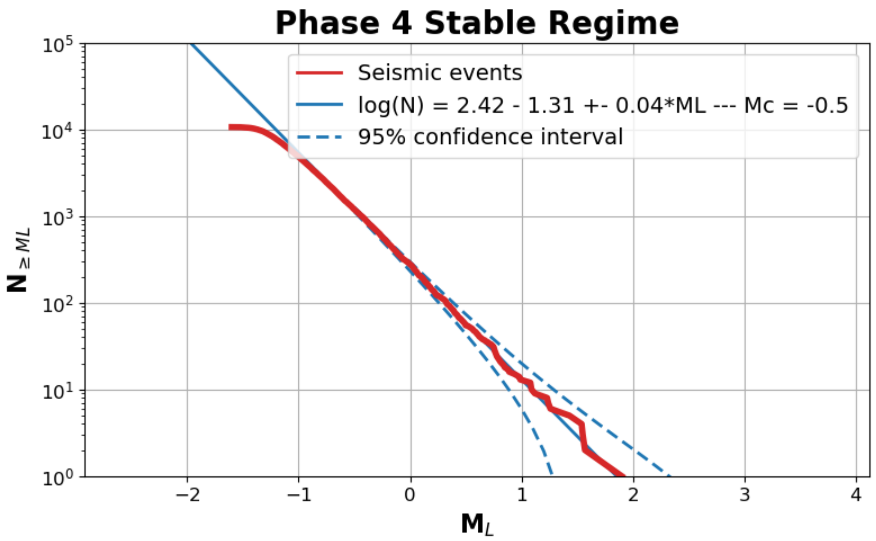
- The *b*-value does not seem to vary despite different injection rates
- The earthquake productivity does not vary outside of uncertainties

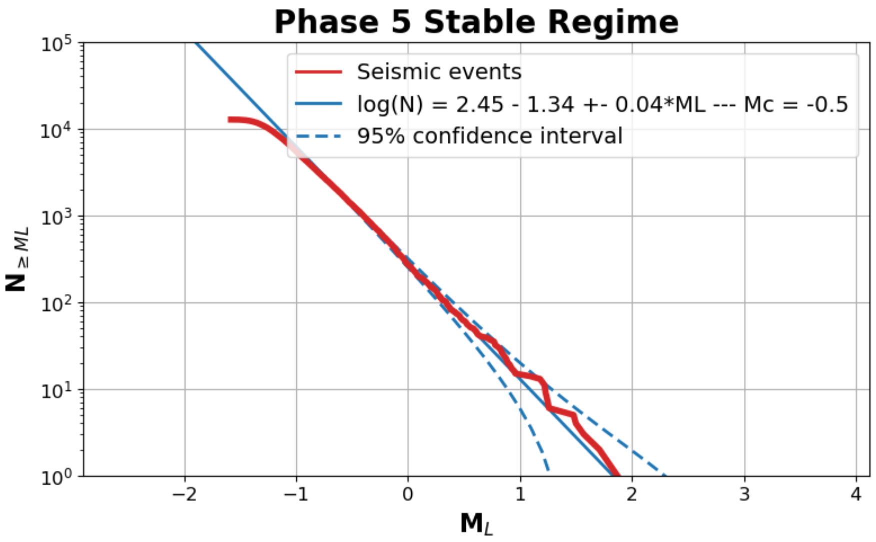
- **This suggests that only the total volume injected impacts on the seismicity**

PARAMETER	PHASE 2	PHASE 4	PHASE 5
Main injection rate (l/min)	600	400	400
Main injection pressure (bar)	900	850	860
Extent of seismicity (m)	99 m	118 m	159 m
Volume injected (m^3)	3448 m^3	3045 m^3	3691 m^3
Seismic productivity (M_L>0 / 100m^3)	8.64 ± 0.49	8.71 ± 0.52	7.71 ± 0.45
GR b-value (M_c = -1)	1.31 ± 0.02	1.23 ± 0.02	1.27 ± 0.02
GR b-value (M_c = -0.5)	1.35 ± 0.04	1.31 ± 0.04	1.34 ± 0.04

