Artificial Intelligence

Lecture 7: Local Search

Credit: Ansaf Salleb-Aouissi, and “Artificial Intelligence: A Modern Approach”, Stuart Russell and Peter Norvig, and “The Elements of Statistical Learning”, Trevor Hastie, Robert Tibshirani, and Jerome Friedman, and “Machine Learning”, Tom Mitchell.

Recap: Search Methods

- **Uniformed search**: Use **no** domain knowledge.
  - BFS, DFS, DLS, IDS, UCS
- **Informed search**: Use a heuristic function that **estimates** how close a state is **to the goal**.
  - Greedy search, A*, IDA*.

Recap: Search Methods

We can organize the algorithms into pairs where the first proceeds by layers, and the other proceeds by subtrees.

(1) Iterate on Node Depth:

- BFS searches layers of increasing node depth.
- IDS searches subtrees of increasing node depth.

(2) Iterate on Path Cost + Heuristic Function:

- A* searches layers of increasing path cost + heuristic function.
- IDA* searches subtrees of increasing path cost + heuristic function.

Recap: Search Methods

Which cost function?

- UCS searches layers of increasing path cost.
- Greedy best first search searches layers of increasing heuristic function.
- A* search searches layers of increasing path cost + heuristic function.

Local search

- Search algorithms seen so far are designed to **explore search spaces systematically.**
- Problems: observable, deterministic, known environments
- where the solution is a sequence of actions.
- Real-World problems are more complex.
- When a goal is found, the path to that goal constitutes a solution to the problem. But, depending on the applications, **the path may or may not matter**.
- If the path does not matter/systematic search is not possible, then consider another class of algorithms.

Local search

- In such cases, we can use iterative improvement algorithms, **Local search**.
- Also useful in pure **optimization problems** where the goal is to find the best state according to an **optimization function**.
- **Examples:**
  - Integrated circuit design, telecommunications network optimization, etc.
  - 8-queen: what matters is the final configuration of the puzzle, not the intermediary steps to reach it.

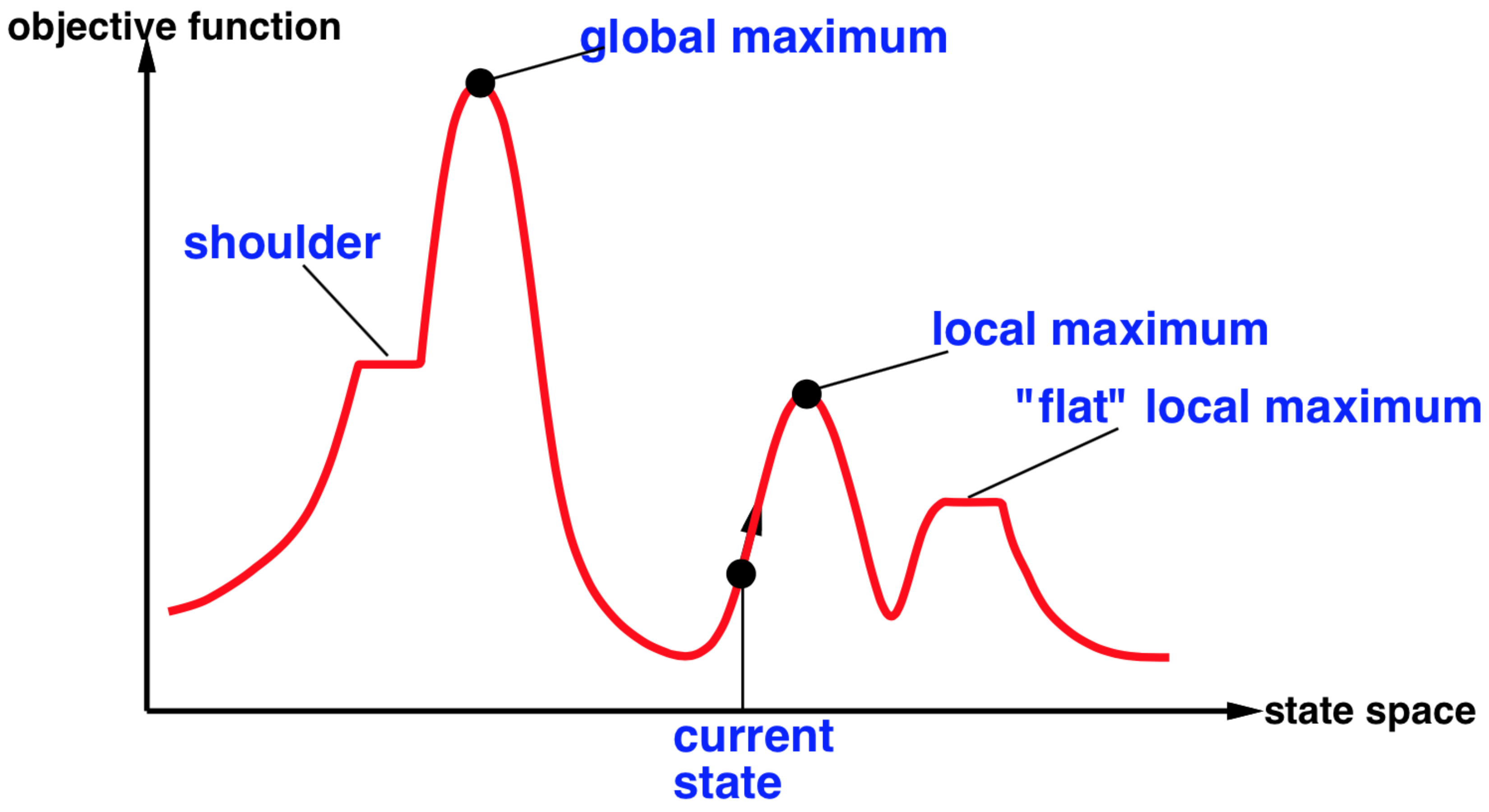
Local search

- **Idea**: keep a single “current” state, and try to improve it.
- Move only to neighbors of that node.
- **Advantages**:
  1. No need to maintain a search tree.
  2. Use very little memory.
  3. Can often find good enough solutions in continuous or large state spaces.

Local search

- **Local Search Algorithms:**
  - Hill climbing (steepest ascent/descent).
  - Simulated Annealing: inspired by statistical physics.
  - Local beam search.
  - Genetic algorithms: inspired by evolutionary biology.

Local search



State space landscape

Hill climbing

- Also called **greedy local search**.
- Looks only to immediate good neighbors and not beyond.
- Search moves uphill: moves in the direction of increasing elevation/value to find the top of the mountain.
- Terminates when it reaches a **peak**.
- Can terminate with a local maximum, global maximum or can get stuck and no progress is possible.
- A **node is a state and a value**.

Hill climbing: Pseudo-code

```
function Hill-Climbing(initialState)
    returns State that is a local maximum

    initialize current with initialState

    loop do
        neighbor = a highest-valued successor of current

        if neighbor.value ≤ current.value:
            return current.state

        current = neighbor
```

Hill climbing

Other variants of hill climbing include

- **Sideways moves** escape from a plateau where best successor has same value as the current state.
- **Random-restart** hill climbing overcomes local maxima: keep trying! (either find a goal or get several possible solution and pick the max).
- **Stochastic** hill climbing chooses at random among the uphill moves.

Hill climbing

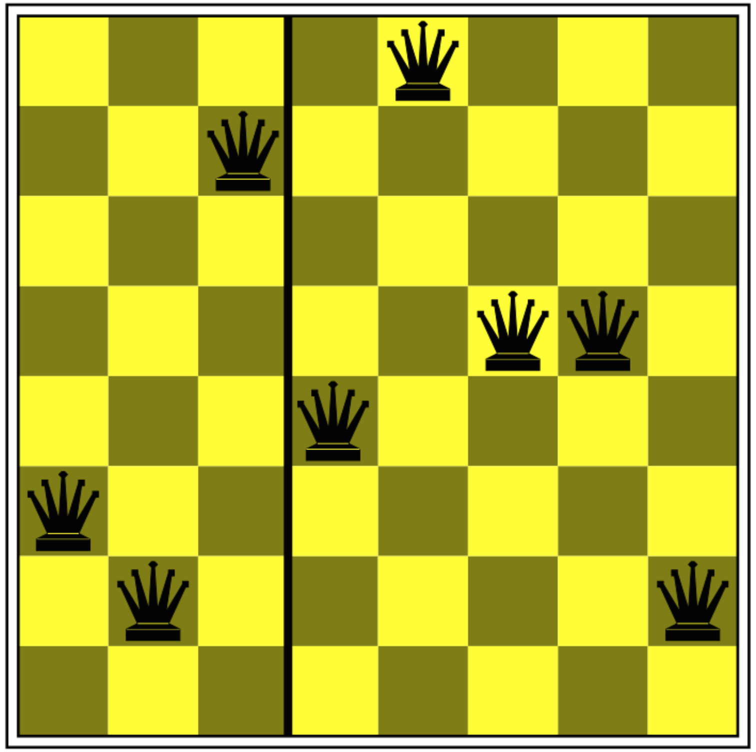
- **Hill climbing** effective in general but depends on shape of the landscape. Successful in many real-problems after a reasonable number of restarts.
- **Local beam search** maintains *k* states instead of one state. Select the *k* best successor, and useful information is passed among the states.
- **Stochastic beam search** choose *k* successors are random. Helps alleviate the problem of the states agglomerating around the same part of the state space.

