

11A.

a) reflexive.  $\forall (a,b) \in \mathbb{Z} \times \mathbb{Z}, \frac{b-a}{n} \in \mathbb{Z}, (\frac{b-a}{n}, \frac{b-a}{n}) \in \mathbb{Z}$ .

b) symmetric.  $\forall (a,b) \in \mathbb{Z} \times \mathbb{Z}, \frac{b-a}{n} \in \mathbb{Z} \Rightarrow (\frac{b-a}{n}, b-a) \in \mathbb{Z}$

c) transitive.  $\forall (a,b) \in \mathbb{Z} \times \mathbb{Z}, (\frac{b-a}{n}, a) \in \mathbb{Z} \Rightarrow (\frac{b-a}{n}, b) \in \mathbb{Z}$ .

$$b. i). \frac{(b+b') - (a+a')}{n} = \frac{b+b' - a - a'}{n} = \frac{b-a' + b-a}{n}$$

$$= \frac{b'-a'}{n} + \frac{b-a}{n} = \frac{b+b' - (a+a')}{n} \in \mathbb{Z}.$$

so  $a \equiv b \pmod{n}$  and  $(a' \equiv b')$ ,  $a+a' \equiv (b+b') \pmod{n}$  is true.

$$ii). (\frac{b-a}{n}) \times a' = \frac{ba' - aa'}{a'n} \in \mathbb{Z}$$

$$\Rightarrow \frac{ba' - aa'}{n} \in \mathbb{Z} \cdot a'$$

$$\Rightarrow \frac{ba' - aa'}{n} \in \mathbb{Z}$$

$$(\frac{b'-a'}{n}) \times b = \frac{bb' - a'b}{nb} \in \mathbb{Z}$$

$$\Rightarrow \frac{bb' - a'b}{n} \in \mathbb{Z} \cdot b$$

$$\Rightarrow \frac{bb' - ba'}{n} \in \mathbb{Z}$$

$$\frac{ba' - aa'}{n} + \frac{bb' - ba'}{n} = \frac{bb' - aa'}{n} \in \mathbb{Z}.$$

$$\therefore a \times a' \equiv b \times b' \pmod{n}.$$

$$iii) \forall b-a \in \mathbb{Z}$$

$$\therefore b+a \in \mathbb{Z}$$

$$(b-a)(b+a) = b^2 - a^2$$

$$(b^2 - a^2)(b^2 + a^2) = b^4 - a^4$$

$$\therefore a^4 \equiv b^4 \pmod{n}.$$

$$c. \cancel{4^6 \equiv 1 \pmod{7}}$$

$$\cancel{4^9 \equiv 4 \pmod{7}}$$

$$\cancel{4^{10} \equiv 1 \pmod{7}}$$

$$\cancel{4^{10} \equiv 4 \pmod{7}}$$

$$\frac{1-46}{7} = 585$$

$$\frac{1-4^{1000}}{7} \in \mathbb{Z}.$$

$$\therefore a^c \equiv b^c \pmod{n}$$

$$\therefore n=1$$