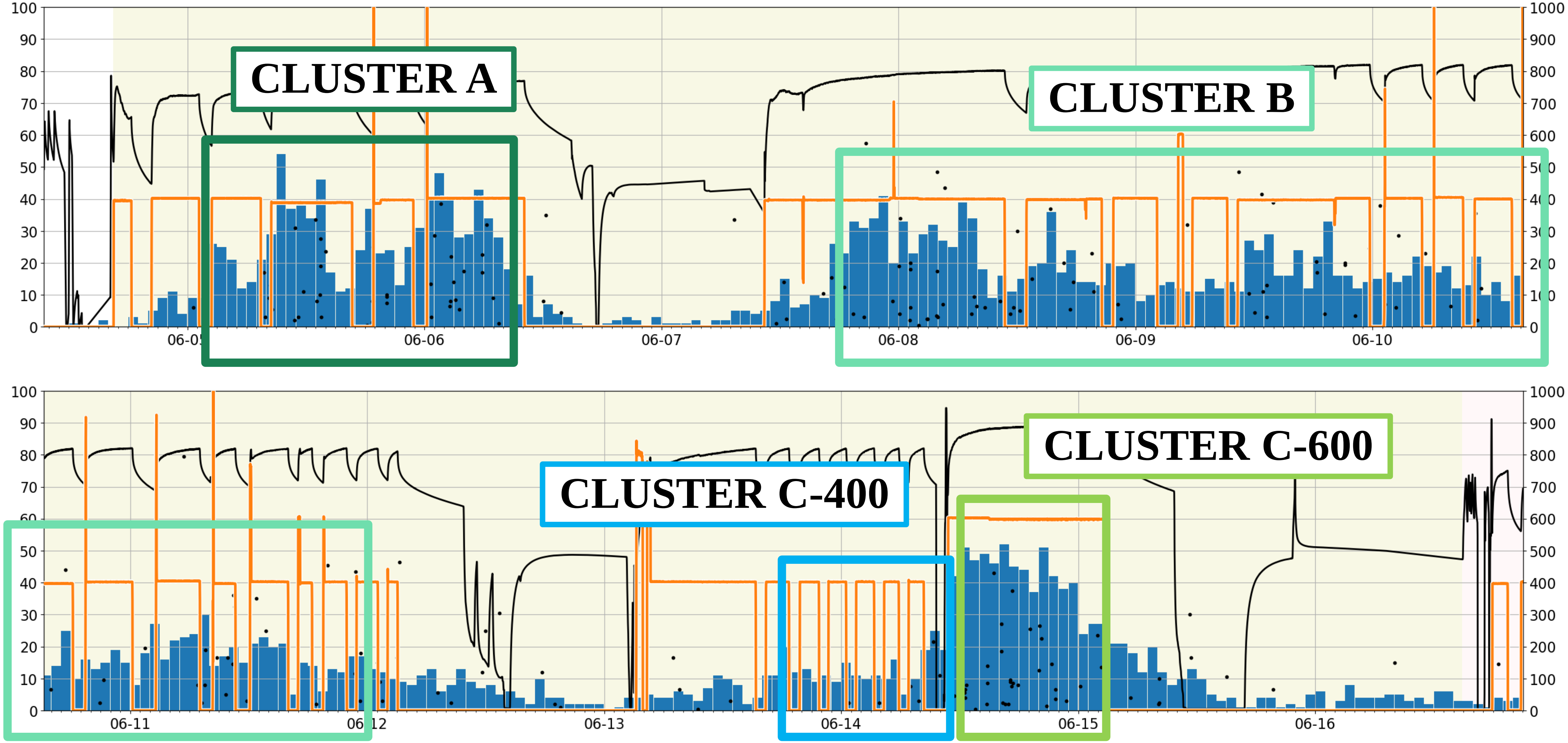
Gutenberg-Richter with M_c = -0.5



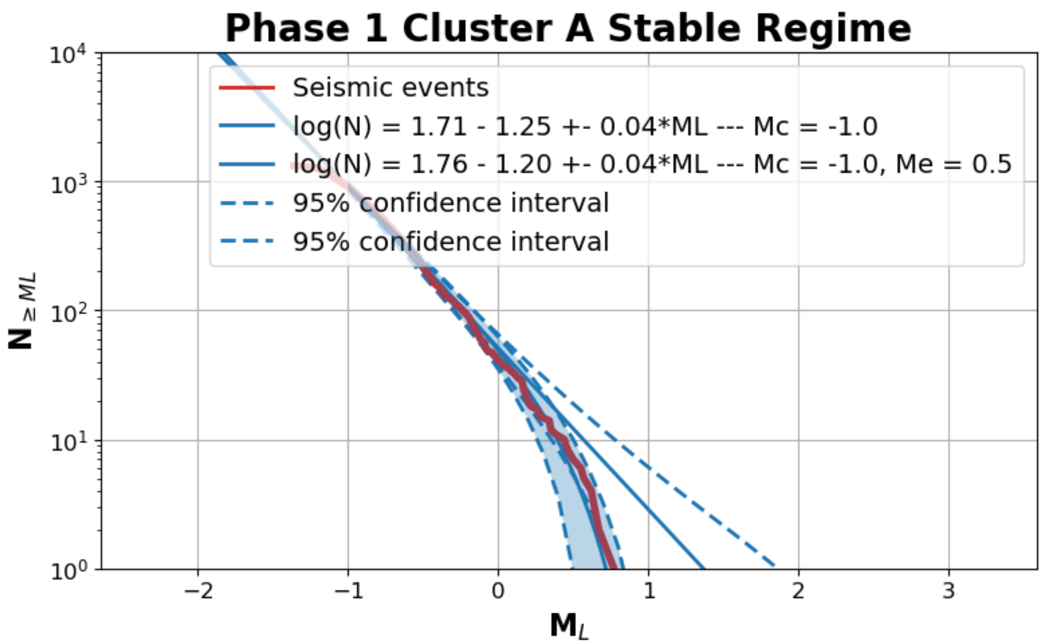


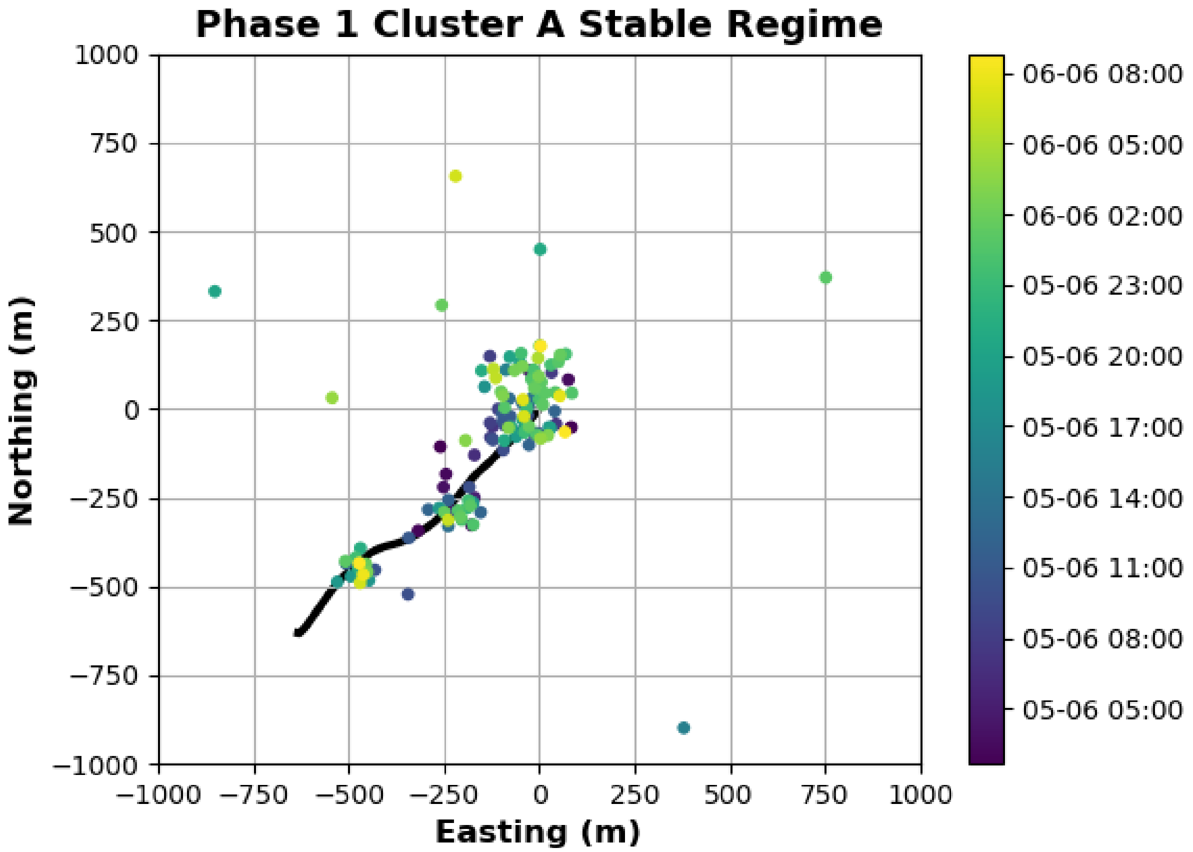


Analysis of the Different Clusters of Phase 1



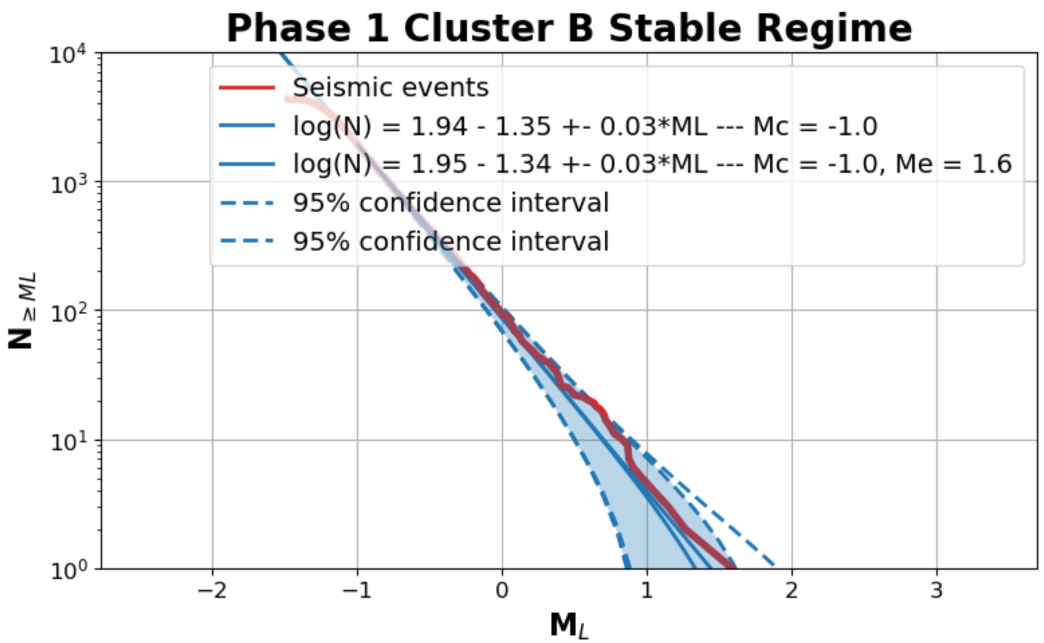
Phase 1 Cluster A Stable Regime

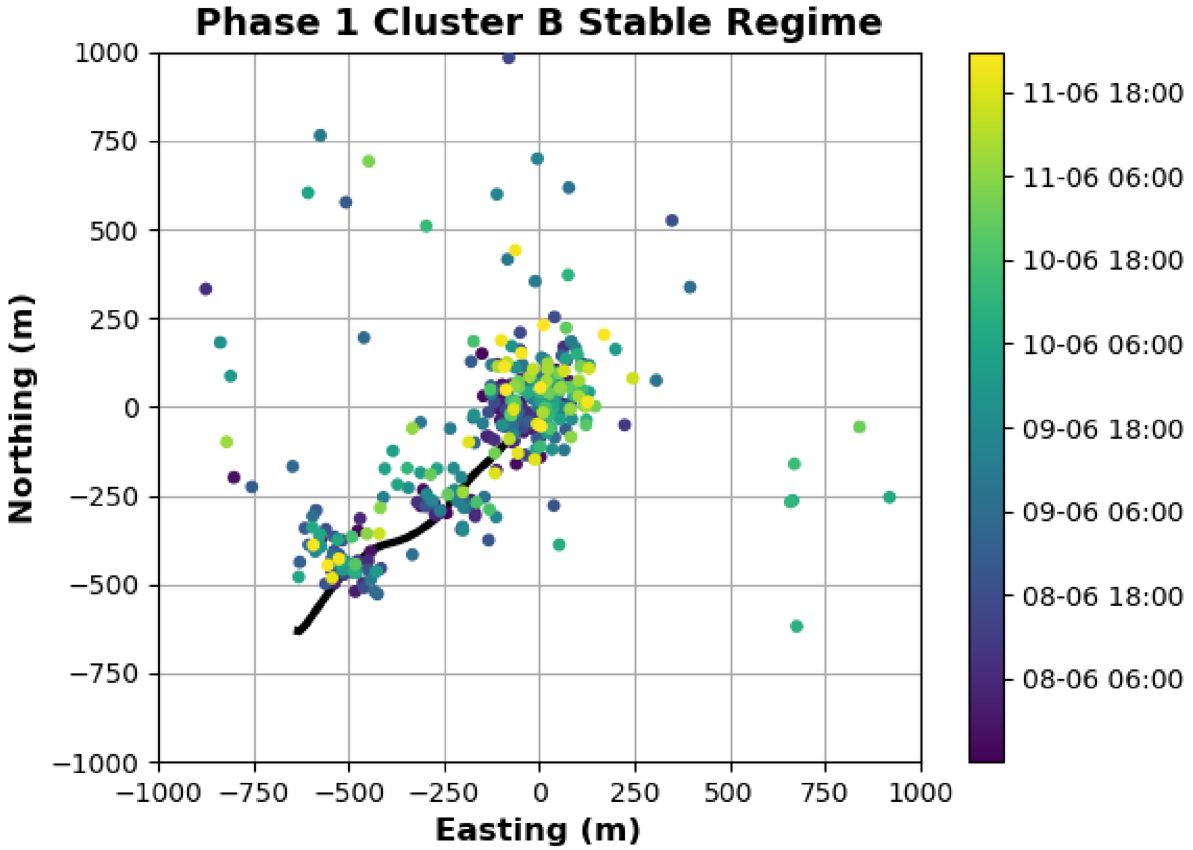




PARAMETER	VALUE
Dates	2018-06-05 02h to 2018-06-06 10h
Extent of seismicity	63 m
Volume injected	645 m^3
Eq productivity	6.12 ± 0.93 M_L>0 / 100m^3
GR b-value	1.25 ± 0.04

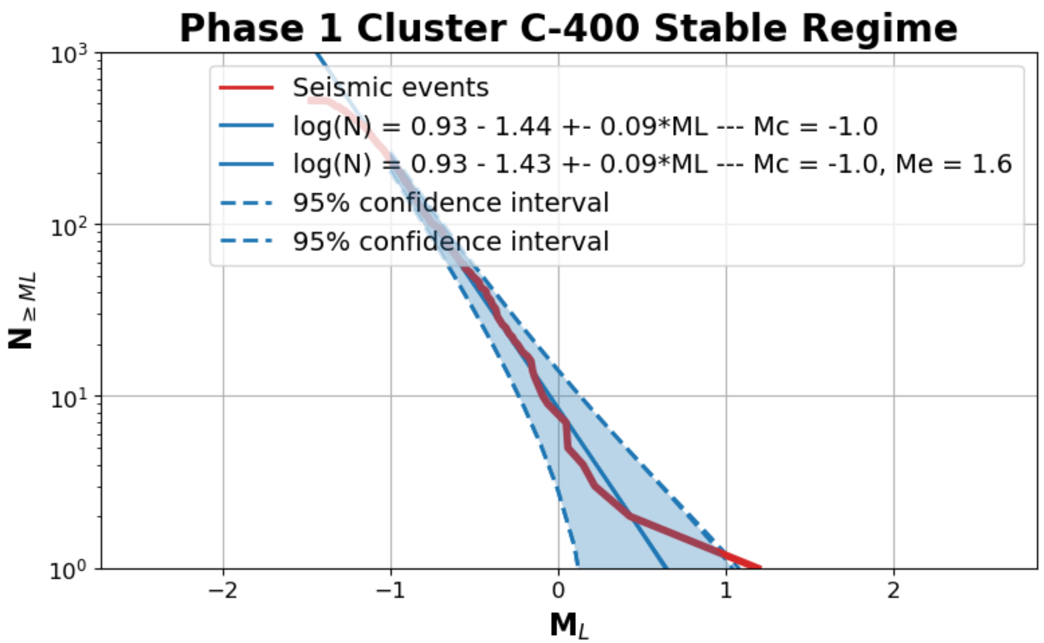
Phase 1 Cluster B Stable Regime



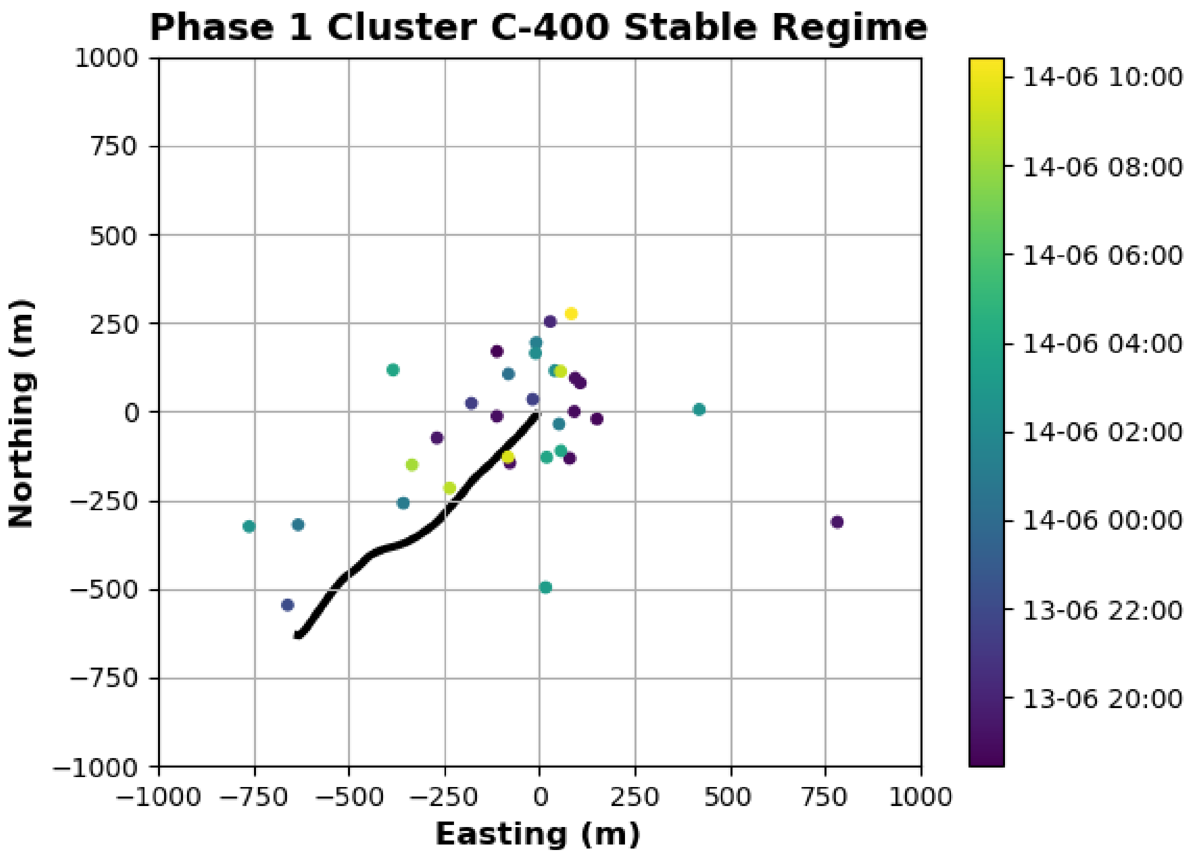


PARAMETER	VALUE
Dates	2018-06-07 18h to 2018-06-12 0h
Extent of seismicity	77 m
Volume injected	1950 m^3
Eq productivity	4.9 ± 0.49 M_L>0 / 100m^3
GR b-value	1.35 ± 0.03

Phase 1 Cluster C-400 Stable Regime



PARAMETER	VALUE
Dates	2018-06-13 18h to 2018-06-14 11h
Extent of seismicity	147 m
Volume injected	240 m^3
Eq productivity	3.22 ± 1.15 $M_L>0$ / 100m^3
GR b-value	1.44 ± 0.09



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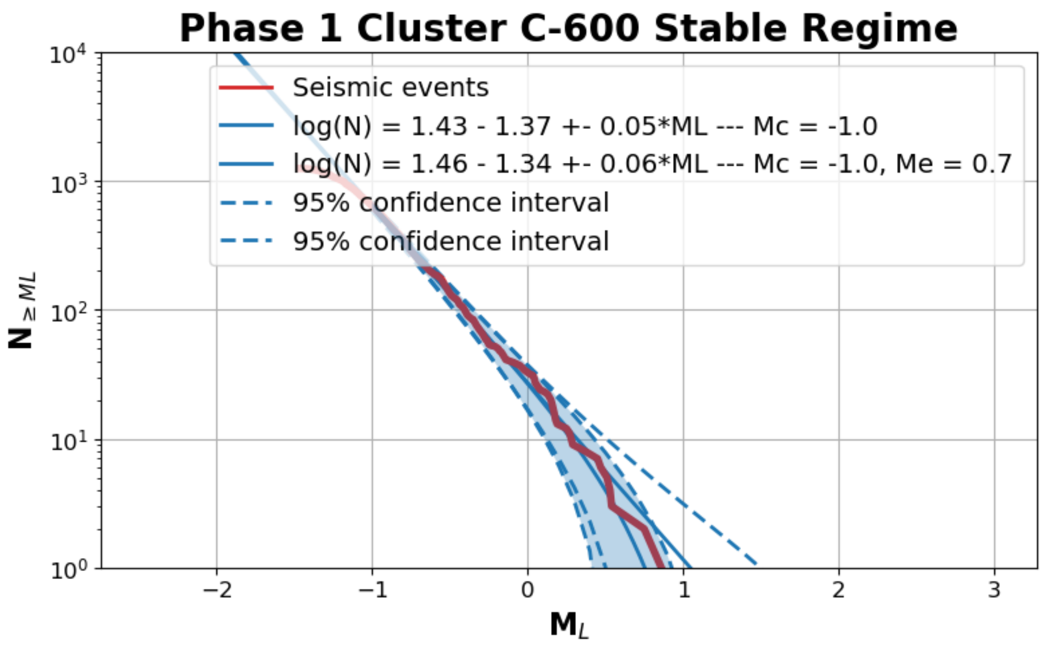
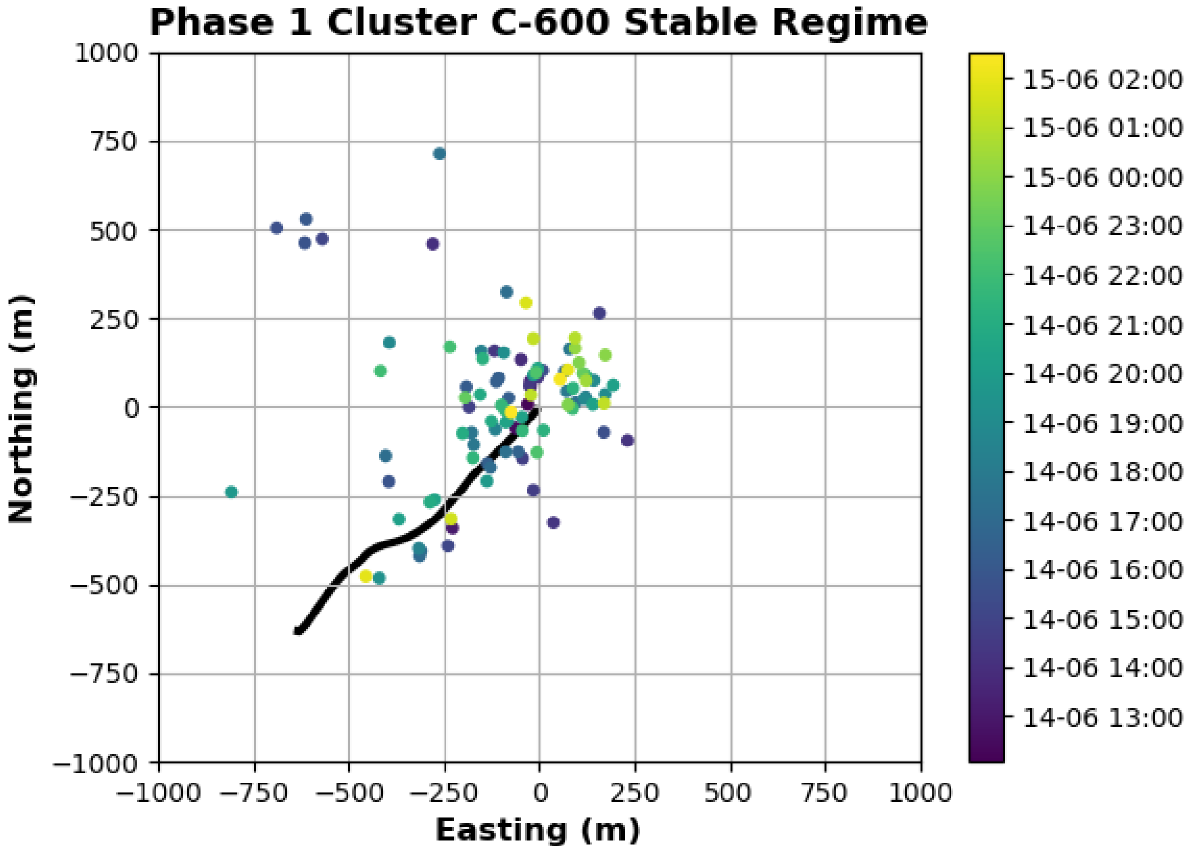
Phase 1 Cluster C-600 Stable Regime





PARAMETER	VALUE
Dates	2018-06-14 12h to 2018-06-15 3h
Extent of seismicity	108 m
Volume injected	539 m^3
Eq productivity	4.77 ± 0.93 $M_L>0$ / $100m^3$
GR b-value	1.37 ± 0.05

