Computer Vision

CS308

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SUSTech CS Vision Intelligence and Perception

Week 6

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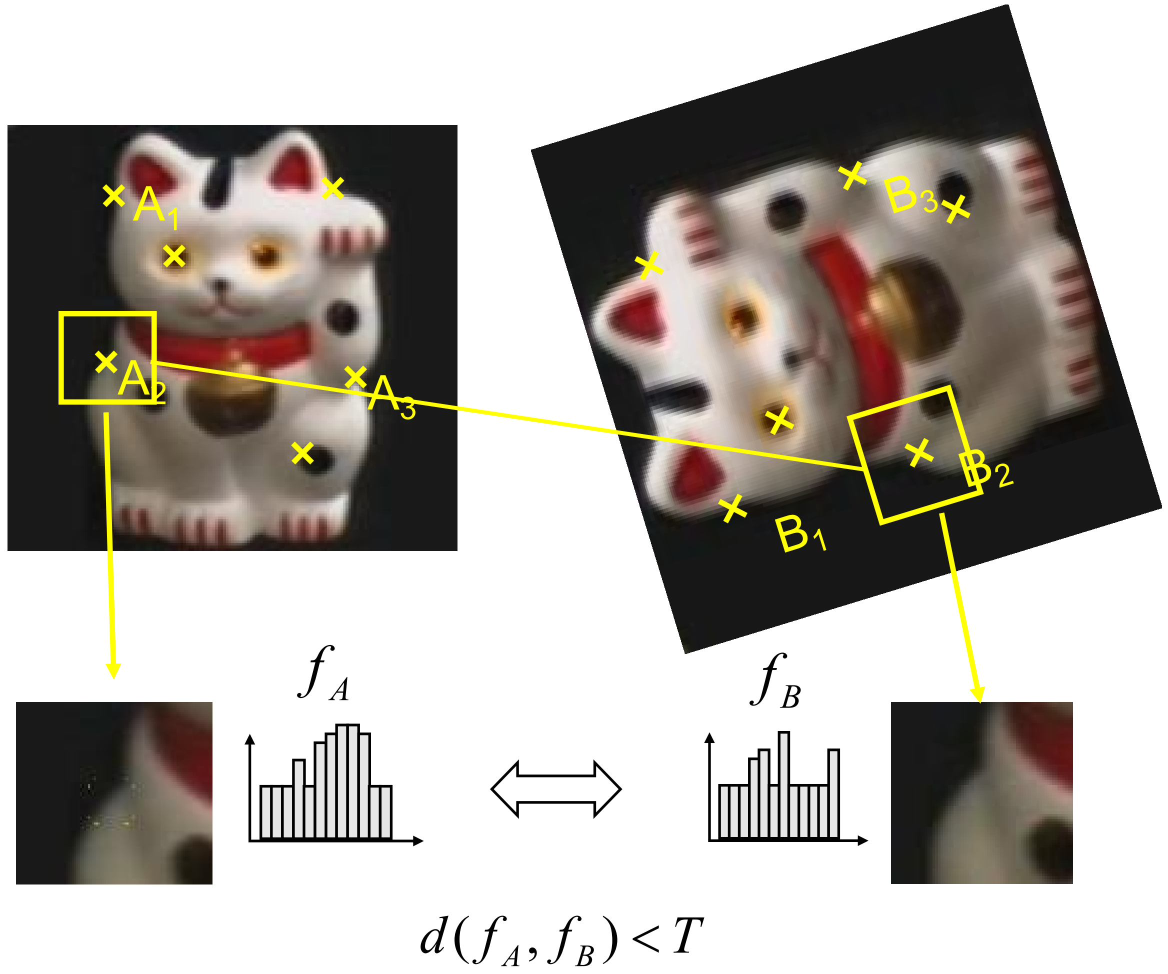
Content

- Brief Review
- Fitting Techniques
  - ➢ Least Squares
  - ➢ Total Least Squares
- Random Sample Consensus (RANSAC)
- Hough Voting
- Image Alignment

Brief Review

Overview of Keypoint Matching

- Steps
  - Find a set of distinctive keypoints
  - Define a region around each keypoint
  - Compute a local descriptor from the region
  - Match local descriptors
- Goals
  - Detect points that are repeatable and distinctive



Fitting Techniques

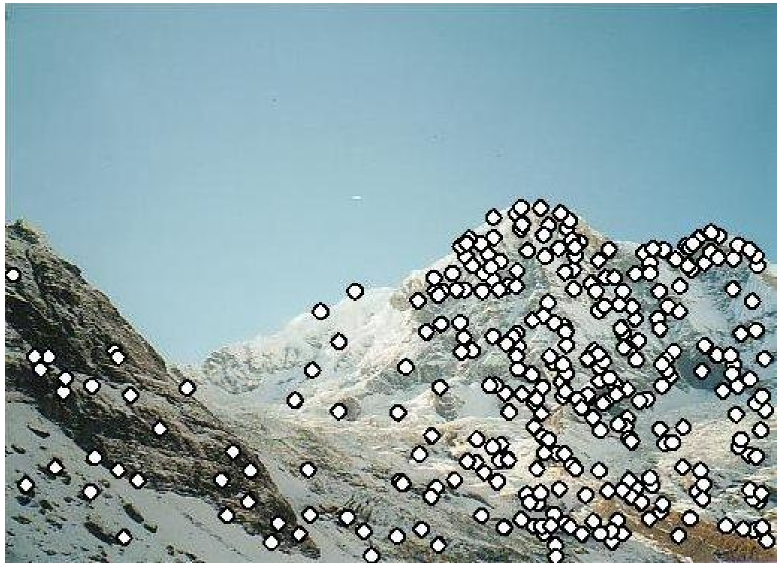
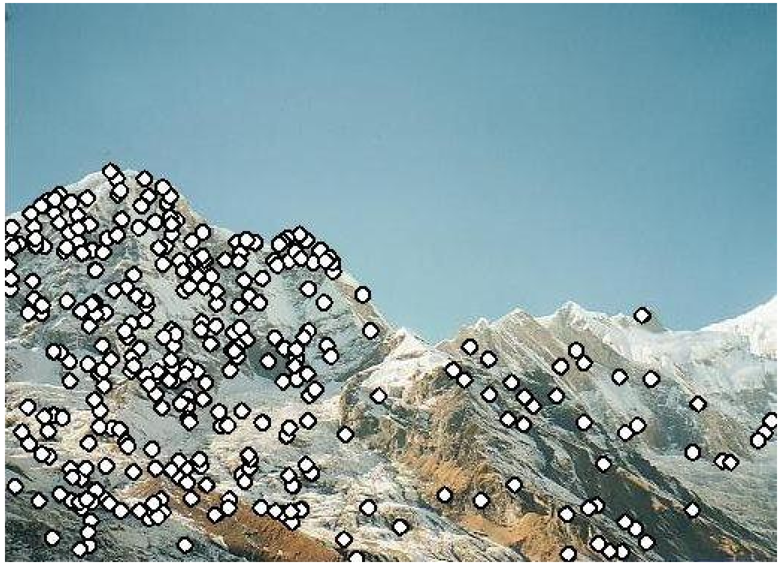
How Do We Build Panorama?

- We need to match (align) images

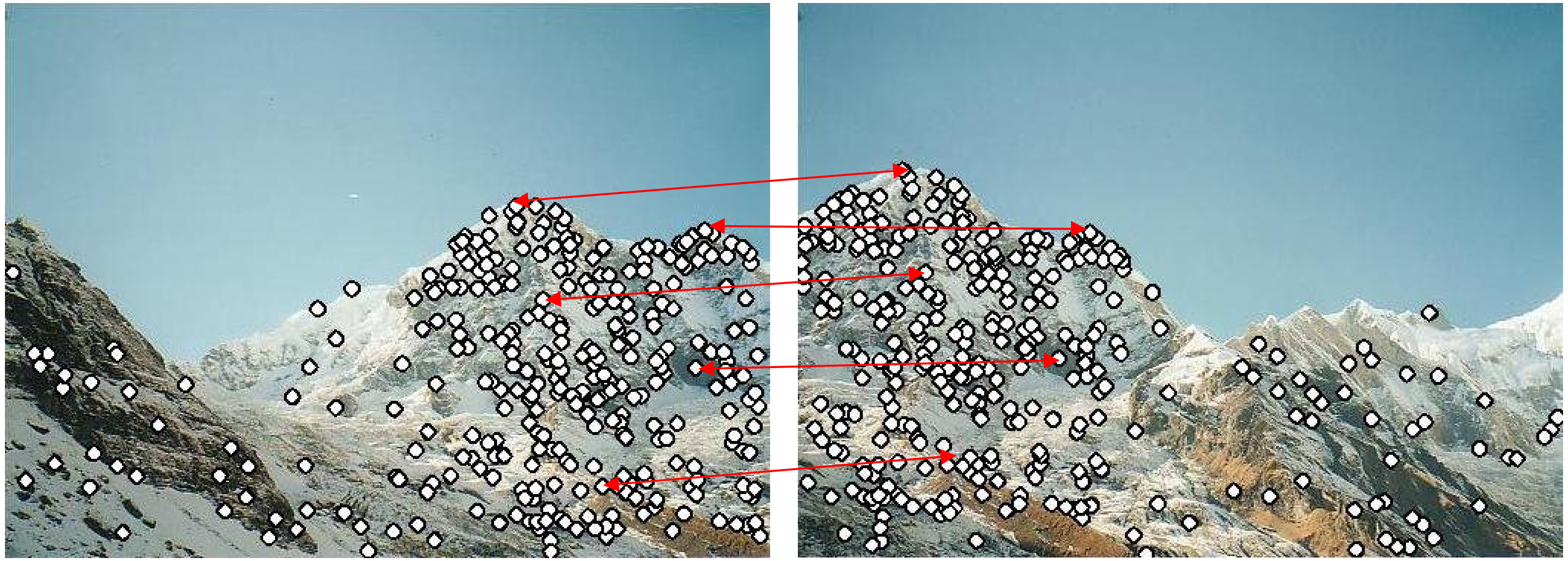
Matching with Features

- Steps
  - Detect feature points in both images

Matching with Features

- Steps
  - Detect feature points in both images
  - Find corresponding pairs



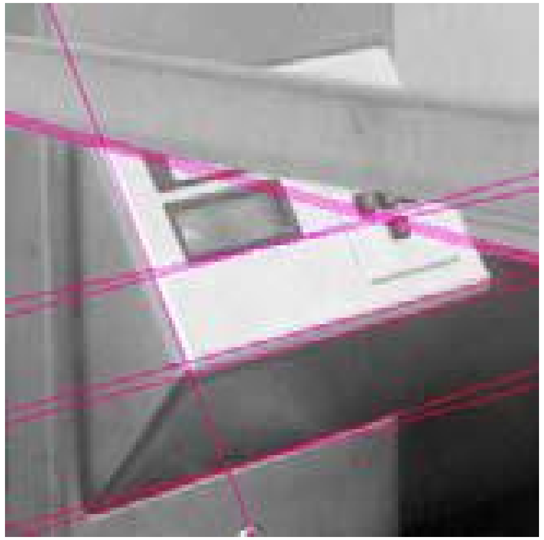
Matching with Features

- Steps
  - ➢ Detect feature points in both images
  - ➢ Find corresponding pairs
  - ➢ Use these pairs to align images

Previous Lecture

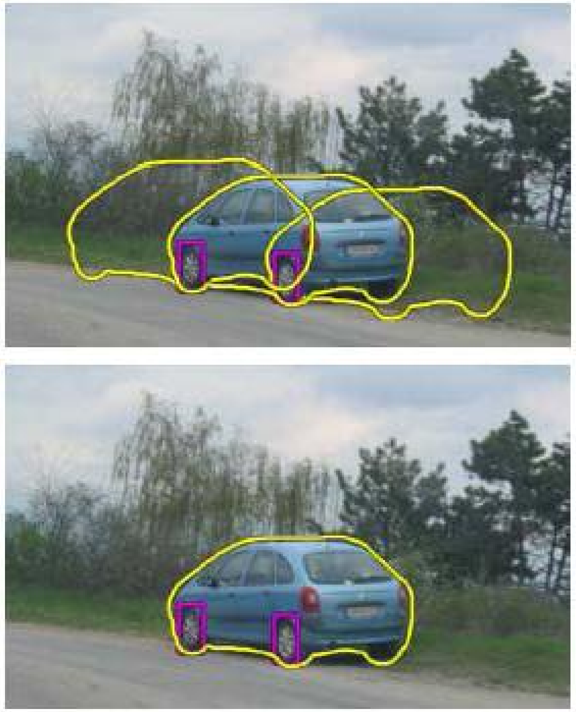
Fitting: Building a Model for a Set of Features

- Choose a parametric model to represent a set of features



Simple model: lines

Simple model: circles



Complicated model: car

Fitting: Issues

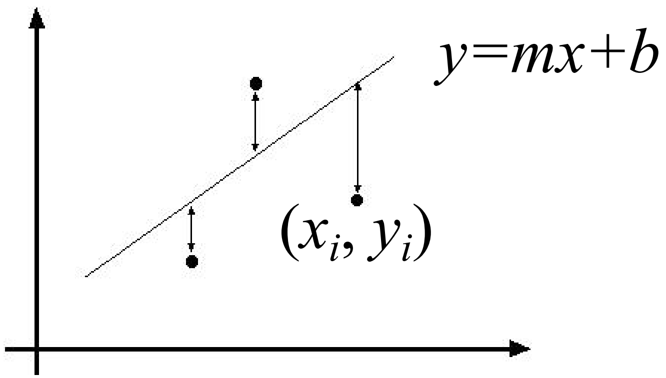
- Case study: Line detection
  - Noise in the measured feature locations
  - Extraneous data: clutter (outliers), multiple lines
  - Missing data: occlusions

Fitting: Issues

- If we know which points belong to the line, how do we find the "optimal" line parameters?
  - Least squares
- What if there are outliers?
  - Robust fitting, RANSAC
- What if there are many lines?
  - Voting methods: RANSAC, Hough transform
- What if we're not even sure it's a line?
  - Model selection

Line Fitting: Ordinary Least Squares

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
- Find (m, b) to minimize



$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$

We know which points belong to the line

Line Fitting: Ordinary Least Squares

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
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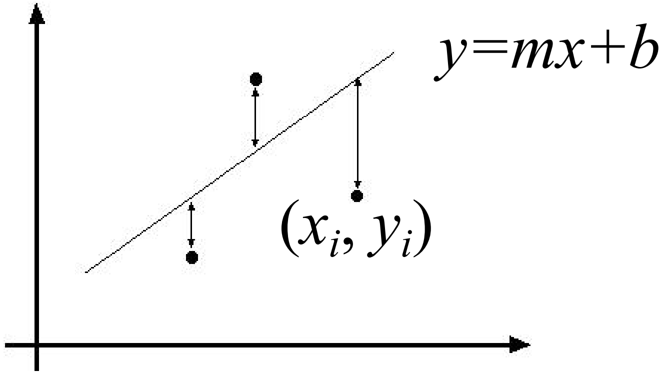


$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$

$$E = \sum_{i=1}^{n} \left(y_i - \begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right)^2 = \left\| \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} - \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right\|^2 = \|Y - XB\|^2$$

$$= (Y - XB)^T (Y - XB) = Y^T Y - 2(XB)^T Y + (XB)^T (XB)$$

Line Fitting: Ordinary Least Squares

- Normal equations: least squares solution to $XB=Y$

$$\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0 \qquad \boxed{X^T XB = X^T Y}$$

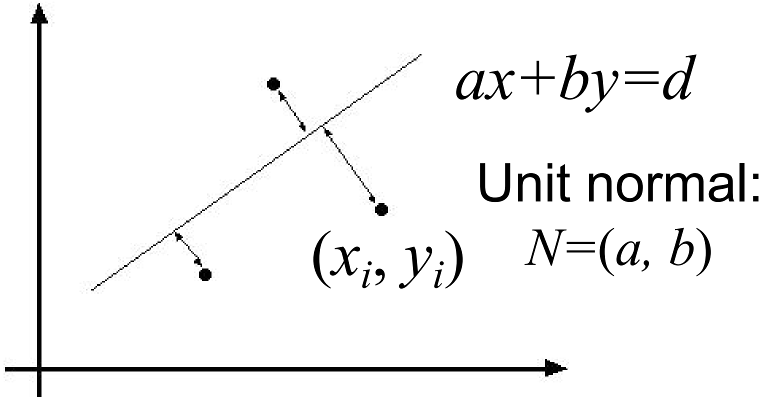
- Problem with "vertical" least squares
  - Not rotation-invariant
  - Fails completely for vertical lines $X^T X$

Total Least Squares

- Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$:

$$|ax_i + by_i - d|$$

$$\boxed{E = \sum_{i=1}^{n} (ax_i + by_i - d)^2}$$



Total Least Squares

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} -2(ax_i + by_i - d) = 0 \;\Rightarrow\; d = \frac{a}{n}\sum_{i=1}^{n} x_i + \frac{b}{n}\sum_{i=1}^{n} x_i = a\bar{x} + b\bar{y}$$

$$E = \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2 = \left\| \underbrace{\begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{N} \right\|^2 = (UN)^T(UN)$$

$$\boxed{\frac{dE}{dN} = 2(U^T U)N = 0}$$

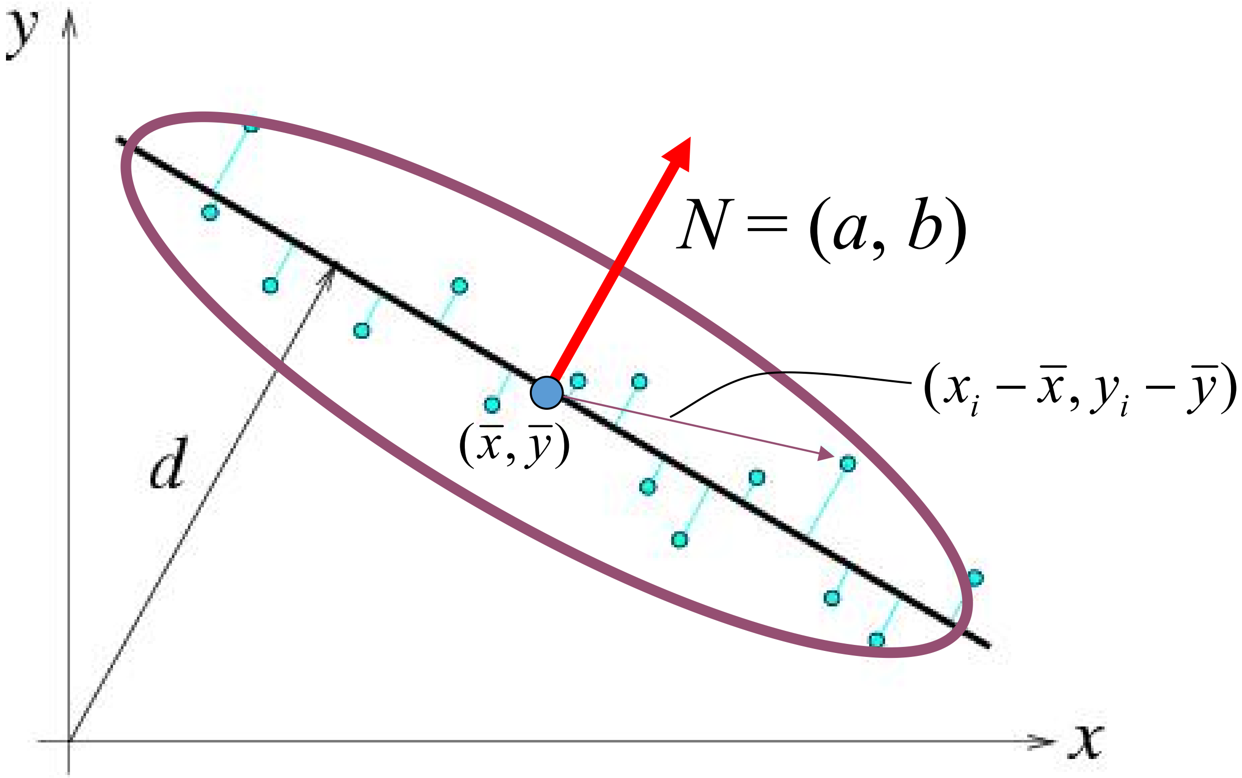
- Solution to $(U^T U)N = 0$, subject to $\|N\|^2 = 1$: eigenvector of $U^T U$ associated with the smallest eigenvalue (least squares solution to *homogeneous linear system* $UN = 0$)

