

(a)

```
Price of 1 year maturity bond: 97.561
Price of 2 year maturity bond: 94.996
Price of 3 year maturity bond: 92.3185
Price of 5 year maturity bond: 86.2609
Price of 10 year maturity bond: 70.8919
Price of 30 year maturity bond: 30.8319
```

Price of 1-year maturity bond is the highest. It's reasonable since the longer the time to maturity, the higher the interest rate tends to be, and price of bond should be lower.

(b)

By approximated Macaulay duration formula:

$$\text{Approximate Macaulay Duration} = \frac{P_- - P_+}{2 \times P_0 \times \Delta \text{Yield}} (1 + \text{Yield})$$

And I set ΔYield as 0.001.

```
Apprximated Macaulay Duration of 1 year maturity bond: 1
Apprximated Macaulay Duration of 2 year maturity bond: 2
Apprximated Macaulay Duration of 3 year maturity bond: 3.00001
Apprximated Macaulay Duration of 5 year maturity bond: 5.00003
Apprximated Macaulay Duration of 10 year maturity bond: 10.0002
Apprximated Macaulay Duration of 30 year maturity bond: 30.0046
```

For zero coupon bond, duration is equal to the maturity of bond.

Price has a negative relationship with bond yields.

(c)

```
Price of 1 year maturity bond with 3% coupon: 100.488
Price of 2 year maturity bond with 3% coupon: 100.77
Price of 3 year maturity bond with 3% coupon: 100.854
Price of 5 year maturity bond with 3% coupon: 100
Price of 10 year maturity bond with 3% coupon: 95.8417
Price of 30 year maturity bond with 3% coupon: 82.708
```

If yield is smaller than coupon rate (3%), that are 1-year, 2-year, 3-year maturity bond, prices are below 100.

If yield is equal to coupon rate (3%), that is 5-year maturity bond, price is equal to 100.

If yield is above coupon rate (3%), that are 10-year, 30-year maturity bond, prices are above 100.

Bond price has a negative relationship with yield to maturity. A bond will trade at a premium when it offers a coupon (interest) rate that is higher than the current interest rate. It will trade at a discount when it offers a coupon rate that is lower than current interest rates. When coupon rate is equal to interest rate, it will trade at a par value.

(d)

```
Apprximated Macauly Duration of 1 year maturity bond with 3% coupon: 1
Apprximated Macauly Duration of 2 year maturity bond with 3% coupon: 1.97099
Apprximated Macauly Duration of 3 year maturity bond with 3% coupon: 2.91388
Apprximated Macauly Duration of 5 year maturity bond with 3% coupon: 4.71713
Apprximated Macauly Duration of 10 year maturity bond with 3% coupon: 8.75498
Apprximated Macauly Duration of 30 year maturity bond with 3% coupon: 19.1058
```

Same approximated Macauly duration formula and $\Delta Yield$ as part(b)

Bond with coupon has a lower duration compared with zero coupon bond. When a coupon is added to the bond, sensitivity of bond price over yield decrease and the bond's duration will always be less than the maturity date. The larger the coupon, the shorter the duration becomes.

(e)

By approximated convexity formula:

$$\text{Approximate Convexity} = \frac{P_- + P_+ - 2 \times P_0}{P_0 \times (\Delta Yield)^2}$$

And I set $\Delta Yield$ as 0.001.

```
Apprximated Convexity of 1 year maturity bond: 1.90363
Apprximated Convexity of 1 year maturity bond with 3% coupon: 1.90363
Apprximated Convexity of 2 year maturity bond: 5.69977
Apprximated Convexity of 2 year maturity bond with 3% coupon: 5.58951
Apprximated Convexity of 3 year maturity bond: 11.3774
Apprximated Convexity of 3 year maturity bond with 3% coupon: 10.9423
Apprximated Convexity of 5 year maturity bond: 28.278
Apprximated Convexity of 5 year maturity bond with 3% coupon: 26.1525
Apprximated Convexity of 10 year maturity bond: 102.687
Apprximated Convexity of 10 year maturity bond with 3% coupon: 85.8298
Apprximated Convexity of 30 year maturity bond: 859.907
Apprximated Convexity of 30 year maturity bond with 3% coupon: 460.943
```

These convexities are positive.

(f)

```
Initial value of portfolio is: -0.112522
```

(g)

```
Duration of portfolio is: 50.1395
Convexity of portfolio is: -1361.07
```

Convexity is negative, and its absolute value is higher than duration

(h)

When yield increase by 1%, value of portfolio is: -0.0501082

(i)

When yield decrease by 1%, value of portfolio is: -0.159611

I may not want to own this portfolio since its value is lower

(j)

Cashflow of year 1: 22.3301
Cashflow of year 2: 21.6797
Cashflow of year 3: 21.0483
Cashflow of year 4: 20.4352
Cashflow of year 5: 19.84

(k)

Price of amortizing bond is: 105.333
Duration of amortizing bond is: 2.94092

Amortizing bond's price is higher than (zero) coupon bond and its duration is lower than (zero) coupon bond.

The primary advantage of amortized bond is that with each payment, the borrower builds equity in the asset. After the final payment, the borrower owns the asset. If the loan has a fixed interest rate, the borrower's payment amount never varies. The main disadvantage is that the monthly payments can be high since both principal and interest are paid. If the amortized bond has a high interest rate and interest rates drop, the borrower is stuck with an excessively high rate.