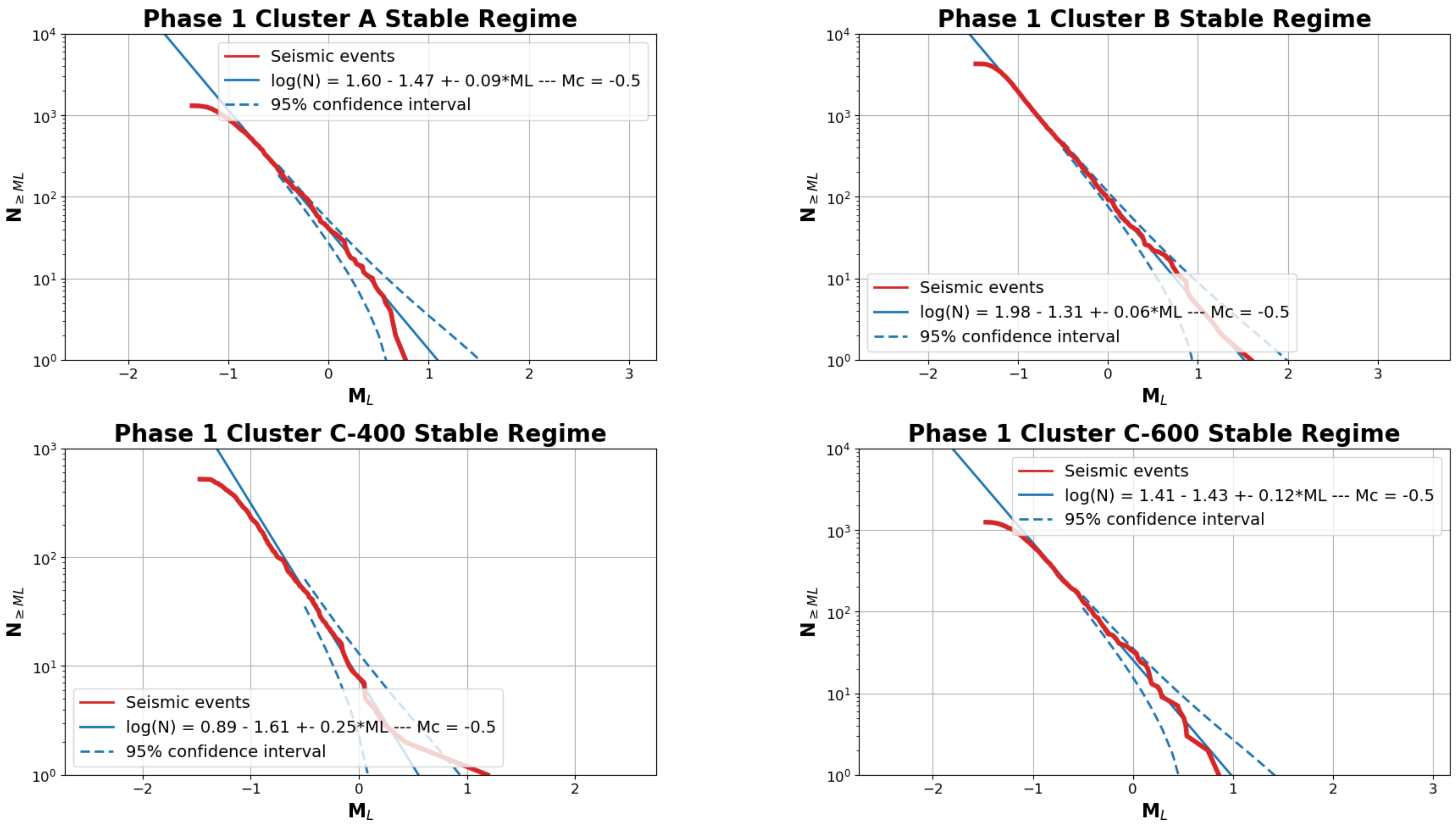
ARUP

Stable Regimes Phase 1

- The b-value does not vary despite different injection rates
- The earthquake productivity does not vary outside of uncertainties

PARAMETER	CLUSTER A	CLUSTER B	CLUSTER C-600
Main injection rate (l/min)	400	400	600
Main injection pressure (bar)	750	820	900
Extent of seismicity (m)	63 m	76 m	124 m
Volume injected (m^3)	645 m^3	1829 m^3	539 m^3
Seismic productivity (M_L>0 / 100m^3)	6.12 ± 0.93	4.86 ± 0.51	4.83 ± 0.88
GR b-value (M_c = -1)	1.25 ± 0.04	1.35 ± 0.03	1.37 ± 0.05
GR b-value (M_c = -0.5)	1.47 ± 0.09	1.33 ± 0.07	1.44 ± 0.12

Gutenberg-Richter with M_c = -0.5



2.3.2 Well Bleed-offs

Well Bleed-Offs

- Nine bleed-offs were reported during the stimulation

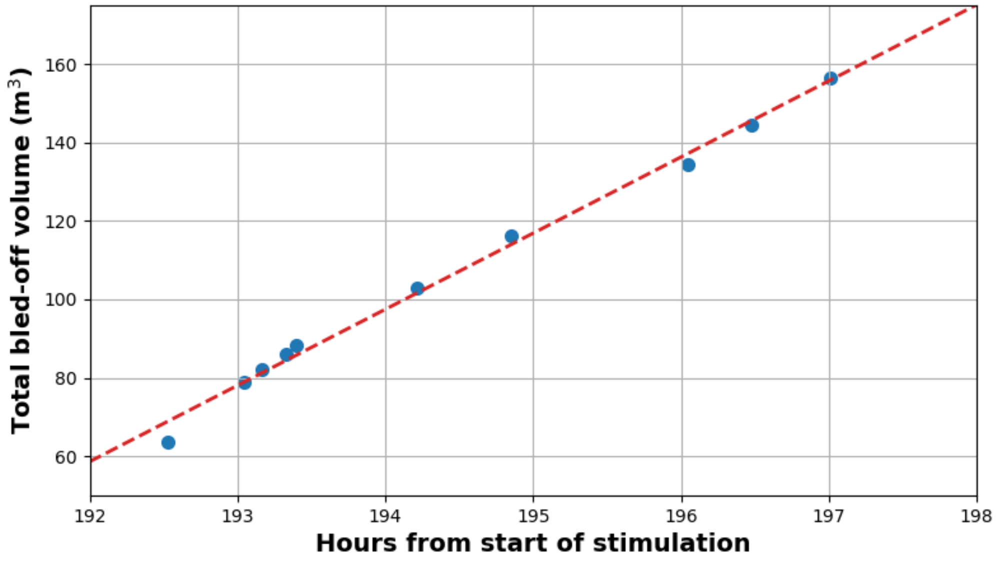
End of bleed-off	Volume bled-off
04/06 13:04	13.8 m^3
06/06 17:38	36 m^3
07/06 10:22	6.2 m^3
12/06 14:28	100.5 m^3
13/06 02:30	12.2 m^3
15/06 16:58	200 m^3
16/06 14:53	6.5 m^3
24/06 01:47	~100 m^3
12/07 13:28	~17 m^3

Only these two bleed-offs have volume measurements during the bleed-offs. The other points only provide the total volume bled-off.

Can be used to estimate flow rate during well bleed-off.

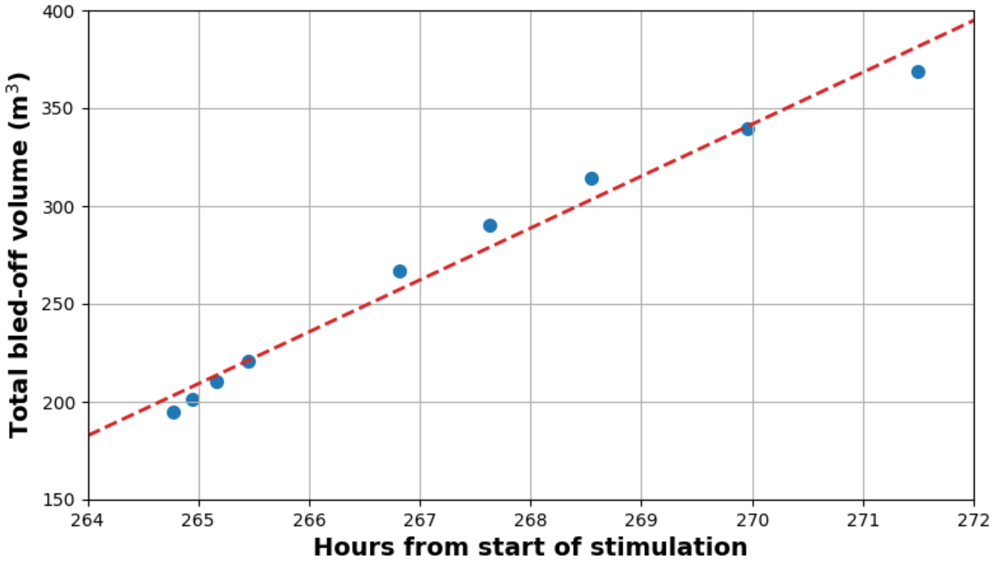
Well Bleed-Offs

12/06 bleed-off



Rate = 320 l/min

15/06 bleed-off



Rate = 440 l/min

- Bleed-off rate ~ 400 l/min.
- Slight curvature of the total volume with time, especially in the second case.

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Well Bleed-Offs

- With an 1/t decay of the bleed-off flow rate:

$$r(t) = \frac{r_0}{1+t/t_a},$$

- The total volume ejected during flow back is:

$$V(t) = r_0 t_a \ln\left(1 + \frac{t}{t_a}\right),$$

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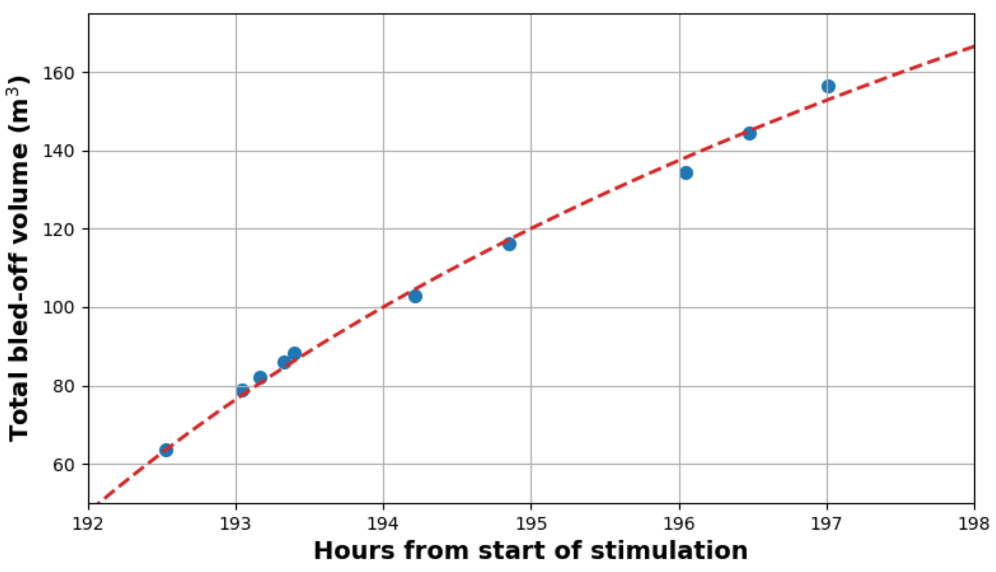
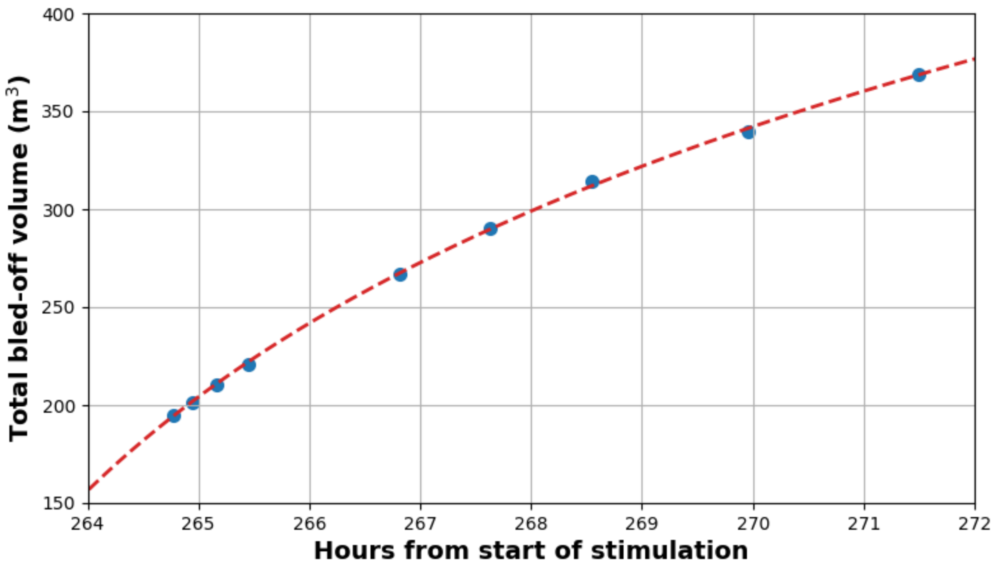
Well Bleed-Offs





r_0 = 475 l/min
t_a = 4.6h

r_0 = 738 l/min
t_a = 4.0h

- Slight curvature of the total volume with time, especially in the second case (fitting procedure detailed in Appendix A4).

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3. Predictive Modelling

Can we predict seismicity rate given an injection rate?

3.3.1 Predictive Model and Parameters

Background

- Seismicity seems to react mostly to volume injected
- The earthquake productivity seems to be constant throughout most of the stimulation (except for phase 1)
- Seismicity seems to decay following an Omori law when injection stops

Background

- Seismicity may be predicted from the injection rate by convolving it with the derivative of the Omori law:

$$R(t) = u(t) * g(t) = \int_{-\infty}^{+\infty} u(\tau)g(t-\tau)d\tau$$

where

$$g(t) = -\frac{d}{dt}\left(\frac{R_0}{1 + t/t_a}\right) = \frac{R_0/t_a}{(1 + t/t_a)^2}$$

$u(t)$ is the injection rate
R_0 is the seismicity rate for a constant unit injection rate
t_a is the relaxation time obtained from the decays when injection stops

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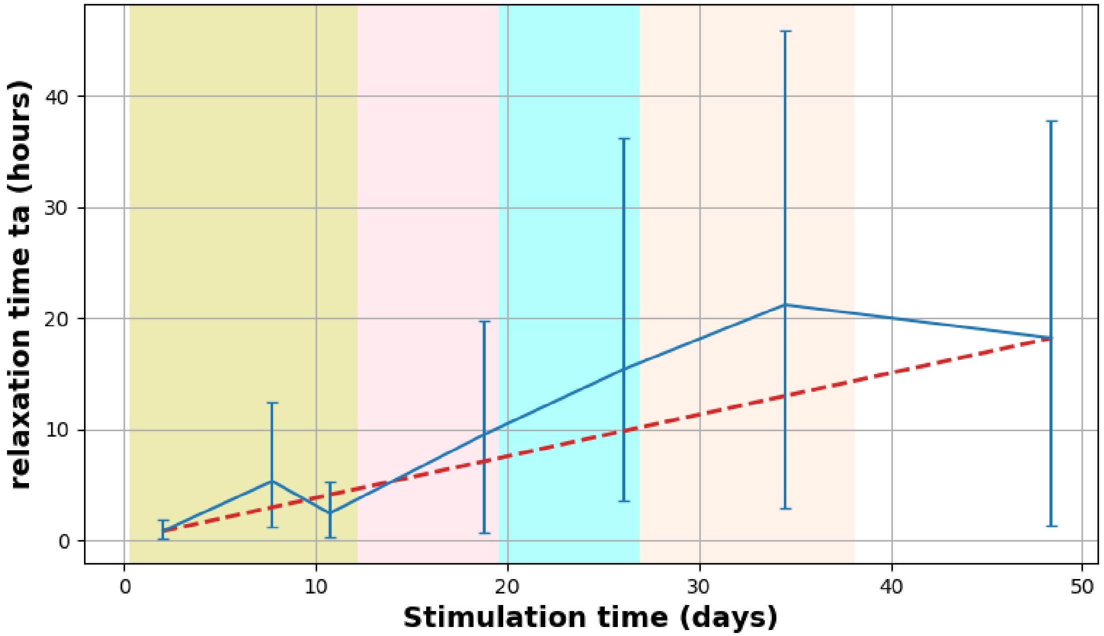
Prediction of t_a for the different phases

- Assume that t_a evolves linearly with time:

$$t_a = 1 + 0.4t,$$

where t is the stimulation time in days

- Take the midpoint of each injection phase



STIMULATION PHASE	AVERAGE TIME FROM START	t_a
Phase 1	6 days	3.5h
Phases 2 & 3	20 days	9h
Phase 4	33 days	14h
Phase 5	43 days	18h

- Here, t_a values are based on final stimulation results

- In practice, t_a can be evaluate as stimulation progresses

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Computation of R_0 for the injection

- Early in the stimulation, one can get:
  - Number of M_L >0 events induced by 100 m^3 of water injected; and
  - *b*-value.