Genetic algorithms

- **Genetic algorithm (GA)** is a variant of stochastic beam search.
- Successor states are generated by combining two parents rather by modifying a single state.
- The process is inspired by **natural selection**.
- Starts with *k* **randomly generated states**, called **population**. Each state is an **individual**.
- An individual is usually represented by a **string** of 0's and 1's, or digits, a finite set.
- The objective function is called **fitness function**: better states have high values of fitness function.

Genetic algorithms

- In the 8-queen problem, an individual can be represented by a **string** digits 1 to 8, that represents the position of the 8 queens in the 8 columns.



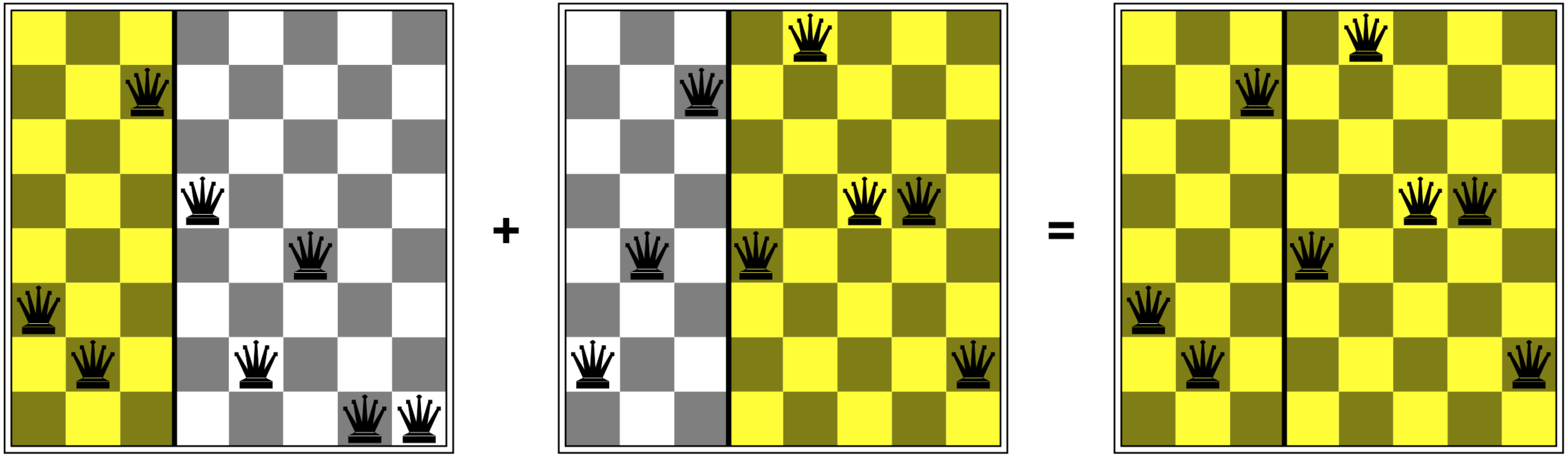
Genetic algorithms

- The objective function is called **fitness function**: better states have high values of fitness function.
- Possible fitness function is the **number of non-attacking pairs of queens.**
- Fitness function of the solution: 28.

Genetic algorithms

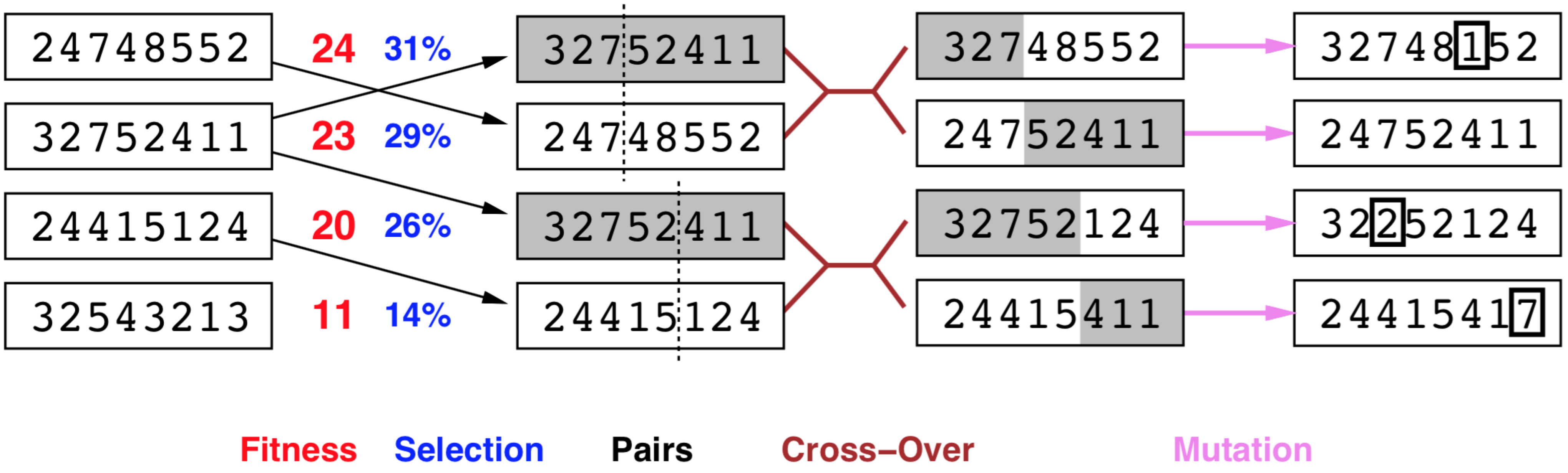
- Pairs of individuals are selected at random for **reproduction** w.r.t. some probabilities.
- A **crossover** point is chosen randomly in the string.
- **Offspring** are created by crossing the parents at the crossover point.
- Each element in the string is also subject to some **mutation** with a small probability.

Genetic algorithms



Genetic algorithms

Generate successors from pairs of states.



Genetic algorithms: Pseudo-code

```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     for i = 1 to SIZE(population) do
        x ← RANDOM-SELECTION(population, FITNESS-FN)
        y ← RANDOM-SELECTION(population, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) then child ← MUTATE(child)
        add child to new_population
     population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
```

```
function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x); c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

To be continued

Simulated Annealing

- Hill Climbing → Simulated Annealing

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  current ← problem.INITIAL
  while true do
    neighbor ← a highest-valued successor state of current
    if VALUE(neighbor) ≤ VALUE(current) then return current
    current ← neighbor
```

Simulated Annealing

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  $current \leftarrow problem$.INITIAL
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if T = 0 **then return** $current$
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
    if $\Delta E < 0$ **then** $current \leftarrow next$ //for maximization
    else $current \leftarrow next$ only with probability $e^{-\Delta E/T}$

To be continued

Further Studies on Metaheuristics

- **[Question]** What are the differences between a heuristic and a metaheuristic?

