# *Learning to Trade via Direct Reinforcement*

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# Reinforcement Learning Methods

Value Learning

- Learns a value function $V(state, action)$
- Optimal action is implicit: $a^* = \arg\max_a V(state, a)$
- Ex: Q-Learning, TD-Learning, Advantage Updating
- Suitable in environments where rewards are not immediate: Grid World, Checkers

# Reinforcement Learning Methods

Value Learning

- Learns $V(state, action)$
- Optimal action is implicit.
- Ex: Q-Learning
- Suitable for delayed rewards

Direct Reinforcement Learning

- No value function
- Learns action policy directly
- Ex: Recurrent Reinforcement Learning (RRL) given by Moody
- Suitable in environments where rewards are immediate: Control problems, stock markets

# Computational Finance Methods

Typical Approach

1) Learn to predict future prices
2) Take into account transaction costs, risk, etc...
3) Make trading decision

Direct Approach

1) Learn trading strategy

# Notation

- Our agent trades fixed quantities of a security $z$.

- The price series is $\{z_1, z_2, \ldots z_t, \ldots z_T\}$ and corresponding price changes $r_t = z_t - z_{t-1}$

- At each time step, our position is
  $$F_t \in \{long, neutral, short\} = \{+1, 0, -1\}$$

# Notation (cont.)

- We wish to learn a trading strategy $F_t = F(\theta_t; F_{i<t}, I_t)$
  $\theta$ is the set of parameters we are learning
  $I_t = \{z_{i \leq t}, etc...\}$ is our **Information** at time $t$

- For example, a system with $m+1$ "autoregressive inputs":
  $F_t = sign(uF_{t-1} + v_0 r_t + v_1 r_{t-1} + \cdots v_m r_{t-m} + w)$
  $\theta = \{u, v_i, w\}$

- Whatever functional form used, $F(\theta)$ must be differentiable $(dF/d\theta)$.

# Profit & Wealth

- Daily Return: $R_t = r_t \cdot F_{t-1} - \delta |F_t - F_{t-1}|$
  $\delta$ is the transaction cost

- Total Profits: $P_T = \sum_{t=1}^{T} R_t$
  i.e. no reinvesting

- Wealth: $W_T = W_0 + P_T$

- Utility: $U_T = U(R_1, \ldots R_T; W_0)$
  Ex: $U_T = W_T$ measures profit without regard to risk

# Measuring Utility: Sharpe Ratio

- Let $U_T$ be the Sharpe Ratio: $S_T = \frac{Average(R_t)}{StandardDeviation(R_t)}$
- To maximize $U_T$, we will need an expression for $dU_T/dR_t$.
- But $dS_T/dR_t$ is hard to work with.
- Instead, Moody comes up with an approximation...

# Approximating the Sharpe Ratio

- First, let us define "exponential moving estimates" $A_t = E(R_t)$ and $B_t = E(R_t^2)$.

$$A_t = A_{t-1} + \eta \Delta A_t \quad \left(\frac{dA_t}{d\eta} = \Delta A_t = R_t - A_{t-1}\right)$$

$$B_t = B_{t-1} + \eta \Delta B_t \quad \left(\frac{dB_t}{d\eta} = \Delta B_t = R_t^2 - B_{t-1}\right)$$

- $\eta^{-1}$ is the size of the window. (Moody sets $\eta = 0.01$.)

- So now we define the Sharpe Ratio in terms of these estimates.

$$S_t = \frac{A_t}{(B_t - A_t^2)^{1/2}} \approx \frac{Ave(R_t)}{Std(R_t)}$$

# Approximating the Sharpe Ratio (cont.)

- Now Taylor expand $S_t$ about $\eta$ ( not obvious why, but it helps the algebra ).

$$S_t \approx S_{t-1} + \eta \frac{dS_t}{d\eta}|_{\eta=0} + O(\eta^2)$$

- Ignore $O(\eta^2)$. Notice only the center term depends on $R_t$.

This lets us say $\frac{dS_t}{dR_t} \approx \frac{d}{dR_t}\eta D_t$

$$D_t \equiv \frac{dS_t}{d\eta} = \frac{d}{d\eta}\frac{A_t}{(B_t - A_t^2)^{1/2}}$$

$$D_t \equiv \frac{B_{t-1}\Delta A_t - \frac{1}{2}A_{t-1}\Delta B_t}{(B_{t-1} - A_{t-1}^2)^{3/2}}$$

# Measuring Utility: Sharpe Ratio (cont.)

- Finally, we arrive at

$$\frac{dD_t}{dR_t} = \frac{B_{t-1} - A_{t-1}R_t}{(B_{t-1} - A_{t-1}^2)^{3/2}}$$

- Hence we've gotten our expression :

$$\frac{dU_t}{dR_t} = \frac{dS_t}{dR_t} \approx \eta \frac{dD_t}{dR_t}$$

- Notice the largest improvement to $U_t$ is when

$$R_t^* = B_{t-1}/A_{t-1}$$

- So the Sharpe Ratio penalizes gains larger than $R_t^*$!