- Here, we got $R_0 \approx 8\ M_L{>}0\ /\ 100m^3$ and b = 1.27*
  - $R_0 = 0.08\ M_L{>}0\ /\ m^3$
  - Injection rate in m^3/min: $1m^3/min = 60m^3/hour$
  - So, $R_0 = 4.8\ M_L{>}0\ /\ 60m^3 = 4.8\ M_L{>}0\ /\ hour$
  - So, $4.8{\times}10^{1.27}\ M_L{>}{-1}\ /\ hour \approx 90\ M_L{>}{-1}\ /\ hour$

$R_0 \approx 90\ M_L{>}{-1}$ **events/hour if injecting at 1 m^3/minute**

*with $M_c = -1$, slightly different from with $M_c = -0.5$.

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Computation of R_0 for the injection

- In practice, R_0 is computed during the injection, by fitting the seismicity rate predicted by convolution in the past seven days to the observed rate:
  - t_a is estimated from the pauses in injection;
  - The convolution function $g(t)$ is computed for an array of values of R_0;
  - The seismicity rate is computed for each value of R_0 for the past seven days;
  - The best value of R_0 is computed by MLE, comparing the predicted rate to the measured rate recorded during the last 7 days.

Computation of the b-value and M_e

- Both the b-value and M_e (if considering a Tapered Gutenberg-Richter distribution) can be computed from the previous seismicity, or taken as default values from the stimulation of OTN-3.

- Both the b-value and M_e are mainly used to predict the maximum magnitude M_{max}.

- In both the GR and TGR models, these values vary slightly with the magnitude of completeness used to fit the MF distribution, due to the curvature of the MF distribution at low magnitudes.

Computation of the b-value and M_e

- As described earlier, we recommend computing the b-value and M_e using a high magnitude of completeness M_c ($M_c \sim 0$), for two reasons:
  - Using the higher magnitudes values will model better the MF distribution around the maximum magnitude;
  - In practice during the stimulation, the catalogue available will most likely have a $M_c \sim 0$.

3.3.2 Including the Well Bleed-offs

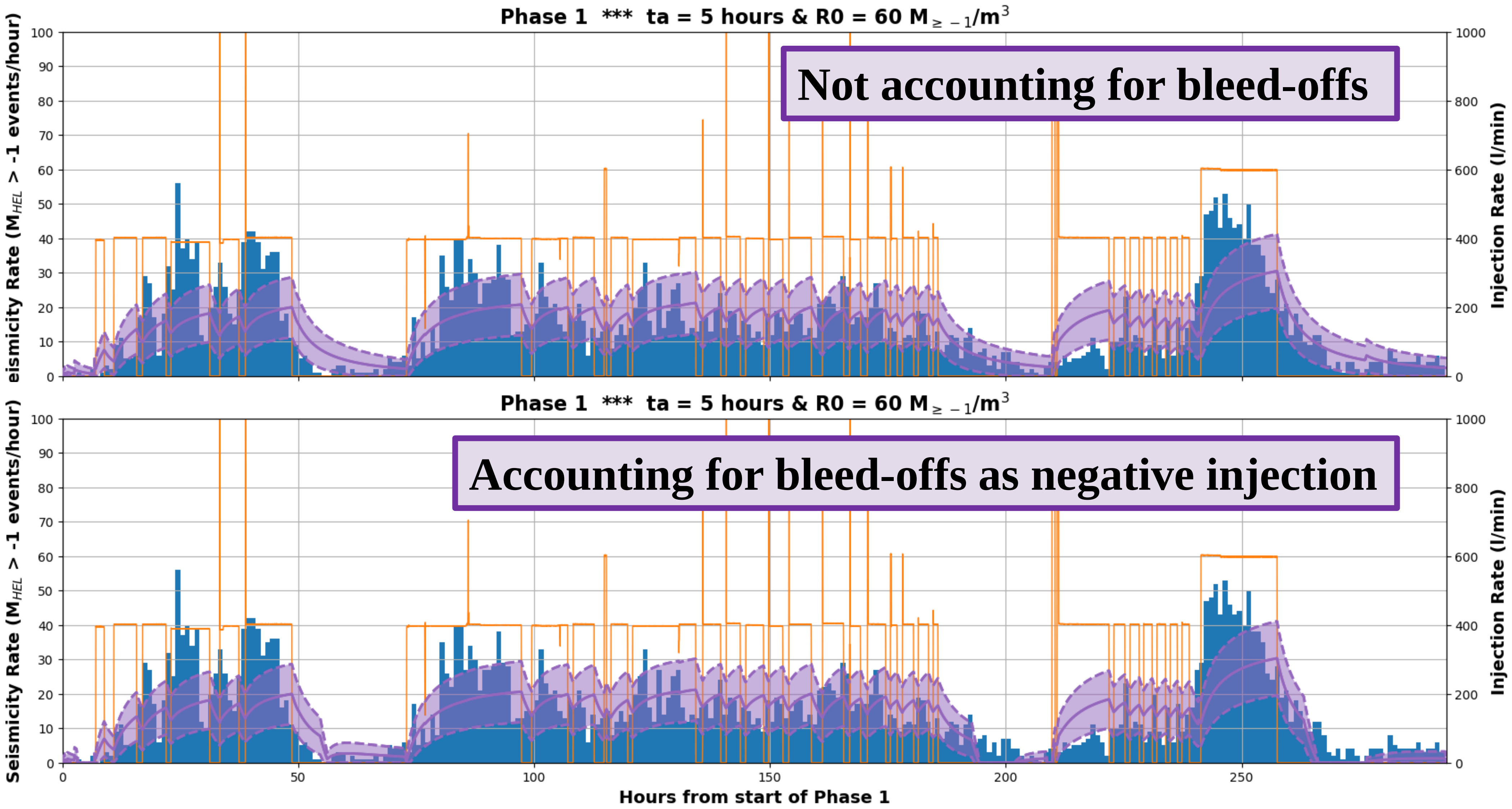
Including the Well Bleed-Offs

There are two possibilities to include the well bleed-offs in the predictive modelling:

1. Consider it equivalent to a negative injection.
   - Total volume known
   - The rate of out flow can be taken at 400 l/min, according to the previous analysis

2. Consider that it effectively delays the start injection after the bleed-off, until the volume ejected has been reinjected.

Bleed-Off as Negative Injection



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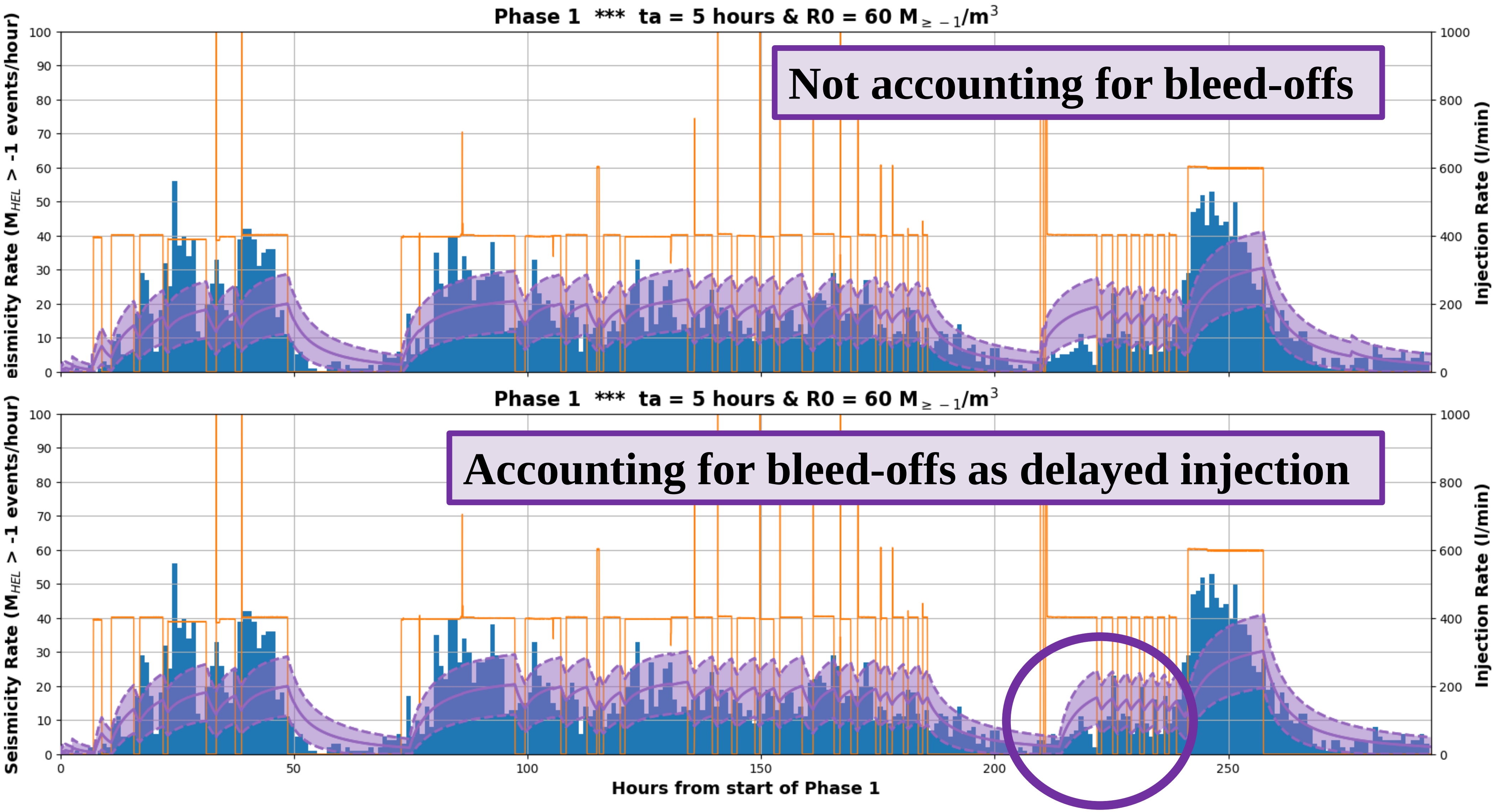
Bleed-Off as Delayed Injection



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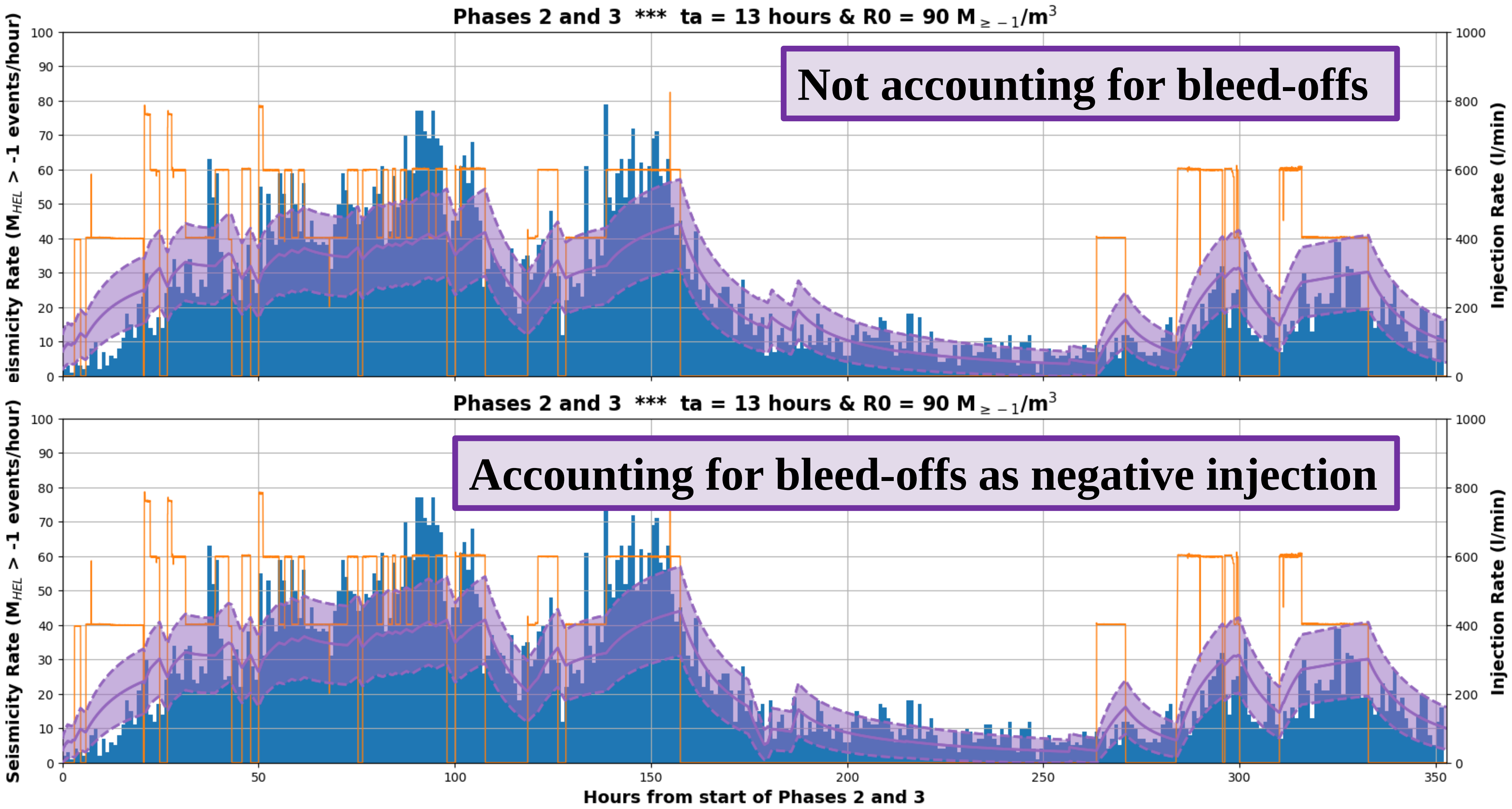
Bleed-Off as Negative Injection



