Name:
Student ID:

Q. 1. (1 point) Prove or disprove that the following argument is valid. Please (i) indicate "prove" or "disprove", and then (ii) prove or disprove the validity accordingly.

- Premises: $\forall x(P(x) \to (Q(x) \wedge S(x)))$, $\forall x(P(x) \wedge R(x))$;
- Conclusion: $\forall x(R(x) \wedge S(x))$.

Solution:

27. **Step**	**Reason**
1. $\forall x(P(x) \wedge R(x))$	Premise
2. $P(a) \wedge R(a)$	Universal instantiation from (1)
3. $P(a)$	Simplification from (2)
4. $\forall x(P(x) \to (Q(x) \wedge S(x)))$	Premise
5. $Q(a) \wedge S(a)$	Universal modus ponens from (3) and (4)
6. $S(a)$	Simplification from (5)
7. $R(a)$	Simplification from (2)
8. $R(a) \wedge S(a)$	Conjunction from (7) and (6)
9. $\forall x(R(x) \wedge S(x))$	Universal generalization from (5)

Q. 2. (1 point) Suppose A, B, and C are sets. Show that $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$. Please use set builder notation and logical equivalence.

Solution:

$$
\begin{aligned}
\overline{A \cap B \cap C} &= \{x \mid x \notin A \cap B \cap C\} \\
&= \{x \mid \neg(x \in (A \cap B \cap C)\} \\
&= \{x \mid \neg(x \in A \wedge x \in B \wedge x \in C)\} \\
&= \{x \mid \neg(x \in A) \vee \neg(x \in B) \wedge \neg(x \in C))\} \\
&= \{x \mid x \notin A \vee x \notin B \vee x \notin C\} \\
&= \{x \mid x \in \overline{A} \vee x \in \overline{B} \vee x \in \overline{C}\} \\
&= \{x \mid x \in \overline{A} \cup \overline{B} \cup \overline{C}\} \\
&= \overline{A} \cup \overline{B} \cup \overline{C}
\end{aligned}
$$

Q. 3. (1 point) Prove or disprove that $|(0,1)| = |[0,1]|$: (i) indicates "prove" or "disprove"; (ii) prove or disprove accordingly.

Solution:

EXAMPLE 6 Show that the $|(0, 1)| = |(0, 1]|$.

Solution: It is not at all obvious how to find a one-to-one correspondence between $(0, 1)$ and $(0, 1]$ to show that $|(0, 1)| = |(0, 1]|$. Fortunately, we can use the Schröder-Bernstein theorem instead. Finding a one-to-one function from $(0, 1)$ to $(0, 1]$ is simple. Because $(0, 1) \subset (0, 1]$, $f(x) = x$ is a one-to-one function from $(0, 1)$ to $(0, 1]$. Finding a one-to-one function from $(0, 1]$ to $(0, 1)$ is also not difficult. The function $g(x) = x/2$ is clearly one-to-one and maps $(0, 1]$ to $(0, 1/2] \subset (0, 1)$. As we have found one-to-one functions from $(0, 1)$ to $(0, 1]$ and from $(0, 1]$ to $(0, 1)$, the Schröder-Bernstein theorem tells us that $|(0, 1)| = |(0, 1]|$. ◀

Q. 4. (1 point) Arrange the functions $\sqrt{n}$, $1000 \log n$, $n \log n$, $2n!$, 2^n, 3^n, $n^2/1000$ in a list such that the complexity (i.e., the growth of function) is in ascending order. No need to prove. A list would be sufficient.

Solution: $1000 \log n$, $\sqrt{n}$, $n \log n$, $n^2/1000$, 2^n, 3^n, $2n!$