Assessment 1

Released: Wednesday 12th Feb at 5 pm
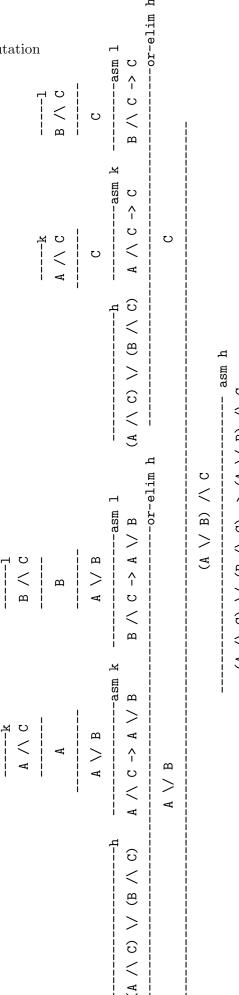### Due: Friday 21st Feb at 5pm

For this assessment, you can use (only) the inference rules from the cheatsheet available on the "Modules/Reference materials" subpage on Canvas! If the question asks you to prove the given statement using intuitionistic logic, then you cannot use DNE or EM. Otherwise if the question doesn't explicitly mention anything then you can use ANY of the inference rules from the cheatsheet.

Please submit your solutions as a single pdf or doc(x) file via Canvas. Lean proof terms should be submitted as machine-readable text, so that it can be copied into Lean.

1. (a) Consider the proof tree in Figure 1 (see page 2). Write the corresponding proof in Lean.
   (b) Consider the proof expression in
       `https://gist.github.com/benediktahrens/ade1d2bc272146a983b87d12f560a687`.
       Draw the corresponding proof tree.

   Use only intuitionistic logic.

2. Check the validity of the logical equivalence $(A \land C) \lor (B \land C) \longleftrightarrow (A \lor B) \land C$ using truth tables.

3. Prove $A \to (A \land B) \lor (A \land \neg B)$

   (a) in natural deduction;
   (b) in Lean.

4. Express $P \land (Q \to R)$

   (a) in CNF;
   (b) in DNF.

```
                                                         -----l
                                                         B /\ C
                                       ----k            --------
                                       A /\ C               C            -----l
                                      --------         -------------asm k B /\ C
                                          C            A /\ C -> C       -------
                                     ------------asm k ------------------   C    --------asm l
          ----k        -----l                                         B /\ C -> C
          A /\ C       B /\ C                      ---------------------------------------or-elim h
         --------     --------                                        C
            A            B
      ----------asm k ---------asm l
      A \/ C -> A \/ B B /\ C -> A \/ B
```

                                                              C

```
-----------------h
(A /\ C) \/ (B /\ C)   A /\ C -> A \/ B    B /\ C -> A \/ B
------------------------------------------------------or-elim h
                       A \/ B
```

```
                      (A \/ B) /\ C
--------------------------------------------asm h
(A /\ C) \/ (B /\ C) -> (A \/ B) /\ C
```

Figure 1: Prooftree