Total Least Squares

- Second moment matrix

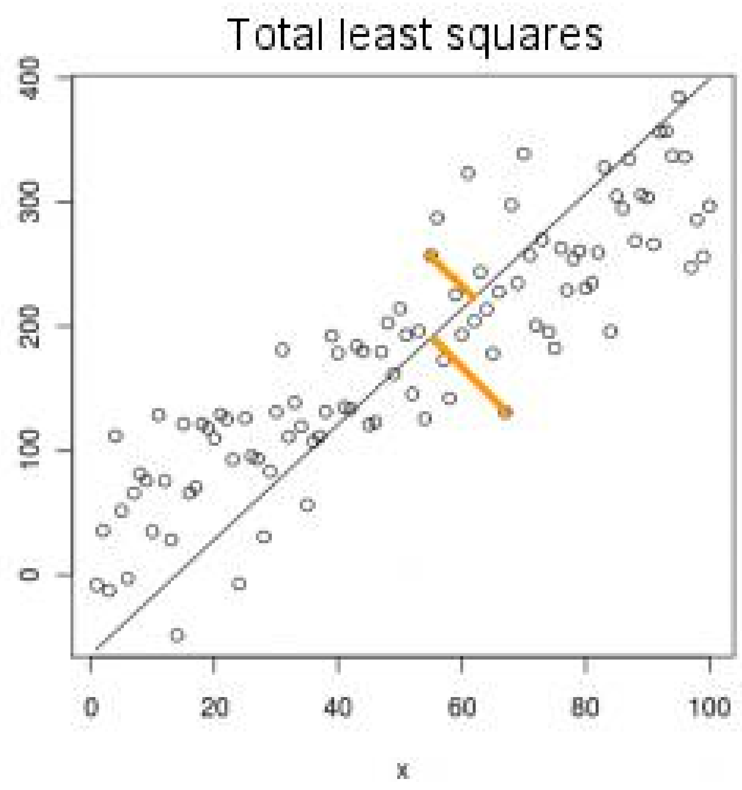
$$U = \begin{bmatrix} x_1 - \overline{x} & y_1 - \overline{y} \\ \vdots & \vdots \\ x_n - \overline{x} & y_n - \overline{y} \end{bmatrix}$$

$$U^T U = \begin{bmatrix} \sum_{i=1}^{n} (x_i - \overline{x})^2 & \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) \\ \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) & \sum_{i=1}^{n} (y_i - \overline{y})^2 \end{bmatrix}$$



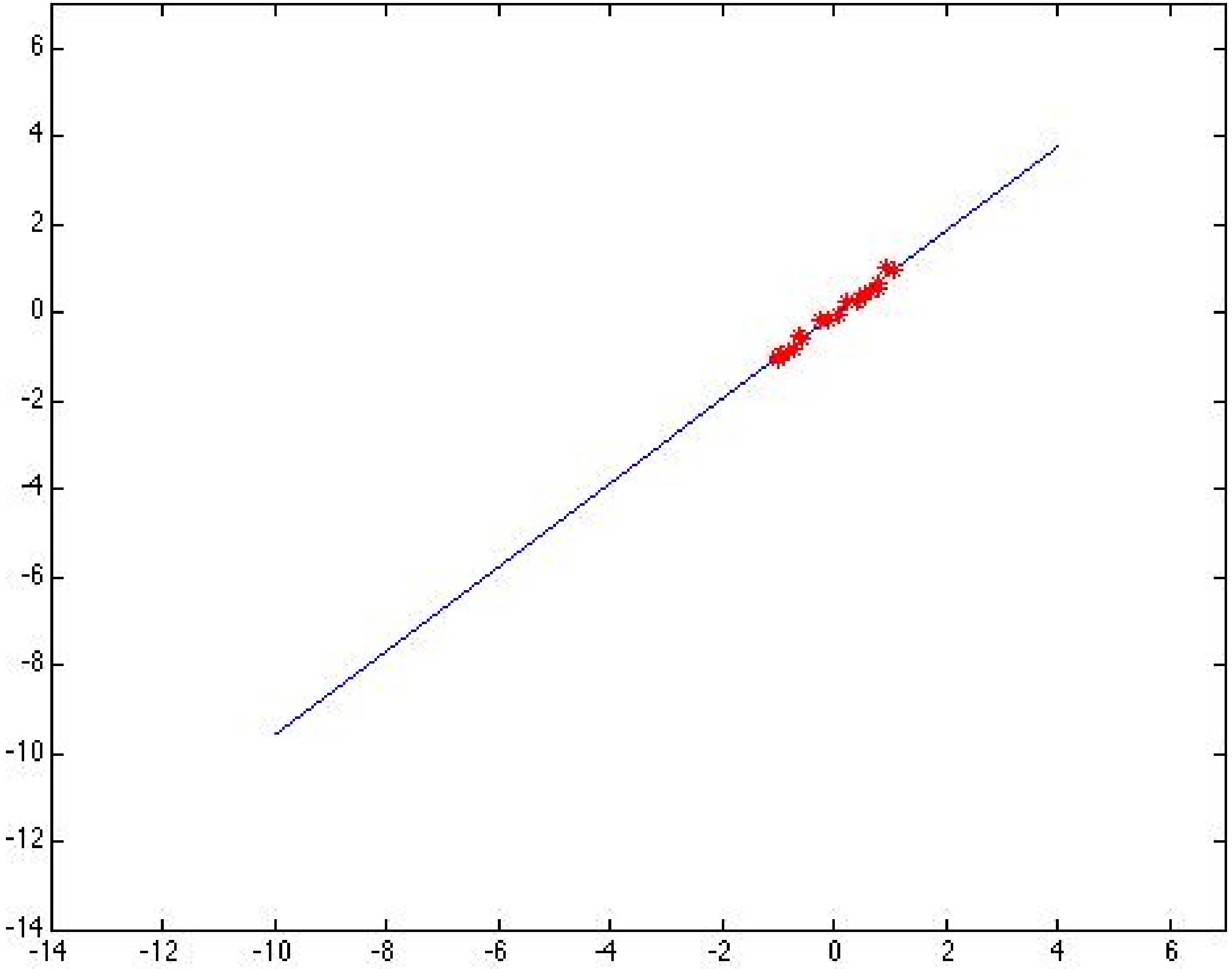
OLS vs. TLS

- The difference between standard OLS regression and "orthogonal" TLS regression



Total Least Squares

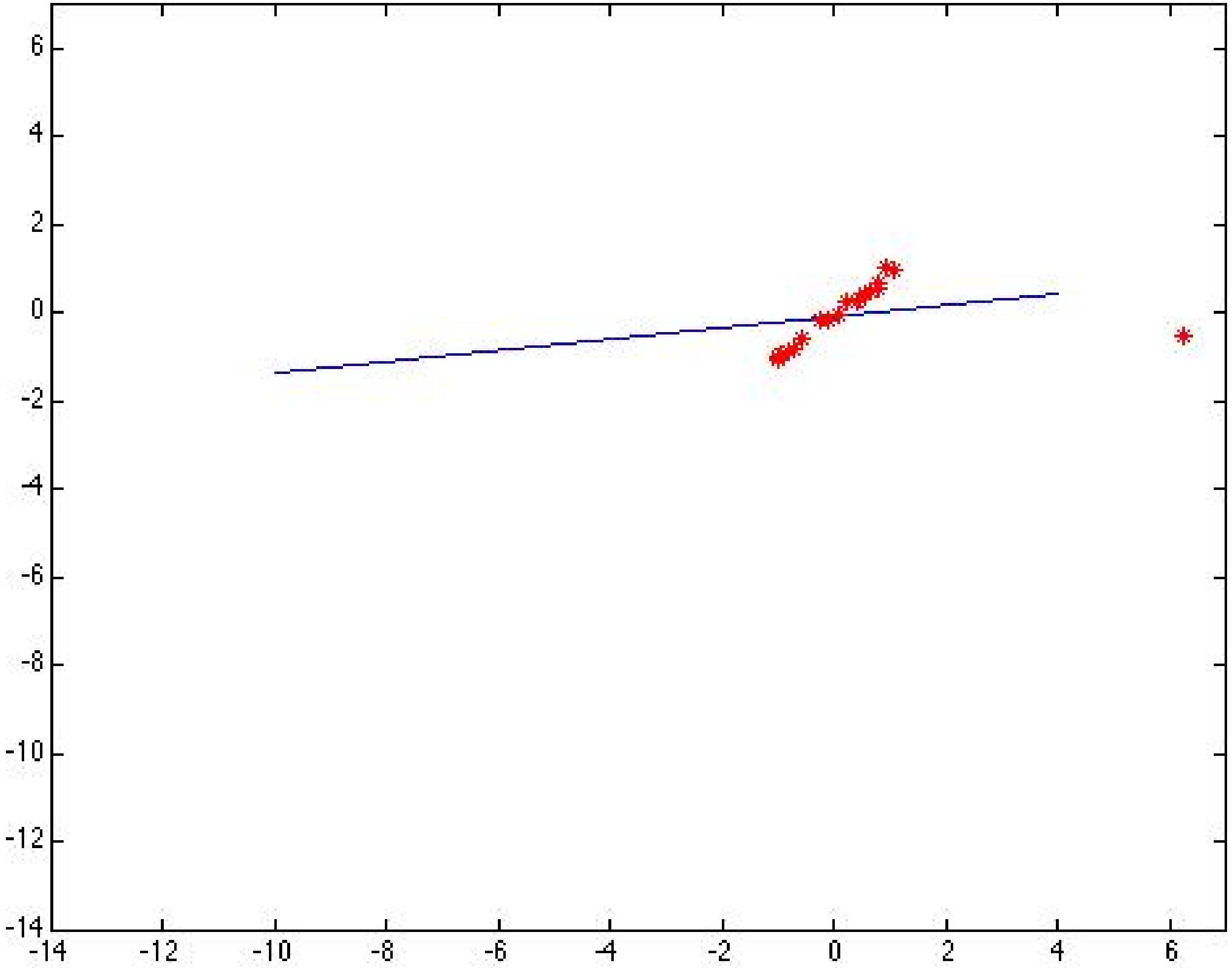
- Robustness to noise: least squares fit to the red points



Total Least Squares

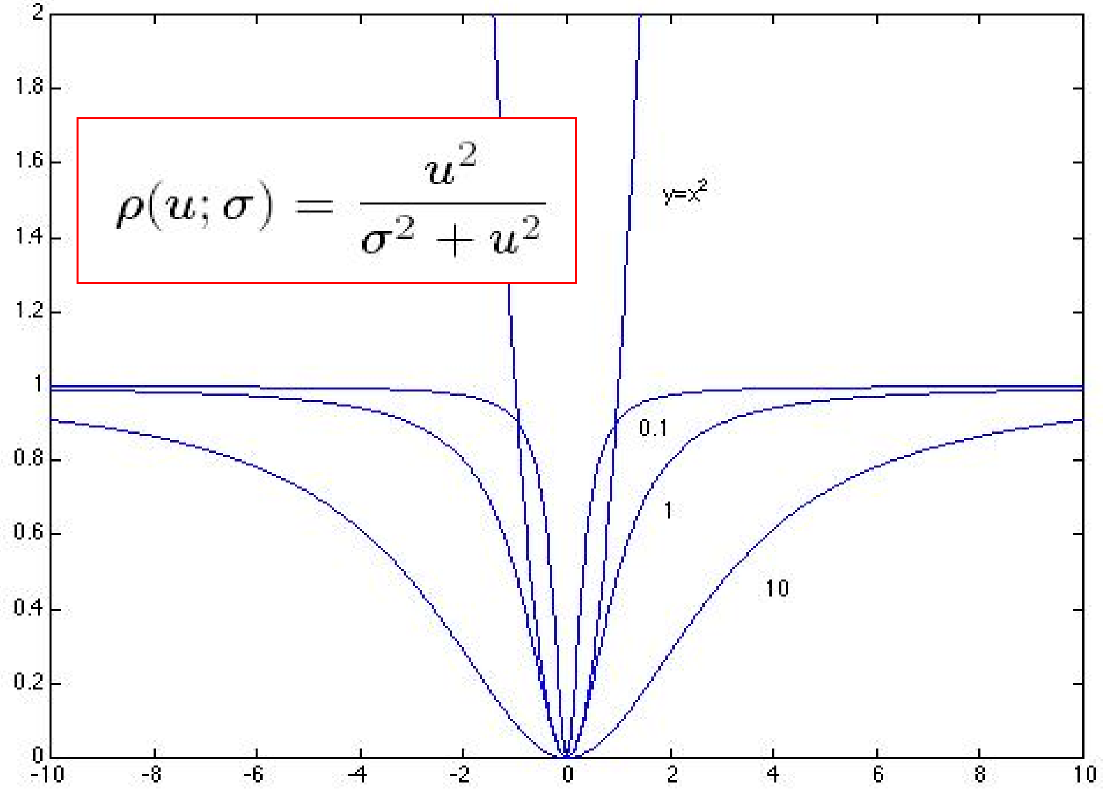
- Robustness to noise: Least squares fit with an outlier

- Problem: squared error heavily penalizes outliers



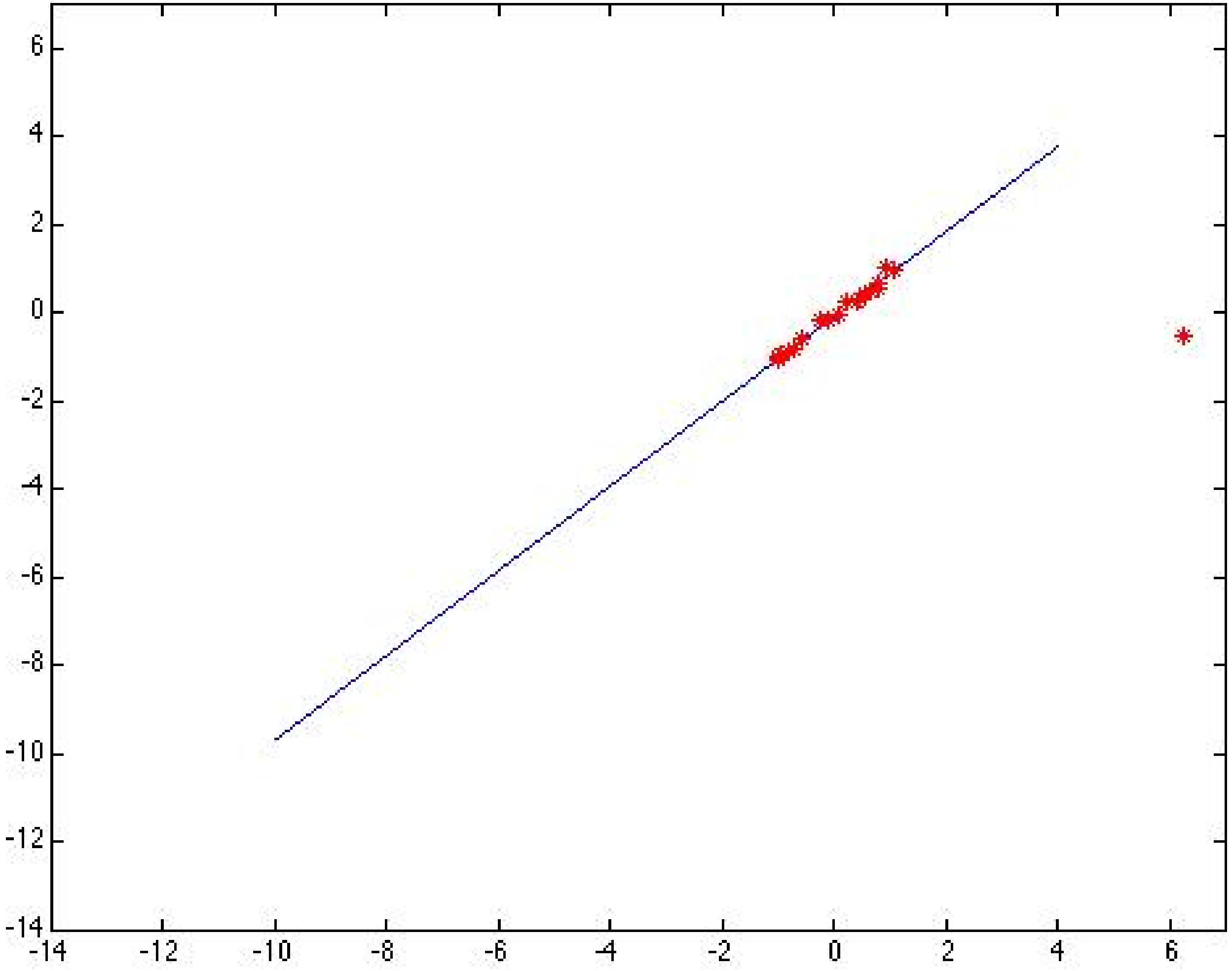
Robust Estimators

- General approach---minimize: $\sum_i \rho(r_i(x_i, \theta); \sigma)$
  $r_i(x_i, \theta)$ – residual of ith point w.r.t. model parameters θ
  ρ – robust function with scale parameter σ
- The robust function ρ behaves like squared distance for small values of the residual u but saturates for larger values of u