# Measuring Utility: Sterling Ratio

- $Sterling_T = \frac{\text{Average Yearly Return}}{\text{Maximum Draw Down}}$

- $MDD = \max_{i<j}(z_i - z_j)$
  where $j - i \leq 1$ year

- Useful for mutual fund managers wanting to minimize displeased customers

- Unfortunately, difficult to deal with analytically

# Measuring Utility: Double Deviation Ratio

- $DDR_T = \frac{Average(R_t)}{DD_T}$

- $DD_T = \left(\frac{1}{T}\sum_{t=1}^{T} min\{R_t, 0\}^2\right)^{1/2}$

- $DDR$ rewards large average positive returns,
  penalizes "risky" (large negative) returns

# Learning Parameters through Gradient Ascent

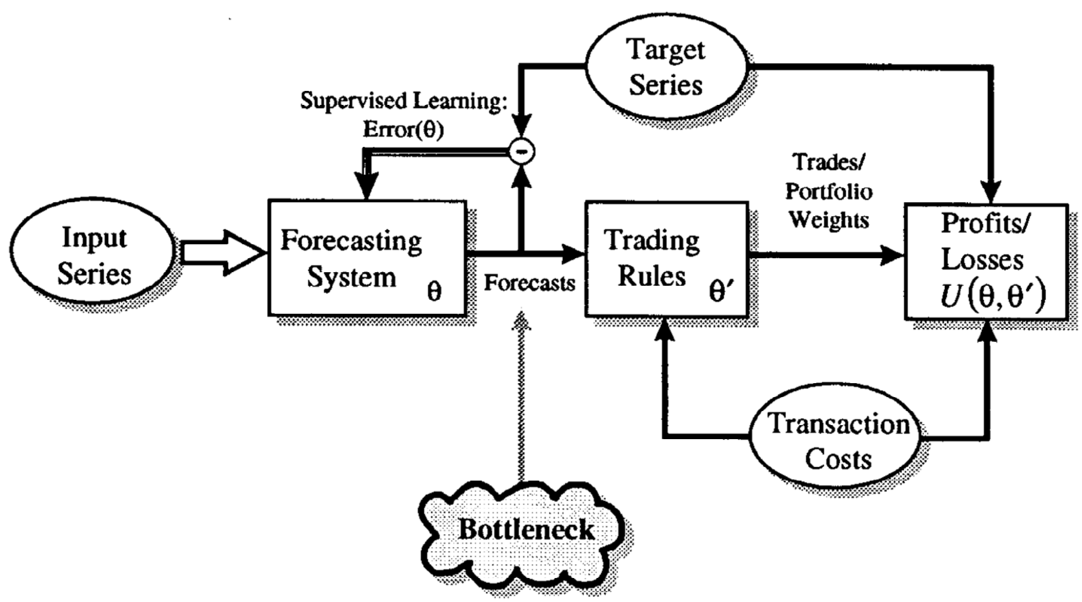
- Given $F_t(\theta)$ we want to adjust $\theta$ to maximize $U_T$:

$$\frac{dU_T(\theta)}{d\theta} = \sum_{t=1}^{T} \frac{dU_T}{dR_t} \left\{ \frac{dR_t}{dF_t} \frac{dF_t}{d\theta} + \frac{dR_t}{dF_{t-1}} \frac{dF_{t-1}}{d\theta} \right\}$$

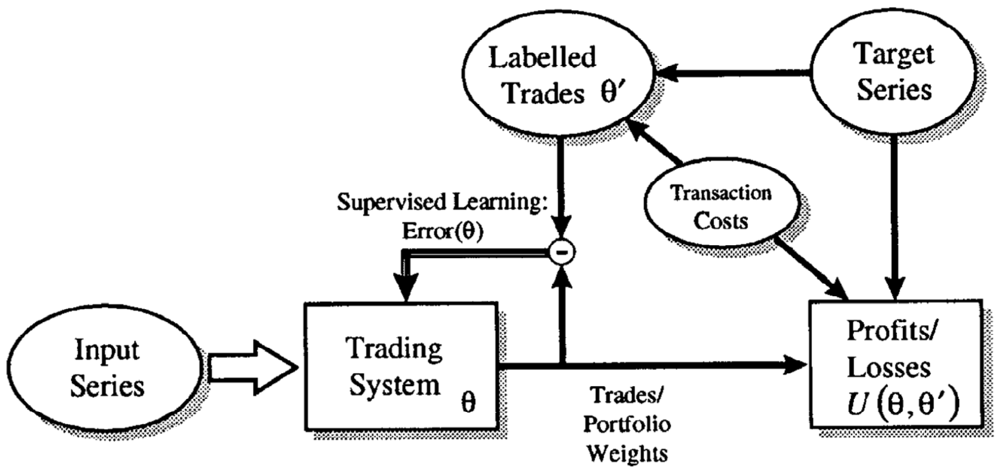
$$\Delta\theta = \rho \frac{dU_T(\theta)}{d\theta} \qquad \text{( } \rho \text{ is the learning rate )}$$

- $\frac{dU_T}{dR_t}$ : we saw this from before
- $\frac{dR_t}{dF_t}$ : easy to determine
- $\frac{dF_t}{d\theta}$ : $\frac{dF_t}{d\theta} = \frac{\partial F_t}{\partial \theta} + \frac{\partial F_t}{\partial F_{t-1}} \frac{dF_{t-1}}{d\theta}$ recursively ...

