Question 1

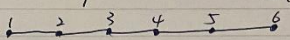
(a)& (c)

1. line: ~~two vectors v_i, v_j~~

~~when $v_i \neq v_j$~~, the Average path $= \dfrac{2\times(N\times0+(N-1)\times1+(N-2)\times2+(N-3)\times3+(N-4)\times4+\dots+0\times N)}{N(N-1)}$

$\Rightarrow \left(=\dfrac{N+1}{3}\right)$

the line we have:



so take $N=6$.

$\therefore$ the average pathlength $= \dfrac{2\times(0+5+8+9+8+5+0)}{5\times6}$

$= \dfrac{2\times35}{30}$

$= \dfrac{7}{3} \doteq 2.333$

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

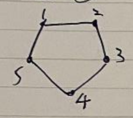
so the diameter $=5$.

2. ring

1) odd ring,

the average pathlength $= \dfrac{\left(\left(\frac{N-1}{2}\right)+\left(\frac{N-1}{2}-1\right)+\dots+0\right)\times2\times N}{N(N-1)} \Rightarrow \left(=\dfrac{\frac{N-1}{2}+1}{2}\right)$

where N is always odd and $\frac{N-1}{2}-n > 0$.



Take $N=5$

the average pathlength $= \dfrac{\left(\left(\frac{5-1}{2}\right)+\left(\frac{5-1}{2}-1\right)+\cancel{\left(\frac{5-1}{2}-2\right)}+0\right)\times2\times5}{5\times4}$

$= \dfrac{(2+1+0)\times2\times5}{20}$

$= \dfrac{30}{20}$

$= \dfrac{3}{2}$

$= 1.5.$

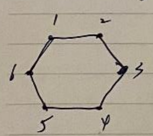
	1	2	3	4	5
1	0	1	2	2	1
2	1	0	1	2	2
3	2	1	0	1	2
4	2	2	1	0	1
5	1	2	2	1	0.

the diameter here is $\dfrac{5-1}{2} = 2$.

2) even ring

the average pathlength $= \dfrac{\frac{N}{2}N + \left(\left(\frac{N}{2}-1\right)+\left(\frac{N}{2}-2\right)+\dots+0\right)\times2\times N}{N(N-1)} \Rightarrow \left(=\dfrac{N^2}{4(N-1)}\right)$.

where N is always even, and $\left(\frac{N}{2}-n\right) > 0$.



Take $N=6$

average pathlength: $\dfrac{\frac{6}{2}\times6 + \left(\left(\frac{6}{2}-1\right)+\left(\frac{6}{2}-2\right)+0\right)\times2\times6}{6\times5}$

$= \dfrac{3+(2+1+0)\times2}{5}$

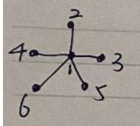
$= \dfrac{3+6}{5}$

$= \dfrac{9}{5} = 1.8$

	1	2	3	4	5	6
1	0	1	2	3	2	1
2	1	0	1	2	3	2
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	2	3	2	1	0	1
6	1	2	3	2	1	0

diameter: $\dfrac{N}{2} = 3$.

3. star, the average path length = $\frac{1+\cancel{(N-1)}+(1+2\times(N-2))\cancel{(N-1)}}{N\cancel{(N-1)}}$



$= \frac{1+1+2(N-2)}{N}$

$= \frac{2+2(N-2)}{N}$.

Take $N=6$, the star's average path length

$= \frac{2+2\times\cancel{8}4}{6}$

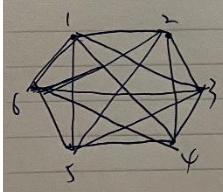
$= \frac{10}{6} \doteq 1.6667$.

	1	2	3	4	5	6
1	0	1	1	1	1	1
2	1	0	2	2	2	2
3	1	2	0	2	2	2
4	1	2	2	0	2	2
5	1	2	2	2	0	2
6	1	2	2	2	2	0

so the diameter here is 2.

4. fully connected.

the average pathlength $= \frac{N\times(N-1)}{N\times(N-1)} = 1$.



	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	1	1	1	1	1
3	1	1	1	1	1	1
4	1	1	1	1	1	1
5	1	1	1	1	1	1
6	1	1	1	1	1	1

the diameter = 1

(b)

These three notations are all Big-O Complexity notations.

- O(1) means the complexity is constant, there is no relation between the input and the cost, which means even the cost increases a lot, the cost will stay the same. An example of O(1) is *Hash*, it can find the target after one calculation.
- O(N) complexity increases the same N times as the number of input increases, it shows as linear, take *for loop* as an example, to find the largest number in N samples, the algorithm will go through all samples N times, so it has the complexity of O(N).
- O(N^2) means when the input increasers N times, the cost will increase N^2 times. One of the examples of this is *Bubble Sort*, it sorts N numbers with N*N times.