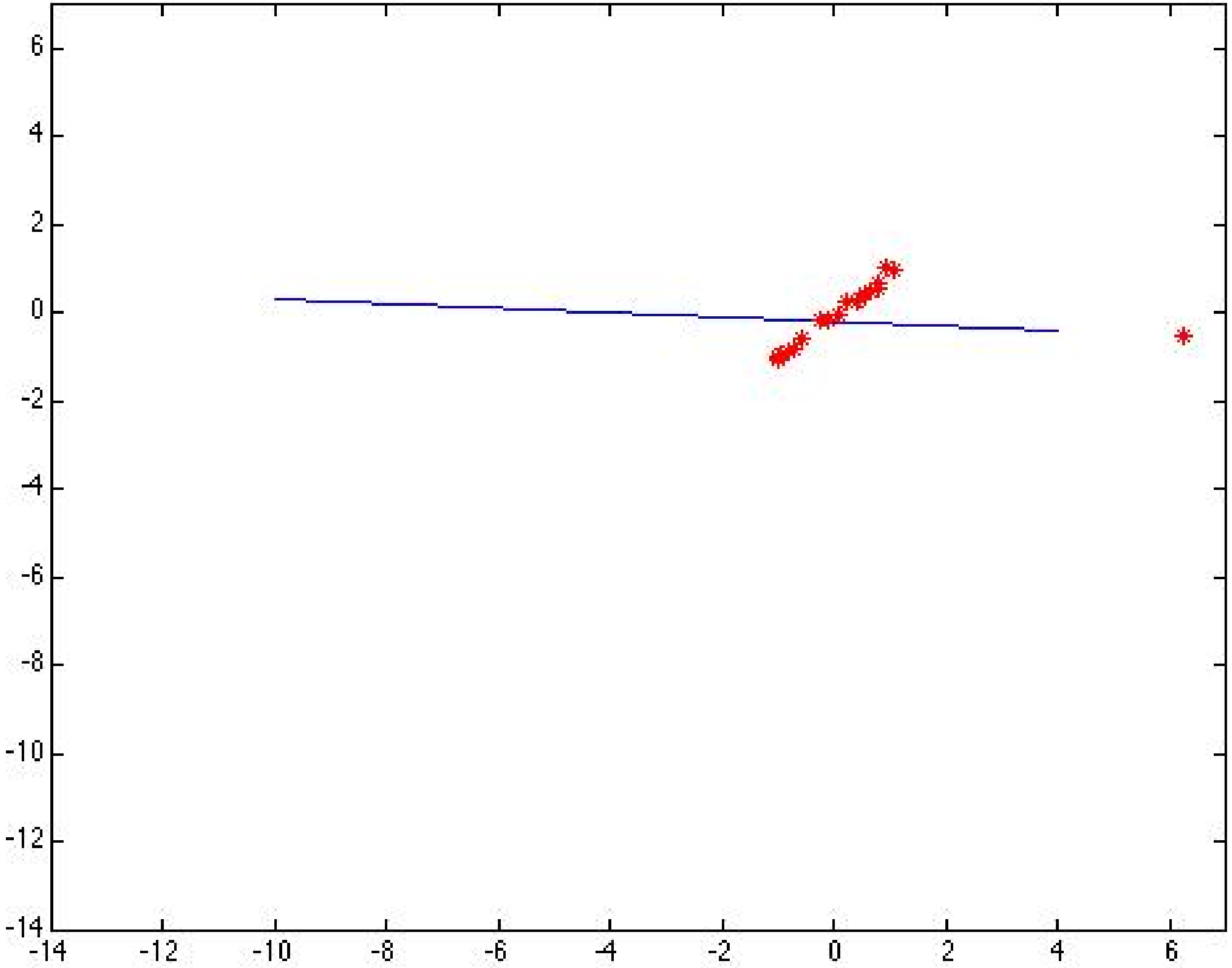
Choosing the Scale: Just Right

- The effect of the outlier is minimized, when choosing a just right scale



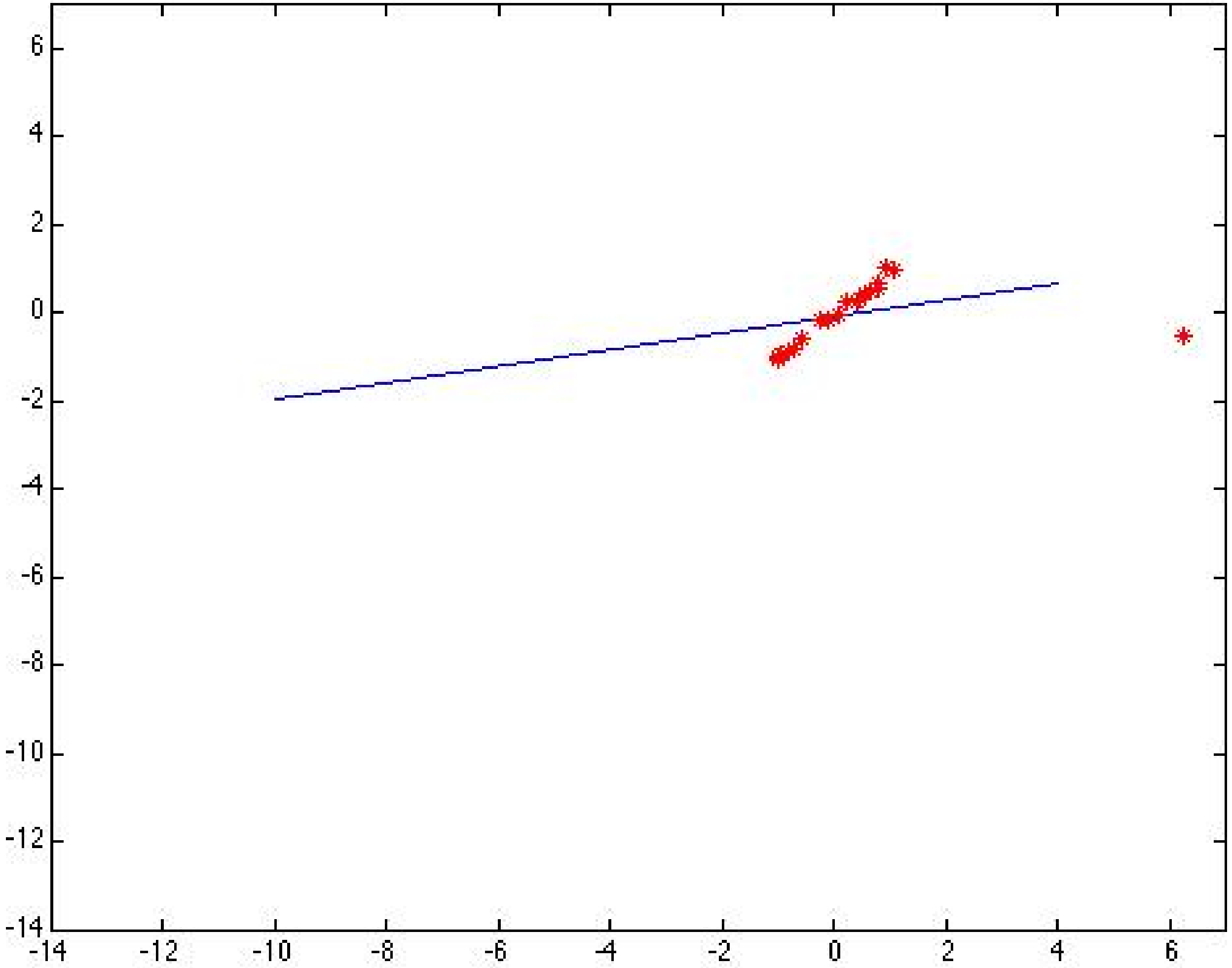
Choosing the Scale: Too Small

- The error value is almost the same for every point and the fit is very poor



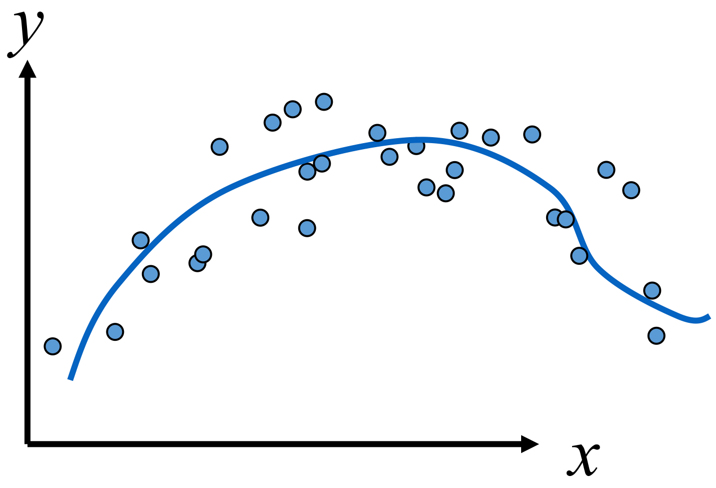
Choosing the Scale: Too Large

- Behaves much the same as least squares



Curve Fitting

- Find Polynomial: $y = f(x) = ax^3 + bx^2 + cx + d$
  - That best fits the given points (x_i, y_i)



- Minimize: $\frac{1}{N}\sum_i [y_i - (ax_i^3 + bx_i^2 + cx_i + d)]^2$
- Using: $\frac{\partial E}{\partial a} = 0$, $\frac{\partial E}{\partial b} = 0$, $\frac{\partial E}{\partial c} = 0$, $\frac{\partial E}{\partial d} = 0$
- Note: $f(x)$ is LINEAR in the parameters (a, b, c, d)

Random Sample Consensus

RANSAC

- Robust fitting (TLS) can deal with a few outliers – what if we have very many?
- Random sample consensus (RANSAC): Very general framework for model fitting in the presence of outliers
- Outline
  - Choose a small subset of points uniformly at random
  - Fit a model to that subset
  - Find all remaining points that are “close” to the model and reject the rest as outliers
  - Do this many times and choose the best model

M. A. Fischler, R. C. Bolles. Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. Comm. of the ACM, Vol 24, pp 381-395, 1981.

RANSAC for Line Fitting

- Algorithm
- Repeat N times:
  - Draw s points uniformly at random
  - Fit line to these s points (TLS)
  - Find inliers to this line among the remaining points (i.e., points whose distance from the line is less than t)
  - If there are d or more inliers, accept the line and refit using all inliers
- End
- Four parameters: s, t, d and N

Choosing the Parameters

- Initial number of points ***s***
  - ➢ Minimum number needed to fit the model
    - ✓2 points

- Distance threshold ***t***
  - ➢ (1) Choose ***t*** so probability for inlier is ***p*** (e.g. 0.95)
  - ➢ (2) Zero-mean Gaussian noise with standard deviation σ: $t^2 = 3.84\sigma^2$

Choosing the Parameters

- Number of times N
  - Choose N so that, with probability p, at least one random sample is free from outliers (e.g. $p=0.99$)

Desired success rate after N times: p

Outlier ratio (Unknown): e

$$\boxed{\left(1-\left(1-e\right)^{s}\right)^{N}=1-p}$$

$$N=\log\left(1-p\right)/\log\left(1-\left(1-e\right)^{s}\right)$$

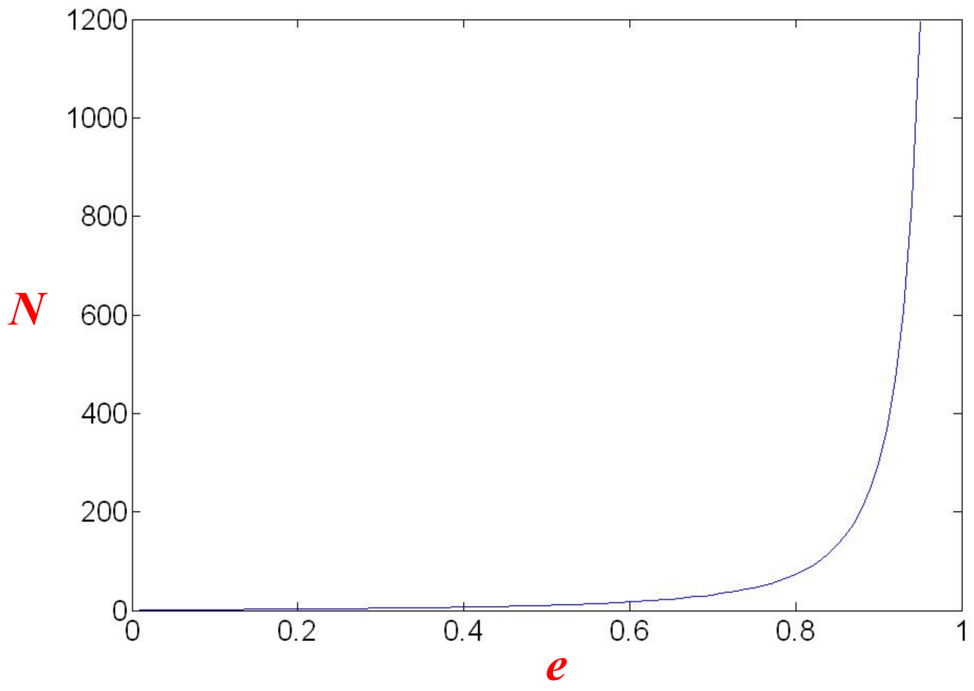
N	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Choosing the Parameters

- Consensus set size d (number of inliers)
  - ➢ Should match expected inlier ratio

$$\left(1-\left(1-e\right)^{s}\right)^{N}=1-p$$

$$N=\log\left(1-p\right)/\log\left(1-\left(1-e\right)^{s}\right)$$



Adaptively determining the number of samples

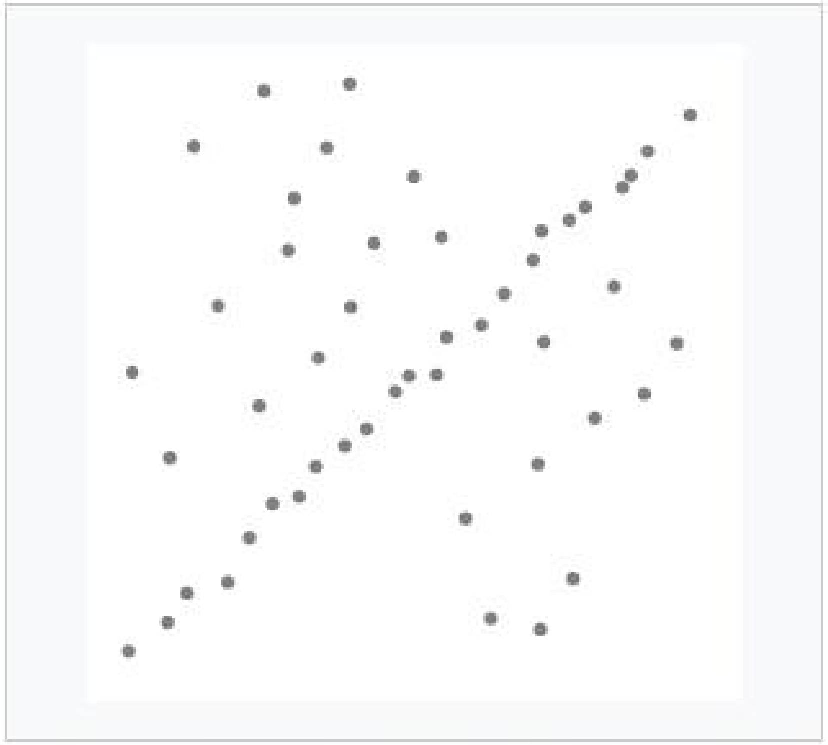
- Inlier ratio e is often unknown a priori, so pick worst case, e.g. 50%, and adapt if more inliers are found, e.g. 80% would yield $e=0.2$
- Adaptive procedure:
  - $N=\infty$, $sample\_count=0$
  - While $N>sample\_count$
    - ✓ Choose a sample (fitting) and count the number of inliers
    - ✓ Set e = 1 – (number of inliers)/(total number of points)
    - ✓ Recompute N from e:

$$N = \log(1-p)/\log\left(1-(1-e)^s\right)$$

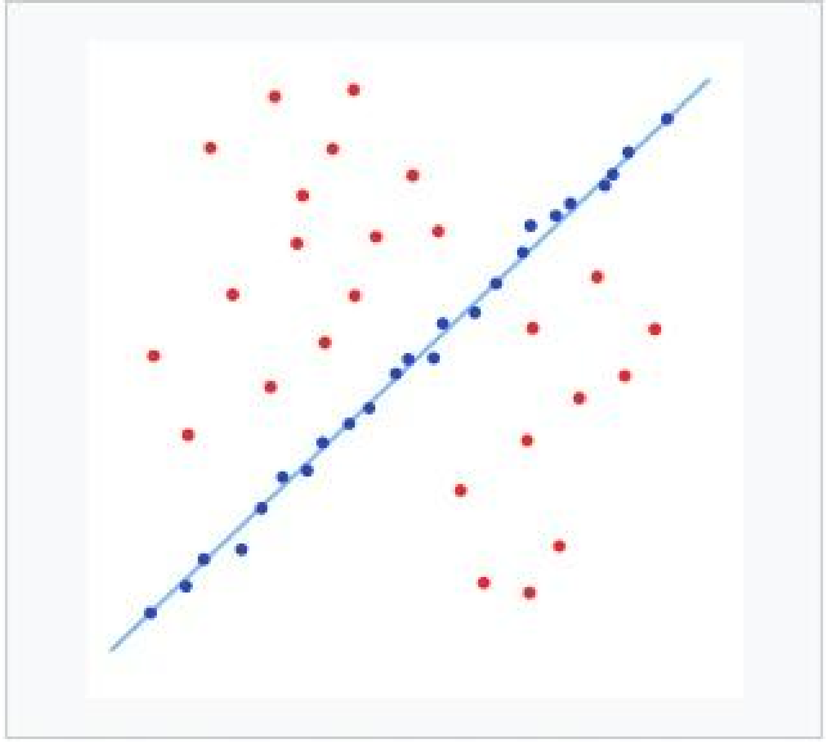
    - ✓ Increment the *sample_count* by 1

RANSAC

- An example



A data set with many outliers for which a line has to be fitted.



Fitted line with RANSAC; outliers have no influence on the result.

RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Lots of parameters to tune
  - Can't always get a good initialization of the model based on the minimum number of samples
  - Sometimes too many iterations are required
  - Can fail for extremely low inlier ratios
  - We can often do better than brute-force sampling

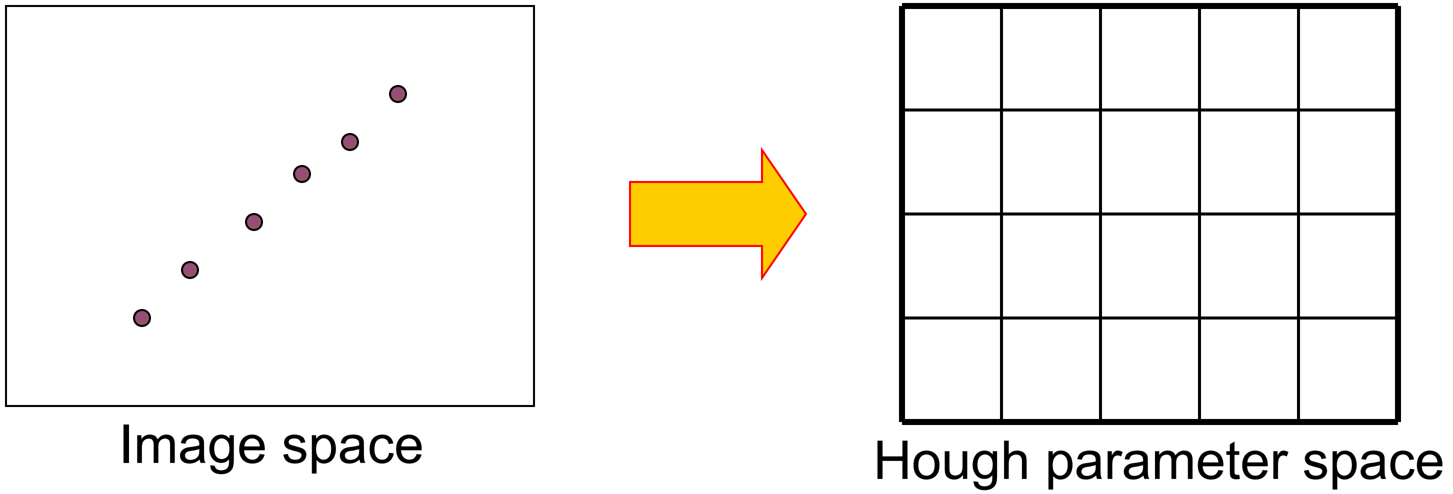
Hough transform

Voting Schemes

- Principal of voting
  - Let each feature (voter) vote for all the models that are compatible with it
  - Hopefully the noise features (voter) will not vote consistently for any single model (nominator)
  - Missing data doesn't matter as long as there are enough features remaining to agree on a good model

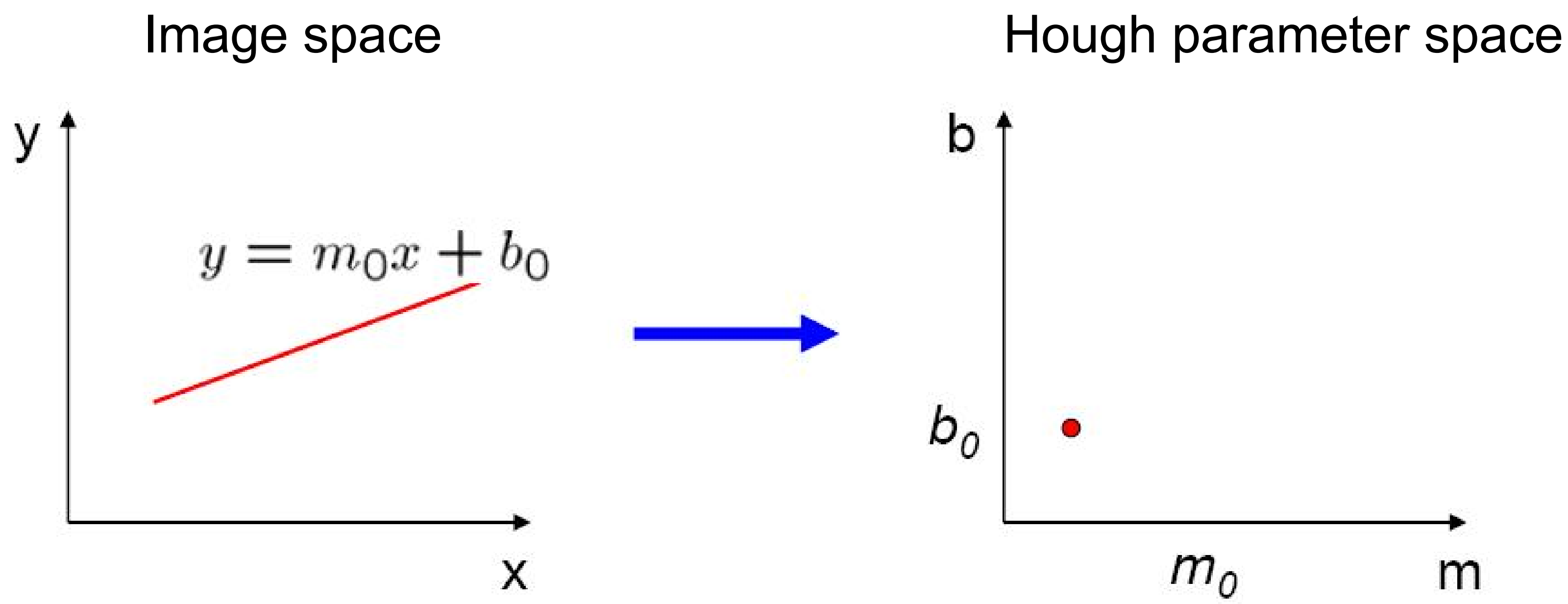
Hough Transform

- An early type of voting scheme
- General outline:
  - Discretize parameter space into bins
  - For each feature point in the image, put a vote in every bin in the parameter space that could have generated this point
  - Find bins that have the most votes



