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Reinhard Klette

# Concise Computer Vision

An Introduction  
into Theory and Algorithms

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*Dedicated to all who have dreams*



Computer vision may count the trees, estimate the distance to the islands, but it cannot detect the fantasies the people might have had who visited this bay

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# Preface

This is a textbook for a third- or fourth-year undergraduate course on Computer vision, which is a discipline in science and engineering.

**Subject Area of the Book** Computer Vision aims at using cameras for analysing or understanding scenes in the real world. This discipline studies methodological and algorithmic problems as well as topics related to the implementation of designed solutions.

In computer vision we may want to know how far away a building is to a camera, whether a vehicle drives in the middle of its lane, how many people are in a scene, or we even want to recognize a particular person—all to be answered based on recorded images or videos. Areas of application have expanded recently due to a solid progress in computer vision. There are significant advances in camera and computing technologies, but also in theoretical foundations of computer vision methodologies.

In recent years, computer vision became a key technology in many fields. For modern consumer products, see, for example apps for mobile phones, driver-assistance for cars, or user interaction with computer games. In industrial automation, computer vision is routinely used for quality or process control. There are significant contributions for the movie industry (e.g. the use of avatars or the creation of virtual worlds based on recorded images, the enhancement of historic video data, or high-quality presentations of movies). This is just mentioning a few application areas, which all come with particular image or video data, and particular needs to process or analyse those data.

**Features of the Book** This text book provides a general introduction into basics of computer vision, as potentially of use for many diverse areas of applications. Mathematical subjects play an important role, and the book also discusses algorithms. The book is not addressing particular applications.

Inserts (grey boxes) in the book provide historic context information, references or sources for presented material, and particular hints on mathematical subjects discussed first time at a given location. They are *additional* readings to the baseline material provided.

The book is *not* a guide on current research in computer vision, and it provides only a *very few* references; the reader can locate more easily on the net by searching for keywords of interest. The field of computer vision is actually so vivid, with countless references, such that any attempt would fail to insert in the given limited space a reasonable collection of references. But here is one hint at least: visit [homepages.inf.ed.ac.uk/rbf/CVonline/](http://homepages.inf.ed.ac.uk/rbf/CVonline/) for a web-based introduction into topics in computer vision.

**Target Audiences** This text book provides material for an introductory course at third- or fourth-year level in an Engineering or Science undergraduate programme. Having some prior knowledge in image processing, image analysis, or computer graphics is of benefit, but the first two chapters of this text book also provide a first-time introduction into computational imaging.

**Previous Uses of the Material** Parts of the presented materials have been used in my lectures in the Mechatronics and Computer Science programmes at The University of Auckland, New Zealand, at CIMAT Guanajuato, Mexico, at Freiburg and Göttingen University, Germany, at the Technical University Cordoba, Argentina, at the Taiwan National Normal University, Taiwan, and at Wuhan University, China.

The presented material also benefits from four earlier book publications, [R. Klette and P. Zamperoni. Handbook of Image Processing Operators. Wiley, Chichester, 1996], [R. Klette, K. Schlüns, and A. Koschan. Computer Vision. Springer, Singapore, 1998], [R. Klette and A. Rosenfeld. Digital Geometry. Morgan Kaufmann, San Francisco, 2004], and [F. Huang, R. Klette, and K. Scheibe. Panoramic Imaging. Wiley, West Sussex, 2008].

The first two of those four books accompanied computer vision lectures of the author in Germany and New Zealand in the 1990s and early 2000s, and the third one also more recent lectures.

**Notes to the Instructor and Suggested Uses** The book contains more material than what can be covered in a one-semester course. An instructor should select according to given context such as prior knowledge of students and research focus in subsequent courses.

Each chapter ends with some exercises, including programming exercises. The book does not favour any particular implementation environment. Using procedures from systems such as `OpenCV` will typically simplify the solution. Programming exercises are intentionally formulated in a way to offer students a wide range of options for answering them. For example, for Exercise 2.5 in Chap. 2, you can use Java applets to visualize the results (but the text does not ask for it), you can use small- or large-sized images (the text does not specify it), and you can limit cursor movement to a central part of the input image such that the  $11 \times 11$  square around location  $p$  is always completely contained in your image (or you can also cover special cases when moving the cursor also closer to the image border). As a result, every student should come up with her/his individual solution to programming exercises, and creativity in the designed solution should also be honoured.