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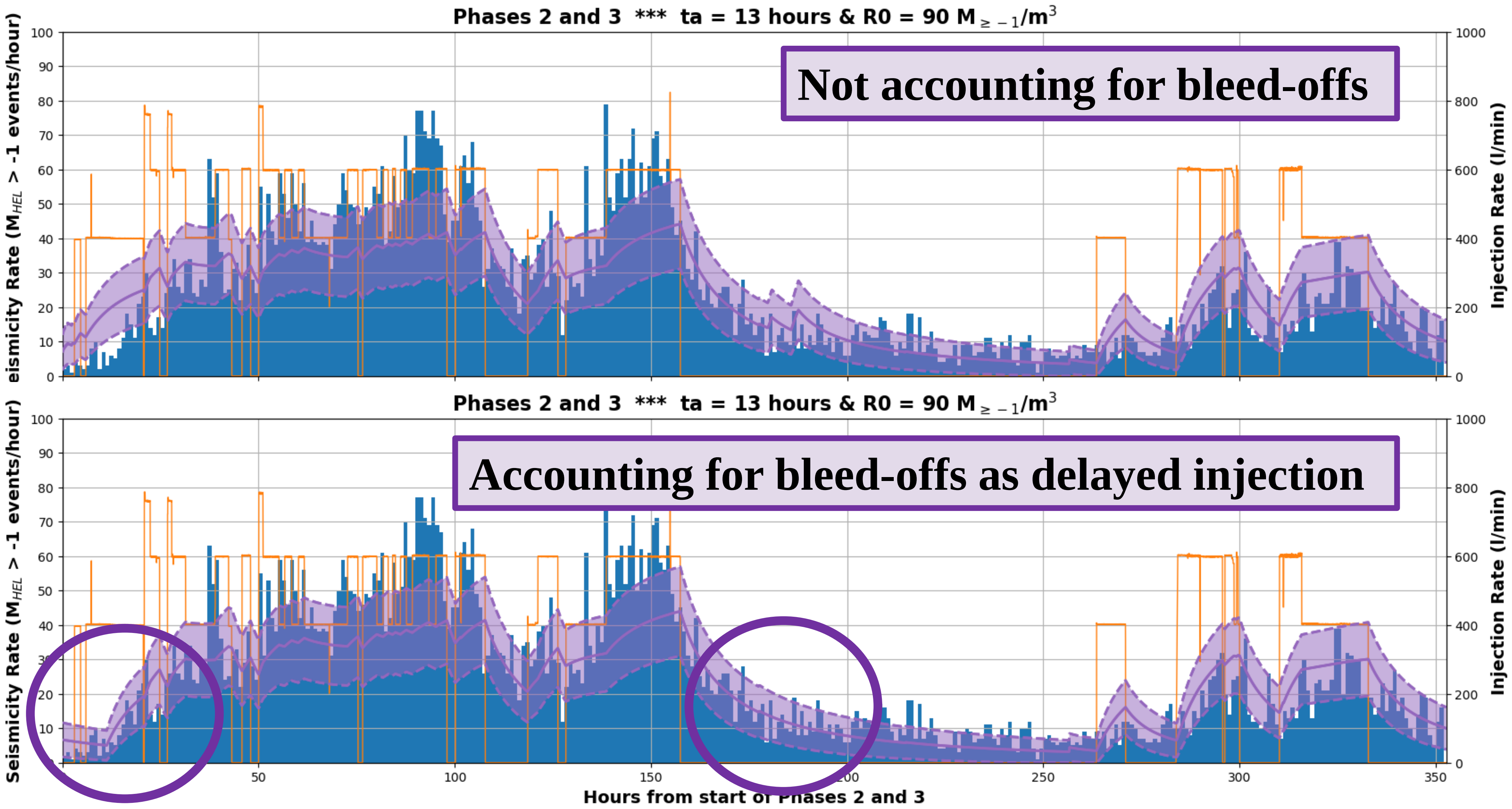
Bleed-Off as Delayed Injection



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Including the Well Bleed-Offs

Well bleed offs should be included as a delay of the subsequent start of injection for three reasons:

- Mathematically, negative injection rates may result in negative seismicity rates, which is physically nonsense;
- Negative injection rates do not reproduce the observed seismicity rate; and
- Delay in injection follows the apparent delay in seismicity rate at the beginning of phase 2.

3.3.3 Predictions for the Different Phases of the Stimulation

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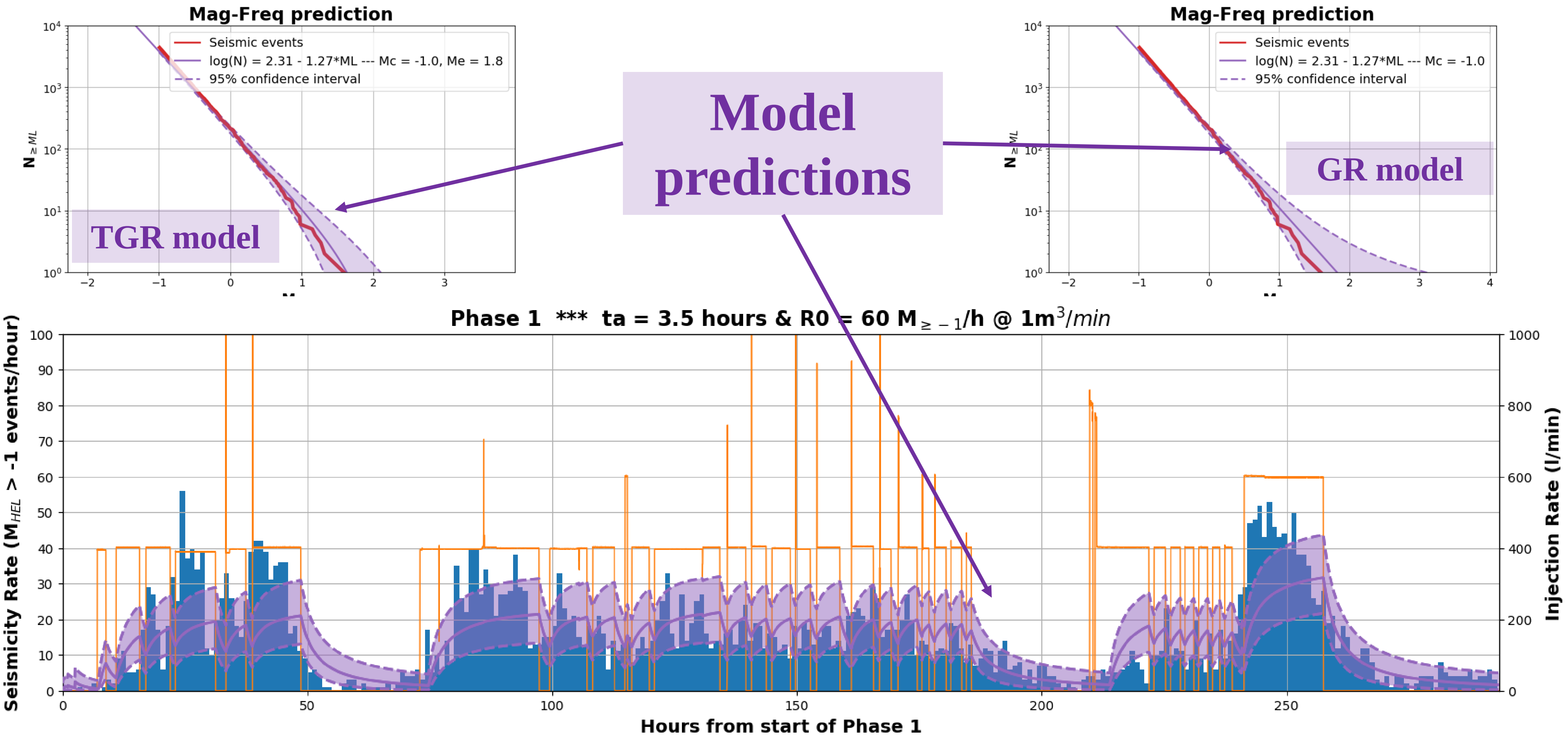
Prediction of Seismicity Rate for Phase 1



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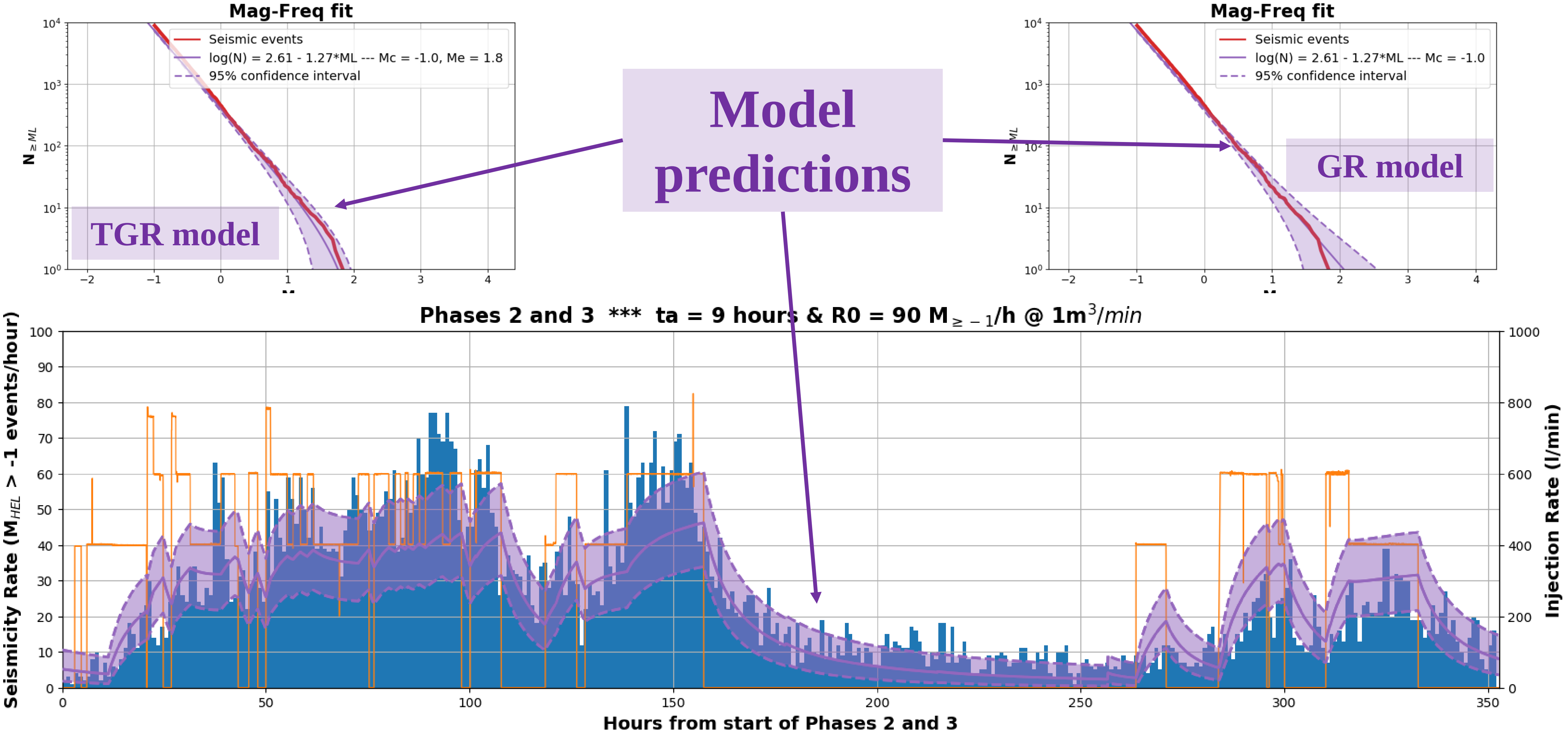
Prediction of Seismicity Rate for Phases 2 and 3



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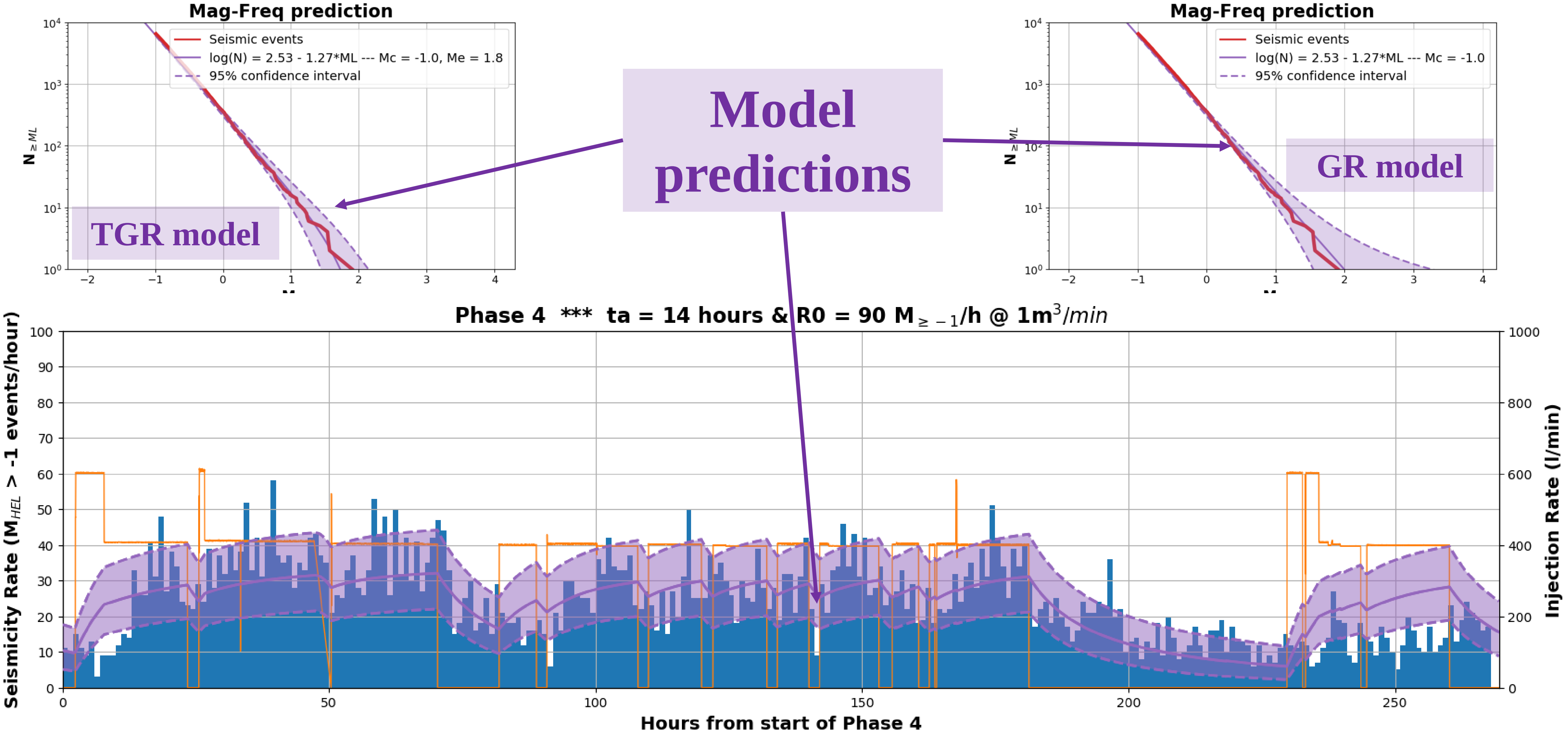
Prediction of Seismicity Rate for Phase 4