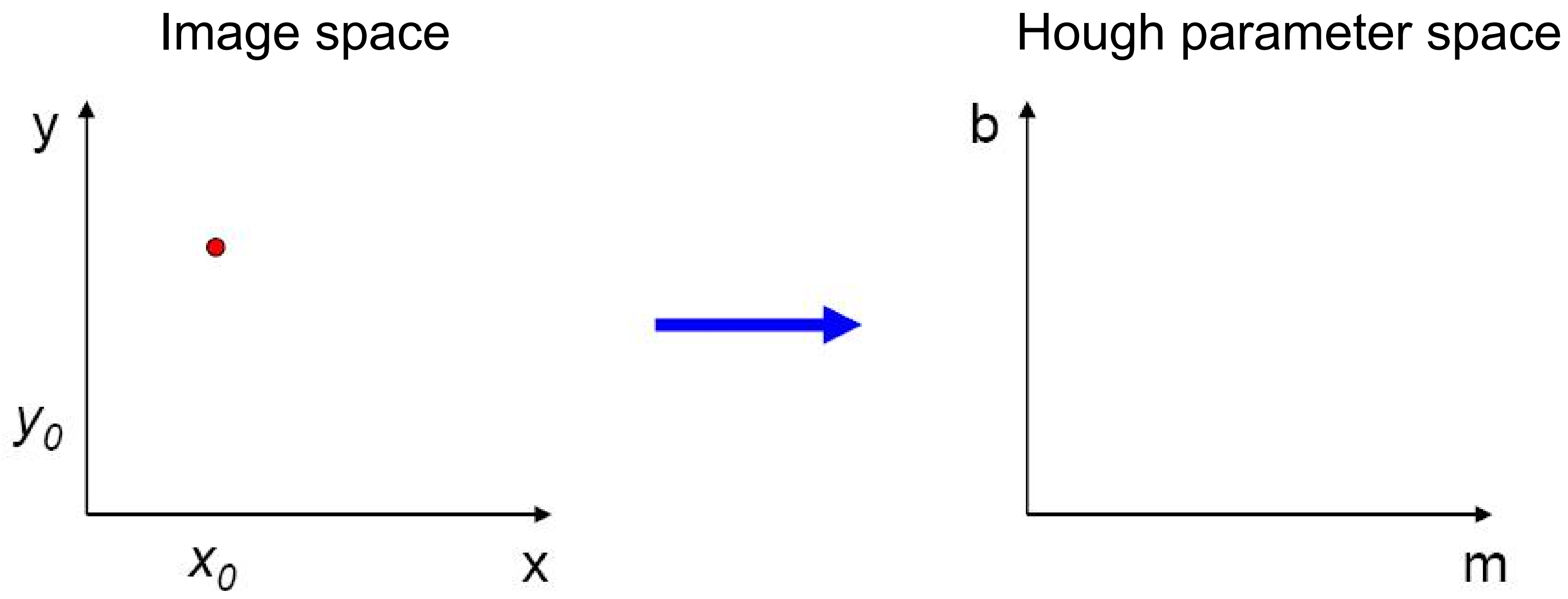
Parameter Space Representation

- A line in the image corresponds to a point in Hough space



Parameter Space Representation

- What does a point (x_0, y_0) in the image space map to in the Hough space?



Parameter Space Representation

- What does a point (x_0, y_0) in the image space map to in the Hough space?
  - Answer: the solutions of $b = -x_0 m + y_0$
  - This is a line in Hough space

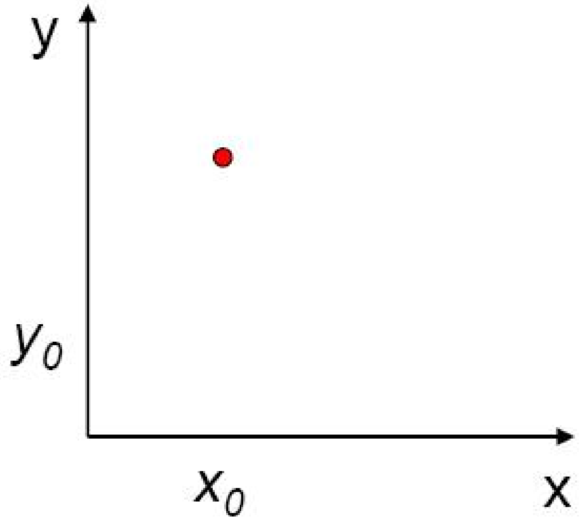
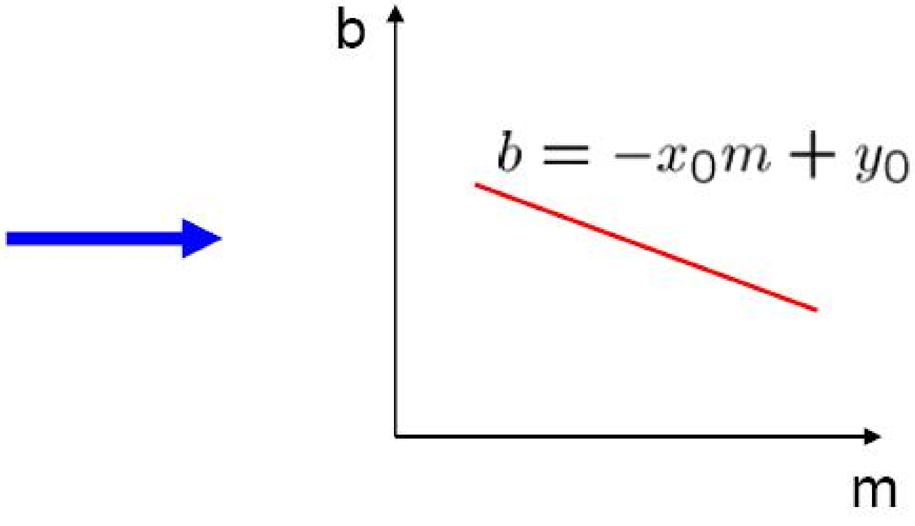


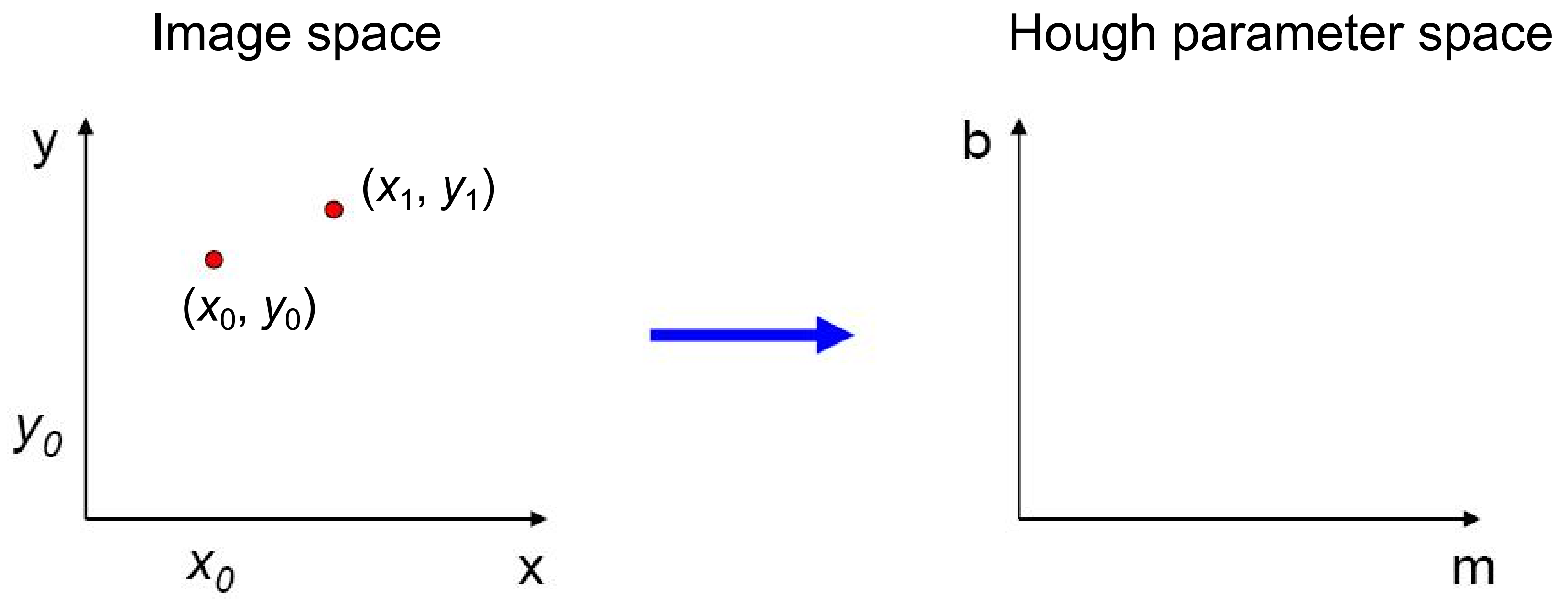
Image space



Hough parameter space

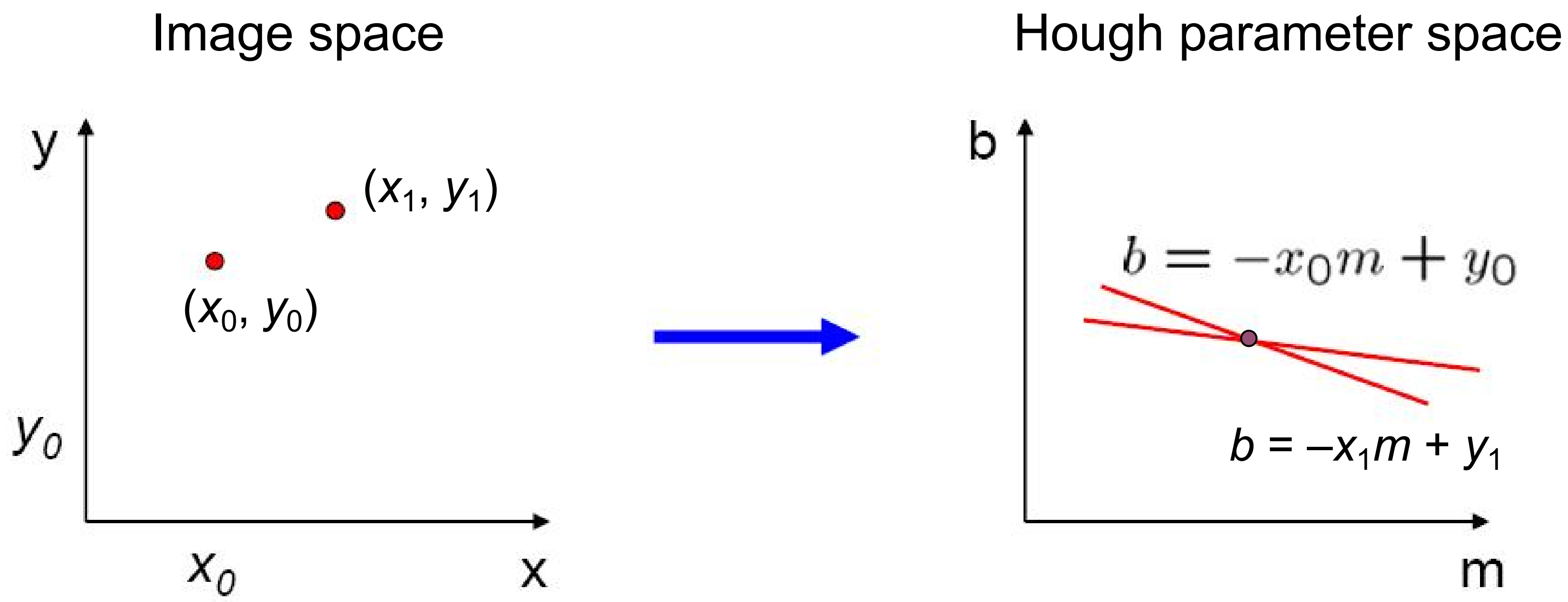
Parameter Space Representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?



Parameter Space Representation

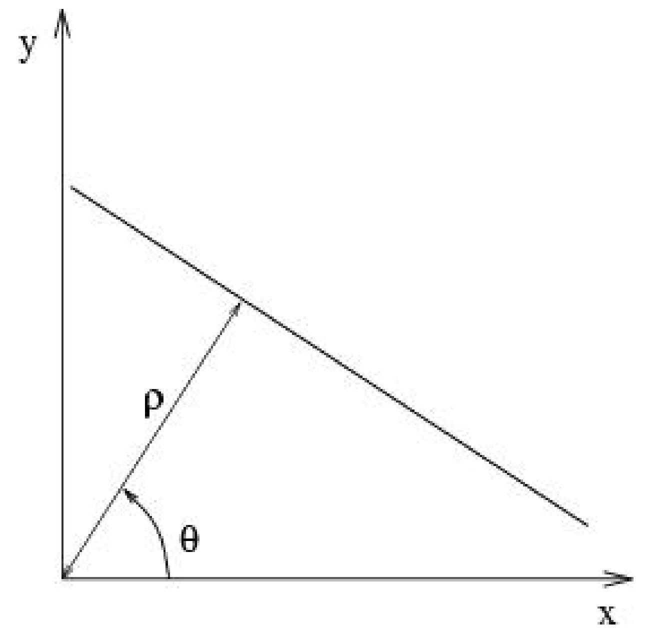
- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?
  - It is the intersection of the lines $b = -x_0 m + y_0$ and $b = -x_1 m + y_1$



Parameter Space Representation

- Problems with the (m,b) space:
  - Unbounded parameter domain
  - Vertical lines require infinite m
- Alternative: polar representation

$$x\cos\theta + y\sin\theta = \rho$$



- Each point will add a sinusoid in the (θ, ρ) parameter space

Algorithm Outline

- Initialize accumulator H to all zeros
- For **each** edge point (x,y) in the image

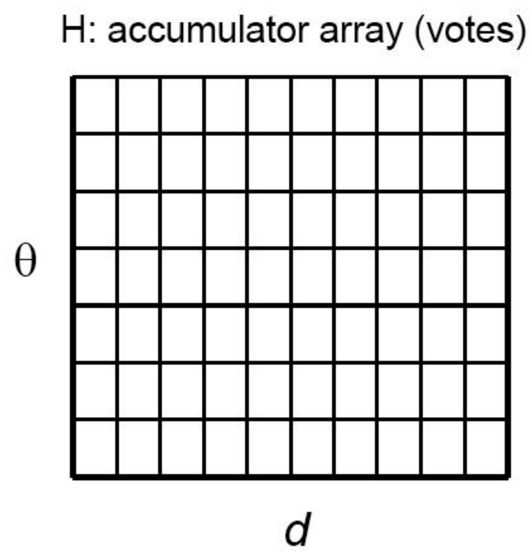
    For $\theta = 0$ to 180

        $\rho = x \cos\theta + y \sin\theta$

        $H(\theta, \rho) = H(\theta, \rho) + 1$

    end

  end
- Find the value(s) of (θ, ρ) where $H(\theta, \rho)$ is a local maximum
    - The detected line in the image is given by $\rho = x \cos\theta + y \sin\theta$



Basic Illustration

- A line

Horizontal axis is θ
Vertical is rho.

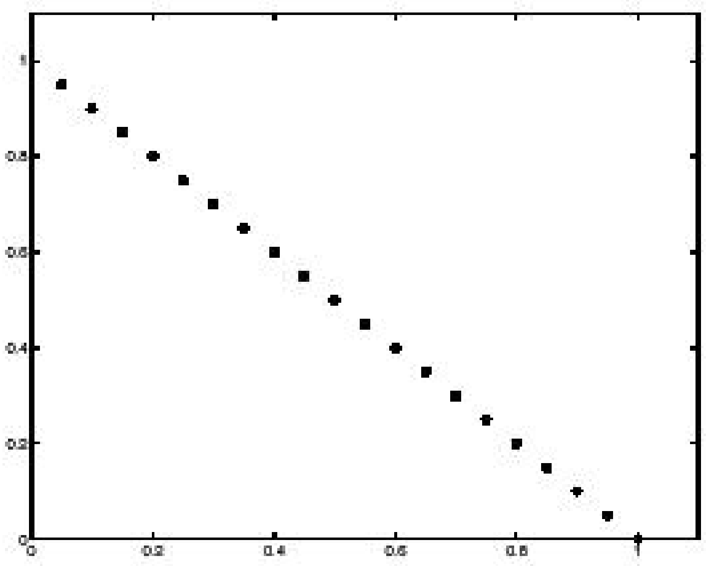
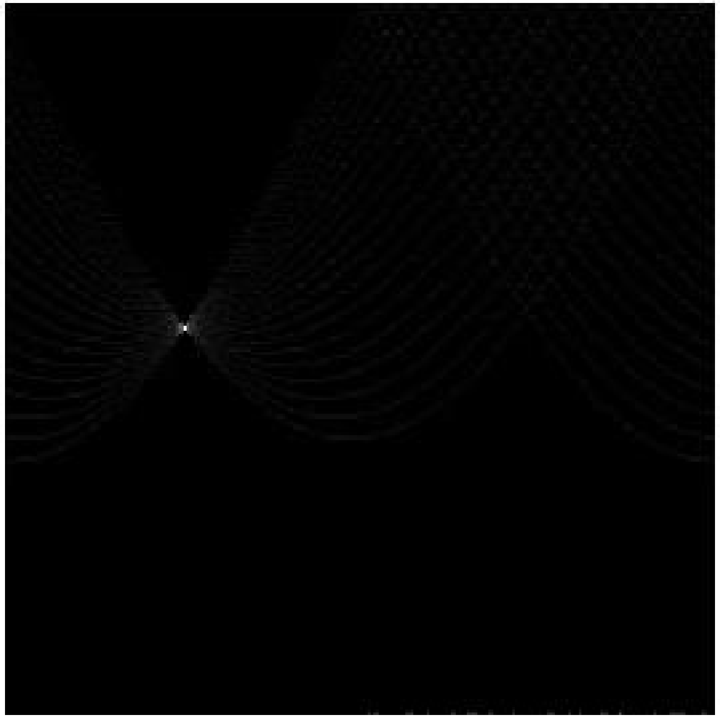


Image space



Votes

Basic Illustration

- A line with noise

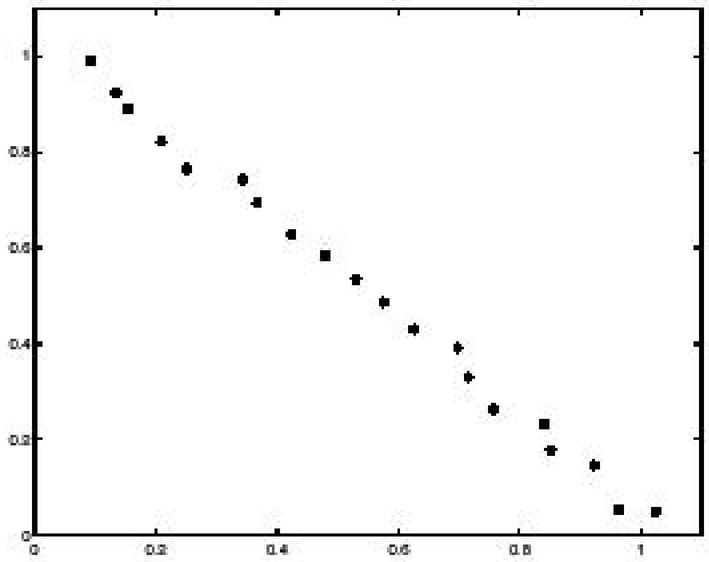
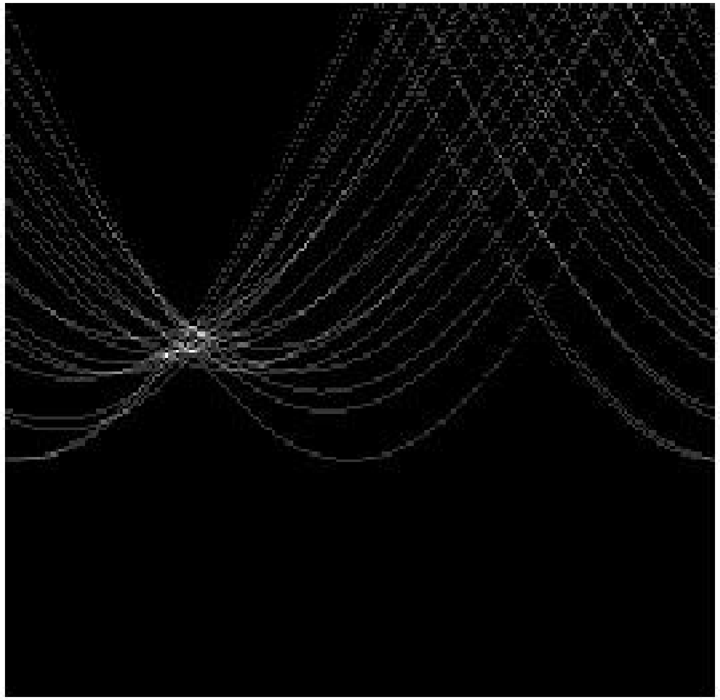