Comparison

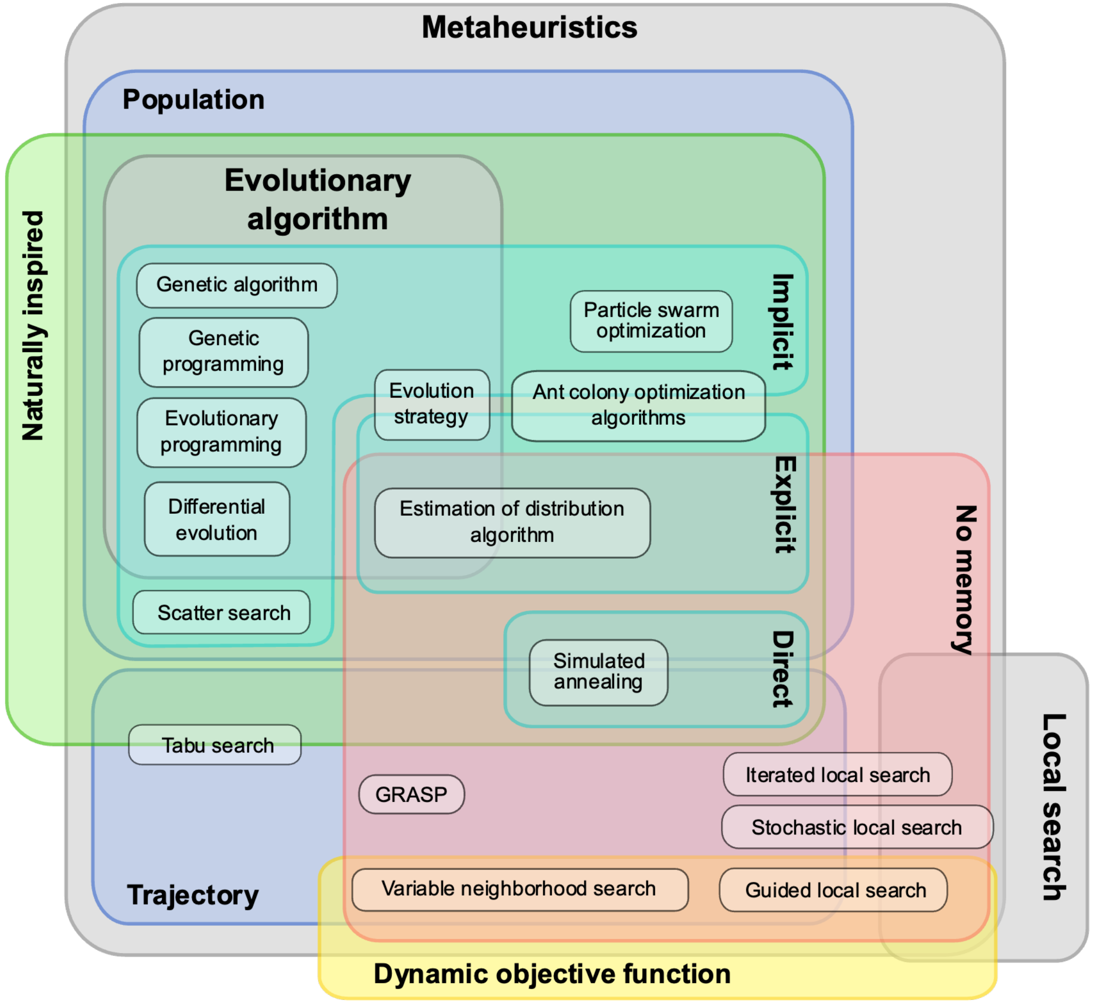
Heuristic

- Problem-specific
- Can be used by a metaheuristic

Metaheuristic

- Problem-independent
- Can use different heuristics or a combination of heuristics

Metaheuristics



“One general law, leading to the advancement of all organic beings, namely, multiply, vary, let the strongest live and weakest die.”

\- Charles Darwin, *The Origin of Species*

Reproduction (crossover)

Mutation

"One general law, leading to the advancement of all organic beings, namely, multiply, vary, let the strongest live and weakest die."

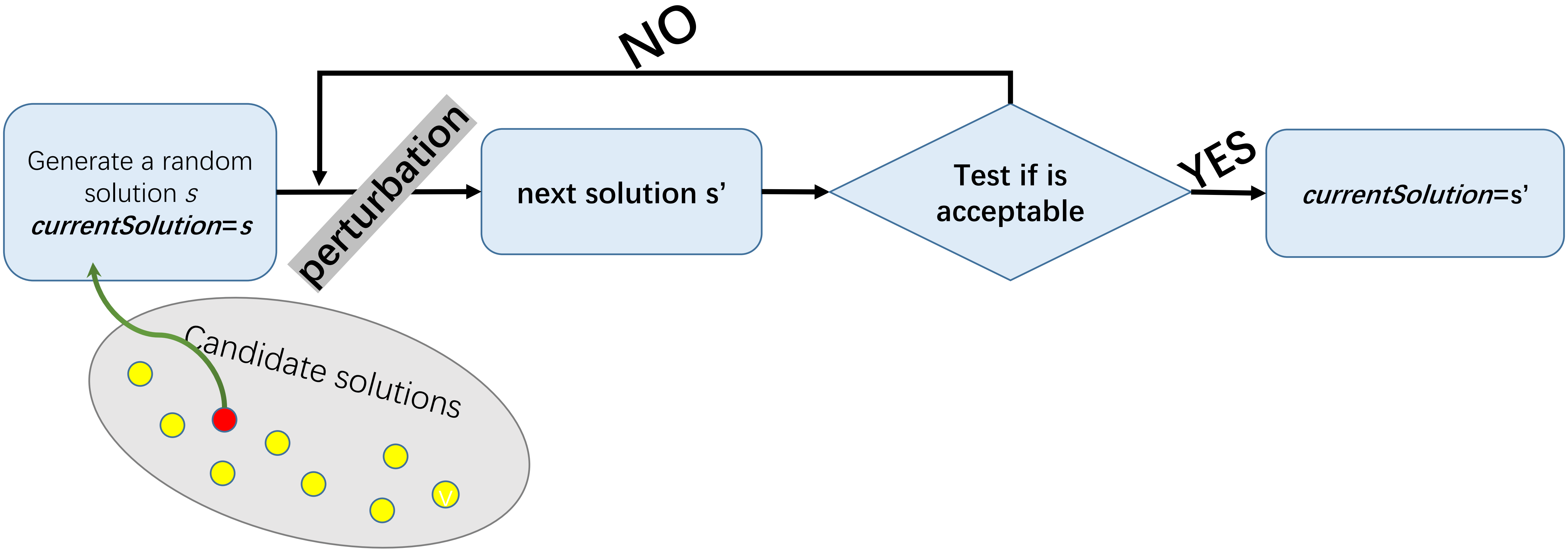
- Charles Darwin, *The Origin of Species*

Selection

Evolutionary Computation

- It is the study of computational systems which use ideas and get inspirations from natural evolution.
- One of the principles borrowed is ***survival of the fittest***.
- Evolutionary computation (EC) techniques can be used in optimisation, learning, and design.
- EC techniques do **not** require rich domain knowledge to use. However, domain knowledge can be incorporated into EC techniques.

Generate-and-Test (G&T)

