Name:

Student ID:

Q. 1. (1 point) Prove or disprove that the following argument is valid. Please (i) indicate "prove" or "disprove", and then (ii) prove or disprove the validity accordingly.

• Premises: $\forall x (P(x) \to (Q(x) \land S(x))), \forall x (P(x) \land R(x));$

• Conclusion: $\forall x (R(x) \land S(x))$.

Solution:

27. Step	Reason
1. $\forall x (P(x) \land R(x))$	Premise
$2. P(a) \wedge R(a)$	Universal instantiation from (1)
3. P(a)	Simplification from (2)
$4. \forall x (P(x) \rightarrow$	Premise
$(Q(x) \wedge S(x)))$	
5. $Q(a) \wedge S(a)$	Universal modus ponens from (3) and (4)
6. S(a)	Simplification from (5)
7. R(a)	Simplification from (2)
8. $R(a) \wedge S(a)$	Conjunction from (7) and (6)
9. $\forall x (R(x) \land S(x))$	Universal generalization from (5)

Q. 2. (1 point) Suppose A, B, and C are sets. Show that $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$. Please use set builder notation and logical equivalence.

Solution:

$$\overline{A \cap B \cap C} = \{x \mid x \notin A \cap B \cap C\}$$

$$= \{x \mid \neg (x \in (A \cap B \cap C))\}$$

$$= \{x \mid \neg (x \in A \land x \in B \land x \in C)\}$$

$$= \{x \mid \neg (x \in A) \lor \neg (x \in B) \land \neg (x \in C))\}$$

$$= \{x \mid x \notin A \lor x \notin B \lor x \notin C\}$$

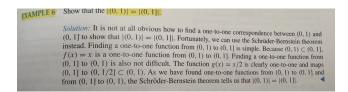
$$= \{x \mid x \in \overline{A} \lor x \in \overline{B} \lor x \in \overline{C}\}$$

$$= \{x \mid x \in \overline{A} \lor B \cup \overline{C}\}$$

$$= \overline{A} \cup \overline{B} \cup \overline{C}$$

Q. 3. (1 point) Prove or disprove that |(0,1)| = |[0,1]|: (i) indicates "prove" or "disprove"; (ii) prove or disprove accordingly.

Solution:



Q. 4. (1 point) Arrange the functions \sqrt{n} , $1000 \log n$, $n \log n$, 2n!, 2^n , 3^n , $n^2/1000$ in a list such that the complexity (i.e., the growth of function) is in ascending order. No need to prove. A list would be sufficient.

Solution: $1000 \log n, \sqrt{n}, n \log n, n^2/1000, 2^n, 3^n, 2n!$