The Factor Tau in the Black-Litterman Model

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Abstract

This paper considers the factor tau (τ) in the Black-Litterman model. It is one of the more confusing aspects of the model, as authors provide contradictory information regarding its use and calibration. We will consider the origin of the mixed-estimation model used in the Black-Litterman Model so that we can develop a richer understanding of τ and its place in the model. We will show that most practitioners can benefit from using an alternative reference model explicitly without τ. For those who want to use τ, we will provide a discussion on its calibration.

JEL Classification: C1, G11

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Introduction

The Black-Litterman Model is a model for estimating asset returns. It has two main features, the use of an informative prior derived from the CAPM equilibrium, and a mixing model that allows the investor to specify views on any linear combination of the assets. The mixed-estimation model was originally developed by Theil (1970). The mixing model allows absolute and relative views, and views may be on any combination of the assets. A summary of the literature and complete details on the Black-Litterman Model can be found in Walters (2008).

This paper will focus on the role of the factor tau (τ). τ is used to scale the investors uncertainty in their prior estimate of the returns. There are several different approaches to calibrating it, or even including it described in the literature. Just to illustrate the difference of opinion, we will look at comments from three authors. He and Litterman (1999) state they set $\tau = 0.05$. Satchell and Scowcroft (2000) state many people use a value of τ around 1. Meucci (2010) proposes a formulation of the Black-Litterman model without τ.

We will use the concept of Reference Models to explain the differences between the various authors. We will start by presenting the Original Reference Model as derived from Theil's mixed estimation approach in Black and Litterman (1991), and further explained in He and Litterman (1999). Next we will present the Alternative Reference Model as proposed by Meucci (2010) which estimates the returns without τ. Finally, we will provide recommendations on whether to include τ, and if so how to calibrate τ.

The Black-Litterman Model

This section will provide an overview of the Black-Litterman Model. The reader can consult one of the references for more details.

We can view the process of using the Black-Litterman Model as having three distinct steps. The first step is the calculation of the informative prior estimate of returns. The prior is derived from the CAPM equilibrium portfolio using the following formula which is the closed form solution to unconstrained Mean-Variance optimization

(1) $$\Pi = \delta \Sigma w$$

Formula (1) is the relationship which we call reverse optimization. Given the risk aversion of the market, δ, the covariance of returns, Σ, and the equilibrium weights, w, we can back out the expected equilibrium returns. Π will be our prior estimate of the mean returns.

The second step is the specification of the investor's views. Here the investor formulates estimated returns and uncertainties for one or more view portfolios, or linear combinations of the assets.

The third step is the mixed estimation process used to blend the prior estimates of returns with the views to create the posterior estimates of the returns along with estimates of the uncertainty of the estimates. We can arrive at the same formulation for the blending process using the standard case of an unknown mean and a known variance from Bayesian theory.

We start assuming that the expected returns are normally distributed. The goal of the Black-Litterman model is to estimate the parameters of this distribution.

(2) $$E(r) \sim N(\mu, \Sigma_\mu)$$

μ Mean return

Σ_μ Covariance matrix for the distribution of returns about the mean

There are two widely used models for the blending process. We call these Reference Models. Each Reference Model contains different assumptions about what parameters to use in order to model formula (2).

The Original Reference Model

This section reviews the Original Reference Model as defined in the papers, Black and Litterman (1991), and He and Litterman (1999).

First we introduce the simple linear model from Theil (1970) for the estimated return.

(3) $\pi = \mu + \epsilon$

Here π is the investors estimate, μ the actual mean return and ε the residual. The core of the Original Reference Model is the concept that the investor is uncertain of their estimate π. Black and Litterman (1991) state, "*The mean is an unobservable random variable*". With μ as a stochastic variable, then ε has a probability distribution. We assume ε is normally distributed with mean 0 and variance Σ_π. Thus, we can state that Σ_π is the variance of the investor's estimate about the mean return μ. Theil (1970) uses the phrase "sampling variance" for Σ_π. Standard error is another name for the square root of sampling variance.

This leads to the following expression for the distribution of the estimated return about the mean return.

(4) $\pi \sim N(\mu, \Sigma_\pi)$

Formula (4) shows the distribution of the estimate of the mean return about the unknown mean return. This is analogous to the situation where we perform the following process. Each of *m* times we draw *n* realizations from the population of returns and compute the mean of each of the *m* samples. Then we view the distribution of the *m* sample means around the population mean. π is the mean of the sample means, and Σ_π is the sampling variance of the distribution of sample means about the population mean. The Central Limit Theory tells us we can expect in the limit as *n* approaches infinity that π approaches μ and Σ_π approaches 0.

In the case of normally distributed samples about the mean, the sampling variance is

(5) $\Sigma_\pi = \dfrac{\Sigma_\mu}{n}$

We assert for simplicity that Σ_π and Σ_μ are independent and uncorrelated, then Σ_r, the variance of the distribution of returns about the estimated mean, π, is given by formula (6).

(6) $\Sigma_r = \Sigma_\mu + \Sigma_\pi$

Given π as our estimate of μ, then Σ_r is essentially our estimate for Σ_μ.

We can check the reference model at the boundary conditions to ensure that it is correct. In the absence of estimation error, e.g. $\varepsilon \equiv 0$, then $\pi = \mu$ and $\Sigma_r = \Sigma_\mu$. As our estimate gets worse, e.g. Σ_π increases, then Σ_r increases as well. This behavior is consistent with our earlier assertion that our posterior estimate of the mean is more precise than either the views or the prior. In addition it is also consistent with the idea that estimates of the variance of a distribution of a financial time series about an estimated mean, can at best approach a lower limit which is the variance of the distribution about the population mean. It cannot go below that value.

