

Q2:

Trapezoid Method:

$$y_{k+1} = y_k + h_k (f(t_k, y_k) + f(t_{k+1}, y_{k+1})) / 2$$

For $y' = \lambda y$:

$$y_{k+1} = y_k + \Delta t (\lambda y_k + \lambda y_{k+1}) / 2$$

$$y_{k+1} = \left(\frac{1 + \Delta t \lambda / 2}{1 - \Delta t \lambda / 2} \right) y_k$$

Growth factor: $G = \frac{1 + \frac{1}{2} \lambda \Delta t}{1 - \frac{1}{2} \lambda \Delta t}$

$$\lim_{\lambda \Delta t \rightarrow \infty} G = \lim_{\lambda \Delta t \rightarrow \infty} \frac{1 + \frac{1}{2} \lambda \Delta t}{1 - \frac{1}{2} \lambda \Delta t} = \lim_{\lambda \Delta t \rightarrow \infty} \frac{\frac{1}{\lambda \Delta t} + \frac{1}{2}}{\frac{1}{\lambda \Delta t} - \frac{1}{2}} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1, \text{ not } L\text{-stable}$$

For the analytical case:

$$G = e^{\lambda t}, \quad \lim_{t \rightarrow \infty} e^{\lambda t} = 0$$