**Supplemental Resources** The book is accompanied by supplemental material (data, sources, examples, presentations) on a website. See [www.cs.auckland.ac.nz/~rklette/Books/K2014/](http://www.cs.auckland.ac.nz/~rklette/Books/K2014/).

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Guanajuato, Mexico  
3 November 2013

Reinhard Klette

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# Symbols

$ S $	Cardinality of a set $S$
$\ \mathbf{a}\ _1$	$L_1$ norm
$\ \mathbf{a}\ _2$	$L_2$ norm
$\wedge$	Logical ‘and’
$\vee$	Logical ‘or’
$\cap$	Intersection of sets
$\cup$	Union of sets
$\square$	End of proof
$a, b, c$	Real numbers
$A$	Adjacency set
$\mathcal{A}(\cdot)$	Area of a measurable set (as a function)
$\mathbf{a}, \mathbf{b}, \mathbf{c}$	Vectors
$\mathbf{A}, \mathbf{B}, \mathbf{C}$	Matrices
$\alpha, \beta, \gamma$	Angles
$b$	Base distance of a stereo camera system
$\mathbb{C}$	Set of complex numbers $a + i \cdot b$ , with $i = \sqrt{-1}$ and $a, b \in \mathbb{R}$
$d$	Disparity
$d_1$	$L_1$ metric
$d_2$	$L_2$ metric, also known as the Euclidean metric
$e$	Real constant $e = \exp(1) \approx 2.7182818284$
$\varepsilon$	Real number greater than zero
$f$	Focal length
$f, g, h$	Functions
$G_{\max}$	Maximum grey level in an image
$\gamma$	Curve in a Euclidean space (e.g. a straight line, polyline, or smooth curve)
$\mathbf{H}$	Hessian matrix
$i, j, k, l, m, n$	Natural numbers; pixel coordinates $(i, j)$ in a window
$I, I(., ., t)$	Image, frame of a sequence, frame at time $t$
$L$	Length (as a real number)

---

$\mathcal{L}(\cdot)$	Length of a rectifiable curve (as a function)
$\lambda$	Real number; default: between 0 and 1
$n$	Natural number
$N$	Neighbourhood (in the image grid)
$N_{cols}, N_{rows}$	Number of columns, number of rows
$\mathbb{N}$	Set $\{0, 1, 2, \dots\}$ of natural numbers
$\mathcal{O}(\cdot)$	Asymptotic upper bound
$\Omega$	Image carrier, set of all $N_{cols} \times N_{rows}$ pixel locations
$p, q$	Points in $\mathbb{R}^2$ , with coordinates $x$ and $y$
$P, Q, R$	Points in $\mathbb{R}^3$ , with coordinates $X, Y$ , and $Z$
$\pi$	Real constant $\pi = 4 \times \arctan(1) \approx 3.14159265358979$
$\Pi$	Polyhedron
$r$	Radius of a disk or sphere; point in $\mathbb{R}^2$ or $\mathbb{R}^3$
$\mathbb{R}$	Set of real numbers
$\mathbf{R}$	Rotation matrix
$\rho$	Path with finite number of vertices
$s$	Point in $\mathbb{R}^2$ or $\mathbb{R}^3$
$S$	Set
$t$	Time; point in $\mathbb{R}^2$ or $\mathbb{R}^3$
$\mathbf{t}$	Translation vector
$T, \tau$	Threshold (real number)
$u, v$	Components of optical flow; vertices or nodes; points in $\mathbb{R}^2$ or $\mathbb{R}^3$
$\mathbf{u}$	Optical flow vector with $\mathbf{u} = (u, v)$
$W, W_p$	Window in an image, window with reference pixel $p$
$x, y$	Real variables; pixel coordinates $(x, y)$ in an image
$X, Y, Z$	Coordinates in $\mathbb{R}^3$
$\mathbb{Z}$	Set of integers