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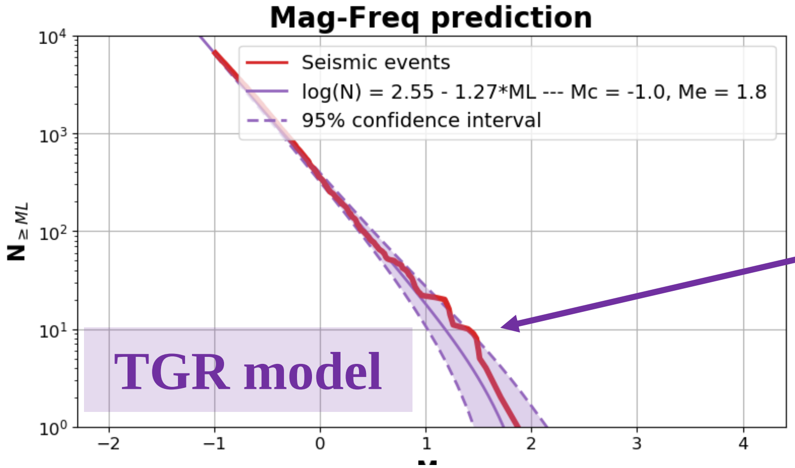
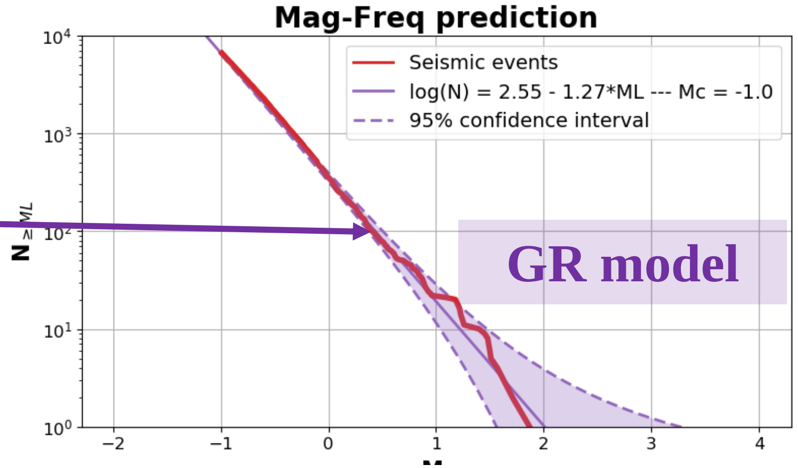
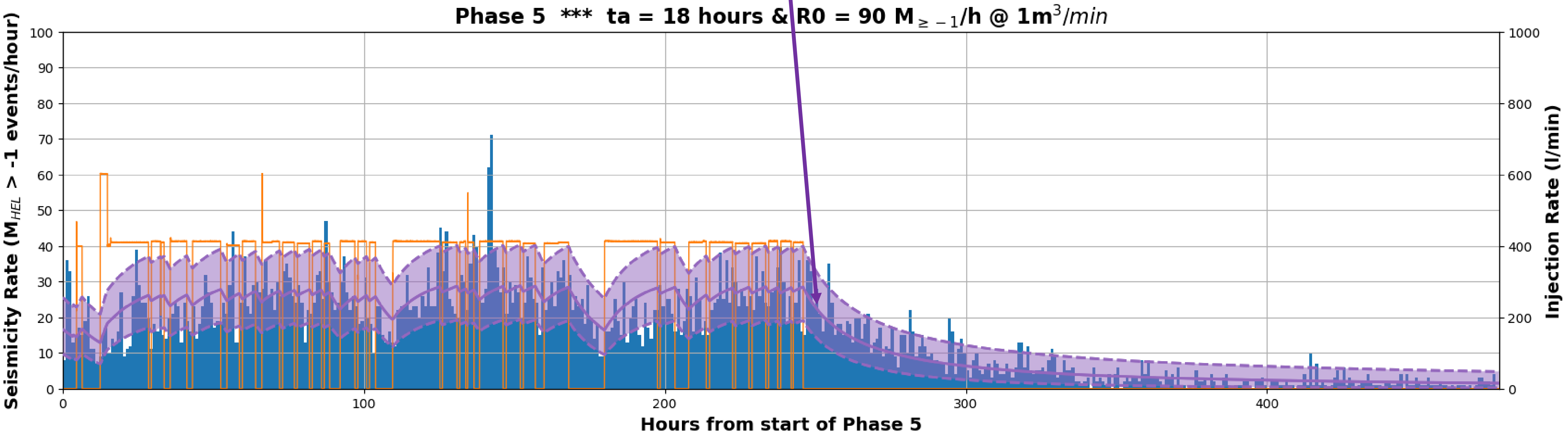
Prediction of Seismicity Rate for Phase 5



Model predictions





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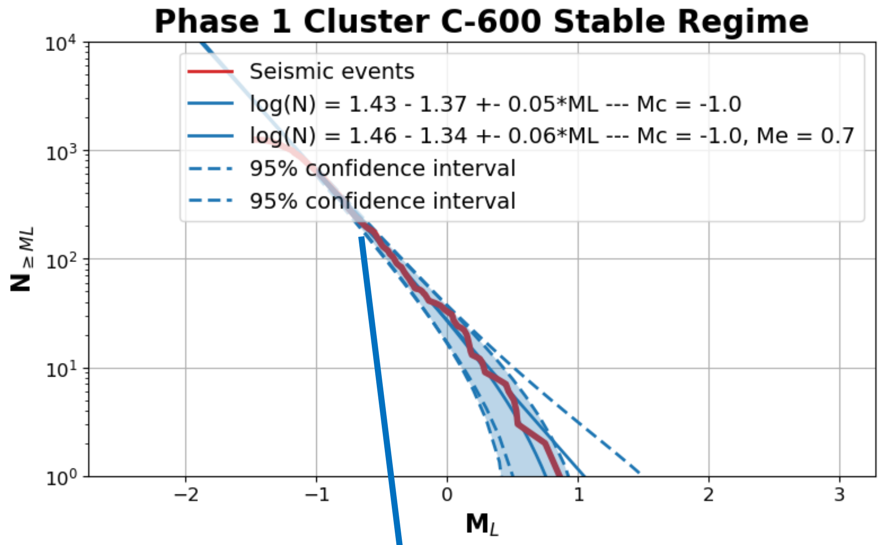
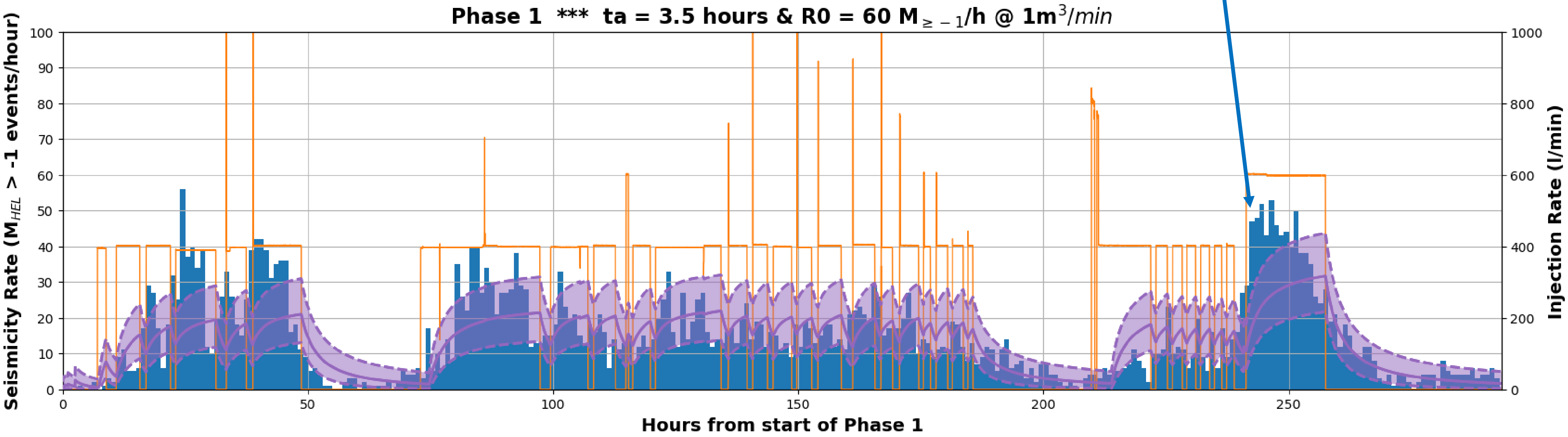
Prediction of Seismicity Rate for Phase 1

- Seismicity during Phase 1, when injecting at 600 l/min, is higher than the prediction.
- This is due to a large number of small events, yielding an apparent cut-off magnitude of 0.7.





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3.3.4 Prediction of the Maximum Magnitude

Maximum Magnitude Models

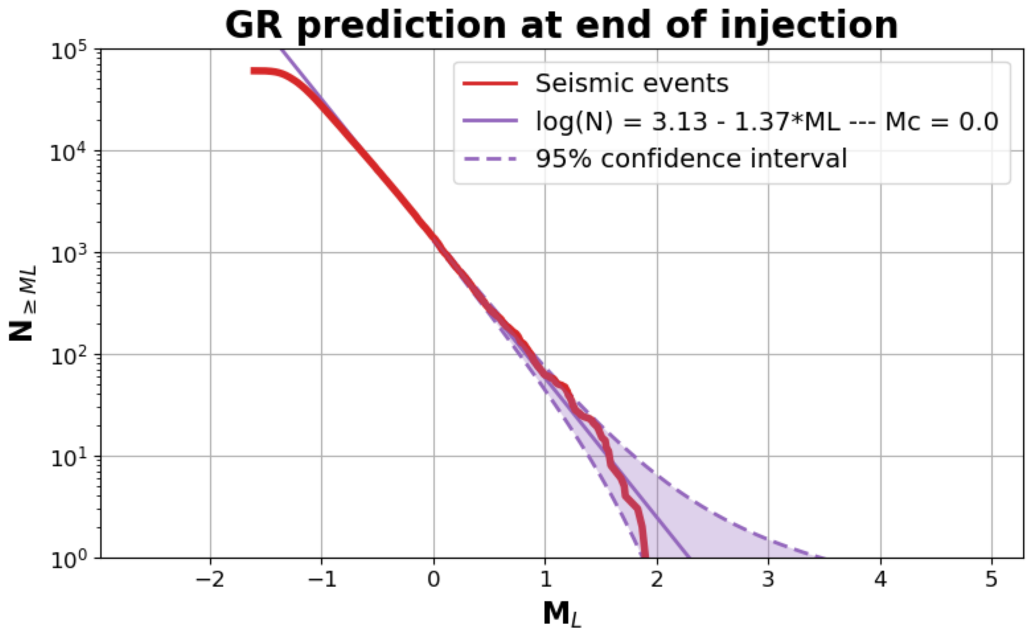
- Depending on the information available for the future injection:
  - Full time series; or
  - Future volume to be injected,

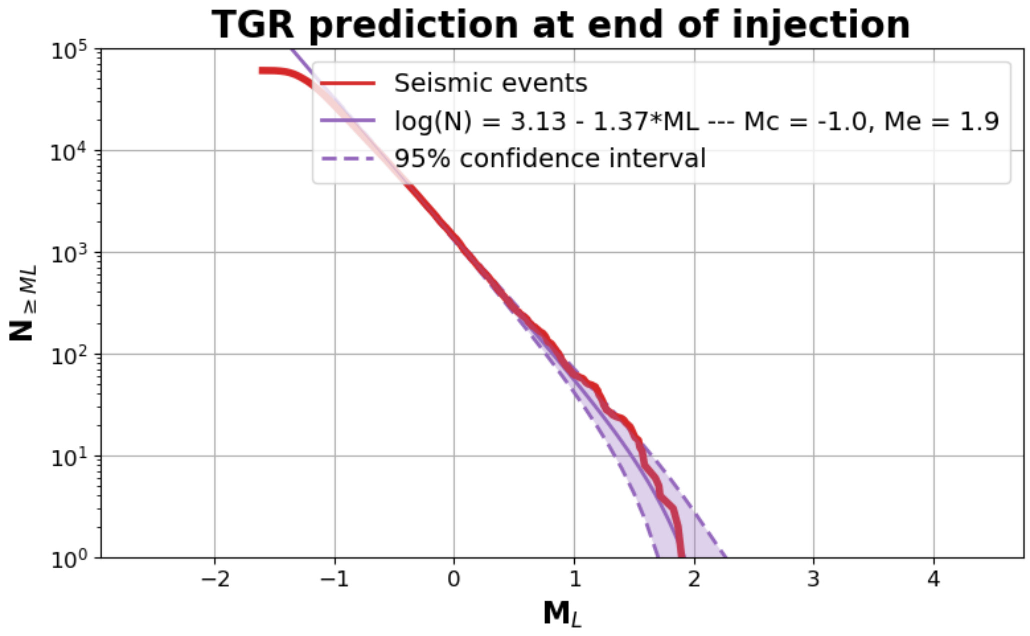
- The total number of events yet to happen, necessary to compute M_{max}, can be respectively estimated from two pieces of information:
  - Predicted future seismicity (better, as it accounts for seismicity during pauses);
  - Earthquake productivity (still provides a good approximation).

- The formulas to compute M_{max} are described in Appendix A2.

Maximum Magnitude Models

- Two models are used to compute the maximum magnitude M_{max} with 95% confidence intervals:
  - Gutenberg-Richter; or
  - Tapered Gutenberg-Richter.

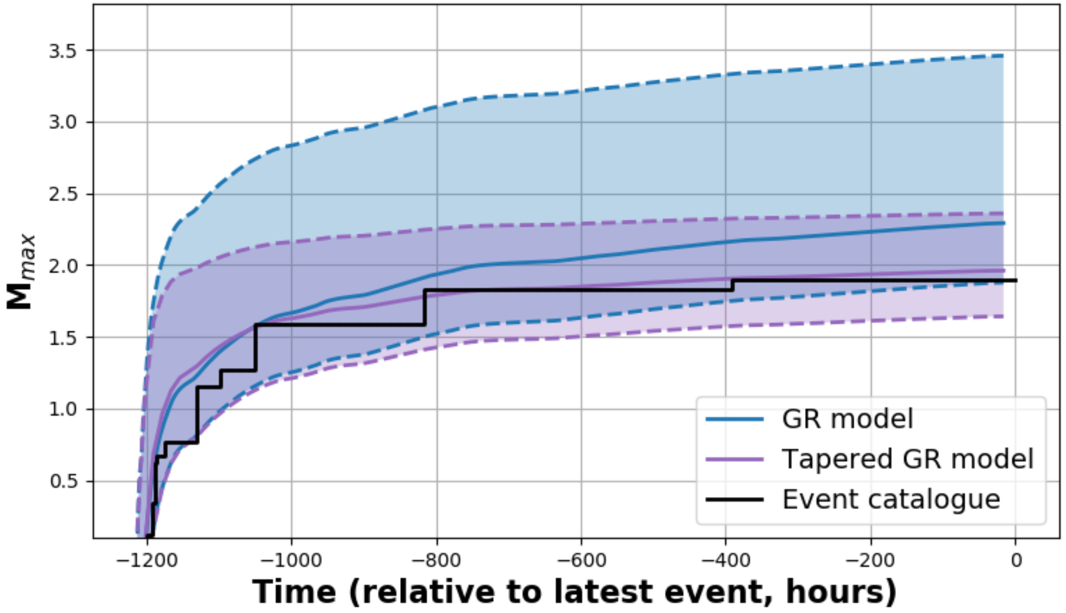




ARUP

Maximum Magnitude vs Time

- If an injection time-series is provided, the evolution of M_{max} can be estimated with time:



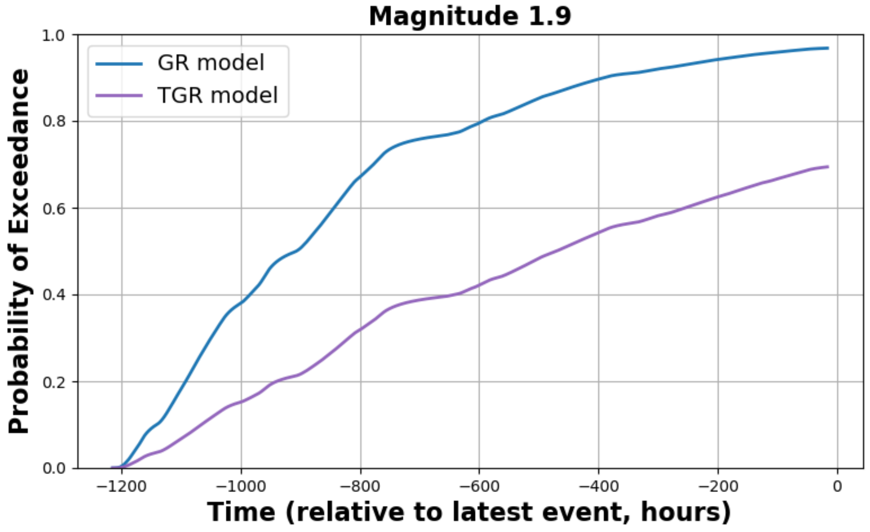
- Both predictions from **GR** and **Tapered-GR** models are provided, with 95% confidence intervals

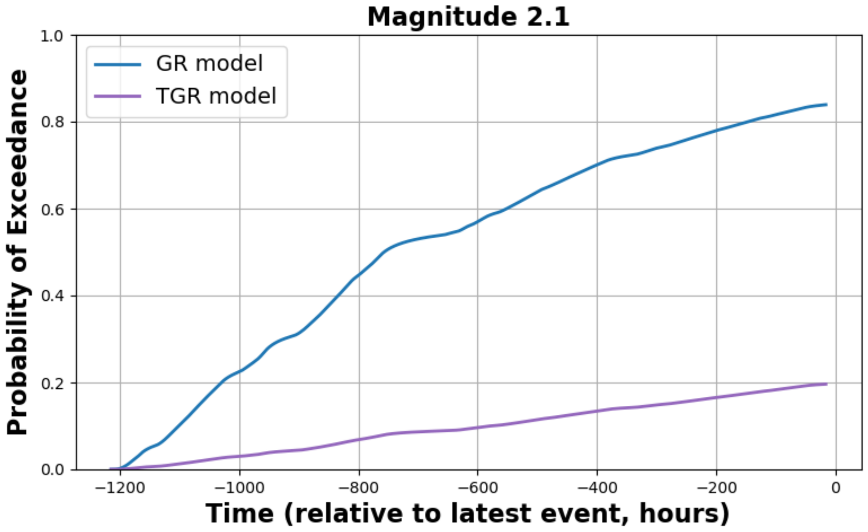
- In this case, the event catalogue is plotted, but in a prediction, the seismicity will be plotted only until it has been recorded.

