

MF 803 HW 4

Sketch of Solutions

October 19, 2018

Claim: in the solutions I won't give you detailed answers for all the questions. Instead, I'm trying to provide intuitions or algorithms for the most crucial and confusing parts.

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(a) The price of n -year zero bond with yield y has the initial value

$$\frac{100}{(1+y)^n}$$

(b)(e) For both duration and convexity, please use the general formulas containing derivatives, otherwise "finite difference" would not be necessary in the calculation.

(c) Whether it's value above or below 100 depends on if the coupon rate is higher than the yield.

(d) Coupon bonds have shorter durations because it requires less time to deliver it's cash flow (since it pays coupon every year).

(f)(g) The initial value of this portfolio is around -0.113. If we only consider the parallel shift of the yield curve, then we can prove (easy algebra, you can try yourself) that the duration/convexity of a portfolio at yield y is just the linear combination of the durations/convexities of its assets at yield y , with weight being the value fraction. Notice that rigorously you should calculate the durations/convexities at same yield level but here the yield curve is quite flat. So you can use the durations/convexities that you calculated above. Applying this fact then you can easily have duration around 49 and convexity around -147. You can also apply finite difference as usual.

(h)(i) By Taylor Expansion we know that

$$dP/P = -M(y)dy + 0.5 * C(y)dy^2$$

where dP is our target and M, C are modified duration and convexity respectively. Plug in the numbers you calculated above. Whether you'd like to hold for this portfolio depends on whether you'd like to hedge the risk of rising interest rates.

(j)(k) The cash flow is constantly 23 for each year (five years in total). The value is

$$\sum_{i=1}^5 \frac{23}{(1 + \text{i-year-yield})^i}$$

Notice that we don't know 4-year yield but since the yield curve is quite flat here we then approximate it by the mean of 3-year yield and 5-year yield. The result is near 105.92.