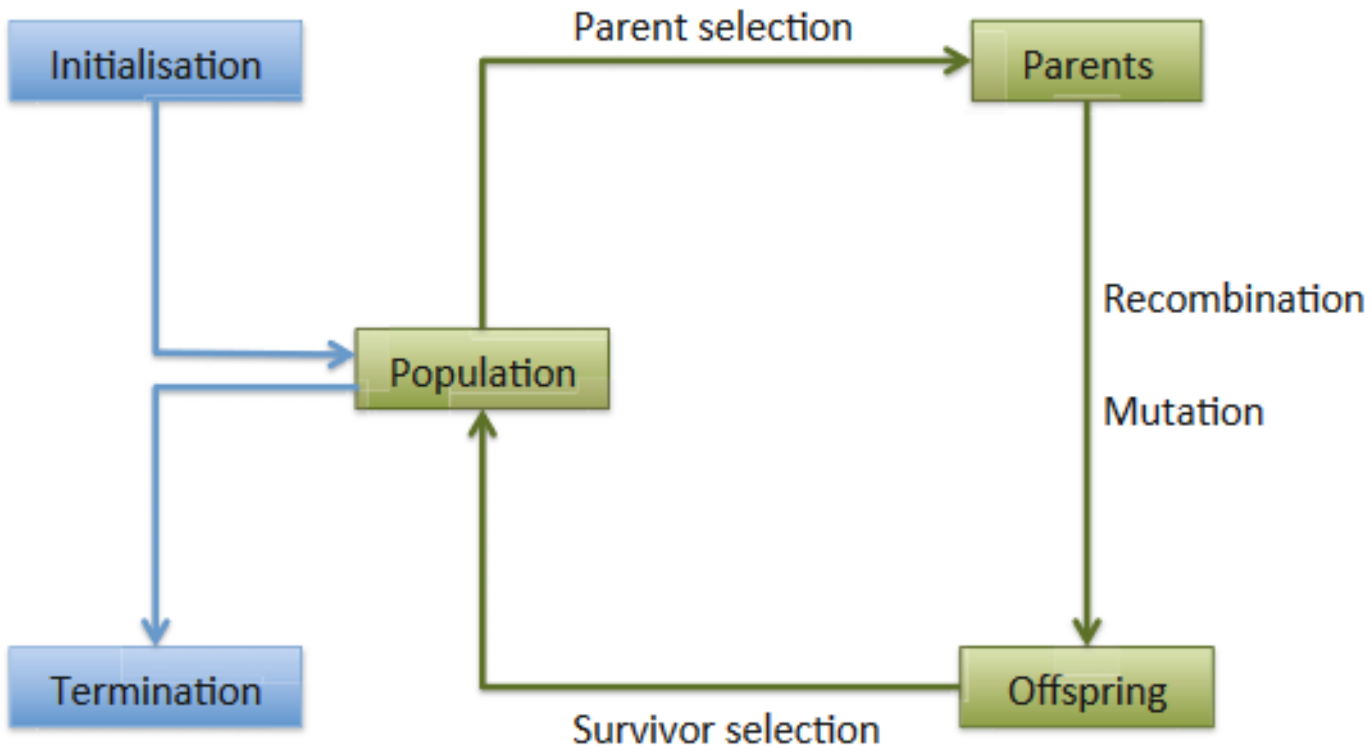


Generate-and-Test: Steps

1. Generate the initial solution at random and denote it as the **current solution**.
2. Generate the **next solution** from the current one by **perturbation**.
3. Test whether the newly generated solution (**next solution**) is acceptable;
    1. Accepted it as the current solution **if yes**;
    2. Keep the current solution unchanged **otherwise**.
4. Go to Step 2 if the current solution is not satisfactory, stop otherwise.

EA: Population-based G&T

- **Generate**: Mutate and/or recombine individuals in a population.
- **Test**: Select the next generation from the parents and offspring.



A Simple Evolutionary Algorithm (EA)

```
Generate the initial population P(0) at random
i ← 0            // Generation counter
WHILE halting criteria are not satisfied
Evaluate the fitness of each individual in P(i)
Select parents from P(i) based on their fitness in P(i)
Generate offspring from the parents using crossover and
        mutation to form P(i + 1)
i ← i + 1
```

So how does this simple EA work?

Illustration Example

Let's use the simple **EA with population size 4** to maximise the function

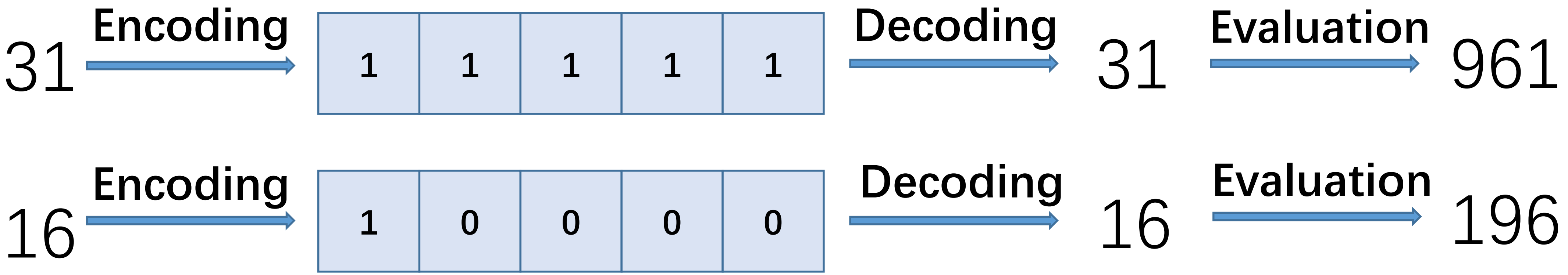
$$f(x) = x^2$$

with x in the *integer* interval $[0, 31]$, i.e., $x = 0, 1, \dots, 30, 31$.

- Population size = 4 ⇔ 4 individuals/chromosomes
- So, what is an **individual** or **chromosome**?

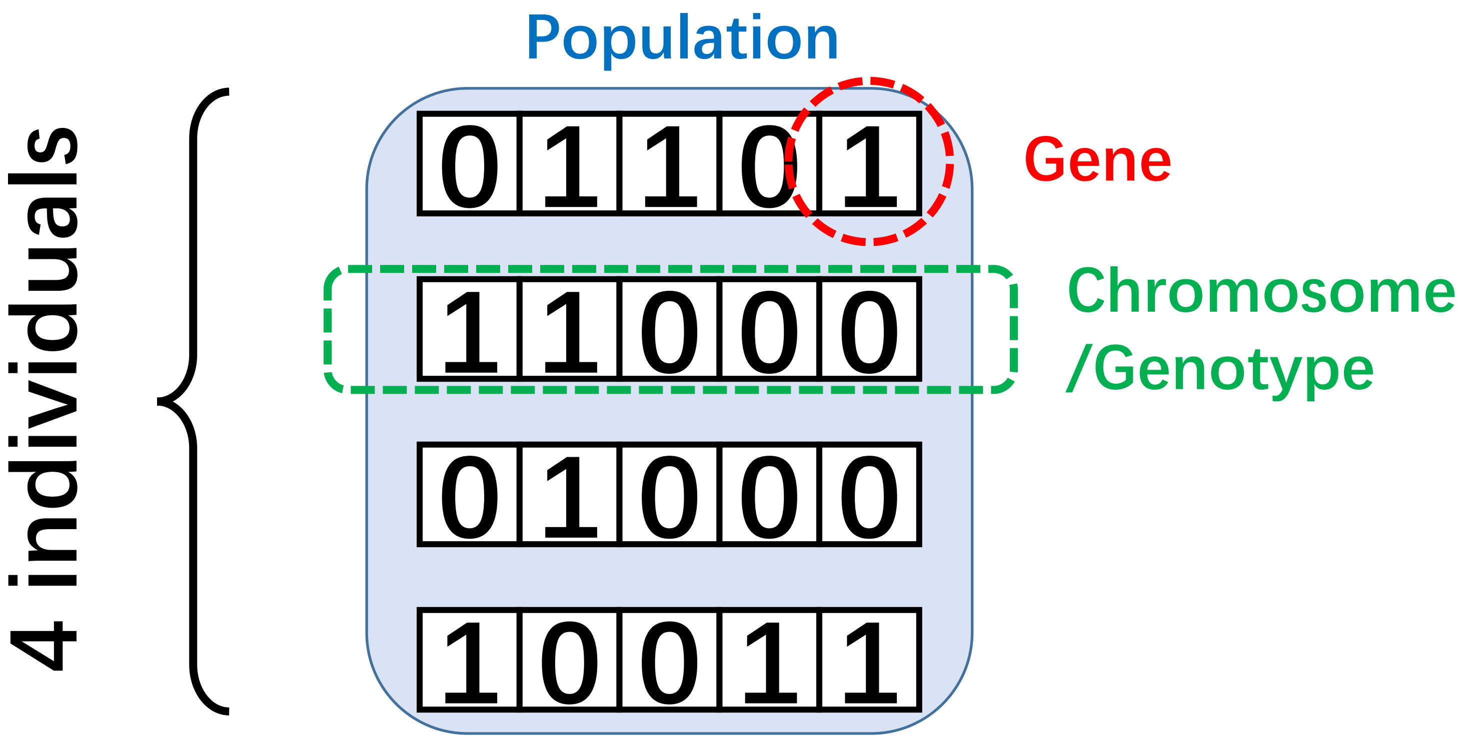
[Example] Encoding and decoding

- **<u>Representation</u>**: The first step of EA applications is ***encoding*** (i.e., the representation of chromosomes).
  - We adopt binary representation for integers.
  - 5 bits are used to represent integers up to 31.
  - Examples:



[Example] EA: Step 1

1. **Initialisation**: Generate initial population at random, e.g., 01101, 11000, 01000, 10011. These are *chromosomes* or *genotypes*.



[Example] EA: Step 2

2. **Evaluation**: Calculate fitness value for each individual.

   a) **Decode** the individual into an integer (called ***phenotypes***):

      01101 -> 13; 11000 -> 24, 01000 -> 8, 10011 -> 19;

   b) **Evaluate** the fitness according to $f(x) = x^2$:

      $\boldsymbol{f(13) = 169, f(24) = 576, f(8) = 64, f(19) = 361.}$

[Example] EA: Step 3-a

3. **Crossover**:
   a) Select two individuals for crossover based on their fitness. If **roulette-wheel selection** is used, then

$$P_i = \frac{f_i}{\sum_j f_j}$$

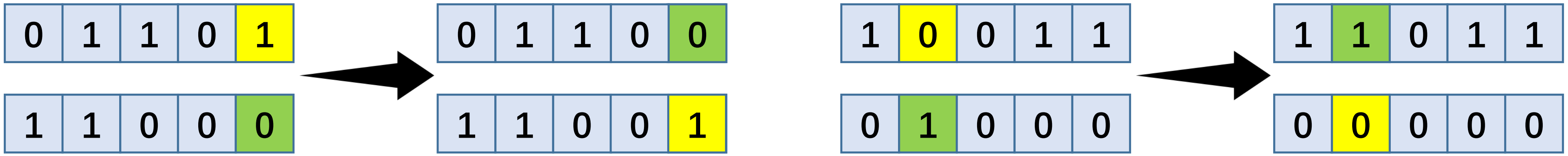
Two offspring are often produced and added to an intermediate population. Repeat this step until the intermediate population is filled.

