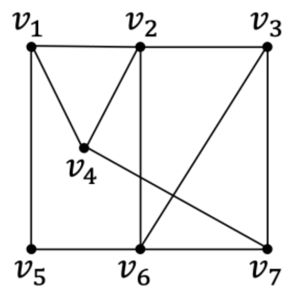
CS201: Discrete Math for Computer Science
2025 Spring Semester Written Assignment #6
For your reference

The assignment needs to be written in English. Assignments in any other language will get zero point. Any plagiarism behavior will lead to zero point.

Q. 1. Consider the following graphs and answer questions. Please explain your answer.



(a) Is this graph bipartite?

(b) What is the edge connectivity?

(c) Does this graph has a Euler circuit?

(d) Is this graph a planar?

(e) What is the chromatic number?

Solution:

(a) It is not bipartite. Consider a subset of vertices v1, v2, v4, where each of vertices is adjacent to the other two vertices in the subset. By coloring v1 as blue and coloring v2 as red, v4 can not be colored with either blue or red.

(b) The edge connectivity is 2. The removal of v1, v5 and v5, v6 disconnects the above graph. The edge connectivity cannot be smaller than 2, as we cannot disconnect the graph by removing one edge.

(c) No, because there are multiple vertices with odd degree.

(d) Yes. This can be proven by drawing the graph in a way without crossing (Omitted).

(e) The chromatic number is 3. This can be achieved by coloring v1, v6 as red, v2, v5, v7 as blue, and v3, v4 as green. The chromatic number cannot be smaller than 3, because vertices in v1, v2, v4 form a K3 and should be colored with three different colors.

Q. 2. Suppose that G is an undirected graph on a finite set of n vertices. Prove the following

(a) If every vertex of G has degree 2, then G contains a cycle.

(b) If G is disconnected, then its complement is connected.

Solution:

(a) Assume for contradiction that G has no cycle, and consider the longest path P in G (one must exist, since the graph is finite). Let v be the final vertex in P – since v has degree 2, it must have two edges e_1 and e_2 incident on it, of which one, say e_1, is the last edge of the path P. Then e_2 cannot be incident on any other vertex of P since that would create a cycle $(v, e_2, [\text{section of } P \text{ ending in } e_1], v)$. So e_2 an its other endpoint are not part of P, and can be appended to P to give a strictly longer path, which contradicts our choice of P. Hence, G must contain a cycle.

(b) Let $\overline{G}$ denote the complement of G. Consider any two vertices u, v in G. If u and v are in different connected components in G, then no edge of G connects them, so there will be an edge $\{u, v\}$ in $\overline{G}$. If u and v are in the same connected component in G, then consider any vertex w in a different connected component (since G is disconnected, there must be at least one other connected component). By our first argument, the edges $\{u, w\}$ and $\{v, w\}$ exist in $\overline{G}$, so u and v are connected by the path (u, w, v). Hence, any two vertices are connected in $\overline{G}$, so the whole graph is connected.