MF 803 HW 3
Sketch of Solutions

October 6, 2018

Claim: in the solutions I won't give you detailed answers for all the questions. Instead, I'm trying to provide intuitions or algorithms for the most crucial and confusing parts.

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(b) You should expect both of them are highly autocorrelated and S&P is even highlier. The reason is that VIX is not a tradeable asset so it's more mean-reverting. S&P on the other hand is tradeable, and therefore we are less likely to find mean reversion due to market efficiency.

(c) You should expect a significant negative correlation, which implies that the implied volatiliy is not constant cross stock prices. But remember one of the assumptions for B-S model is that volatility should be constant. Then this means that investors are willing to pay an extra "premium" when markets go south.

(e) A rolling 90-day realized volatility is the sum of squared log daily returns of S&P for last 90 days. Notice that VIX Index is quoted annually so you need to annualized your realized volatility to have a reasonable comparison. Generally the premium is positive except for a few of periods including 2008 crisis. The premium is high when market is doing well and is low when shocks hit. Actually, some literatures believe that the volatility premium has prediction power over S&P 500 (see Bollerslev, Tauchen and Zhou, 2009).

(g) The payoff of this 1-month straddle is just the absolute difference between S&P 500 today and S&P 500 one month later. If you hate converting time series data, you can use 21 days gap to get an approximate result.

(h) You should expect a negative relationship between them. The intuition is that the payoff of this straddle is positively related to the REALIZED volatility but the option prices that you need to pay is positively related to the IMPLIED volatility. So the P&L of this straddle is negatively related to volatility premium.

Just in case you're interested in this very popular topic, I will explain it in a more rigorous way below:

Suppose risk-free rate is zero such that the stock price follows the standard SDE

$$dS_t = S_t(\sigma_t dB_t)$$

and the realized volatility in period $[0, T]$ is defined as

$$V_{0,T} = (\int_0^T \sigma_t^2 dt)/T$$

and the model-free definition of implied volatility is

$$I_{0,T} = E_t^*[V_{0,T}]$$

By Ito's Lemma and Taylor-Expansion one can show that

$$dlogS_t = dS_t/S_t - 0.5\sigma_t^2 dt$$

$$logS_T = logS_0 + (S_T - S_0)/S_0 - \int_0^{S_0} (K - S_T)^+/K^2 dK - \int_{S_0}^{\infty} (S_T - k)^+/K^2 DK$$

Then using formulas above one can show that

$$I_{0,T} = 2/T[\int_0^{S_0} P(K)/K^2 dK + \int_{S_0}^{\infty} C(K)/K^2 dK]$$

which is highly positive related to $P(S_0) + C(S_0)$. The relation between payoff of straddle and realized volatility is trivial.