In our example:

$$P_1(13) = \frac{169}{1170} = 0.14,\ P_2(24) = \frac{576}{1170} = 0.49,\ P_3(8) = \frac{64}{1170} = 0.06,\ P_4(19) = \frac{361}{1170} = 0.31$$

[Example] EA: Step 3-b

b) Examples of **crossover**



Now the intermediate population is 01100, 11001, 11011, 00000.

EA: Steps 4-5

4. Apply **mutation** to individuals in the intermediate population with a *small* probability. A simple mutation is bit-flipping. For example, we may have the following new population $P(1)$ after random mutation:

Example:

0	1	1	0	1		1	1	0	1	1
1	1	0	0	1		1	0	0	0	0

5. Go to step 2 if not stop.

Different Evolutionary Algorithms

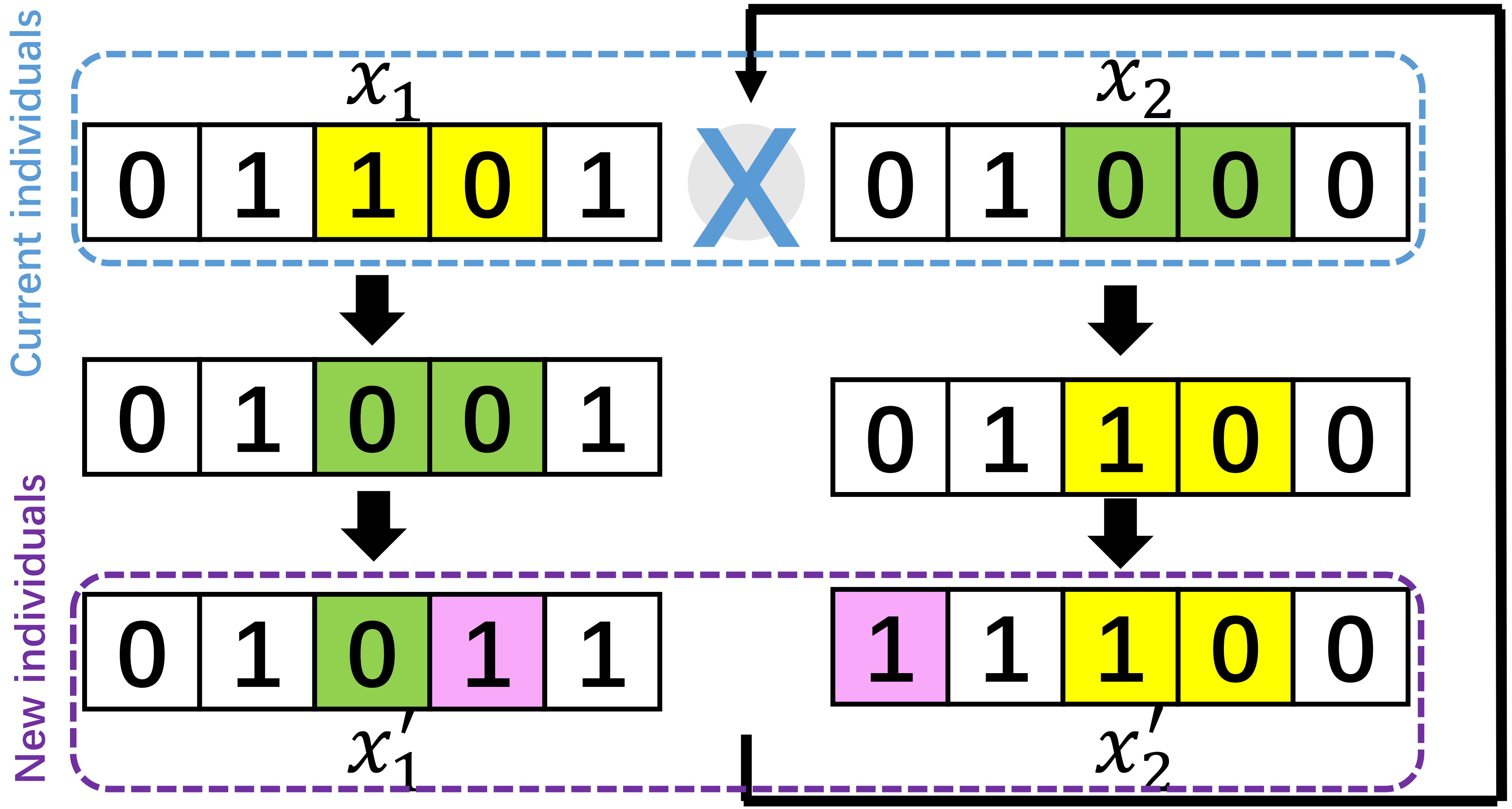
- There are several well-known EAs with different
  - historical backgrounds,
  - representations,
  - variation operators,
  - and selection schemes.

In fact, EAs refer to **a whole family of algorithms**, not a single algorithm.

EA families

- Genetic Algorithms (GAs)
- Evolutionary Programming (EP)
- Evolution Strategies (ES)
- Genetic Programming (GP)
- …

Genetic Algorithms (GAs)



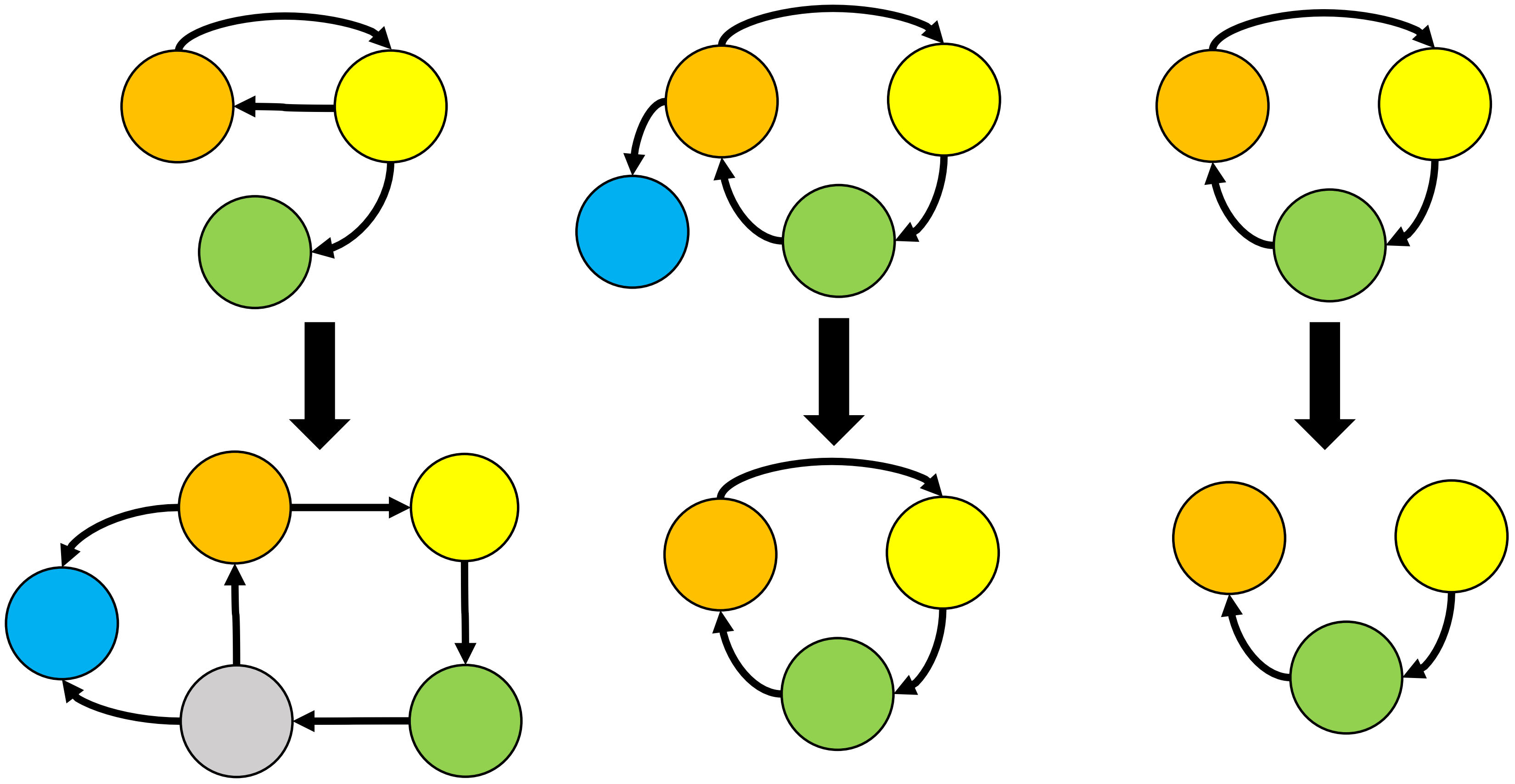
Genetic Algorithms (GAs)

- First formulated by Holland for adaptive search and by his students for optimisation from mid 1960s to mid 1970s.
- Binary strings have been used extensively as individuals (***chromosomes***).
- Simulate **Darwinian evolution**.
- Search operators are only applied to the ***genotypic*** representation (chromosome) of individuals.
- Emphasise the role of **recombination** (***crossover***). Mutation is only used as a background operator.
- Often use **roulette-wheel** selection.