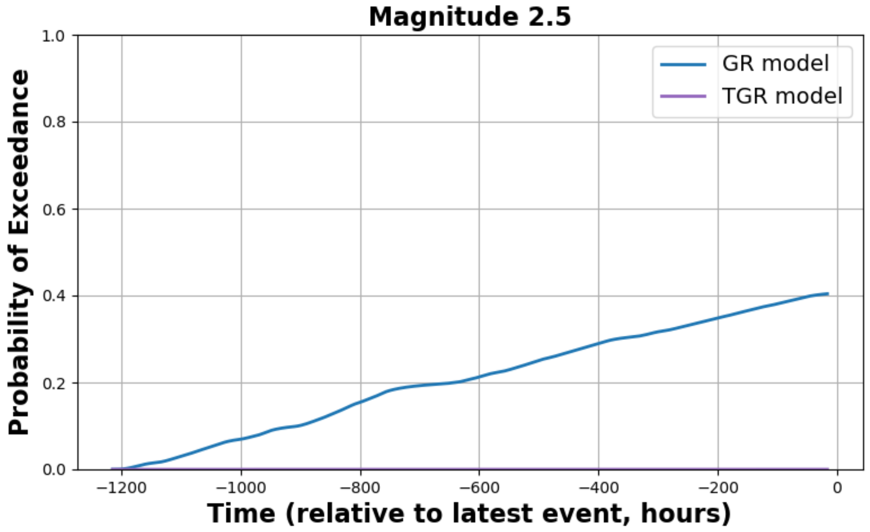
Probability of Exceedance vs Time

- The probability of exceedance through time can be estimated for given magnitudes.

- Results are provided both for the **GR** and **Tapered-GR** models

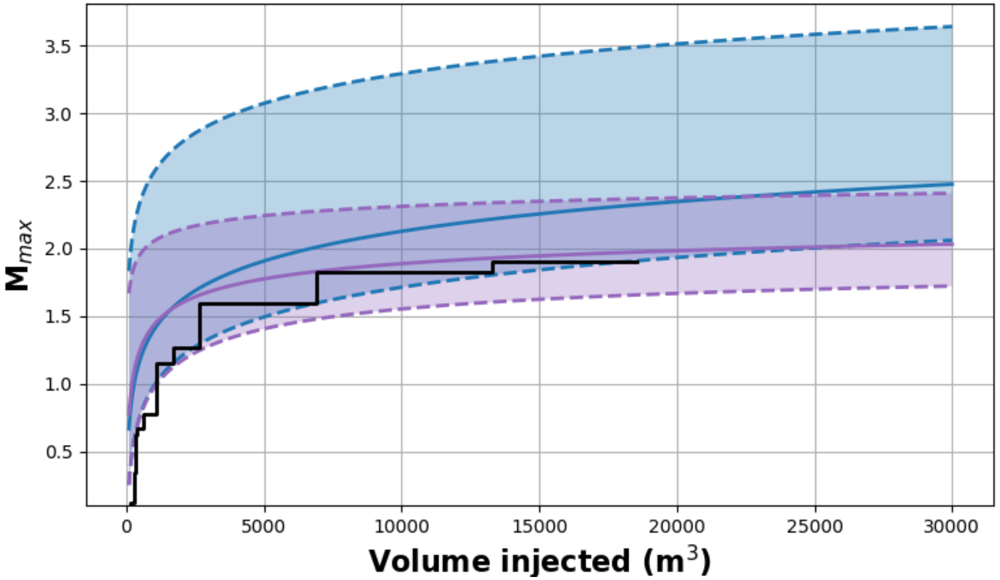






Maximum Magnitude vs Volume

- If no injection time-series is available, the evolution of M_{max} can still be estimated as a function of the total volume injected and to be injected in the future:



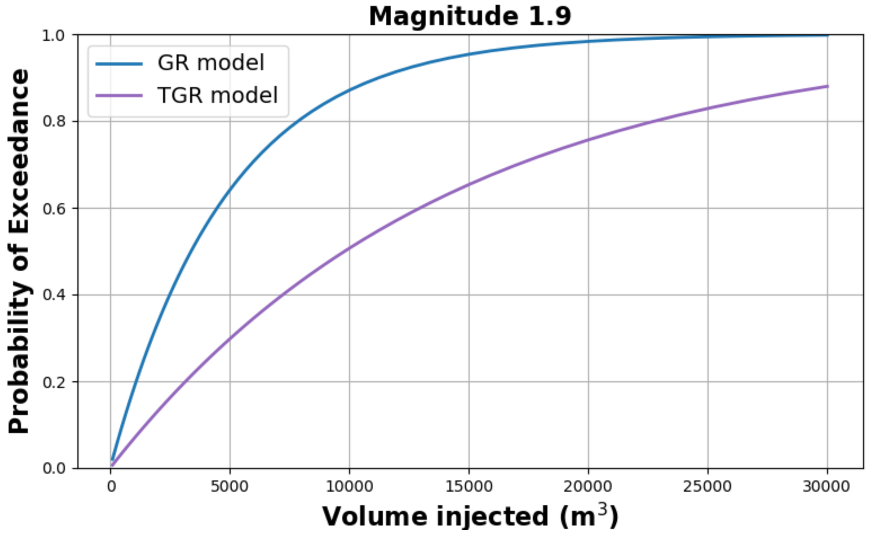
- Both predictions from **GR** and **Tapered-GR** models are provided, with 95% confidence intervals

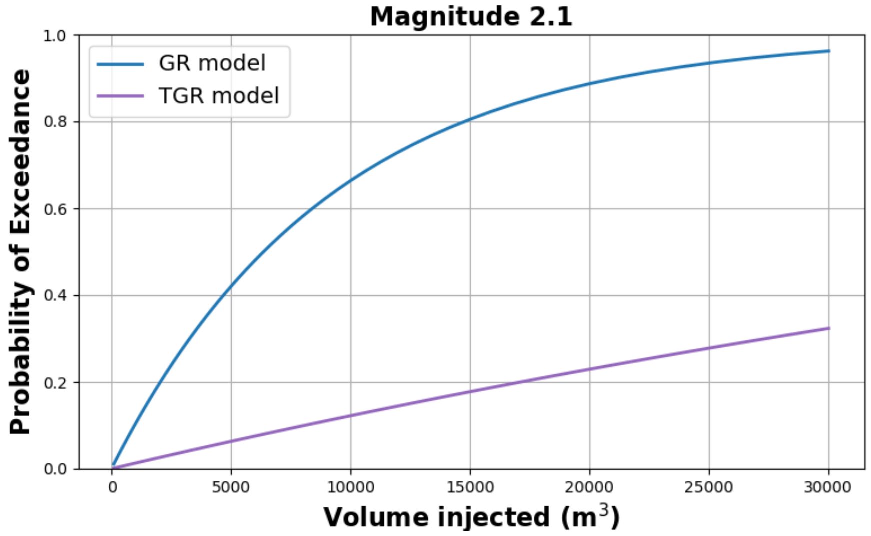
- In this case, the event catalogue is plotted, but in a prediction, the seismicity will be plotted only until it has been recorded.

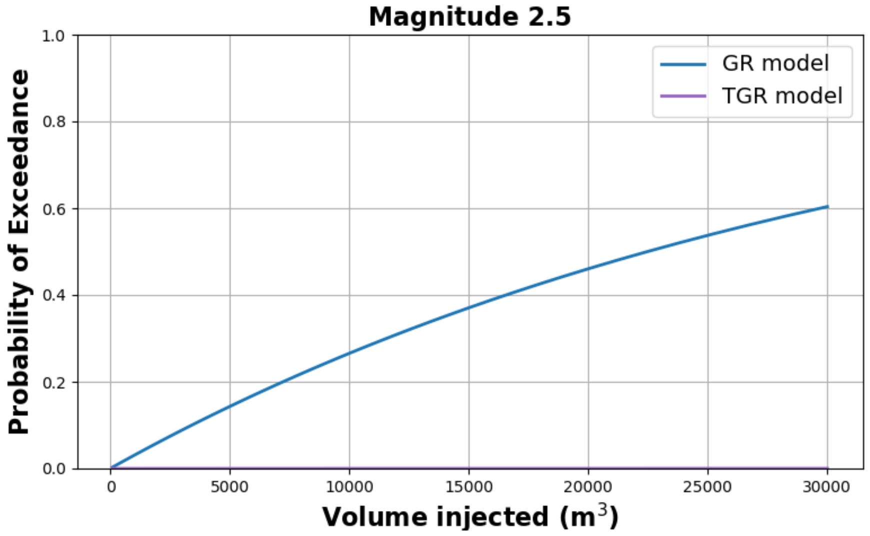
ARUP

Probability of Exceedance vs Volume

- The probability of exceedance with the volume injected can be estimated for given magnitudes.
- Results are provided both for the **GR** and **Tapered-GR** models







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Appendices

A1. Computation of Uncertainties on MLE Parameters

As a reminder, if $l(\theta)$ is the log-likelihood of parameter θ, the MLE θ is by definition $\hat{\theta}$, such that:

$$l(\hat{\theta}) = l_{max}$$

95% confidence intervals on MLE parameter θ are estimated as the interval of θ at which:

$$l(\theta_{95\%}) \geq l_{max} - 1.92.$$

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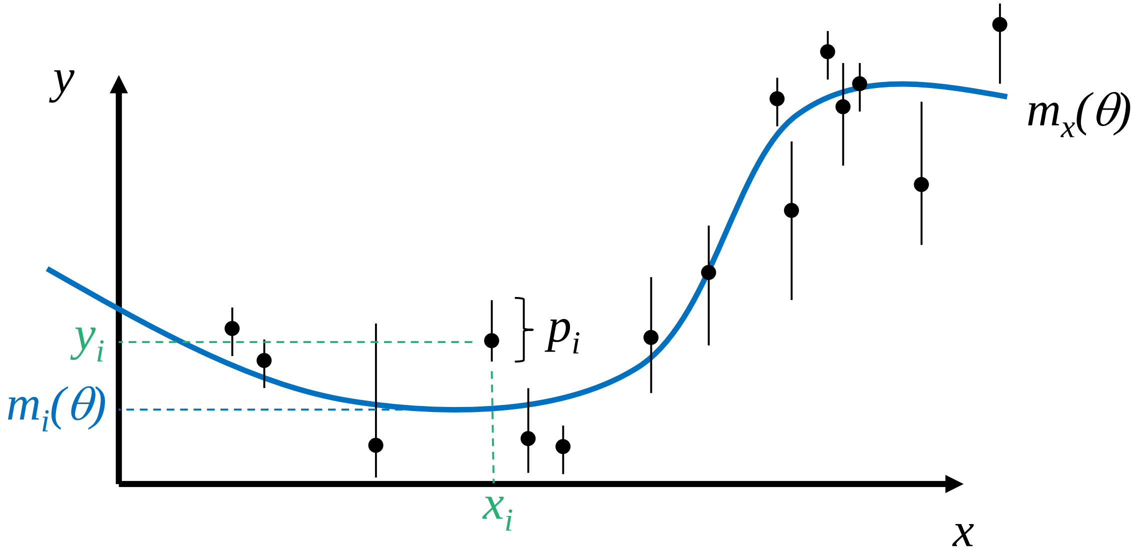
Computation of Uncertainties on MLE Parameters

- Likelihood $L(\theta)$:
  - We have data points at locations (x_i, y_i) with uncertainties described by the PDF p_i
  - We have a model m defined by parameter set θ



- Probability that model represents data point (x_i, y_i) is $p_i(y_i - m_i)$.

- Probability that model represents all of the data points is the likelihood:

$$L(\theta) = \prod_i p_i(y_i - m_i(\theta))$$

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Computation of Uncertainties on MLE Parameters

- Log-likelihood *l*: $$l(\theta) = lnL(\theta) = \sum_i ln(p_i(\theta))$$
- Optimal model parameter $\hat{\theta}$ is the one that maximizes $L(\theta)$ and therefore $l(\theta)$ so that

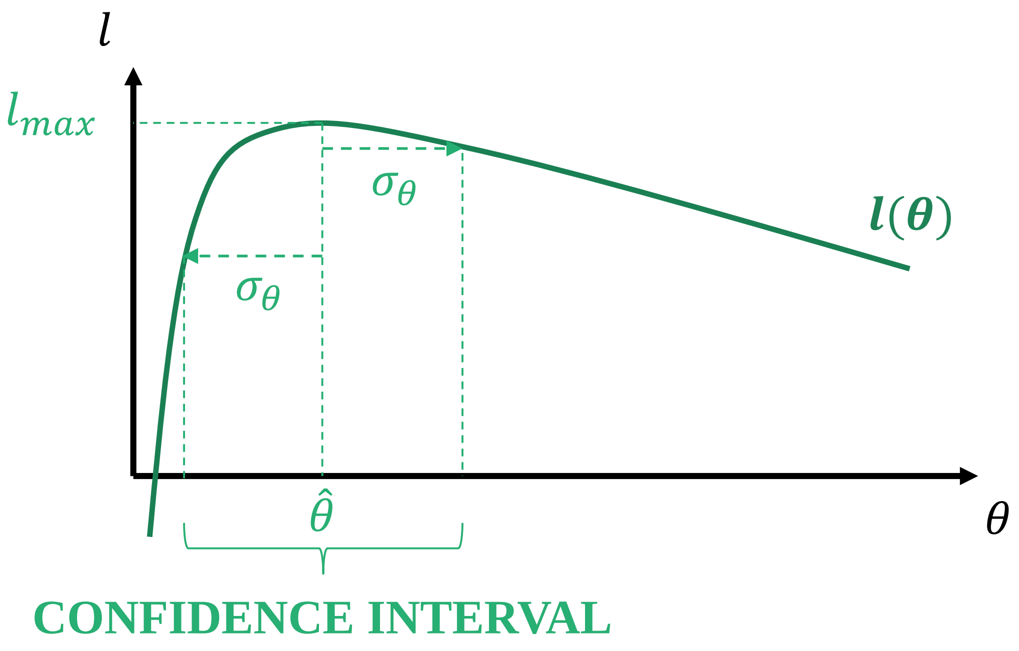
$$\frac{dl}{d\theta}(\hat{\theta}) = 0$$

- In practice, the standard error on $\hat{\theta}$ is given by the approximation:

$$\sigma_\theta = \sqrt{-\frac{1}{\frac{d^2 l}{d\theta^2}(\hat{\theta})}}$$

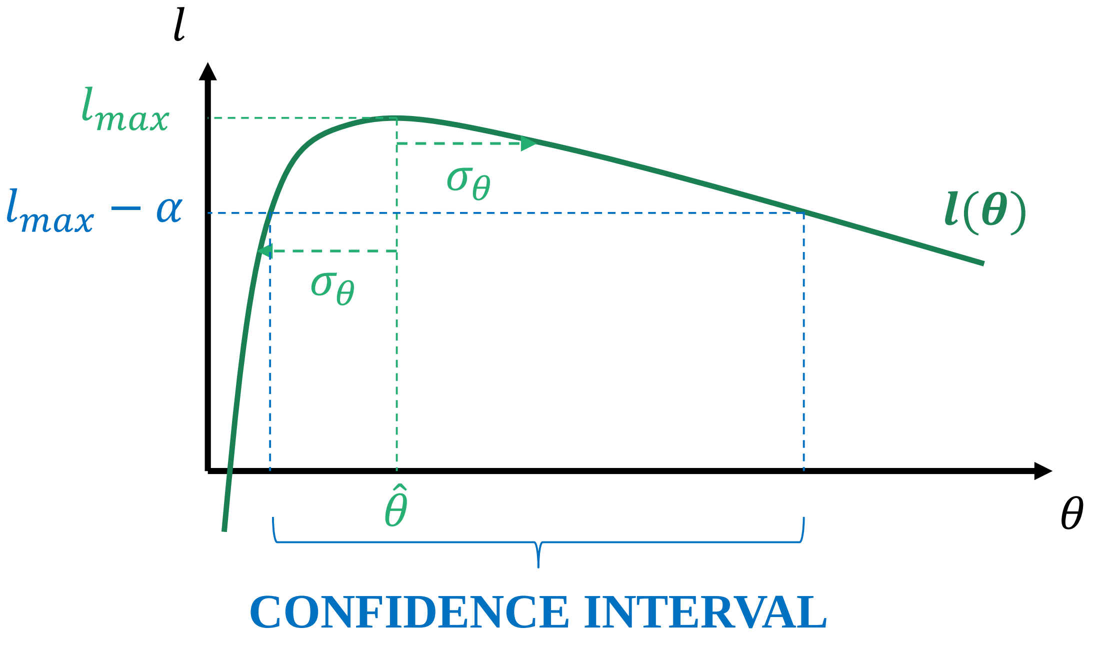
Entirely defined by the curvature of the log-likelihood at $\hat{\theta}$.

