

5A2.

a). $X = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + s \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+2s \\ 1+s \end{pmatrix}$
 let the normal form be $ax + by = d$.

let $a = -1$

$b = 2$

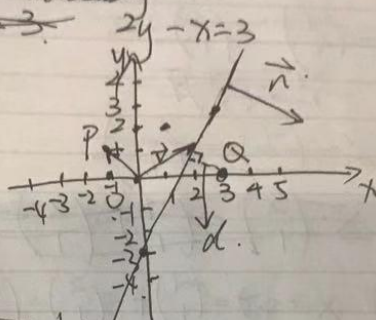
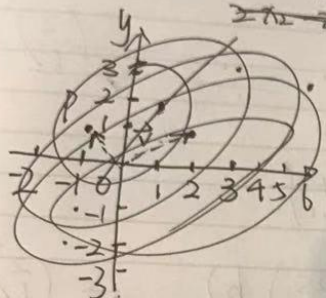
$d = -x + 2y$

$-(-1+2s) + 2(1+s) = -x + 2y$

$1 - 2s + 2 + 2s = -x + 2y$

$3 = -x + 2y$

b).



c. Distance = $\left| \frac{-1 \cdot 1 + 2 \cdot 1 - 3}{\sqrt{(-1)^2 + 2^2}} \right|$

$= \left| \frac{-3 - 3}{\sqrt{5}} \right|$

$\vec{n} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\frac{d - \langle \vec{n}, Q \rangle}{|\vec{n}|} = \frac{\langle \vec{n}, P \rangle - \langle \vec{n}, Q \rangle}{|\vec{n}|}$

$\vec{OP} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$= \frac{\langle \vec{n}, \vec{OP} \rangle}{|\vec{n}|}$

$= \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$= \frac{4+2}{\sqrt{5}}$

$= \frac{6}{\sqrt{5}}$

d. $Q' = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \frac{3 - (-3)}{5} \times \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \frac{6}{5} \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 - \frac{6}{5} \\ \frac{12}{5} \end{pmatrix} = \begin{pmatrix} \frac{9}{5} \\ \frac{12}{5} \end{pmatrix}$

$$\begin{aligned}
 \text{e.g. } Q'' &= \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 2 \times \frac{3 - (-3)}{\sqrt{5}} \cdot \frac{\begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\sqrt{5}} \\
 &= \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 2 \times \frac{6}{5} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix} \\
 &= \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \frac{12}{5} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix} \\
 &= \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{12}{5} \\ \frac{24}{5} \end{pmatrix} \\
 &= \begin{pmatrix} \frac{3}{5} \\ \frac{24}{5} \end{pmatrix}.
 \end{aligned}$$