Sketch of the simple GA

Representation	Bit-strings
Recombination	1-Point crossover
Mutation	Bit flip
Parent selection	Fitness proportional - implemented by Roulette Wheel
Survival selection	Generational

Evolutionary Programming (EP)



Evolutionary Programming (EP)

- First proposed by Fogel *et al.* in mid 1960s for simulating intelligence.
- **Finite state machines** (FSMs) were used to represent individuals, although real-valued vectors have always been used in numerical optimisation.
- It is closer to **Lamarckian evolution**.
- Search operators (mutations only) are applied to the ***phenotypic*** representation of individuals.
- It does ***not*** use any recombination.
- Usually use **tournament** selection.

Evolution Strategies (ES)

$$[x_0^{(1)}, x_1^{(1)}, \ldots, x_{d-1}^{(1)}] \qquad [x_0^{(2)}, x_1^{(2)}, \ldots, x_{d-1}^{(2)}]$$

$$[x'^{(1)}_0, x'^{(1)}_1, \ldots, x'^{(1)}_{d-1}] \qquad [x'^{(2)}_0, x'^{(2)}_1, \ldots, x'^{(2)}_{d-1}]$$

Evolution Strategies (ES)

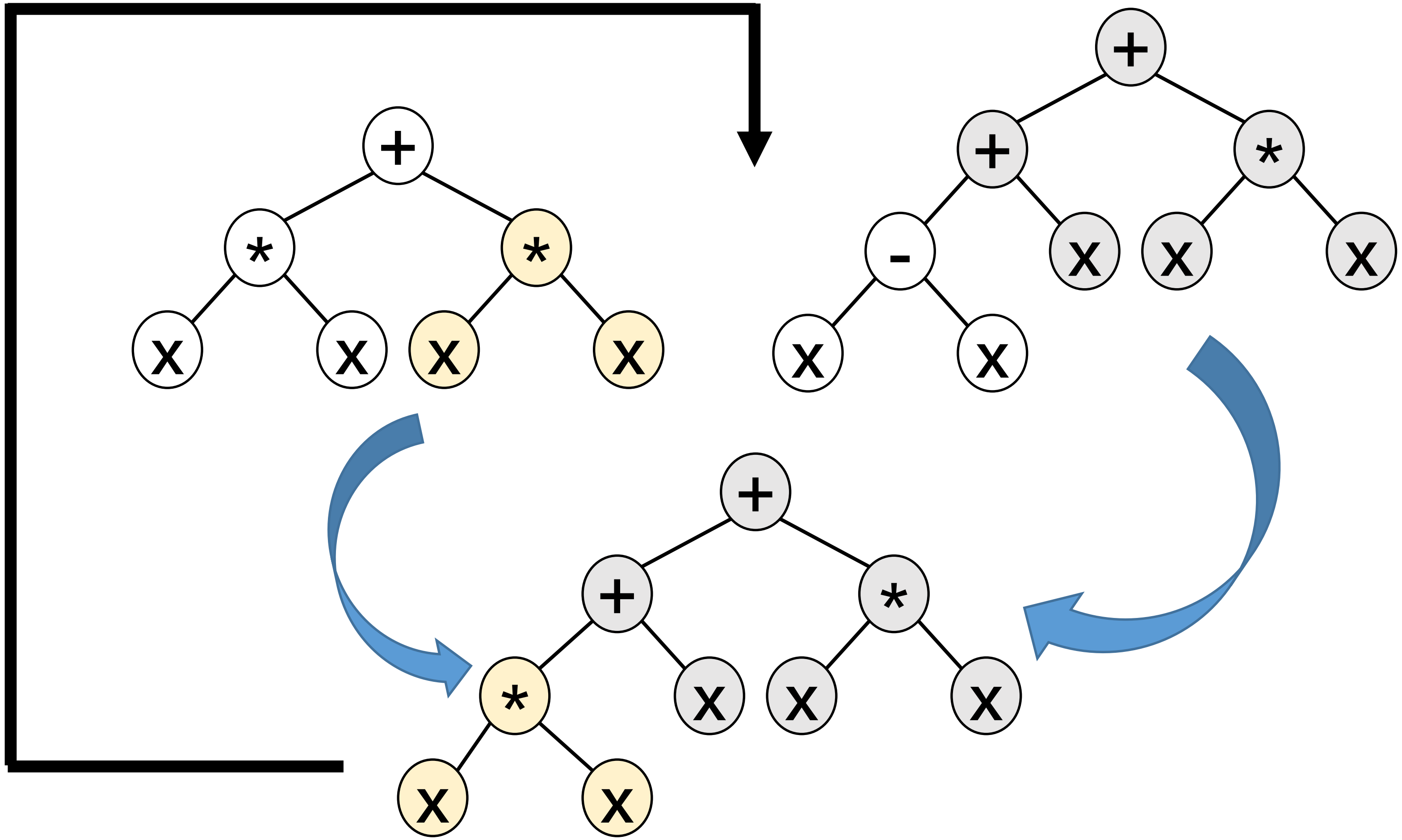
- First proposed by Rechenberg and Schwefel in mid 1960s for numerical optimisation.
- **Real-valued vectors** are used to represent individuals.
- They are closer to **Larmackian evolution**.
- They do have recombination.
- They use ***self-adaptive mutations***.

Sketch of the simple ES

Representation	Real-valued vectors
Recombination	Discrete or intermediary
Mutation	Gaussian perturbation
Parent selection	Uniform random
Survivor selection	Deterministic elitist replacement by (μ, λ) or $(\mu + \lambda)$
Speciality	Self-adaptation of mutation step sizes

- μ: parents size
- λ: offspring size

Genetic Programming (GP)