Computation of Uncertainties on MLE Parameters



- For skewed l, as is the case most of the time, the curvature at $\hat{\theta}$ does not describe well the curvature of $l(\theta)$ within the range of the confidence intervals ($\hat{\theta} \pm \sigma_\theta$, $\hat{\theta} \pm 2\sigma_\theta$, etc.).

Computation of Uncertainties on MLE Parameters



- For skewed l, as is the case most of the time, the curvature at $\hat{\theta}$ does not describe well the curvature of $l(\theta)$ within the range of the confidence intervals ($\hat{\theta} \pm \sigma_\theta$, $\hat{\theta} \pm 2\sigma_\theta$, etc.).

- Intuitively, we would prefer to define the confidence intervals bounds as points where the likelihood is the same

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Computation of Uncertainties on MLE Parameters

- The following approximation of the standard error on $\hat{\theta}$ is based on a linearization of $\frac{dl}{d\theta}$ around $\hat{\theta}$:

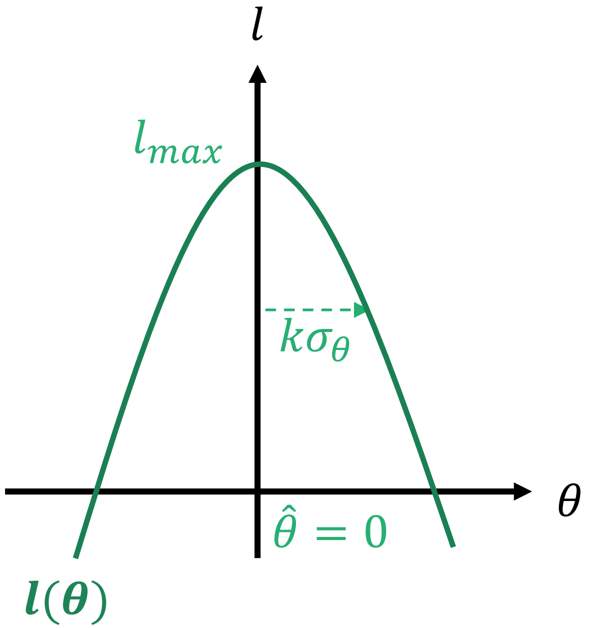
$$\sigma_\theta = \sqrt{-\frac{1}{\frac{d^2 l}{d\theta^2}(\hat{\theta})}}$$

- It therefore approximates $l(\theta)$ by a quadratic expression around $\hat{\theta}$.

- If $l(\theta)$ is quadratic, it is symmetric, and the confidence intervals defined by $l(\hat{\theta} \pm k\sigma_\theta)$ indeed correspond to the same values: $l(\hat{\theta} + k\sigma_\theta) = l(\hat{\theta} - k\sigma_\theta)$.

Computation of Uncertainties on MLE Parameters

- For a simple quadratic log-likelihood:
  - $l(\theta) = l_{max} - \beta\theta^2$
  - $\hat{\theta} = 0$
  - $\sigma_\theta = \sqrt{-\frac{1}{\frac{d^2 l}{d\theta^2}(0)}} = \sqrt{\frac{1}{2\beta}}$



- So, for a confidence interval defined by $\hat{\theta} \pm k\sigma_\theta$:
  - $l(\hat{\theta} \pm k\sigma_\theta) = l(k\sigma_\theta) = l\left(\frac{k}{2\beta}\right) = l_{max} - \frac{k^2}{2}$

- A 95% confidence interval, corresponding to $\hat{\theta} \pm 1.96\sigma_\theta$, corresponds to a log-likelihood:

$$l(95\%) = l_{max} - 1.92.$$

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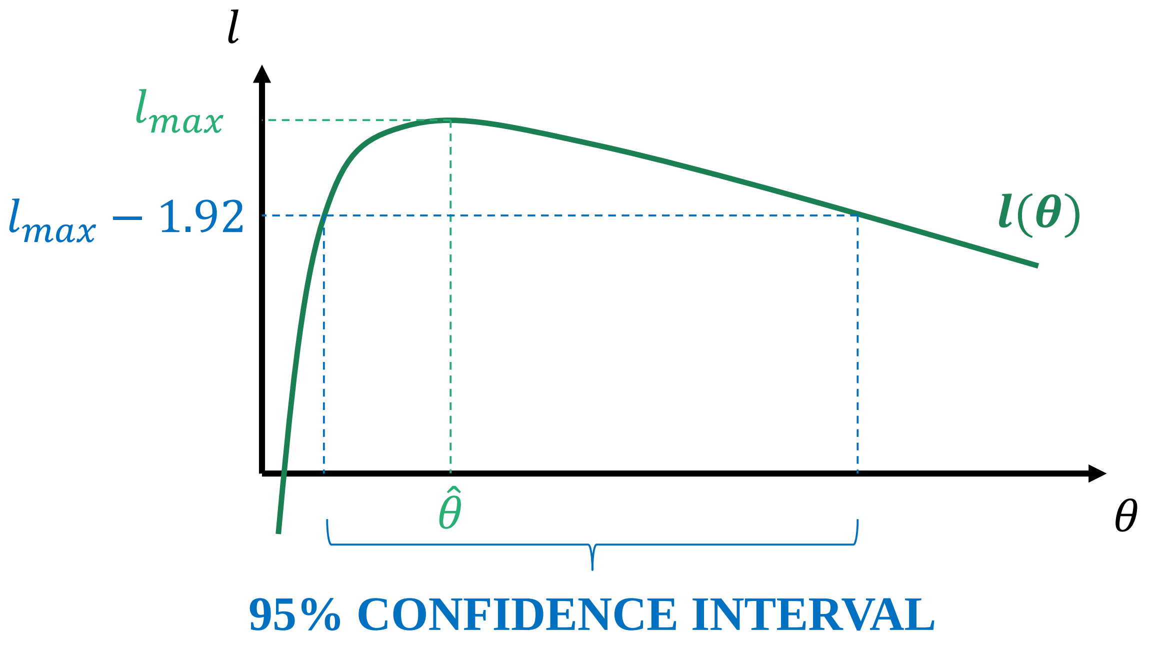
Computation of Uncertainties on MLE Parameters



- We therefore estimate the 95% confidence interval on $\hat{\theta}$ as the interval over which:

$$l(\theta_{95\%}) \geq l_{max} - 1.92.$$

ARUP

A2. PDF of Maximum Magnitude

If event of magnitude M has a return period T, the probability that $M_{max} \geq M$ during time t is the probability to observe at least one event of magnitude $\geq M$ during time t:

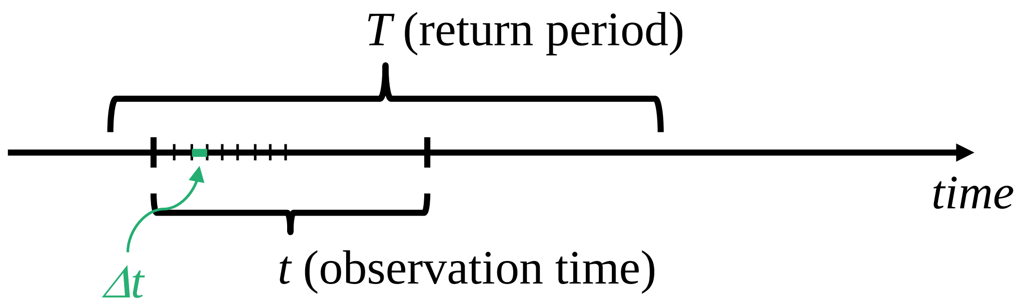
$$\mathrm{P}(M_{max} \geq M) = P_{exc}(M) = 1 - e^{-\frac{t}{T}} = 1 - e^{-N(M)}$$

The PDF of M_{max} is simply:

$$p(M_{max}) = -\frac{d\mathrm{P}(M_{max} \geq M)}{dM} = -\frac{dN}{dM}\, e^{-N(M)}.$$

ARUP

Probability of Exceedance of a Given Magnitude



- Assume that events of magnitude $\geq M$ are a Poisson process with return period T. What is the probability to have it happen during time t?
- Divide t into $t = n \times \Delta t$, such that $\Delta t \ll T$.
- Probability to observe an event during Δt: $\Delta t/T$.
- Probability to not observe an event during Δt: $1 - \Delta t/T$.
- Probability to not observe an event during $2\Delta t$:$\left(1-\frac{\Delta t}{T}\right) \times \left(1-\frac{\Delta t}{T}\right)$. (Poisson process)
- Probability to not observe an event during $t = n\Delta t$: $\left(1-\frac{\Delta t}{T}\right)^n$. (Poisson process)
- Probability to observe an event during $t = n\Delta t$: $P_{exc} = 1-\left(1-\frac{\Delta t}{T}\right)^n$.

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Probability of Exceedance of a Given Magnitude

- $P_{exc} = 1 - \left(1 - \frac{\Delta t}{T}\right)^n = 1 - e^{n\ln\left(1-\frac{\Delta t}{T}\right)} \approx 1 - e^{-n\frac{\Delta t}{T}} = 1 - e^{-\frac{t}{T}}.$

- In the case of the stimulation, if t is the duration of the stimulation, one has:

$$N_{\geq M} = t/T,$$

where $N_{\geq M}$, the number of events with magnitude greater than M can be obtained from the magnitude-frequency (MF) relationship.

- So $P_{exc} = 1 - e^{-N(\geq M)}$

ARUP

PDF of Mmax

- The probability that M_{max} is in the interval $[M, M + dM]$ can be computed from the probability of exceedance:

$$p(M)dM = P_{exc}(M) - P_{exc}(M + dM)$$

$$= -\frac{dP_{exc}(M)}{dM}dM$$

- So that:

$$p(M) = -\frac{dP_{exc}(M)}{dM}$$

ARUP

Gutenberg-Richter case

- For a MF following the Gutenberg-Richter (GR) law:

$$log_{10}N(\geq M) = a - bM,$$

where M_c is the magnitude of completeness.

- So

$$N(\geq M) = 10^{a-bM},$$

where $N(M \geq 0) = 10^a$ is the number of events above magnitude 0.

Gutenberg-Richter case

- The probability of exceedance (or probability that $M_{max} \geq M$):

$$P_{exc}(M) = P(M_{max} \geq M) = 1 - e^{-10^{a-bM}}.$$

- The PDF of M_{max} is:

$$p(M_{max}) = b\ln 10 \times 10^{a-bM} \times e^{-10^{a-bM}}.$$

Tapered Gutenberg-Richter case

- For a MF following the tapered Gutenberg-Richter (TGR) law:

$$N(\geq M) = N_c e^{-\beta(M-M_c)} \exp\left[\frac{10^{3/2M_c} - 10^{3/2M}}{10^{3/2M_e}}\right],$$

where M_e is the corner magnitude for the roll-off at large magnitudes.

- Using a $= \log(N(M \geq 0))$, this can be rewritten as:

$$N(\geq M) = 10^{a-bM} \exp\left[10^{-\frac{3}{2}M_e} - 10^{\frac{3}{2}(M-M_e)}\right],$$

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Tapered Gutenberg-Richter case

- The probability of exceedance (or probability that $M_{max} \geq M$):

$$P_{exc}(M) = P(M_{max} \geq M) = 1 - \exp\left(10^{a-bM}\exp\left[10^{-\frac{3}{2}M_e} - 10^{\frac{3}{2}(M-M_e)}\right]\right).$$

- The PDF of M_{max} is:

$$p(M_{max}) = \ln 10\left(b + \frac{3}{2}10^{\frac{3}{2}(M_{max}-M_e)}\right) \times e^{-N(\geq M_{max})}.$$

A3. Computation of t_a when injection stops

ARUP

Computation of t_a when Injection Stops

- The seismicity rate at time t_i, taken at the number of $M \geq -1$ events per hour is either:
  - Modelled: $\lambda_i = \lambda(t_i) = \frac{\lambda_0}{1+t_i/t_a}$
  - Measured: $R_i = R(t_i)$

- The seismicity rate is assumed to follow a Poisson process with a different rate λ_i at each time t_i, so that the probability of the observed rate R_i at time t_i is:

$$p_i = \frac{{\lambda_i}^{R_i} e^{-\lambda_i}}{R_i!}$$

- The likelihood of the observed seismicity rate at all times is therefore:

$$\mathcal{L} = \prod_i p_i$$

Computation of t_a when Injection Stops

- The log-likelihood is:

$$\ell(t_a) = \ln\mathcal{L} = \sum_i R_i \ln\lambda_i(t_a) - \sum_i \lambda_i(t_a) - \sum_i \ln(R_i!)$$

- The value of t_a that maximizes the log-likelihood is then the MLE of t_a.

- The confidence intervals on t_a are then computed with the function $\ell(t_a)$ following the method described in appendix A1.

A4. Computation of t_a for bleed-off

ARUP

Computation of t_a for Bleed-Offs

- Bleed-off rate $r(t)$ such that:

$$r(t) = \frac{r_0}{1+t/t_a}$$

- Cumulative bled-off volume $V(t)$ is therefore simply:

$$V(t) = \int r(t)dt = V_0 \ln\left(1 + \frac{t}{t_a}\right),$$

where $V_0 = r_0 t_a$.

- Calling V_i the measured volume at time t_i and $V(t_i)$ the modelled volume, the error function is

$$\chi^2 = \sum_i \left(V_i - V(t_i)\right)^2 = \sum_i \left(V_i - V_0 \ln\left(1 + \frac{t_i}{t_a}\right)\right)^2.$$

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Computation of t_a for Bleed-Offs

- χ^2 needs to be minimized with respect to the two unknown parameters V_0 and t_a, which will be solution of the two equations:

$$\frac{\partial \chi^2}{\partial V_0} = 0 \qquad (1)$$

$$\frac{\partial \chi^2}{\partial t_a} = 0 \qquad (2)$$

- Equation (1) gives:

$$\sum_i \ln\left(1+\frac{t_i}{t_a}\right)\left[V_i - V_0 \ln\left(1+\frac{t_i}{t_a}\right)\right] = 0,$$

- So that V_0 can be expressed as a function of t_a:

$$V_0 = \frac{\sum_i V_i \ln\left(1+\frac{t_i}{t_a}\right)}{\left[\ln\left(1+\frac{t_i}{t_a}\right)\right]^2}.$$

Computation of t_a for Bleed-Offs

- Finally, expressing the parameter V_0 as a function t_a, the error function χ^2 only depends on t_a.

- The error function χ^2 is therefore minimized with respect to t_a by simple grid search on t_a to find the best value for t_a.