6. (1) Using monomial basis

$$Ax = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Thus, $p(t) = t^2$

(2) Using Lagrange basis

$$p(t) = y_1 \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)} + y_2 \frac{(t-t_1)(t-t_3)}{(t_2-t_1)(t_2-t_3)} + y_3 \frac{(t-t_1)(t-t_2)}{(t_3-t_1)(t_3-t_2)}$$

$$= \frac{t(t-1)}{2} + \frac{t(t+1)}{2} = t^2$$

Thus, $p(t) = t^2$

(3) Using Newto basis

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & t_2-t_1 & 0 \\ 1 & t_3-t_1 & (t_3-t_1)(t_3-t_2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Thus, $p(t) = 1 - (t+1) + (t+1)(t) = t^2$

In conclusion, these three methods give the same polynomial