Image space



Votes

Basic Illustration

- Scattered points

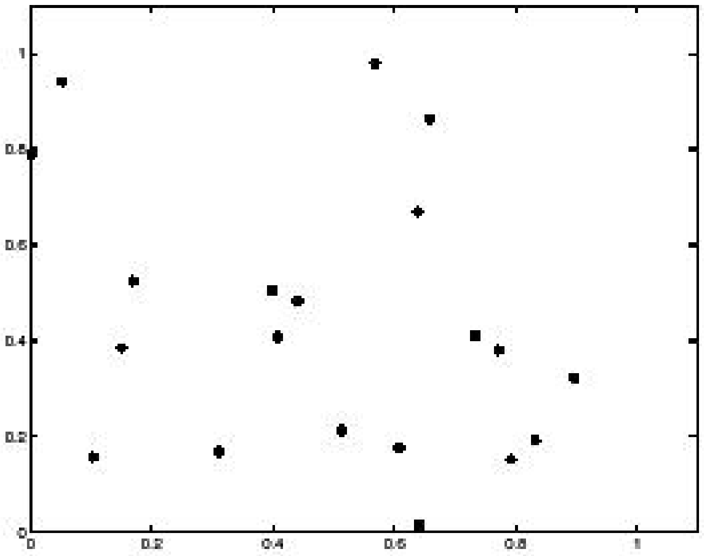
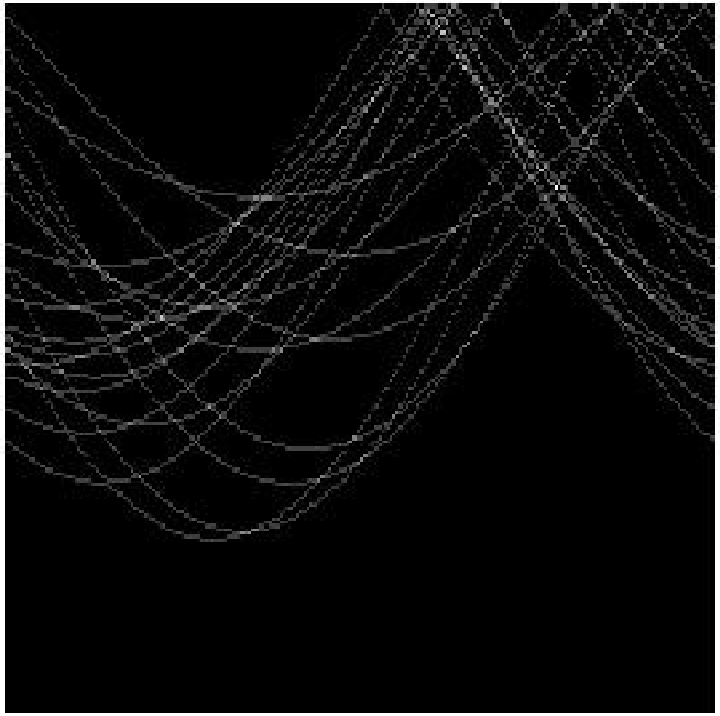


Image space



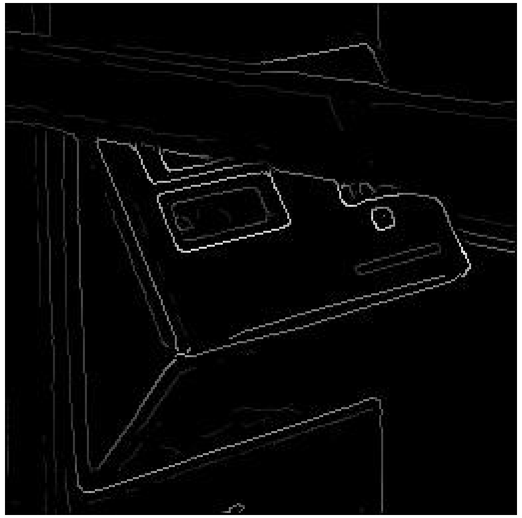
Votes

Mechanics of the Hough transform

- Difficulties
  - How big should the cells be? (too big, and we merge quite different lines; too small, and noise causes lines to be missed)
- How many lines?
  - Count the peaks in the Hough array
  - Treat adjacent peaks as a single peak
- Which points belong to each line?
  - Search for points close to the line
  - Solve again for line and iterate

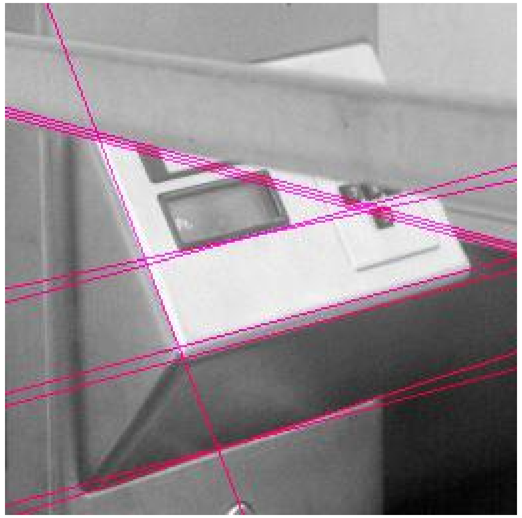
Real World Example

Original



Edge Detection

Parameter Space

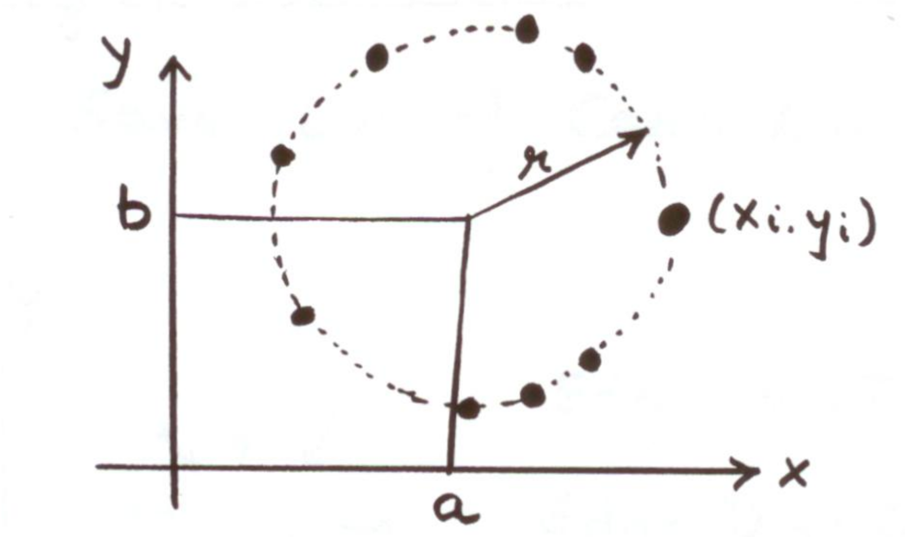


Found Lines

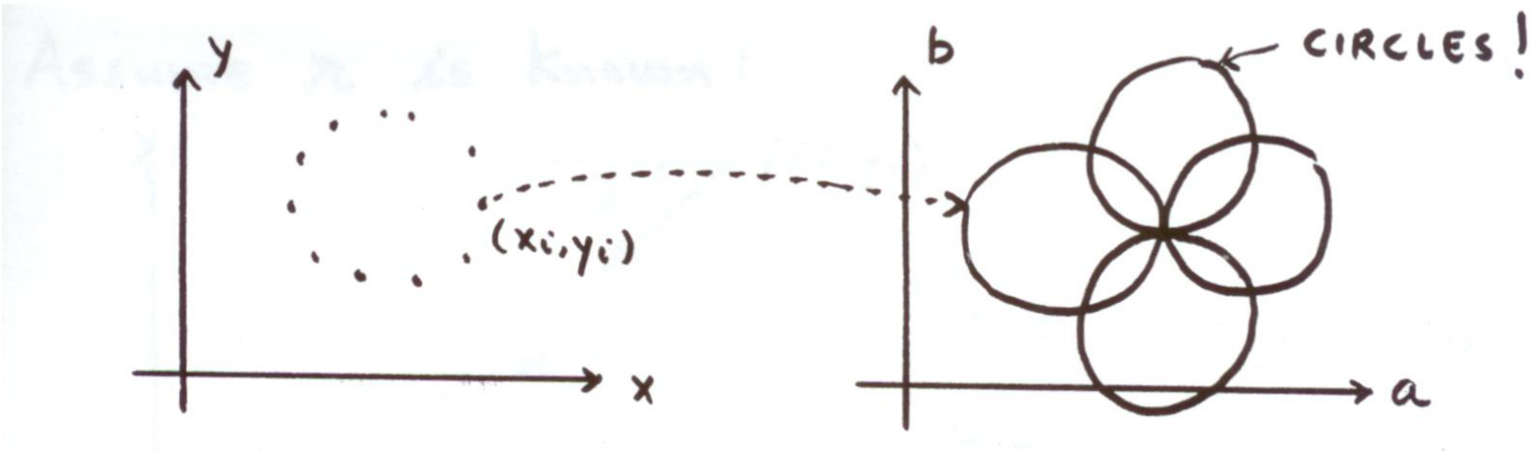
Finding Circles by Hough Transform

- Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$



- If radius is known:
  - ➢ 2D Hough Space
- Accumulator Array: $A(a,b)$

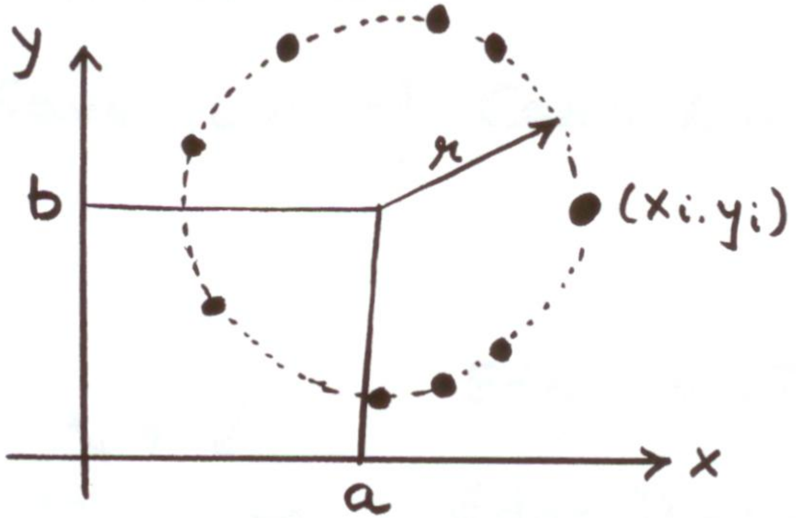


Finding Circles by Hough Transform

- Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- If radius is not known:
  - ➢ 3D Hough space!
- Use Accumulator array: $A(a,b,r)$



- What is the surface in the Hough space?

Finding Circles by Hough Transform

- Hough transform for circles

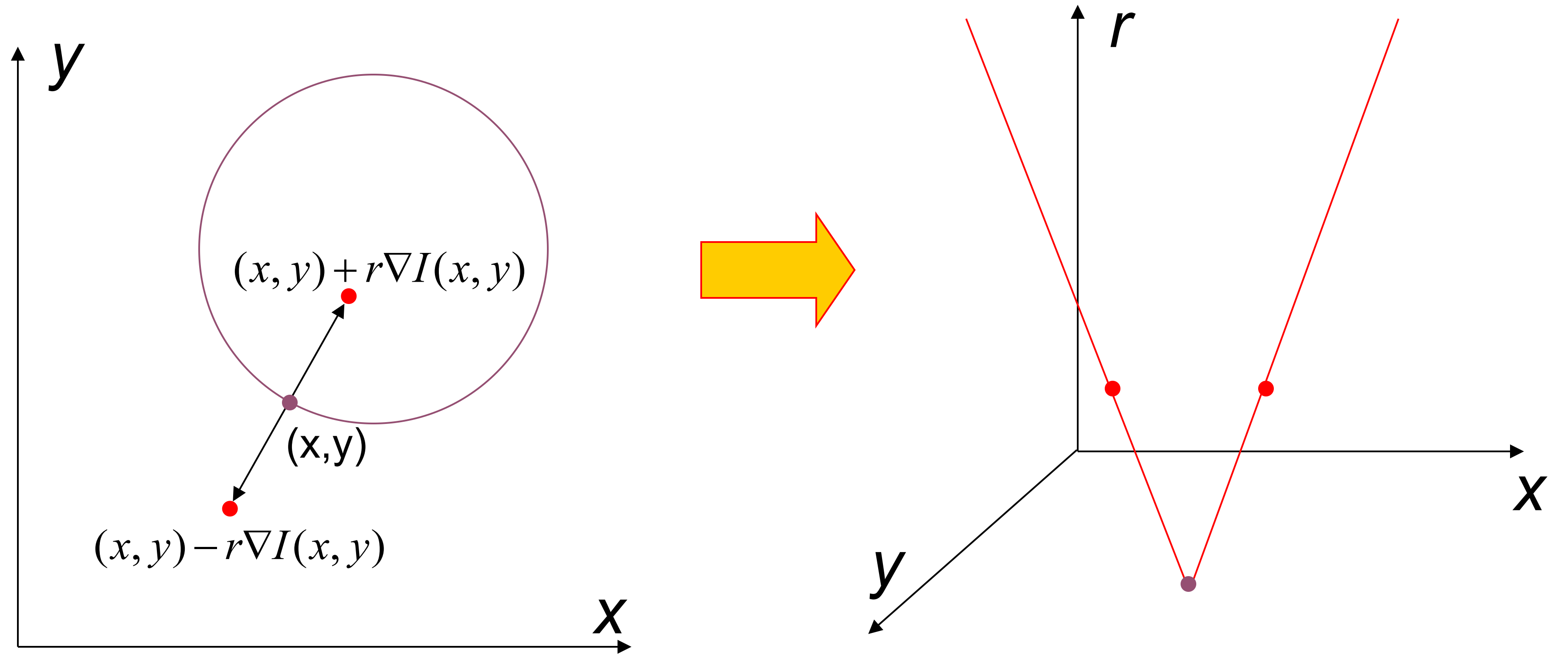
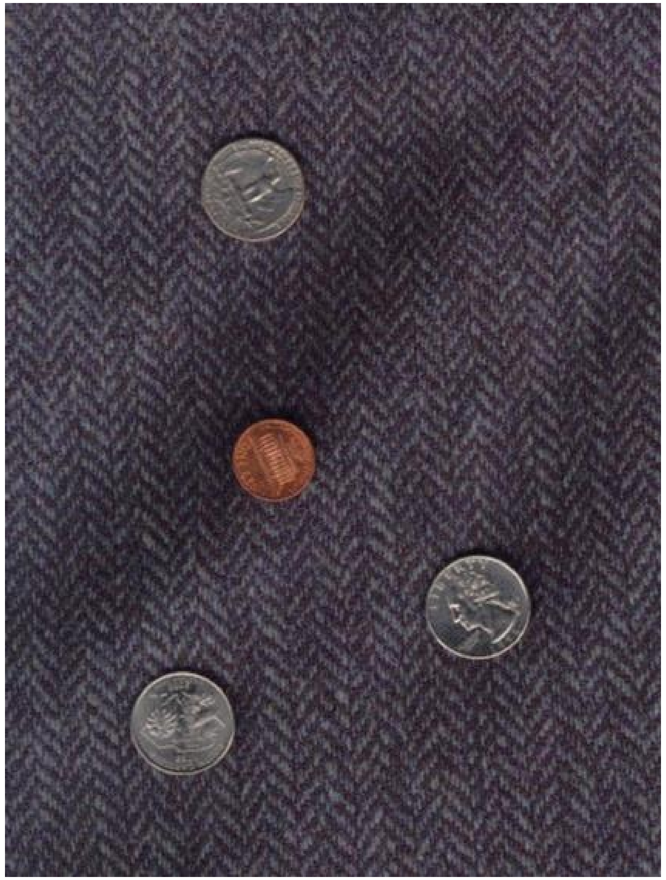


Image space

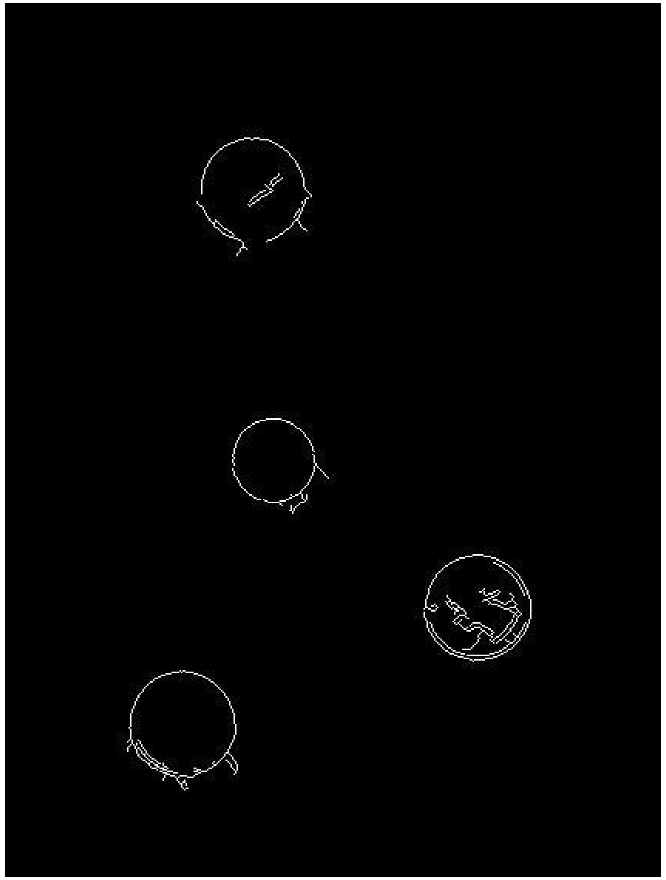
Hough parameter space

Finding Coins

Original

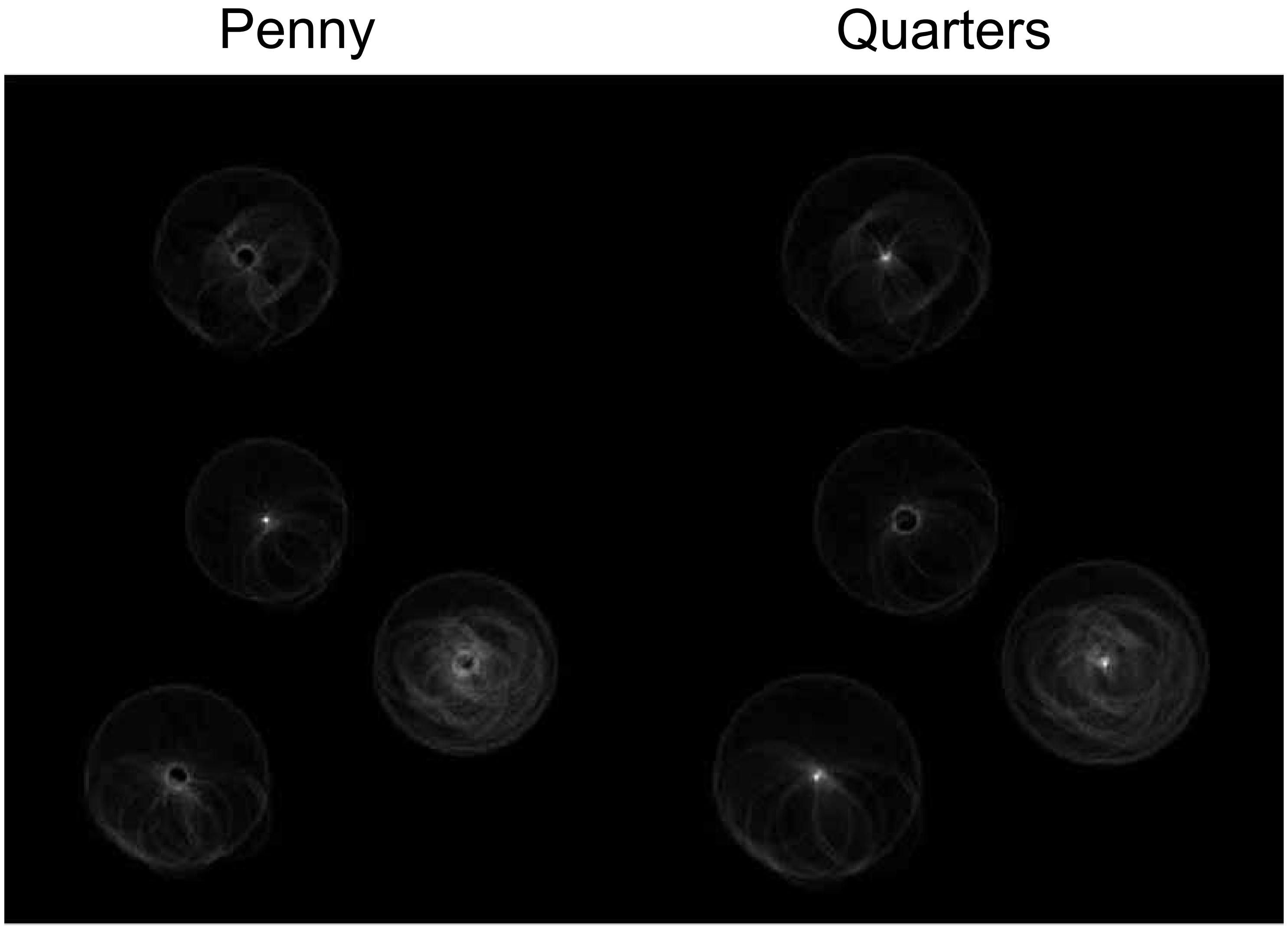


Edges (note noise)