Here, for the above graphs, they also have different complexity:

Line: O(N)

The Odd ring: O(N)

The Even Ring: O(N^2)

Star: O(N)
Fully connected: O(1)

Question 2

The results of five graphs in matlab terminal are shown below

```
Line
Connection status
     1     1     1     1     1     1

Total number of links
     5

Average length
    2.3333

Diameter
     5
```

```
Ring_even
Connection status
     1     1     1     1     1     1

Total number of links
     6

Average length
    1.8000

Diameter
     3
```

```
Ring_odd
Connection status
     1     1     1     1     1

Total number of links
     5

Average length
    1.5000

Diameter
     2
```

```
Star
Connection status
     1     1     1     1     1     1

Total number of links
     5

Average length
    1.6667

Diameter
     2
```

```
Fully connected
Connection status
     1     1     1     1     1     1

Total number of links
    15

Average length
     1

Diameter
     1
```

(a)
The average path length of the line is about 2.3333;
The average path length of the even ring is about 1.8000;
The average path length of the odd ring is about 1.5000;
The average path length of the star is about 1.6667;
The average path length of the fully connected is about 1;
(b)
The diameter of the line is 5;
The diameter of the even ring is 3;
The diameter of the odd ring is 2;
The diameter of the star is 2;

The diameter of the fully connected is 1;

(c1)

The total number of links of the line is 5;

The total number of links of the even ring is 6;

The total number of links of the odd ring is 5;

The total number of links of the star is 5;

The total number of links of the fully connected is 15;

(c2)

To check whether the G is connected or not, I used function *conncomp()*, it will show '1' if the graph is connected. From the four results of 'connection states' above, we can know that all G are connected.

Question 3

(a)

```
>> mutation_add(ring, 3)

pos =

     0     1     0     0     0     1
     1     0     1     1     0     0
     0     1     0     1     0     0
     0     1     1     0     1     0
     0     0     0     1     0     1
     1     0     0     0     1     0


     1     1     1     1     1     1
```

```
>> mutation_add(ring_odd, 3)

pos =

     0     1     1     0     1
     1     0     1     0     0
     1     1     0     1     0
     0     0     1     0     1
     1     0     0     1     0


     1     1     1     1     1
```

The above two different results can show a link was added randomly. The first graph used the even ring graph, and the second used the odd ring graph. Both the parameters of (graph,seed) can be changed. To prove each of them is still fully connected, the function *conncomp()* also used here, the results of it were also showed status 1, so they are fully connected graphs.

(b)

```
>> mutation_remove(ring, 55)

pos =

     0     1     0     0     0     0
     1     0     1     0     0     0
     0     1     0     1     0     0
     0     0     1     0     1     0
     0     0     0     1     0     1
     0     0     0     0     1     0


     1     1     1     1     1     1
```

```
>> mutation_remove(ring_odd, 33)

pos =

     0     1     0     0     1
     1     0     0     0     0
     0     0     0     1     0
     0     0     1     0     1
     1     0     0     1     0


     1     1     1     1     1
```

Same as question(a), the above two different results both shows a randomly chosen link has been removed, and the graph stays connected by using function *conncomp()* with all results status 1. The first graph used the even ring graph and the second used the odd ring graph which with seed 55 and 33.

(c)

```
pos =

     0     1     1     0     1
     1     0     1     0     0
     1     1     0     1     0
     0     0     1     0     1
     1     0     0     1     0

     1     1     1     1     1


pos =

     0     1     1     0     1
     1     0     1     0     0
     1     1     0     0     0
     0     0     0     0     1
     1     0     0     1     0

     1     1     1     1     1
```

The result shows a random rewriting of the combination of (a) and (b), result of function *conncomp()* showed it is still connected. I calculated this combination by firstly adding a link then removing another link.

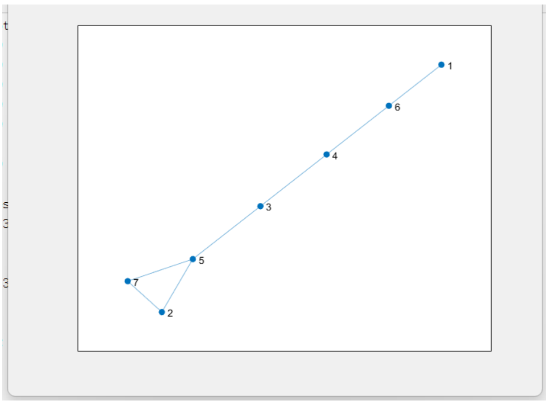
Question 4

(a)&(b)

The SA algorithm was used for this question. To ensure the graph is connected, we should let the diagonal all be zero and the matrix should be symmetrical. Then I put a judgement of the function *conncomp()* to reach this goal. The fitness function uses here is (*diameter – average path length)*, For the probabilities, p1 was set to 0.3, p2 was set to 0.4 and p3 was set to 0.3. The number of iteration was set to 20.

