1.What is the characteristic polynomial of A?

$A - \lambda I = \begin{bmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{bmatrix}$

$\det(A - \lambda I) = \lambda^2 - 2\lambda - 3 = 0$

2. What are the roots of the characteristic polynomial of A?

By the equation above, r_1=-1, r_2=3.

3.What are the eigenvalues of A?

λ_1=-1, λ_2=3.

4.What are the eigenvectors of A?

λ=-1:

$\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$x=0, then $x = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

λ=3:

$\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}$x=0, then $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

5.Perform one iteration of power iteration on A, using $x_0=[1,1]^T$ as starting vector

$x_1 = Ax_0 = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

6. To what eigenvector of A will power iteration ultimately converge?

It will finally converge to $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

7.What eigenvalue estimate is given by the Rayleigh quotient, using the vector $\mathbf{x=[1,1]^T}$?

$$\lambda = \frac{x^T A x}{x^T x} = 3.5$$

8.To what eigenvector of $\mathbf{A}$ would the inverse iteration ultimately converge?

It will converge to the eigenvector that correspond to the smallest eigenvalue of A, that is $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

9. What eigenvalue of $\mathbf{A}$ would be obtained if inverse iteration were used with shift $\boldsymbol{\sigma=2}$?

It will obtain the eigenvalue that is the closest to σ, that is 3.

10.If QR iteration were applied to $\mathbf{A}$, to what form would it converge: diagonal or triangular? Why?

A is not symmetric, so it will not converge to diagonal, but converge to triangular.