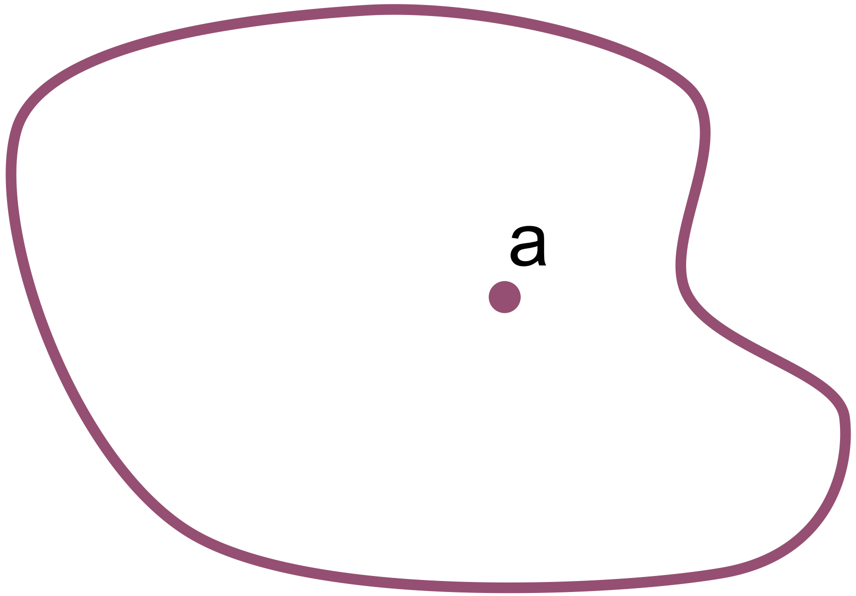
Finding Coins

- Note that because the quarters and penny are different sizes, a different Hough transform (with separate accumulators) was used for each circle size



Generalized Hough Transform

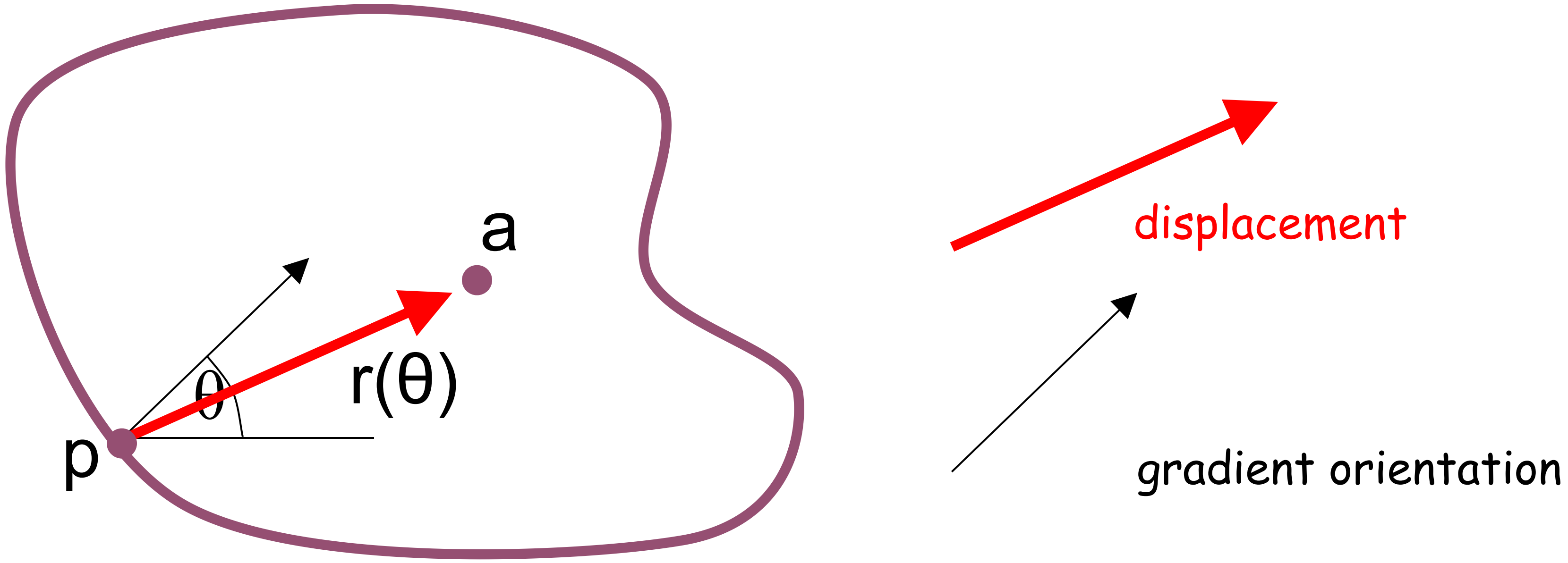
- We want to find a fixed shape **(known)** defined by its boundary points and a reference point



D. Ballard, [Generalizing the Hough Transform to Detect Arbitrary Shapes](), PR 13(2), 1981, pp. 111-122.

Generalized Hough Transform

- We want to find a fixed shape **(known)** defined by its boundary points and a reference point
- For every boundary point p, we can compute the displacement vector $r = a - p$ as a function of gradient orientation θ

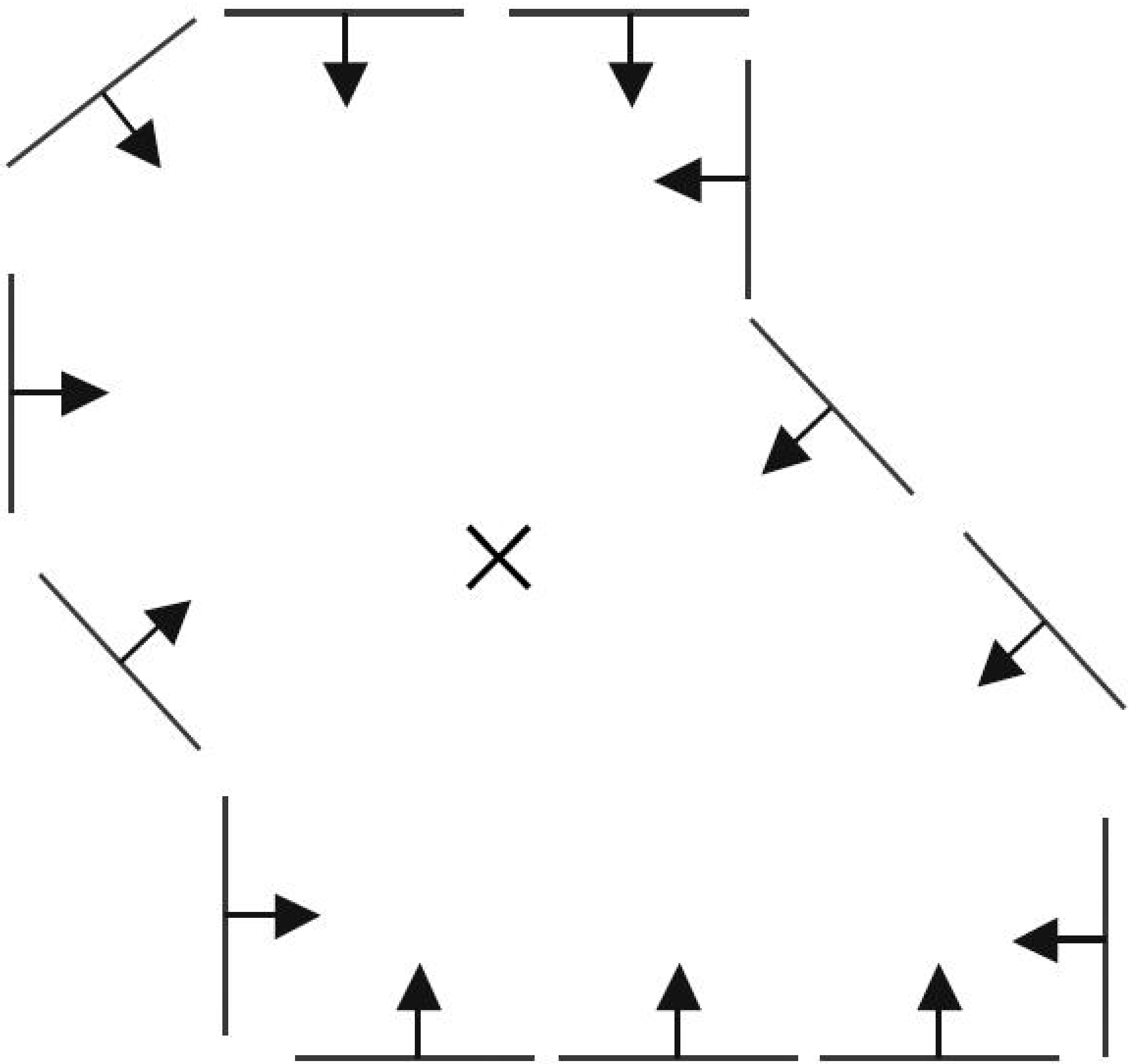


Generalized Hough Transform

- Construct a model for a shape:
  - ➢ Construct a table indexed by θ storing displacement vectors r as function of gradient direction
- Detect using the model
  - ➢ For each edge point *p* with gradient orientation θ:
    - ✓ Retrieve all r indexed with θ
    - ✓ For each $r(\theta)$, put a vote in the Hough space at $p + r(\theta)$
  - ➢ Peak in this Hough space is reference point with most supporting edges
- Assumption: translation is the only transformation here, i.e., orientation and scale are fixed