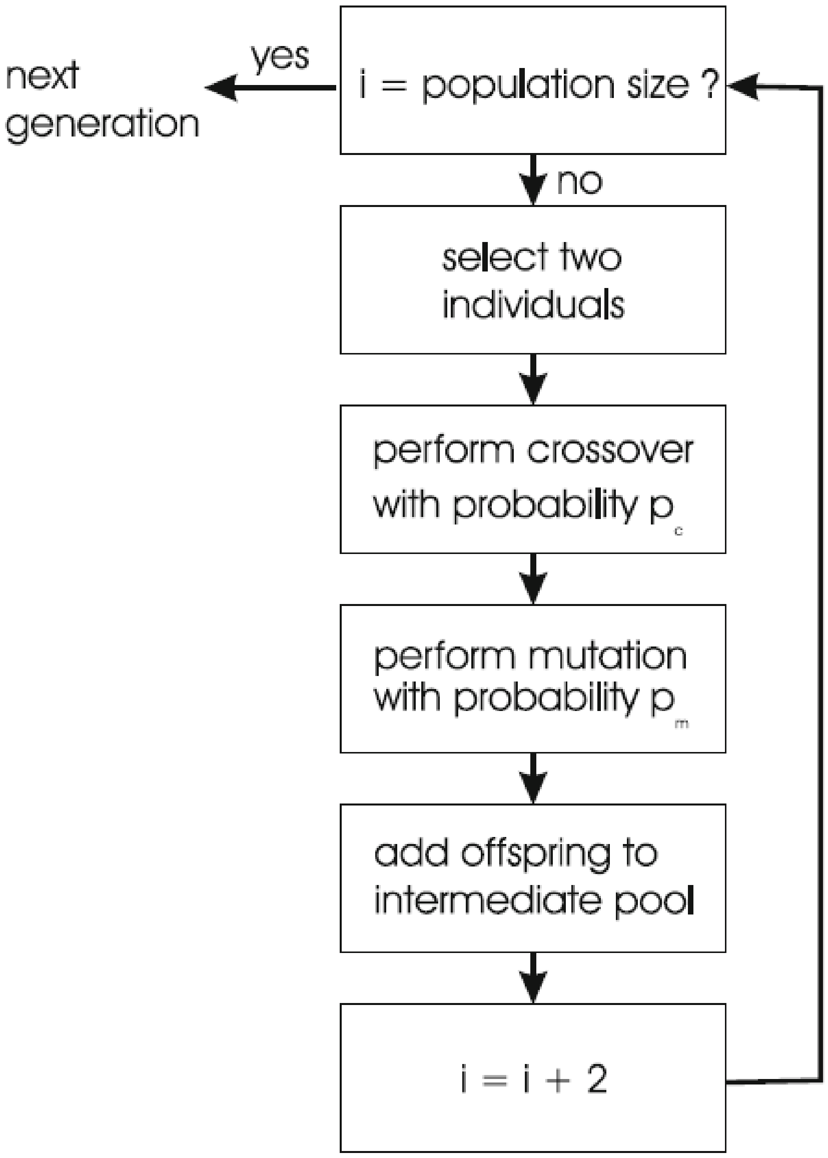


Genetic Programming (GP)

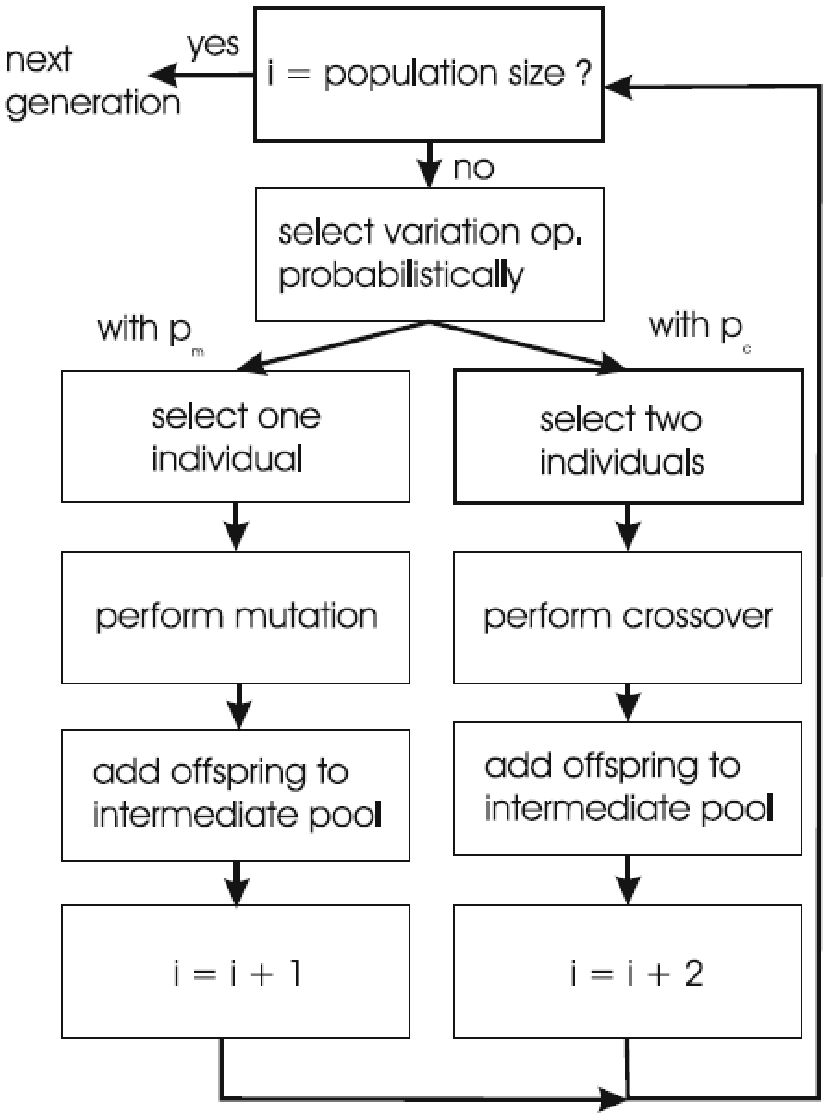
- First used by de Garis to indicate the evolution of artificial neural networks, but used by Koza to indicate the evolution of computer programs.

- **Trees** (especially Lisp expression trees) are often used to represent individuals.

- Both *crossover* and *mutation* are used.



GA loop



GP loop

Preferred Term: Evolutionary Algorithms

- EAs face the same fundamental issues as those classical AI faces, i.e., representation, and search.

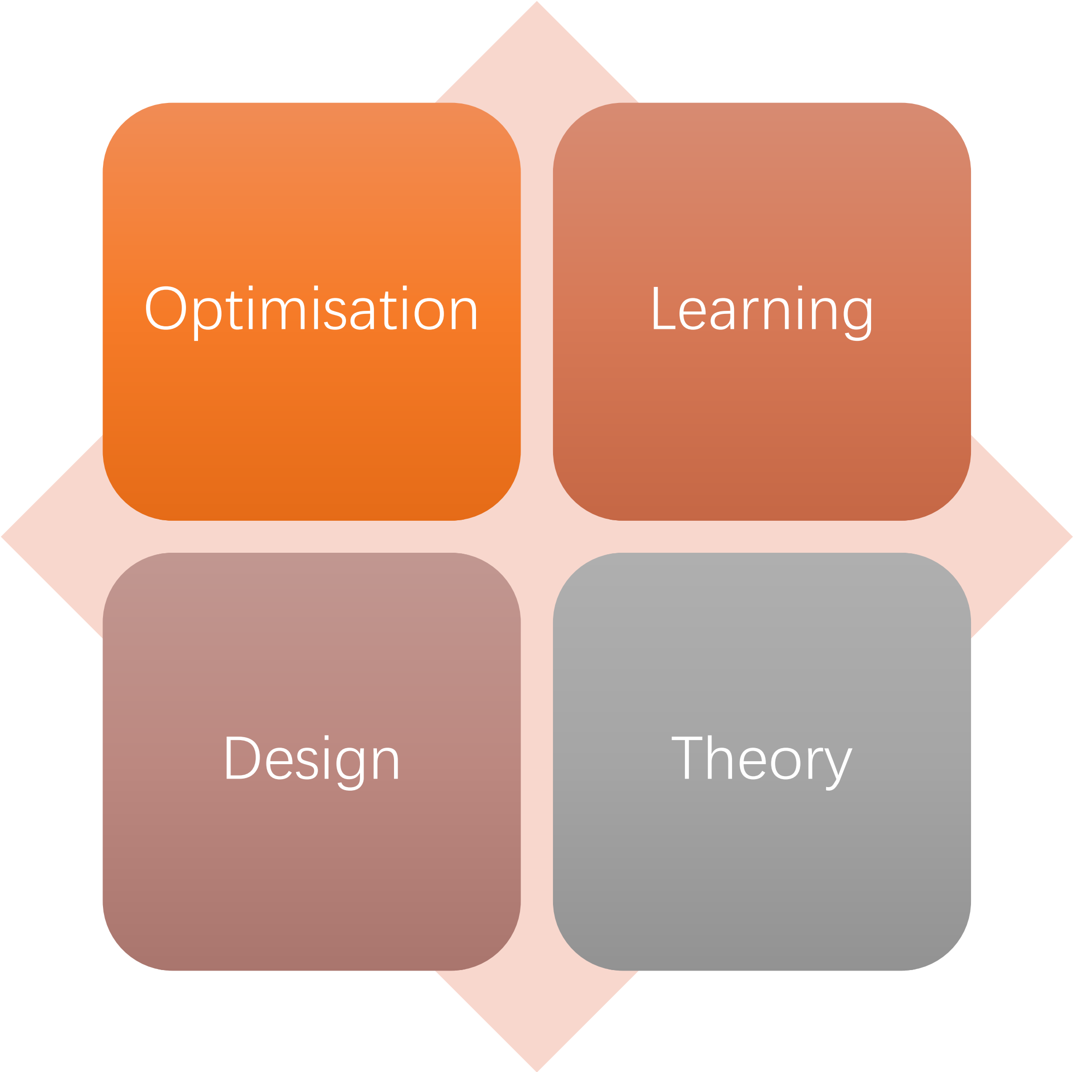
- Although GAs, EP, ES, and GP are different, they are all different variants of **population-based** **generate-and-test** algorithms. They share more similarities than differences!