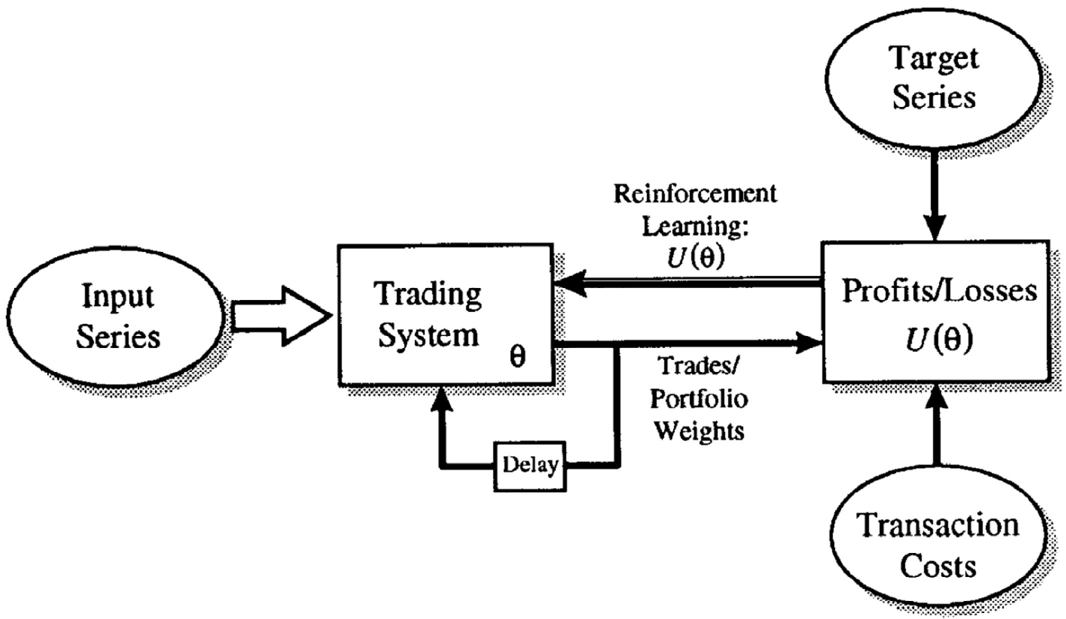
# Training to make forecasts



Training with labeled data (example trades).



# RRL "Direct Reinforcement" Approach



# Empirical Results: Data Sets Used

| DataSet | Goal |
|---|---|
| 1) Artificial Time Series | Show the system can learn |
| 2) Foreign Exchange Data | Show the system can learn a profitable strategy on real data |
| 3) S&P and Treasury Bill | Show RRL is better than Q-Learning |

# 1) Artificial Data

- Data is designed to have a "tradeable structure"

- They generate log-normal random walks, but with autoregressive trends:

  Trend variable: $\beta(t) = \alpha\beta(t-1) + \nu(t)$
  Log price: $p(t) = p(t-1) + \beta(t-1) + \epsilon(t)$

- $\epsilon(t)$, $\nu(t)$ are "noise" terms with zero mean, unit variance

- $\alpha < 1$ sets how the "autoregressiveness"

- $z_t = exp(p(t))$ is used to generate 10,000-point price series.