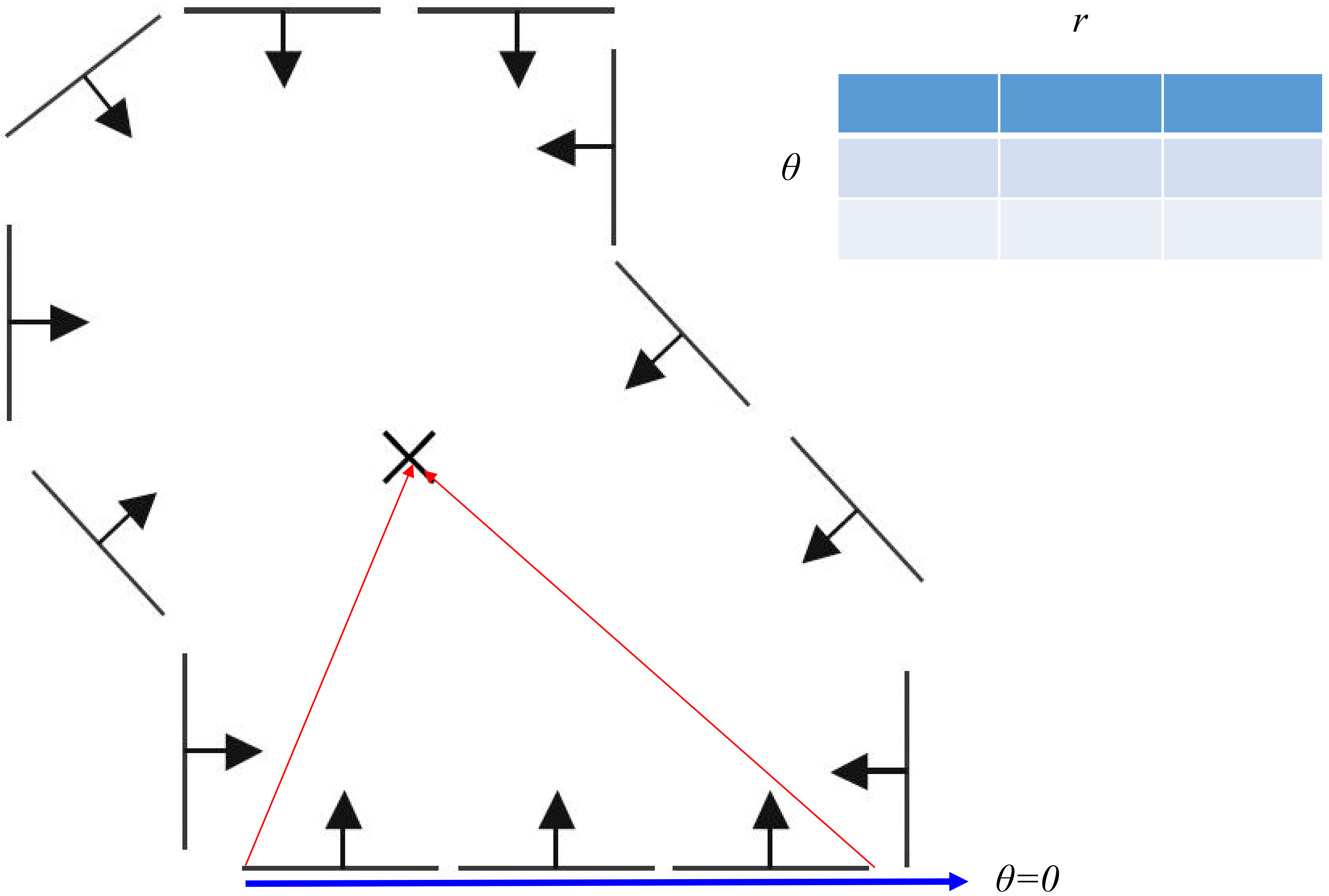
Example: a Known and Fixed Shape

- Model shape
  - Gradient orientation
  - No rotation



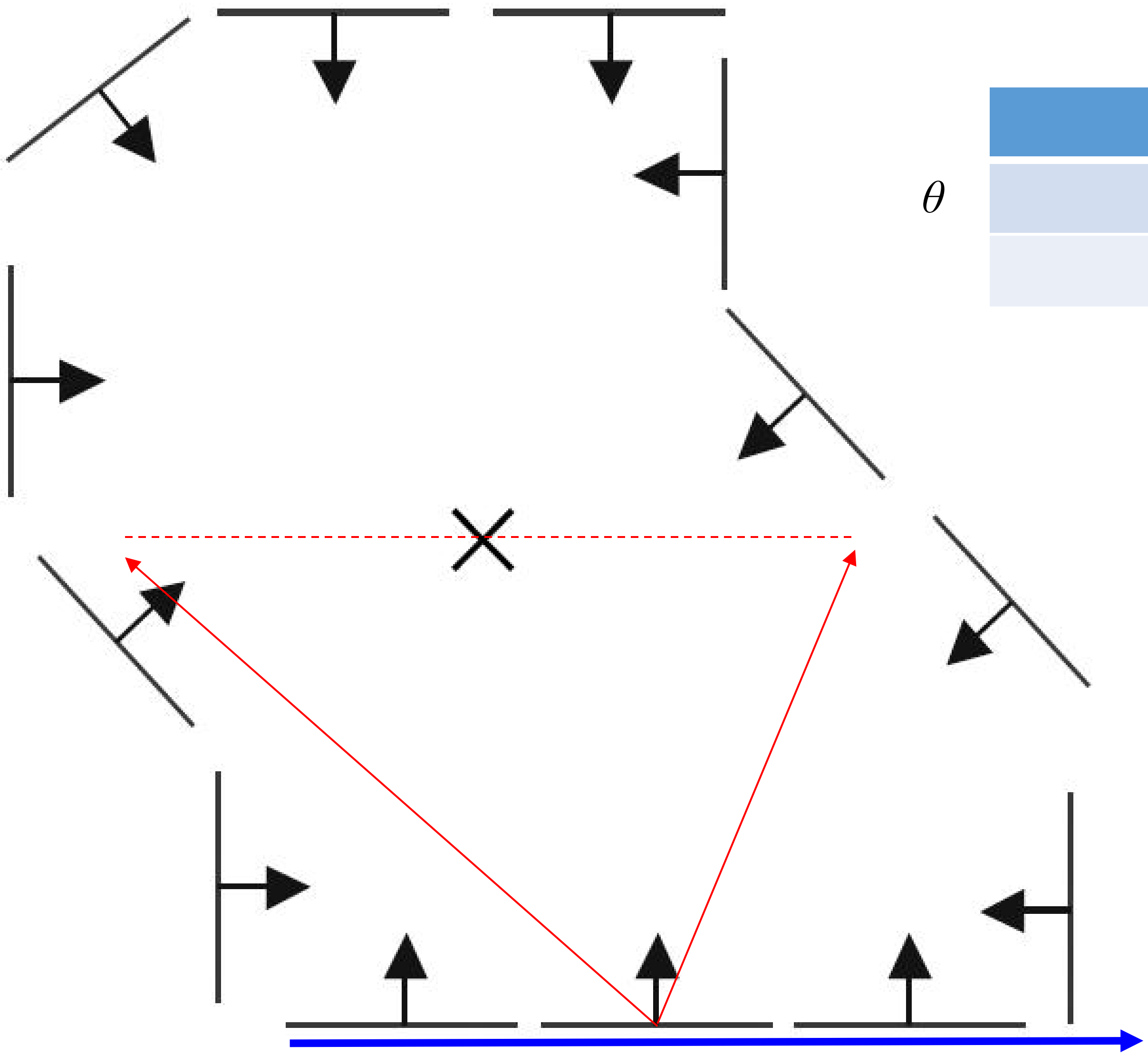
Example: Building a Table

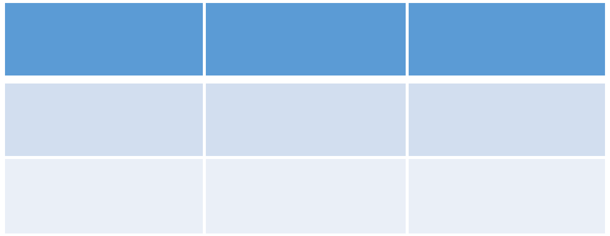
- Displacement vectors for model points



Example: Detection

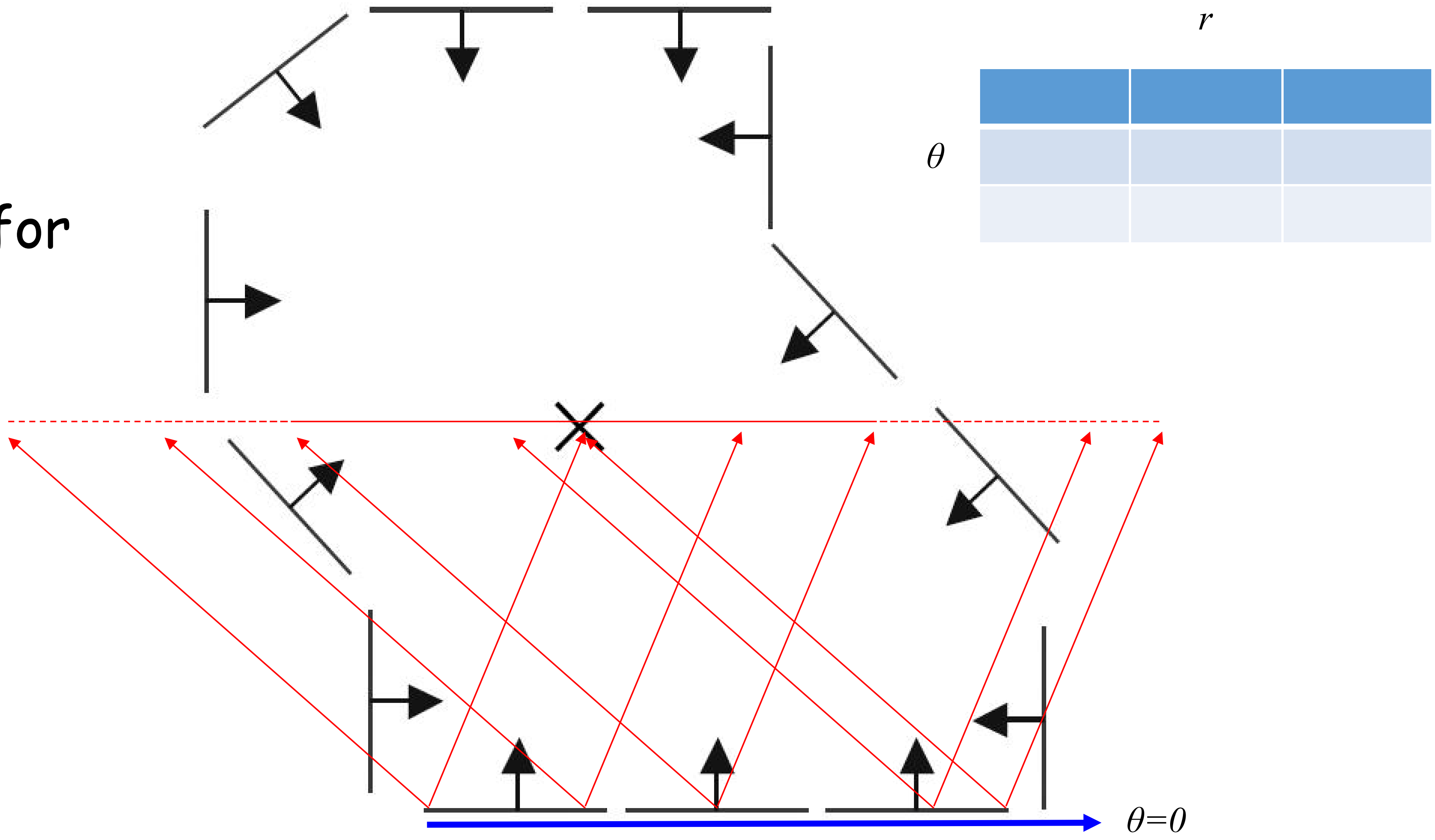
- Range of voting locations for **test** point





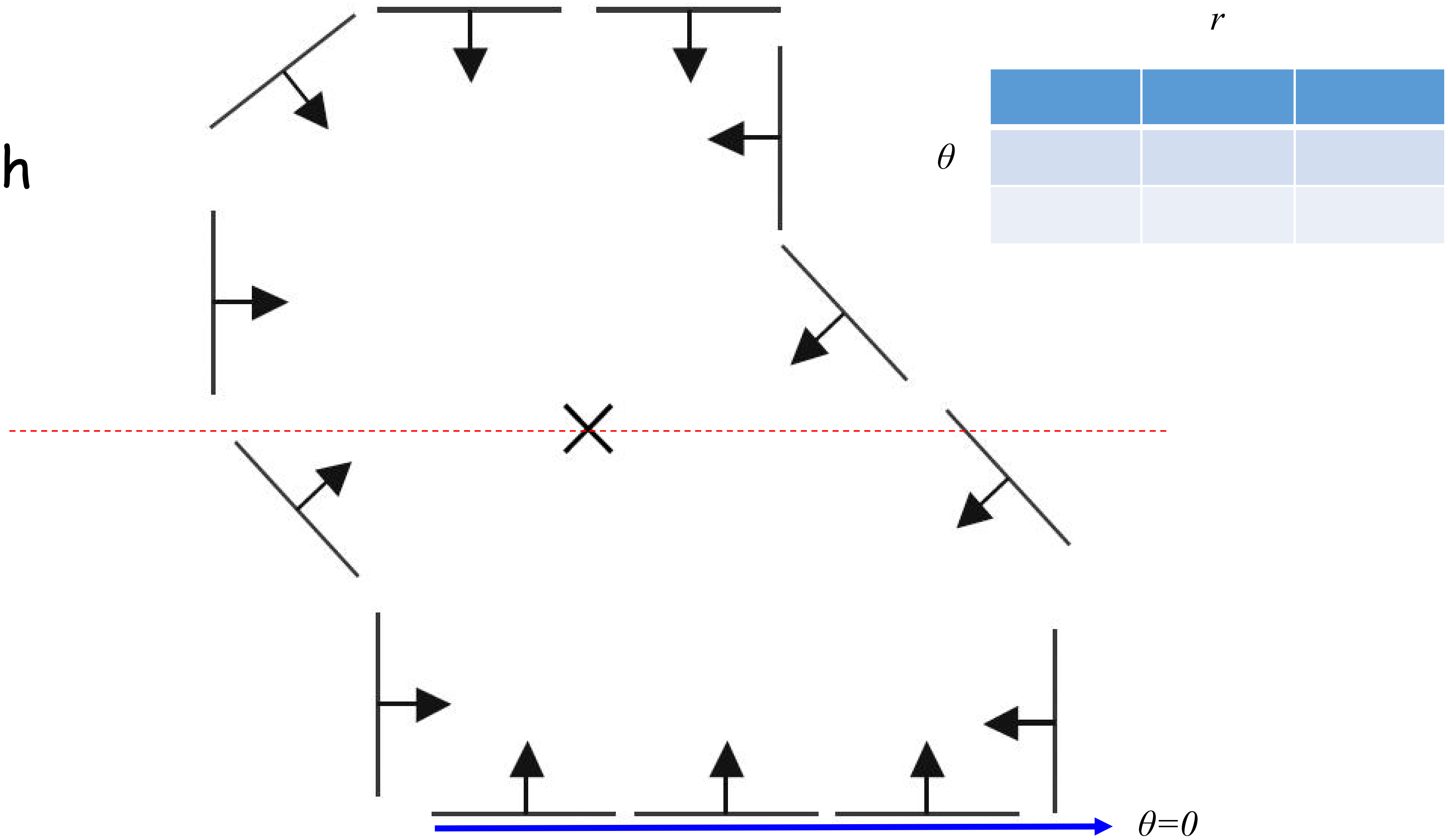
Example: Detection

- Range of voting locations for test point



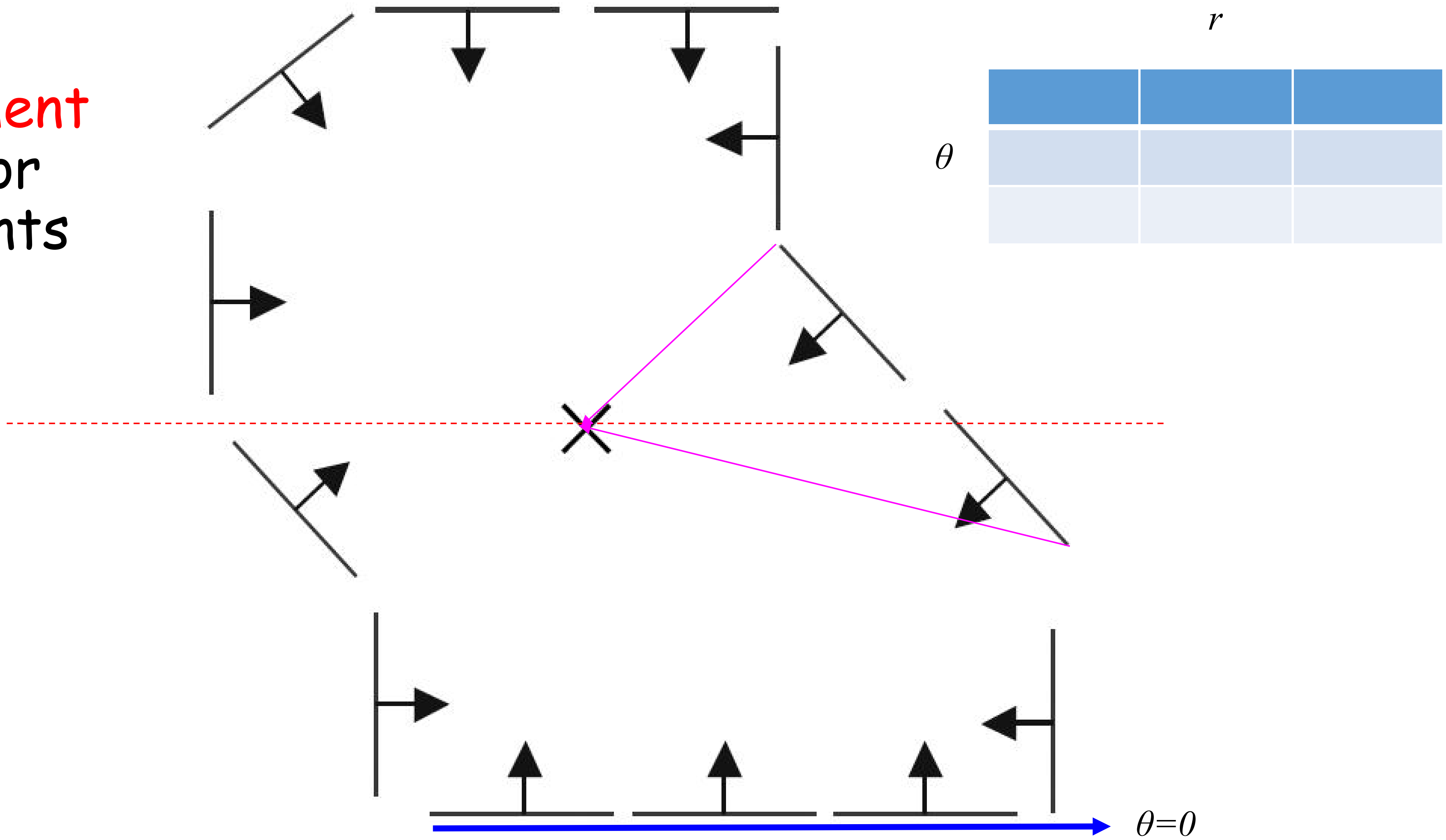
Example: Detection

- Votes for points with $\theta = \uparrow$



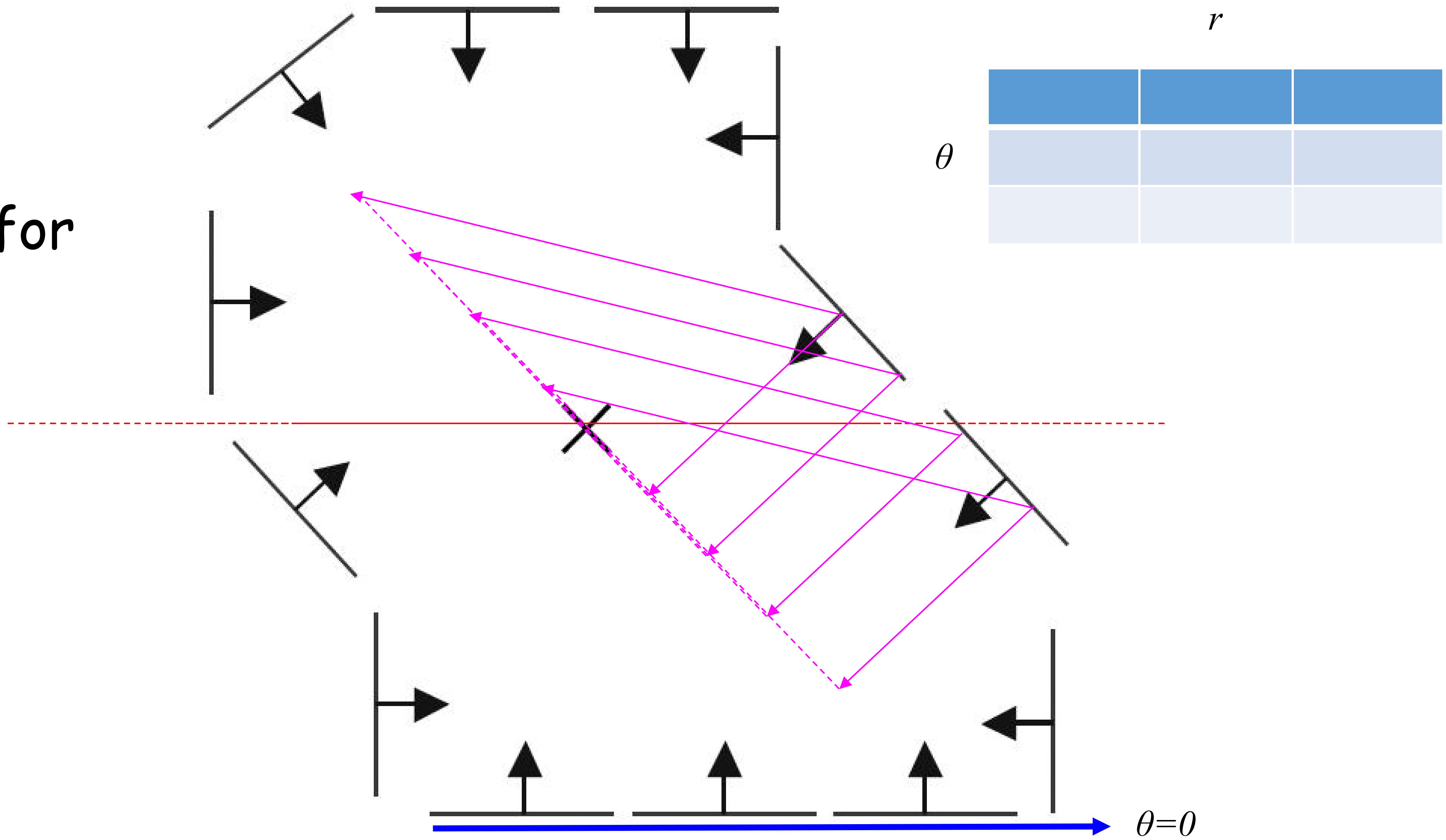
Example: Building a Table

- Displacement vectors for model points



Example: Detection

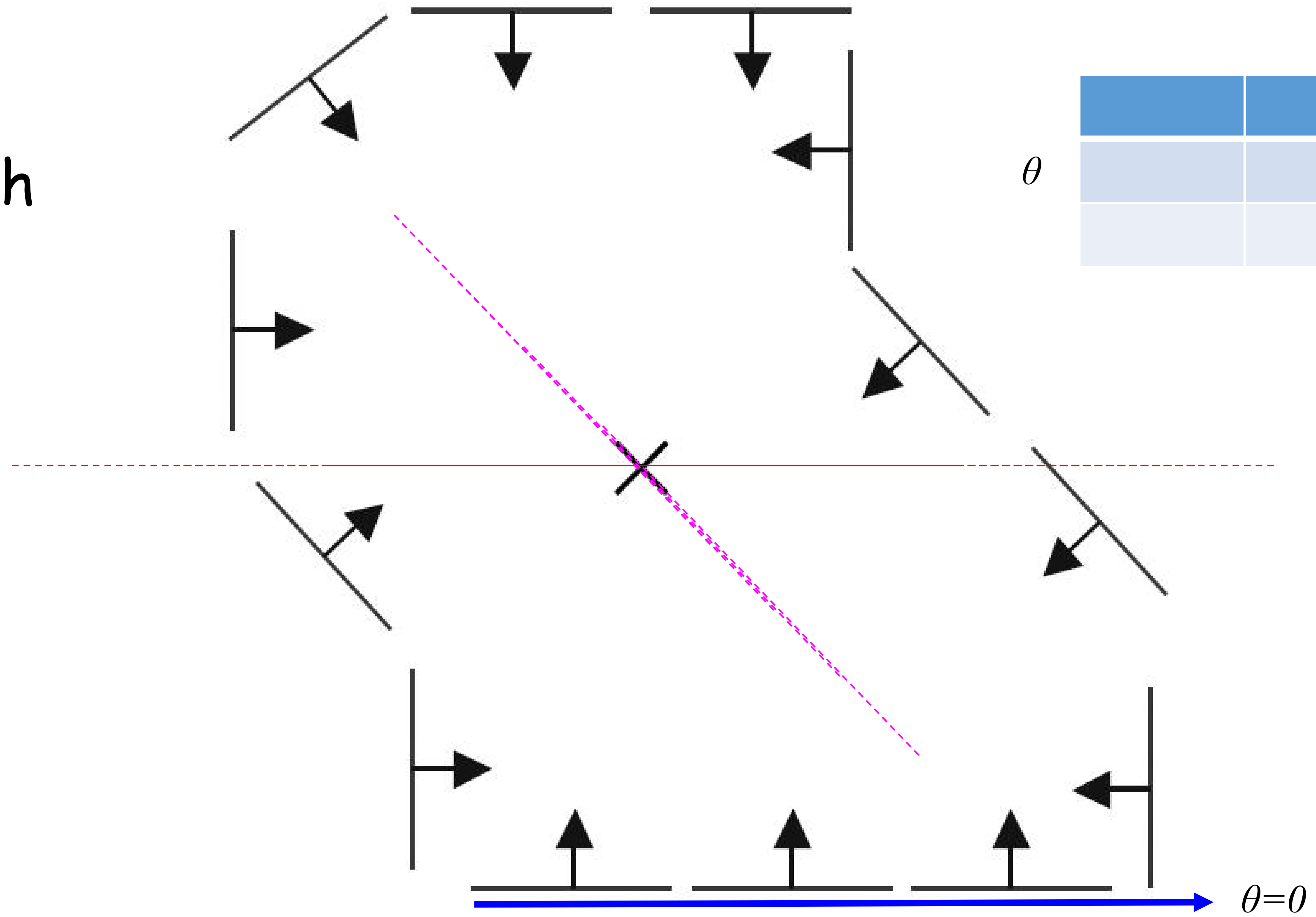
- Range of voting locations for test point



Example: Detection

- Votes for points with

  $\theta = \swarrow$

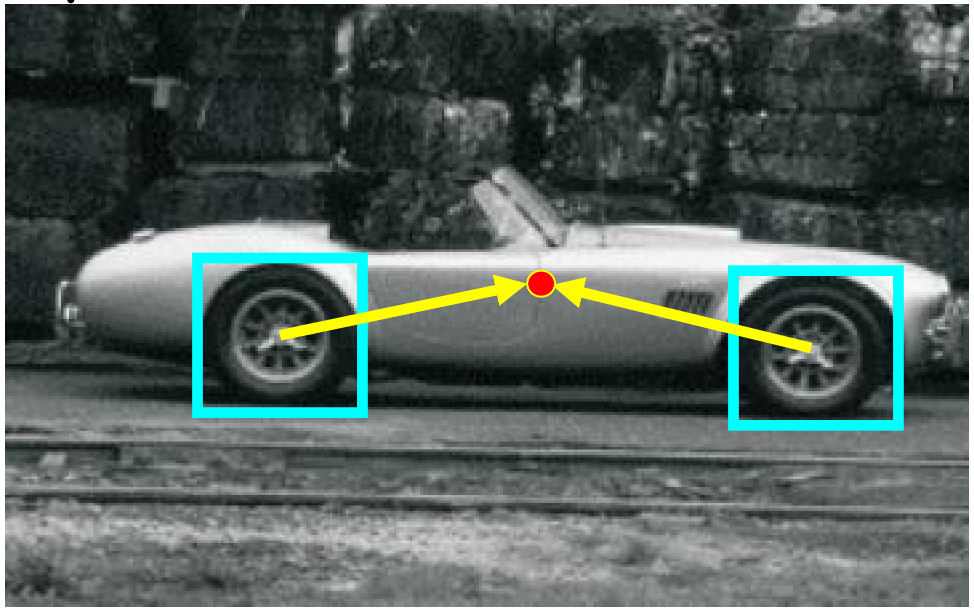


r

θ			

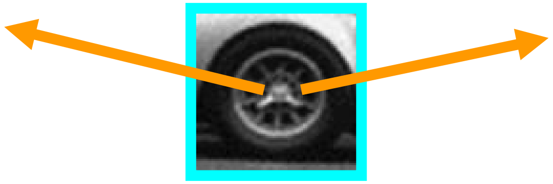
Application in Recognition

- Instead of indexing displacements by gradient orientation, index by "visual codeword"



training image

What is the codeword?



visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Application in Recognition

- Instead of indexing displacements by gradient orientation, index by "visual codeword"

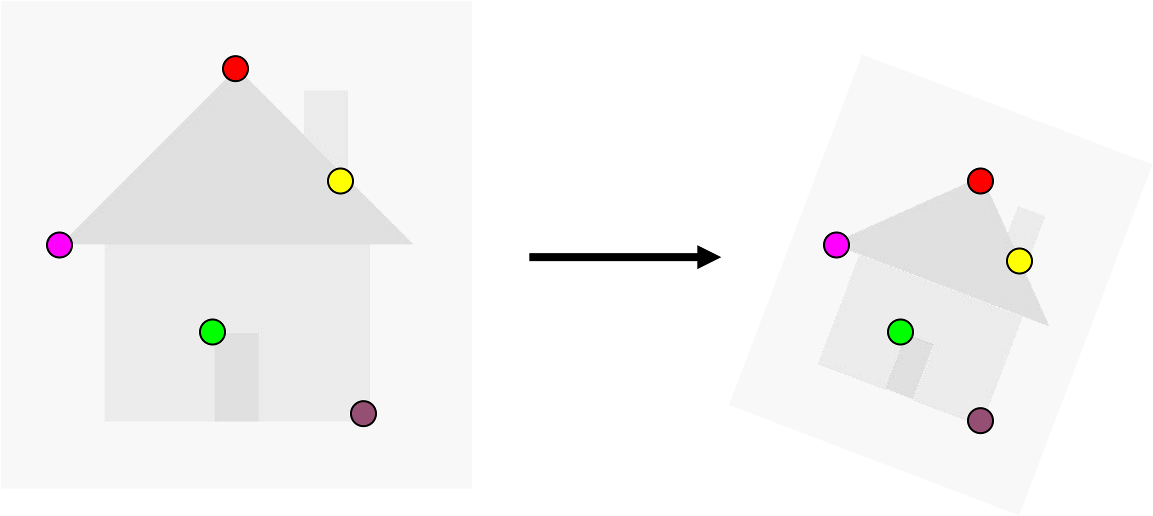
test image

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004

Image Alignment

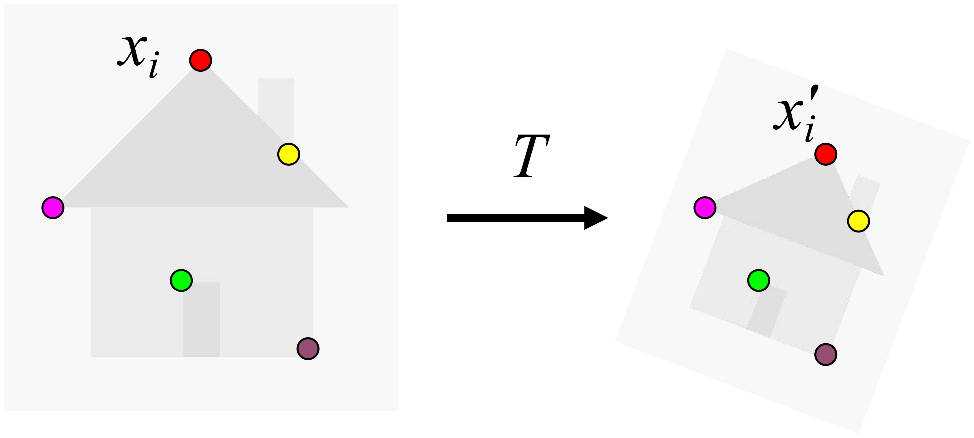
Image Alignment

- Two broad approaches:
  - ➢ Direct (pixel-based) alignment
    - ✓ Search for alignment where most pixels agree
  - ➢ Feature-based alignment
    - ✓ Search for alignment where *extracted features* agree
    - ✓ Can be verified using pixel-based alignment



Alignment as Fitting

- Alignment: fitting a model to a transformation between pairs of features (*matches*) in two images

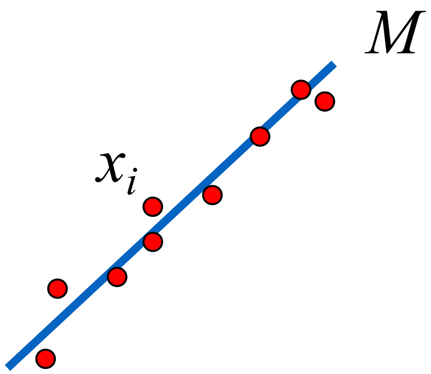


Find transformation T that minimizes

$$\sum_i \text{residual}(T(x_i), x_i')$$

Alignment as Fitting

- Previously: fitting a model to features in one image

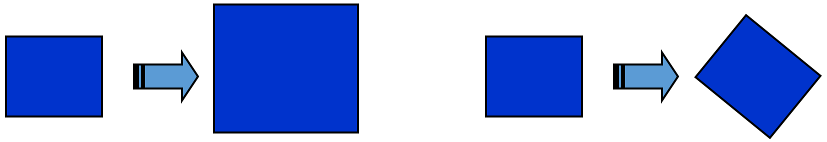


Find model M that minimizes

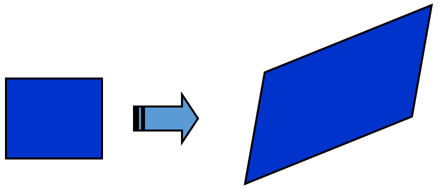
$$\sum_i \text{residual}(x_i, M)$$

2D Transformation Models

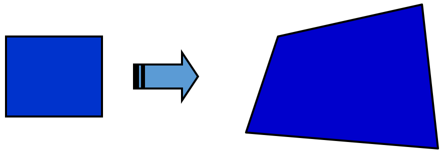
- Similarity (translation, scale, rotation)



- Affine

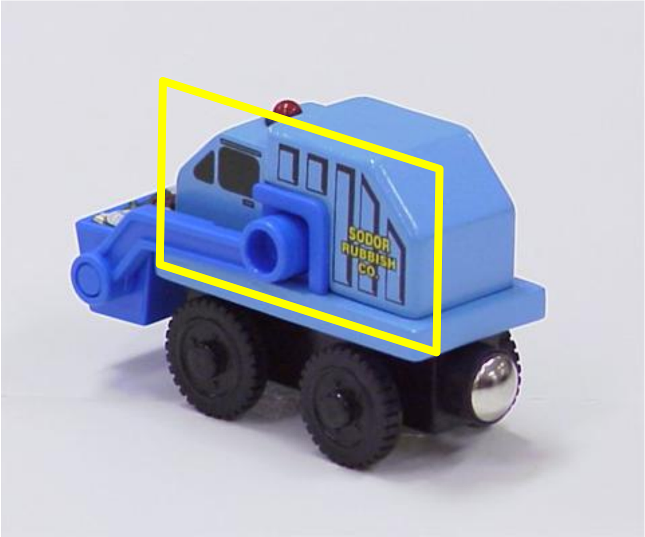


- Projective (homography)



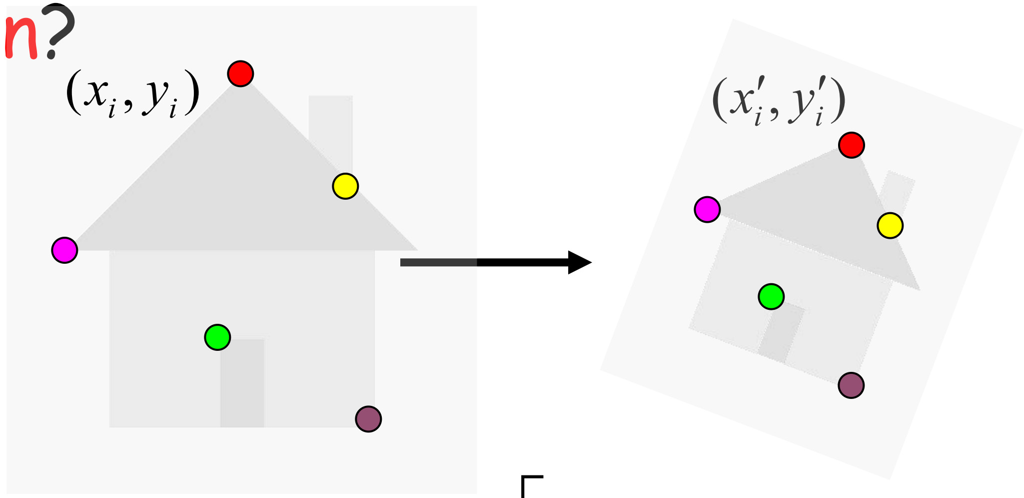
Affine Transformations

- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
- Can be used to initialize fitting for more complex models



Affine Transformations

- Assume we know the correspondences (???), how do we get the transformation?



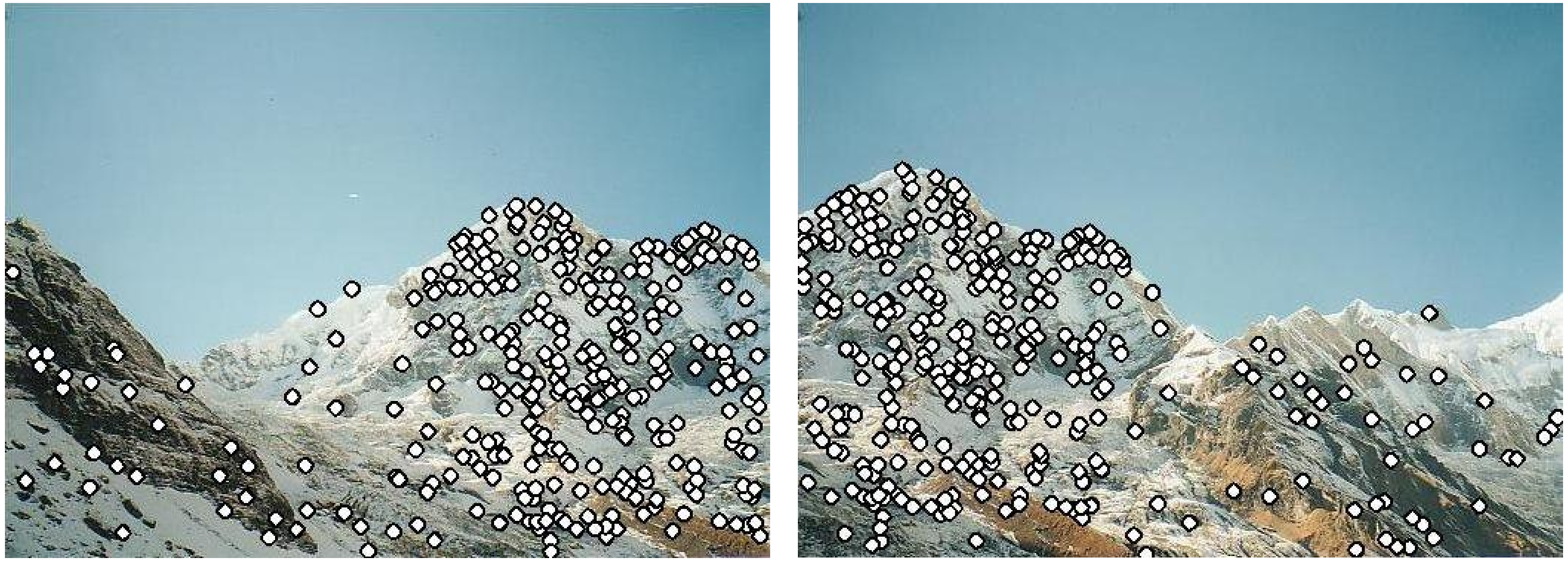
$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \Rightarrow \begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x'_i \\ y'_i \\ \cdots \end{bmatrix}$$

Affine Transformations

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters

$$
\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}
$$

Feature-Based Alignment Outline

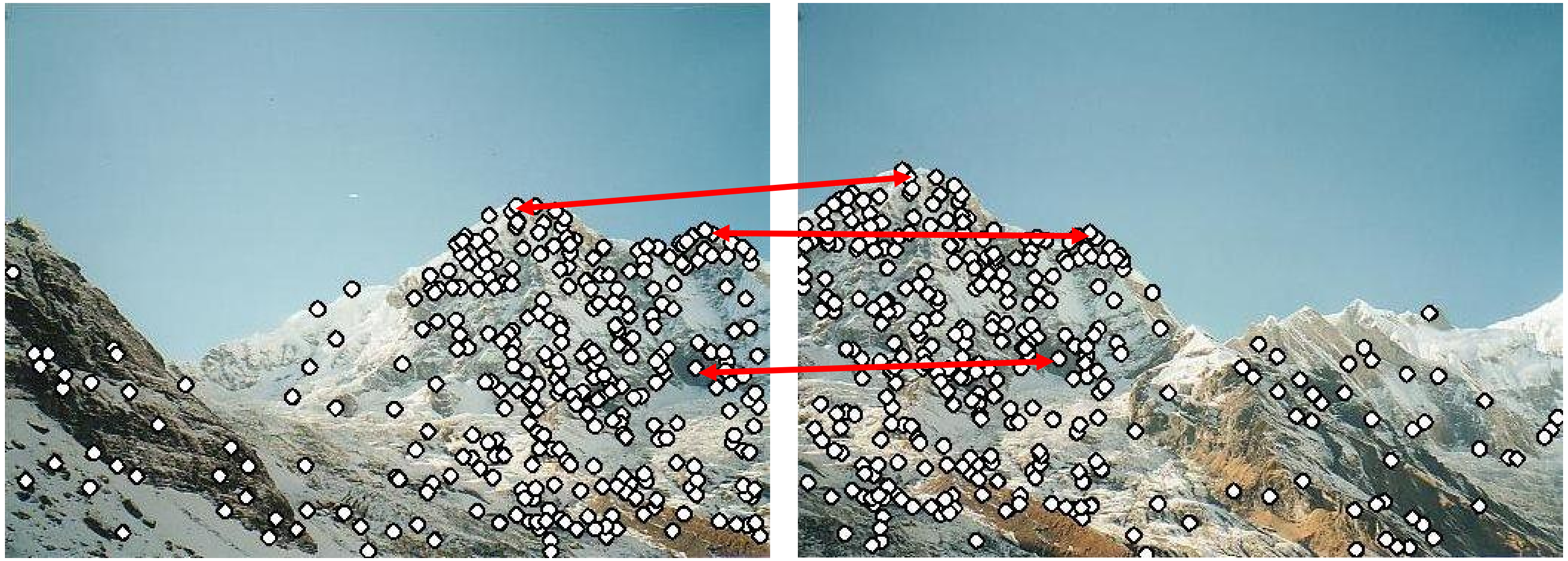


- Extract features

Feature-Based Alignment Outline

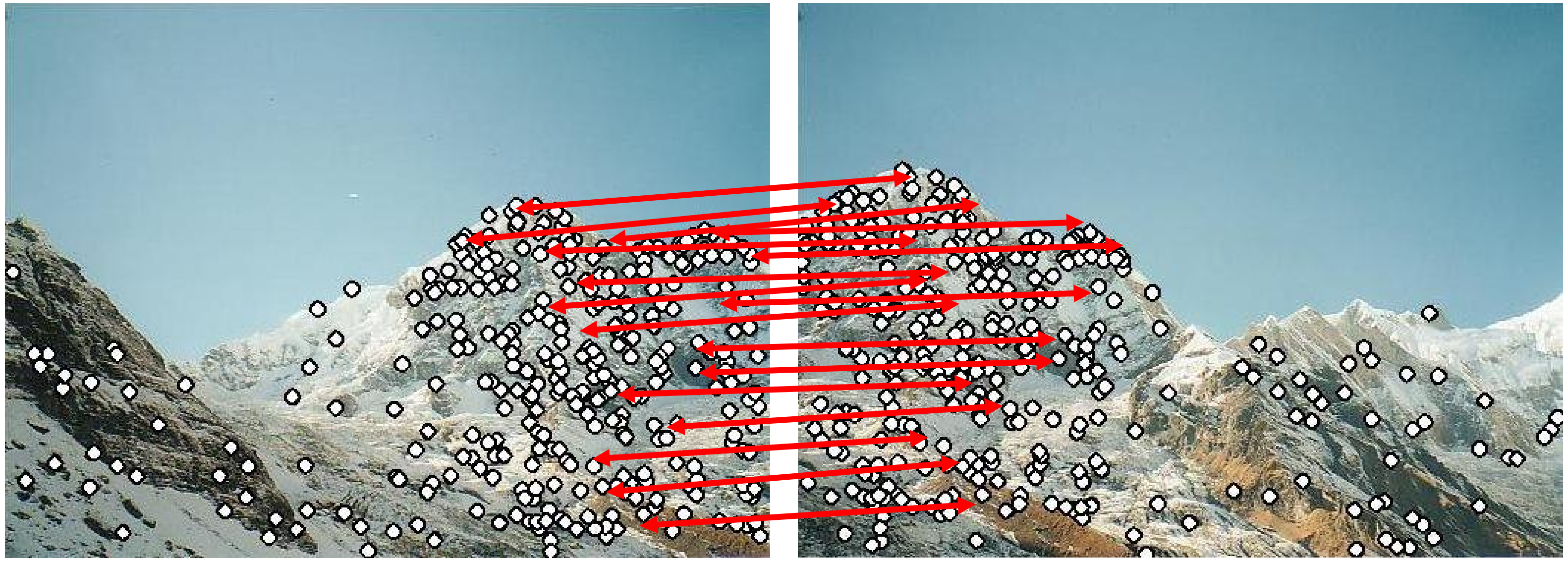
- Extract features
- Compute *putative matches*

Feature-Based Alignment Outline



- Extract features
- Compute *putative matches*
- Loop:
  - ➢ *Hypothesize* transformation *T*

Feature-Based Alignment Outline



- Extract features
- Compute *putative* matches
- Loop:
  - ➢ *Hypothesize* transformation *T*
  - ➢ *Verify* transformation (search for other matches consistent with *T*)

Feature-Based Alignment Outline

- Extract features
- Compute *putative matches*
- Loop:
  - ➢ *Hypothesize* transformation *T*
  - ➢ *Verify* transformation (search for other matches consistent with *T*)

Conclusions

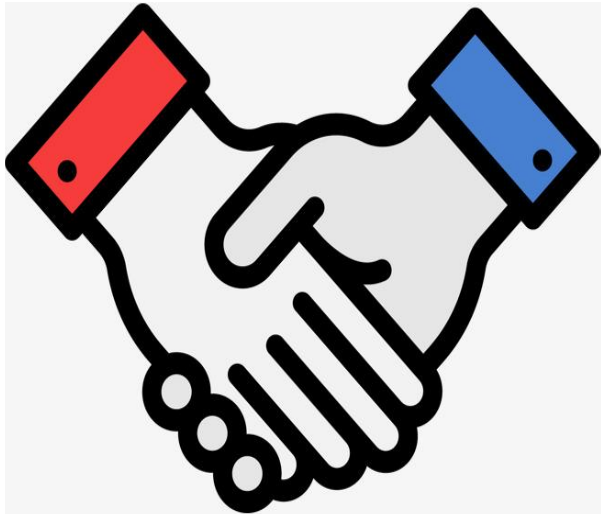
Conclusion

- Fitting techniques
  - Least Squares
  - Total Least Squares

- RANSAC

- Hough Voting

- Alignment as a fitting problem



Thanks

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