- A better and more general term to use is evolutionary algorithms (EAs).

Variations in Operators

- **Crossover/Recombination**: one-point crossover, two-point crossover, uniform crossover, intermediate crossover, etc.
- **Mutation**: bit-flipping, Gaussian mutation, Cauchy mutation, etc.
- **Selection**: roulette wheel selection (fitness proportional selection), tournament selection, rank-based selection (linear and nonlinear), etc.
- **Replacement Strategy**: generational, steady-state (continuous), etc.
- **Specialised Operators**: multi-parent recombination, inversion, order-based crossover, etc.

Major Areas in EC



Evolutionary Optimisation

- Numerical (global) optimisation.
- Combinatorial optimisation (of NP-hard problems).
- Mixed optimisation.
- Constrained optimisation.
- Multi-objective optimisation.
- Optimisation in a dynamic environment (with a dynamic fitness function).

Evolutionary Learning

Evolutionary learning can be used in supervised, unsupervised and reinforcement learning.

- Learning classifier systems (Rule-based systems).
- Evolutionary artificial neural networks.
- Evolutionary fuzzy logic systems.
- Co-evolutionary learning.

Evolutionary Design

EC techniques are particularly good at exploring unconventional designs which are very difficult to obtain by hand.

- Evolutionary design of artificial neural networks.
- Evolutionary design of electronic circuits.
- Evolvable hardware.
- Evolutionary design of (building) architectures.

Some Applications of EA

- High-speed train head design (Japan)

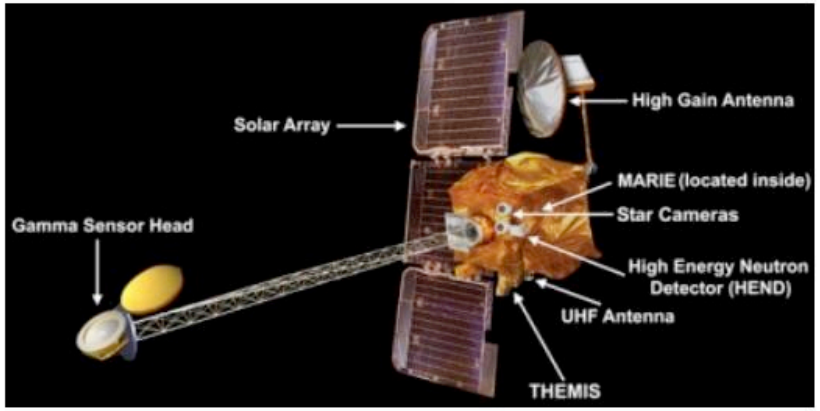
Series 700 Designed by human

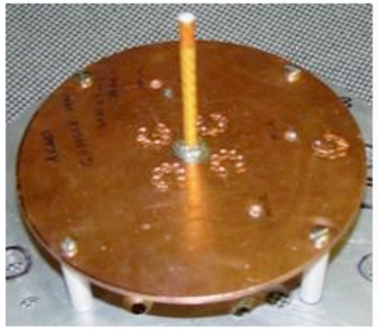
Series N700 Designed by EA

- Save 19% energy…30 increase in the output…

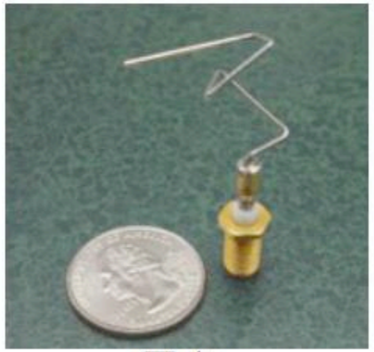
Some Applications of EA

- X-Band Antenna Design (NASA, US)





Human

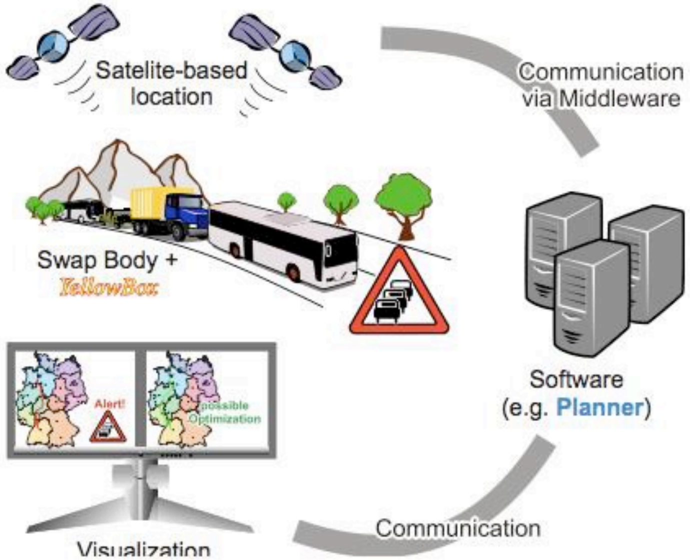


EA

- Increase efficiency from 38% to 93%

Some Applications of EA

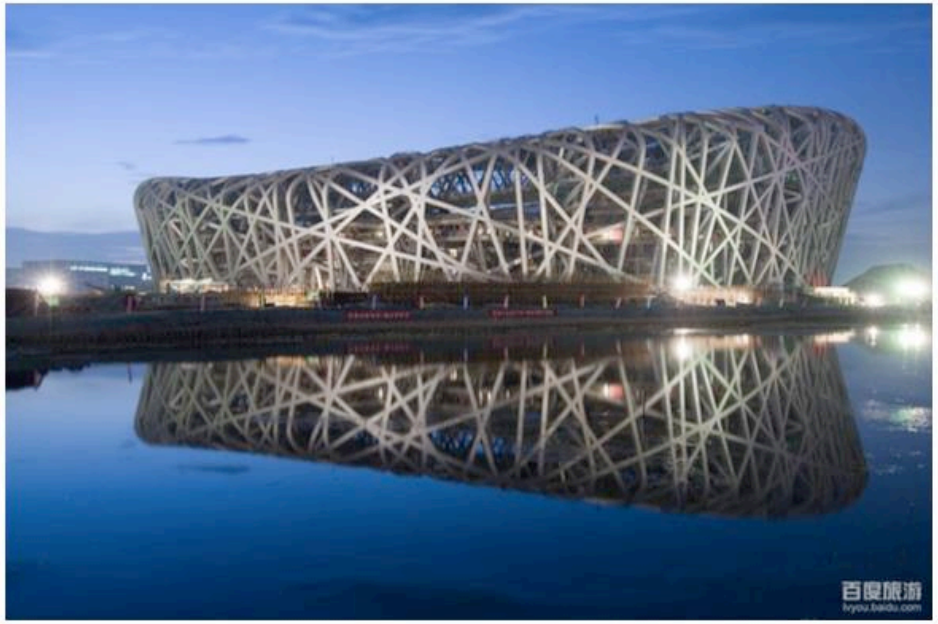
- Transportation Planning System (DHL, Germany)

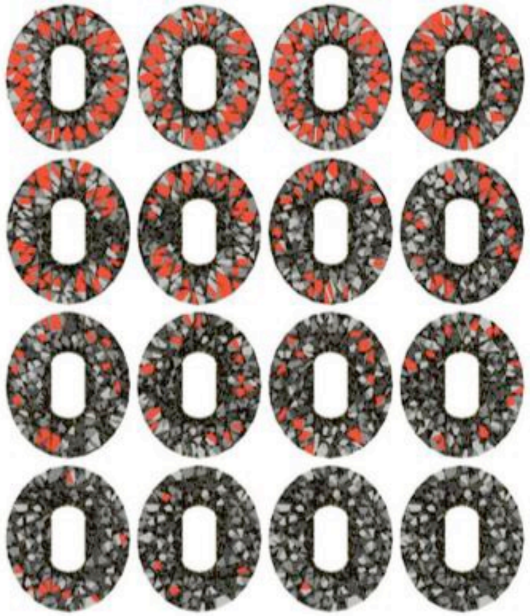


- Save 9% of the transportation costs.

Some Applications of EA

- Birds Nest (China & Switzerland)





- The irregular ordering of the beams poses an insoluble problem for the then-current CAD tools.

Summary

- Evolutionary algorithms can be regarded as population-based generate-and-test algorithms.
- Evolutionary computation techniques can be used in optimisation, learning and design.
- Evolutionary computation techniques are flexible and robust.
- Evolutionary computation techniques are definitely useful tools in your toolbox, but there are problems for which other techniques might be more suitable.

To be continued