SELF-CHECK FOR MID-TERM

1. Write the definitions of (be sure you can write down ALL of them independently):

(1) vector spaces;
(2) subspaces;
(3) sum of vector spaces;
(4) direct sum of vector spaces;
(5) span lists;
(6) linearly independent lists;
(7) a basis of a vector space;
(8) the dimension of a finite dimensional vector space;
(9) linear maps;
(10) $\mathcal{L}(V, W)$, with the operations defined on this vector space;
(11) the null space of a linear map;
(12) the range of a linear map;
(13) injectivity of a linear map (as well as an quivalent condition);
(14) surjectivity of a linear map;
(15) matrix (with respect to bases);
(16) invertible linear maps;
(17) an isomorphism of vector spaces;
(18) isomorphic vector spaces;
(19) the matrix of a vector (with respect to a basis);
(20) operators;
(21) products of vector spaces, with the operations defined on this vector space;
(22) affine subsets;
(23) quotient spaces, with the operations defined on this vector space;
(24) quotient maps;
(25) linear functionals;
(26) dual spaces;
(27) a dual basis (with respect to a basis);
(28) dual maps;
(29) annihilators;
(30) the rank of a matrix;

(31) the degree of a polynomial;
(32) $\mathcal{P}_m(\mathbb{F})$;
(33) invariant subspaces;
(34) eigenvalues and eigenvectors;
(35) restriction operators and quotient operators;
(36) $p(T)$ with some $p \in \mathcal{P}(\mathbb{F})$;
(37) the matrix of an operator (with respect to a basis);
(38) upper-triangular matrices (as well as some equivalent conditions);
(39) eigenspaces;
(40) diagonalizability of an operator (as well as some equivalent conditions).

2. Think about how to find:
   (1) $\mathcal{M}(T, (v_1, \ldots, v_m), (w_1, \ldots, w_n))$;
   (2) $\mathcal{M}(v)$, with respect to a basis $v_1, \ldots, v_m$;
   (3) $\mathcal{M}(Tv)$, with respect to a basis $w_1, \ldots, w_n$;
   (4) eigenvalues, and the corresponding eigenvectors and eigenspaces;
   (5) given a vector space V and its subspace U, how to find U such that $U \oplus W = V$;
   (6) an isomorphism of two vector spaces (may not be finite-dimentional).

3. Think about how to prove:
   (1) a set with some operations to be a vector space;
   (2) subspace of a vector space;
   (3) a sum is a direct sum (may be more than two summands);
   (4) a list is a span list;
   (5) a list is a linearly independent list;
   (6) a list is a basis;
   (7) a map is a linear map;
   (8) injectivity and surjectivity of a linear map;
   (9) a map to be an isomorphism;
   (10) a list in the dual space to be a dual basis (with respect to a given basis);
   (11) a subspace to be invariant;
   (12) an operator to be diagonalizable.

4. How to use dimesions in your proof if you are dealing with a finite-dimensional case? **Note: when dealing with an infinite-dimensional case, you CANNOT use dimensions any more! Be sure what properties need "finite-dimensional" as a precondition!**

5. GOOD LUCK!