# 1) Artificial Data - System Details

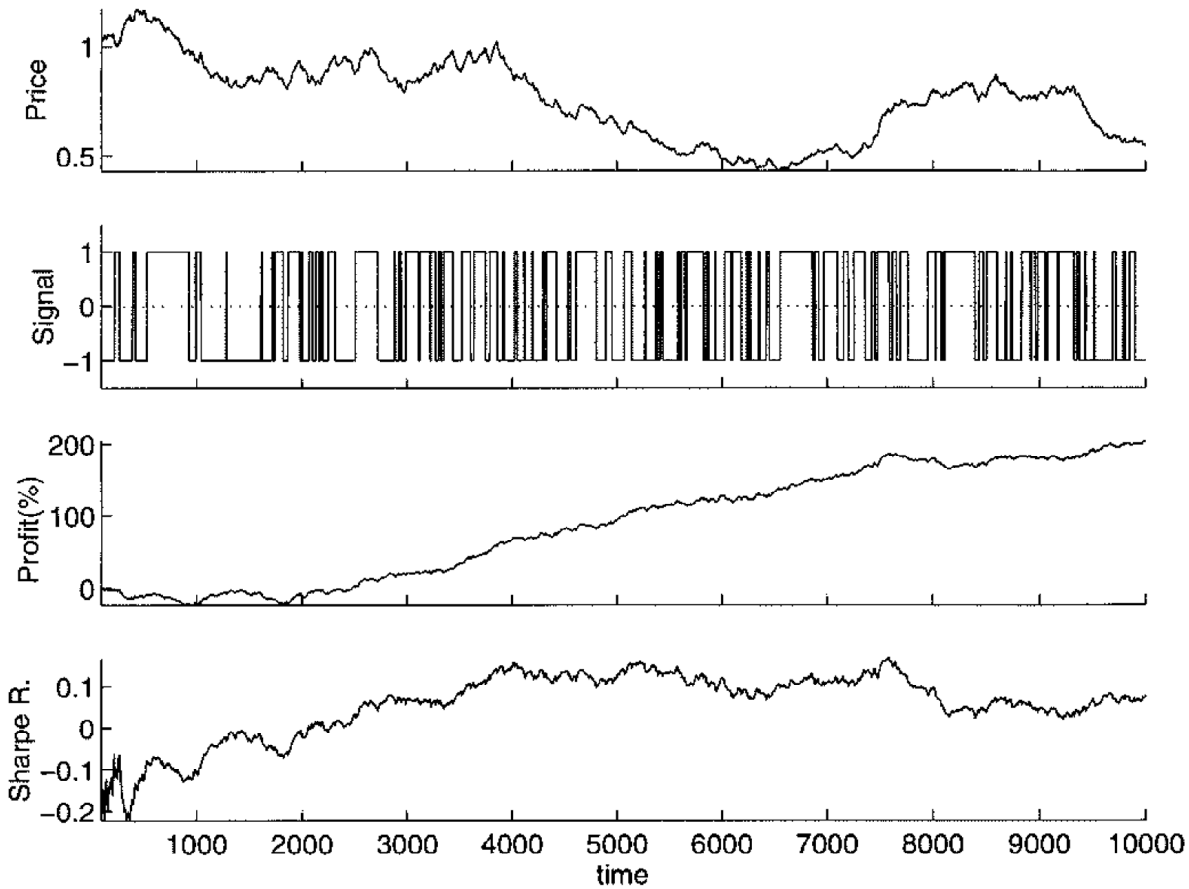
- The trading function has autoregressive inputs (matches the data):

