MA215 Probability Theory

Assignment 04

1. For each of the following statements, say whether true or false. For false statements, give the correct version of the statement.

   (i) $P(A \cap B) = P(A) \times P(B)$ if A, B are independent.

   (ii) $P(A \cup B) = P(A) + P(B)$ if A, B are independent.

   (iii) In a sequence of n independent identical trials, each of which results in either "success" or "failure", with probability θ of success, the number of successes follows a Bernoulli distribution.

2. In five independent tosses of an unbiased coin, find

   (i) the probability that the total number of heads is even;

   (ii) the probability that there are exactly five heads.

   (Note: zero is also a even number.)

3. A discrete random variable X has possible values -2, 1, 3, 4 with probabilities satisfying

   $$P(X = -2) = P(X = 1) = 2P(X = 3) = 3P(X = 4).$$

   Find the probability mass function and the (cumulative) distribution function of X, and graph them both.

4. The following table shows the probability mass function of a discrete random variable X. Plot the (cumulative) distribution function of this random variable.

k	1	2	3	4	5
$P(X = k)$	0.1	0.2	0.4	0.1	0.2

5. Suppose $F(x)$ is the c.d.f. of a random variable X. Show that $F(x)$ has the following properties:

   (i) $0 \leqslant F(x) \leqslant 1$;

   (ii) $F(x)$ is an increasing function of x, i.e., $F(x) \leqslant F(y)$ for any $x < y$;

   (iii) $\lim_{x \to +\infty} F(x) = 1$; $\lim_{x \to -\infty} F(x) = 0$;

   (iv) Show that $F(x)$ is a right-continuous function of $x \in R$: (Just show that if a sequence of real numbers $x_n \downarrow x$, then $\lim_{n \to \infty} F(x_n) = F(x)$).