Name:
Student ID:

Q. 1 (1 point)**.** For the following logic statements, using "Yes" or "No" to answer whether they are tautology/tautologies or not.

(1) $\neg p \to (p \to q)$ Yes

(2) $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ Yes

Q. 2 (1 point)**.** For the following pairs of logic statements, using "Yes" or "No" to answer whether the two are logically equivalent or not.

(1) $p \vee q \vee r$ and $(p \wedge \neg q) \vee (q \wedge \neg r) \vee (r \wedge \neg p) \vee (p \wedge q \wedge r)$ Yes

(2) $\exists x(P(x) \to Q(x))$ and $\forall x P(x) \to \exists x Q(x)$ Both yes and no are reasonable. If both x correspond to the same domain, then "yes"; if their domain can be different, then "no". In future exams, I will make this point clear.

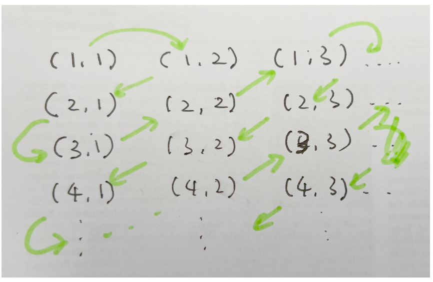
Q. 3 (1 point)**.** Let $f : A \to B$ and $g : B \to C$ be two functions. Use "Yes" or "No" to answer the following.

(1) If the composition $g \circ f$ is a bijection, then (a) must f be one-to-one? (b) must g be onto? Yes

(2) If f is one-to-one and g is onto, then must $g \circ f$ be bijective? No.

Q. 4 (2 points)**.** Show that the set $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable. Note: $\mathbb{Z}^+$ is the set of positive integers; examples of elements in set $\mathbb{Z}^+ \times \mathbb{Z}^+$: $(6, 8)$, $(5, 5)$, ...

To show set $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable, we can show that the elements in this set can be listed in a sequence. Specifically, we can arrange these elements in set $\mathbb{Z}^+ \times \mathbb{Z}^+$ as follows:



The green arrow shows the sequence. That is, first list the pair whose sum is two, then list those pairs whose summmations are three, so and so forth. Because **all** pairs are listed **once**, we show that set $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable.