

Question 1

1. Among n nonempty cells in the base cuboid, for each dimension, if the value is distinct compared with the value in other cells (e.g: $d=3$, $n=2$, $\{a1, b1, c1\}$, $\{a2, b2, c2\}$) and no cells share any values in d dimension, we will get the max number of cells.

That is: $\sum_{k=1}^d \binom{d}{k} n + 1 = (2^d - 1) * n + 1$

2. If all of the base cells only have different value in one dimension (e.g. $d=3$, $n=2$, $\{a1, b1, c1\}$, $\{a1, b1, c1\}$), we will get the minimum number of cells.

When I calculate the minimum, I use the previous max value minus the overlap cells of $d-1$ dimension.

That is: $\left(\sum_{k=1}^d \binom{d}{k} n + 1\right) - \left(\sum_{k=1}^{d-1} \binom{d}{k} (n - 1)\right) = n * 2^d - (n - 1) * 2^{d-1} = (n + 1) * 2^{d-1}$

Question 2

V2 and V3 are same.

Drill down is the process that let the user know more about its sub dimensions. And slice is the process that filter certain information of the dimension. I believe that they are independent process and they will not affect each other.

For instance, $V1$ is $\{A1/A2, B, C, D, *\}$, and there are two cases:

- (1) When we drill down and slice the same dimension: If drill down B first, we get $\{A1/A2, B1/B2, C, D, M\}$, then slice $B=0$, we get $\{A1/A2, B1=0/B2=0, C, D, M\}$. On the other hand, if slice $B=0$ first, we get $\{a1/a2, B=0, C, D, M\}$, then drill down B, we get $\{A1/A2, B1=0/B2=0, C, D, M\}$. V2 and V3 are same.
- (2) When we drill down and slice the s dimension: If drill down B first, we get $\{A1/A2, B1/B2, C, D, M\}$, then slice $C=0$, we get $\{A1/A2, B1/B2, C=0, D, M\}$. On the other hand, if slice $C=0$ first, we get $\{a1/a2, B, C=0, D, M\}$, then drill down B, we get $\{A1/A2, B1/B2, C=0, D, M\}$. V2 and V3 are same.