$$F_t = sign(uF_{t-1} + v_0 r_t + v_1 r_{t-1} + \cdots v_7 r_{t-7} + w)$$

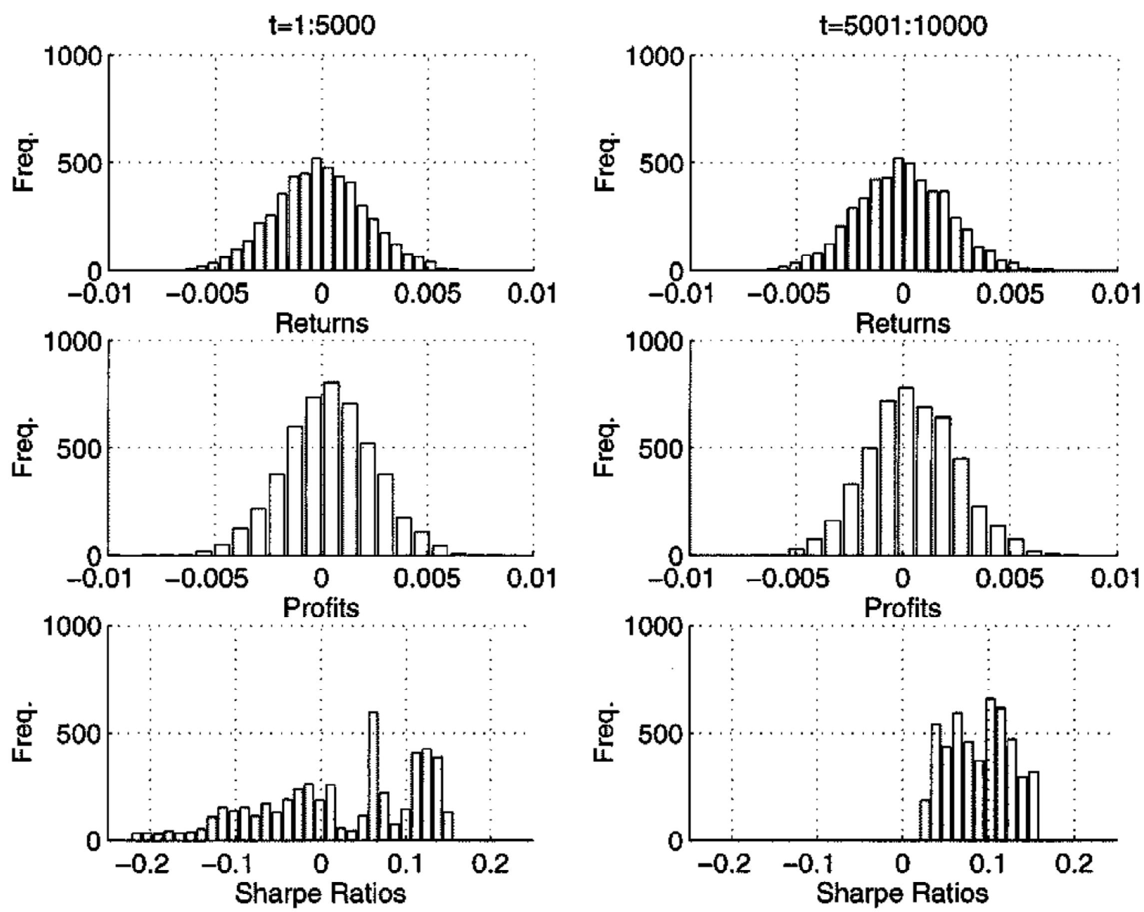
- Transaction cost: $\delta = 0.5\%$

- Learns to maximize the Differential Sharpe Ratio

# Artificial Prices, and Results



# Histograms of Artificial Data and Results



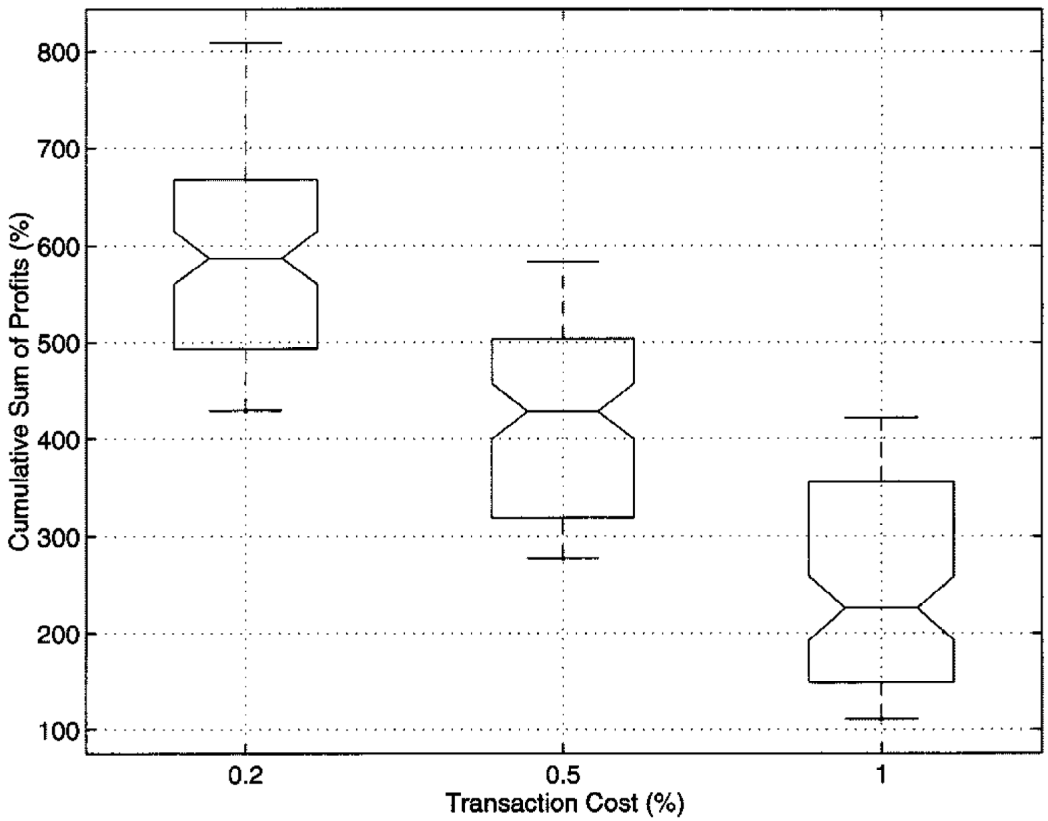
# Artificial Data (continued)

- How do transaction costs affect trading performance?

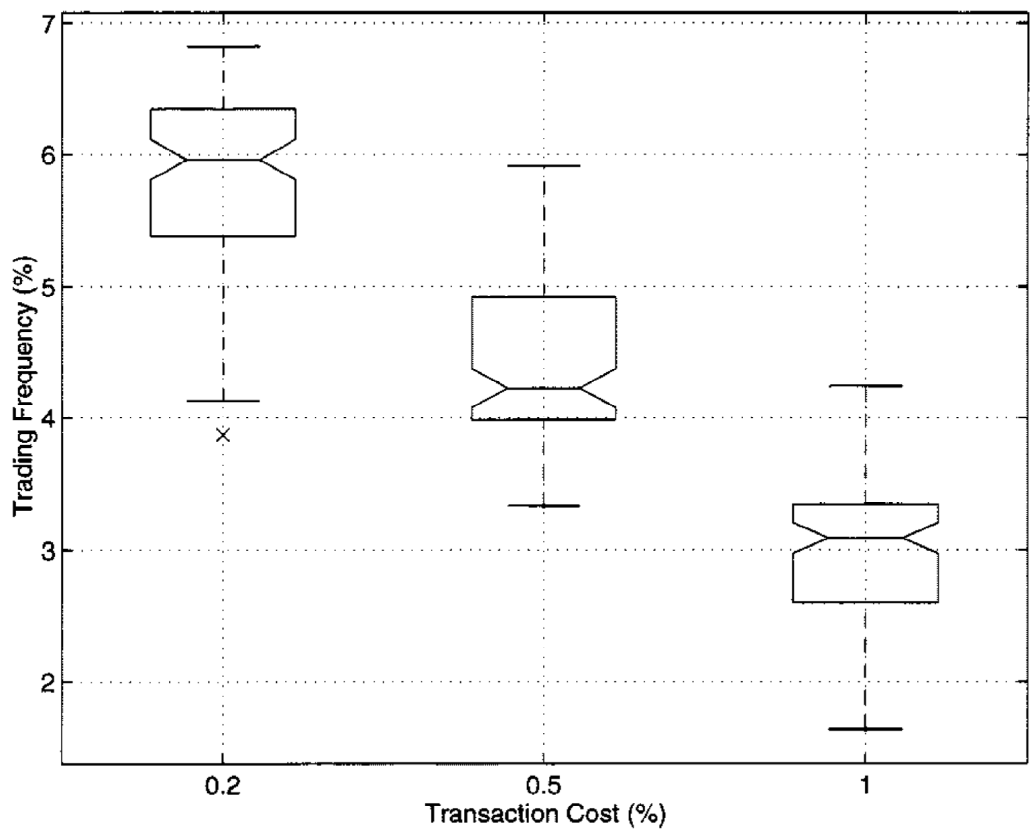
- Repeat the previous experiments 100 times...
  try $\delta = 0.2\%, 0.5\%, 1.0\%$