The three results and graph is shown below:

1. simulated_annealing(7,6)



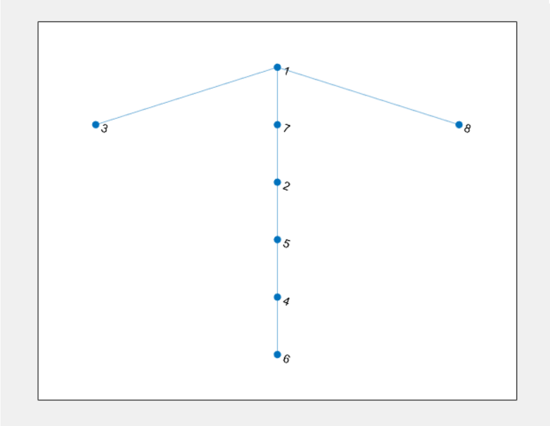

```
The optimal solution obtained by the final search is:
     0     0     0     0     0     1     0
     0     0     0     0     1     0     1
     0     0     0     1     1     0     0
     0     0     1     0     0     1     0
     0     1     1     0     0     0     1
     1     0     0     1     0     0     0
     0     1     0     0     1     0     0

The best fitness is:
    3.3333

f1 is
    1.3333

f2 is
     2
```

2. simulated_annealing(8,6)




```
     1     1     1     1     1     1     1     1

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The optimal solution obtained by the final search is:
     0     0     1     0     0     0     1     1
     0     0     0     0     1     0     1     0
     1     0     0     0     0     0     0     0
     0     0     0     0     1     1     0     0
     0     1     0     1     0     0     0     0
     0     0     0     1     0     0     0     0
     1     1     0     0     0     0     0     0
     1     0     0     0     0     0     0     0

The best fitness is:
     4

f1 is
    1.5000

f2 is
     3
```
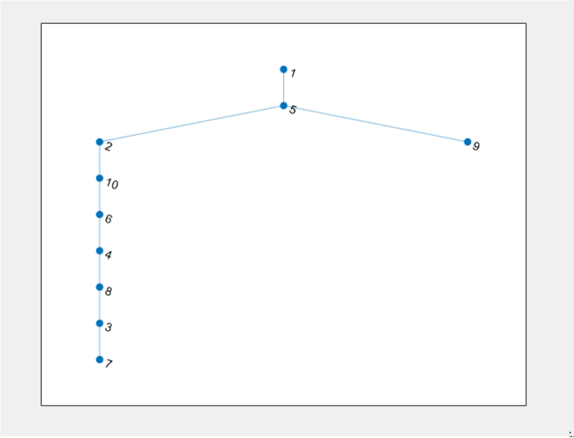

3. simulated_annealing(10,5)




```
     1     1     1     1     1     1     1     1     1     1

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The optimal solution obtained by the final search is:
     0     0     0     0     1     0     0     0     0     0
     0     0     0     0     1     0     0     0     0     1
     0     0     0     0     0     0     1     1     0     0
     0     0     0     0     0     1     0     1     0     0
     1     1     0     0     0     0     0     0     1     0
     0     0     0     1     0     0     0     0     0     1
     0     0     1     0     0     0     0     0     0     0
     0     0     1     1     0     0     0     0     0     0
     0     0     0     0     1     0     0     0     0     0
     0     1     0     0     0     1     0     0     0     0

The best fitness is:
    5.3333

f1 is
    1.4444

f2 is
     2
```

(c)

To maximize the value of (*diameter – average path length),* we should maximize the diameter and minimize the average path length. And from Question 1 & Question 2, they both proved that, among all the graphs we have, the Line has a maximum value of diameter minus average path length. Then we can know that if the graph is more becoming a line, then it is the expected results. As shown in *Part(a)&(b),* the graphs were likely to a line, so the results are expected.

Question 5

(a)

SA was also used here with a different fitness function, and the other parameters stayed the same. Weight chose (except the largest one, if the rest of two has same value, the results will be clearer and better to compare):

```
w1 = [0.6,0.2,0.2];
w2 = [0.2,0.6,0.2];
w3 = [0.2,0.2,0.6];
```

Comparing to Question 4, since I chose three different weights with a larger weight for f1, f2, f3 respectively, so the f(w) (*a1f1 + a2f2 + a3f3 =: f(w))* also changed correspondingly, and the other parameters stayed the same. The results with running Question5.m with different weight w1, w2, w3are shown below:

```
The best fitness is:
    2.6286

f1 is
    1.5238

f2 is
     3
```

```
     0      0      0      0      1      0      0

The best fitness is:
    2.7429

f1 is
    1.5238
```

```
The best fitness is:
    4.3429

f1 is
    1.5238

f2 is
```

The goal of this question was to minimize f(w), and from the results, we can know that when f1 has a larger weight, the least f(w) came out. So f1 will be relatively important here,