- Hypothesis: lower costs should allow:
  - more trading
  - more profits
  - better Sharpe Ratio

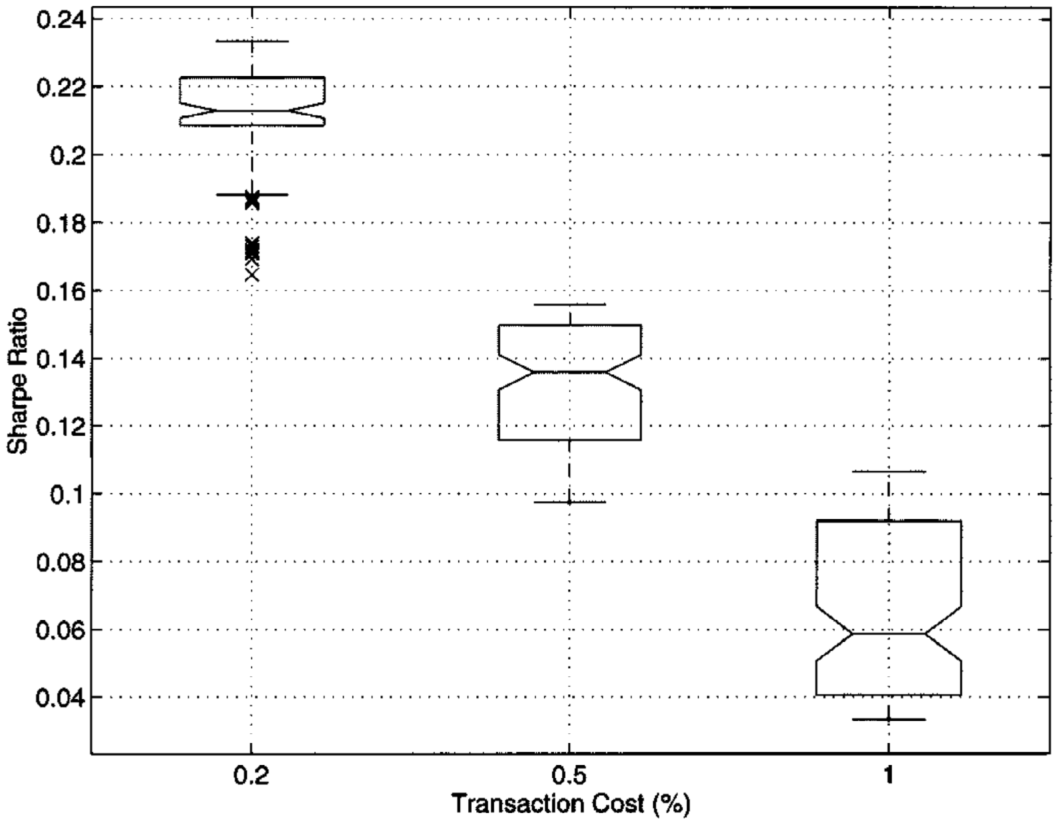
Boxplots of how transaction costs affect **profits**



# Boxplots of how transaction costs affect **trading frequency**



Boxplots of how transaction costs affect **Sharpe Ratio**



## 2) Foreign Exchange Data

- US Dollar vs. British Pound

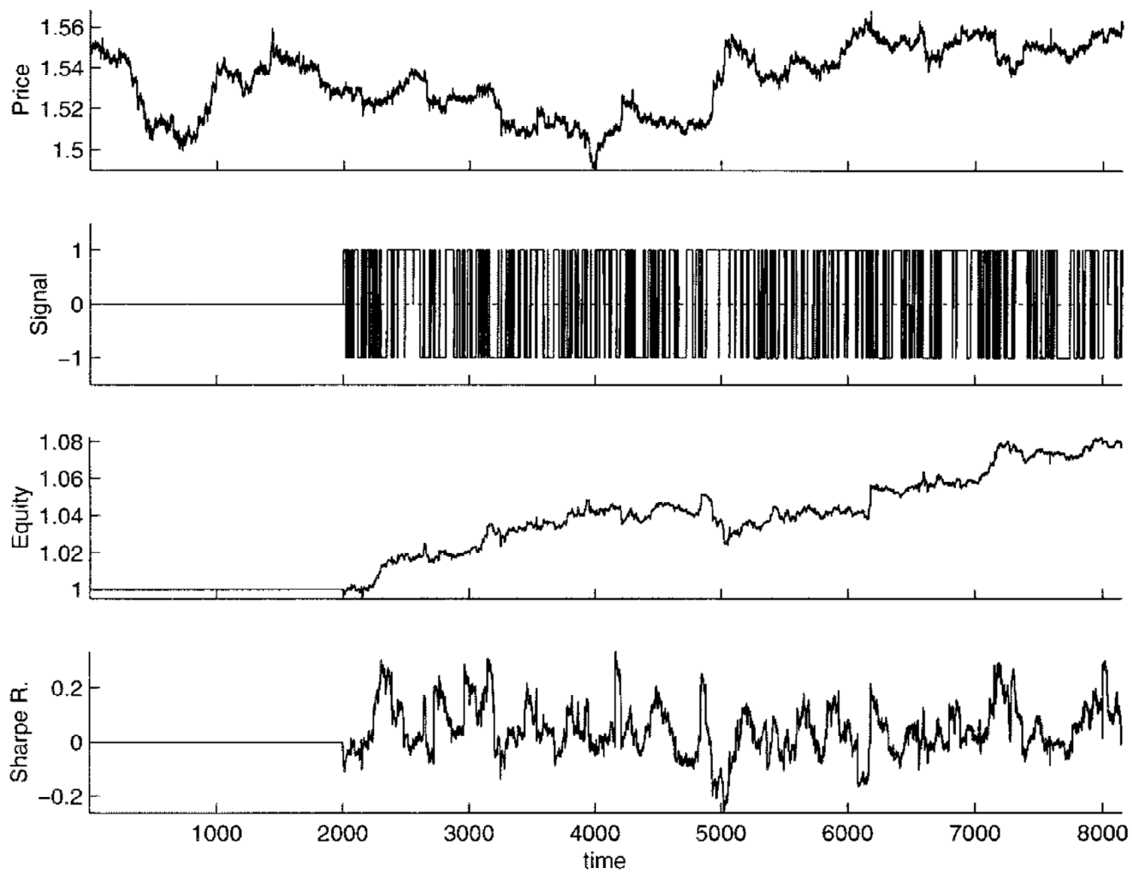
- 8 months of data (half-hour quotes) during 1996

- Same autoregressive inputs as Artificial Data experiment?
  (the paper was unclear)

- The system is trained to maximize the
  Downside Deviation Ratio.

- Transaction cost is the bid-ask spread
  (which has a typical average but is not fixed).

# Foreign Exchange Prices, and Results



# Foreign Exchange Result Summary

- 15% annualized return, Sharpe Ratio of 2.3

- (S&P index gets roughly 15% return, $Sharpe < 1$)

- Trading frequency: trades are made roughly once every 5 hours

- It's difficult to say how well this would have done in real world environment (since you can't simulate all market frictions).

# 3) US Stock Market Data: Q-Trader against RRL-Trader

- S&P vs. Treasury Bill

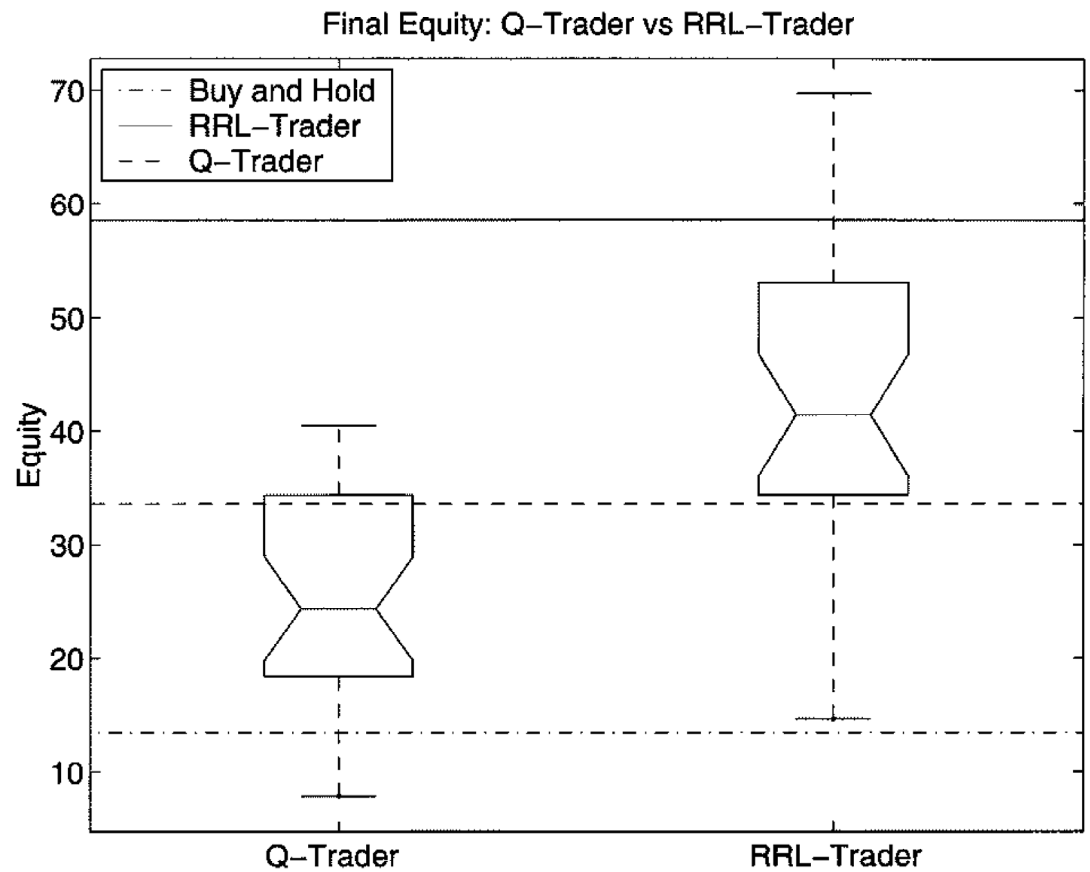
- 25 years of data: 1970-1994

- System is trained on previous 20 years (sliding window)

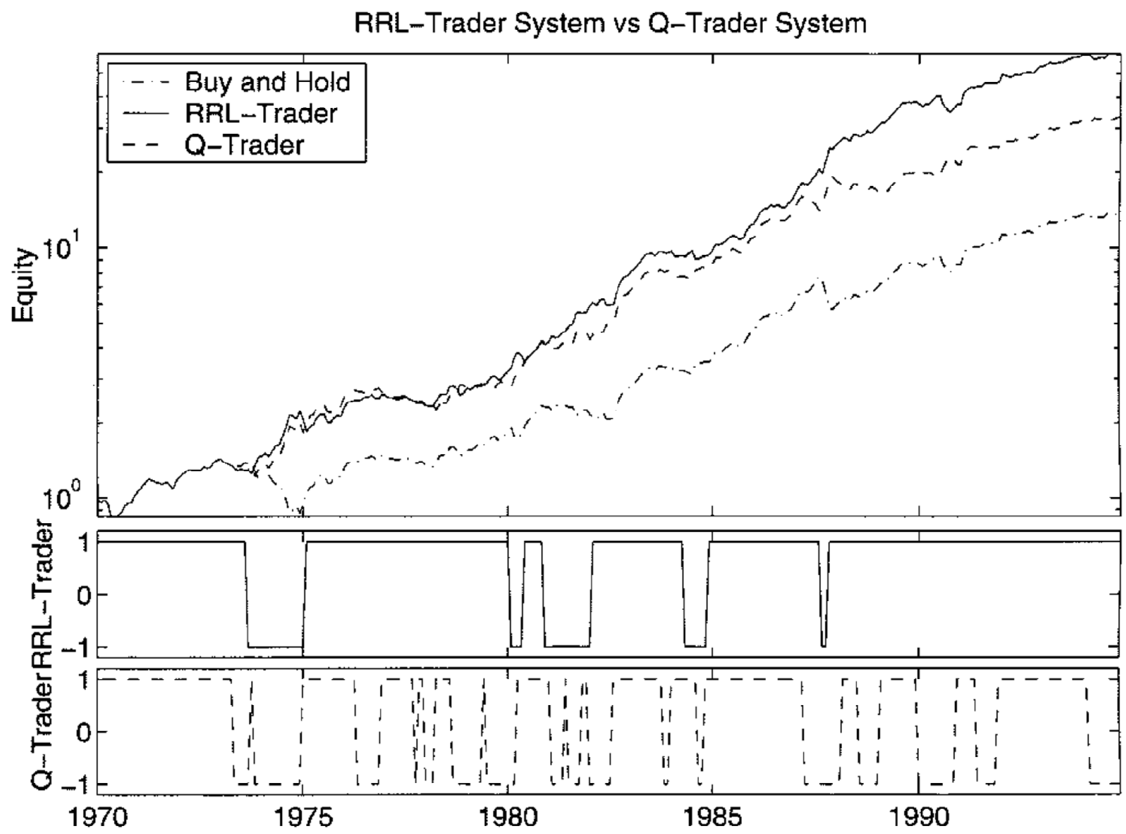
- The Information $I_t$ also includes macroeconomic data

- Also implement Q-Learning (actually, a variant called Advantage Updating) to compare.

# Q-Trader vs. RRL-Trader Results



# Q-Trader vs. RRL-Trader Results (continued)



# Conclusions

- Moody makes the case that RRL is better than Q-Learning for trading since it is a simpler approach.
  (Simpler is better - a recurring theme in this class.)

- I'm not sure I agree personally with some of his methods.

- Moody has set up an interesting method for learning directly, but hasn't addressed the problem of choosing a good trading model.