To further simplify the model we can assert that Σ_π is proportional to Σ_μ where the constant of proportionality is known as τ. This assertion is useful since we usually estimate Σ_μ rather than Σ_π.

(7) $\Sigma_\pi = \tau \Sigma_\mu$

If we combine formulas (5) and (7), we can relate τ and n.

$$\Sigma_\pi = \tau \Sigma_\mu = \frac{\Sigma_\mu}{n}$$

(8) $\tau = \frac{1}{n}$

If we used a statistical process to formulate our prior estimate, then we would have a clear method for calibrating it based on this relationship.

Now we can introduce our expression for the Original Reference Model for the estimated distribution of expected returns.

(9) $E(r) \sim N(\pi, (\Sigma_\mu + \Sigma_\pi)), \ \ \pi \sim N(\mu, \Sigma_\pi)$

Formula (9) represents the complete Original Reference Model which corresponds to our goal as defined in formula (2). This reference model matches up with formulas 8, 9 and 10 in He and Litterman (1999).

We will not show the derivation here as it can be found in several of the references, but the standard expression for the Black-Litterman posterior estimated mean and sampling variance is:

(10) $\hat{\Pi} \sim N([(\tau \Sigma_\mu)^{-1} \Pi + P^T \Omega^{-1} Q][(\tau \Sigma_\mu)^{-1} + P^T \Omega^{-1} P]^{-1}, [(\tau \Sigma_\mu)^{-1} + P^T \Omega^{-1} P]^{-1})$

$\hat{\Pi}$ Posterior estimate of the mean returns

P View selection matrix

Ω Covariance of the estimated view mean returns about the actual view mean returns

Q Estimated mean returns for the views

Ω is a term similar to τΣ, representing the uncertainty of the estimated returns of the views. In this reference model, Ω is not the variance of the distribution of returns of the views.

The discussion of formula (10) is easier in terms of the inverse of the covariance matrix, a term known as precision in the Bayesian literature. We can summarize the posterior estimated mean in formula (10) as the precision weighted average of the prior estimate and the view estimates. The posterior precision is the sum of the prior and view precisions. Both these formulations match our intuition as we expect the precision of our posterior estimate to be more than the precision of either the prior or the views. Second, the mixed estimation process should make use of the precision of the estimates in the weighting of the mixing, e.g. an imprecise estimate should have less impact on the posterior than a precise estimate.

With a small modification to the covariance term we can rewrite (10) using (6) to be an expression for the Black-Litterman posterior estimate of the mean and covariance of returns around the mean.

(11) $E(r) \sim N([(\tau \Sigma_\mu)^{-1} \Pi + P^T \Omega^{-1} Q][(\tau \Sigma_\mu)^{-1} + P^T \Omega^{-1} P]^{-1}, (\Sigma_\mu + [(\tau \Sigma_\mu)^{-1} + P^T \Omega^{-1} P]^{-1}))$

The updated sampling variance of the mean estimate will be lower than either the prior or conditional sampling variance of the mean estimate, indicating that the addition of more information will reduce the uncertainty of the posterior estimates. In Bayesian terms, the posterior estimate is more precise than either the prior or the view estimates.

The variance of the returns about the mean from formula (10) will never be less than the known variance of returns about the mean, Σ_{μ}. This matches our intuition about how the variance of returns can change. Adding more information should reduce the uncertainty (increase the precision) of the estimates, but cannot reduce the covariance beyond that limit. Given that there is some uncertainty in the variance of the returns about the mean, then formula (10) provides a better estimator of the variance of returns about our estimated mean than the known variance about the mean from the equilibrium.

The Alternative Reference Model

The Alternative Reference Model is commonly used in the literature, though usually not explicitly. While it has been essentially described by other authors, Meucci (2010) explicitly described its features. We will further assert that any author who suggests τ with a scale of 1, or does not use an updated posterior variance is using this reference model implicitly.

The Alternative Reference model does not consider uncertainty in any of the parameters, it is rather a shrinkage model for the expected returns. It starts from the position that the investor has some views on the expected returns. In the Black-Litterman Model, these views can be on relative or absolute returns, and can be partial and do not need to cover all assets. Where no view is specified for an asset, or the views are relative, some return to use as a starting point is required. In addition, the investor believes that directly using the returns implied by the views will lead to extreme portfolios. The blending model inside Black-Litterman allows the use of the CAPM equilibrium returns (Π) both as a starting point and a shrinkage target, which results in less extreme portfolios and more stable optimization results.

Given that the investor has no uncertainty about their estimates, they know the covariance matrix of the excess returns is Σ_{μ}. The investor does not need to express views on the covariance matrix, and there is no shrinkage of the covariance matrix.

The mixed-estimation model at the heart of the Black-Litterman Model allows for easy blending of the prior and views. We will show it can also be viewed as a shrinkage model.

The linear form of a shrinkage model for returns is:

(12) $\hat{\Pi} = \delta \Pi + (1-\delta) V$

Here, δ is a scalar shrinkage parameter and V is the investors views on estimated returns.

In the Black-Litterman model, we model the investors views as , a k x 1 vector (Q) of the expected return to each view and a k x n matrix (P) of view portfolios indicating the weight of the assets in each of the view portfolios.

We will replace V from (12) with $P^{-1} Q$ [2]. We also want the shrinkage factor to be a matrix rather than a scalar allowing a different amount of shrinkage to be applied to each view. This gives us an updated model.

2 Note that if P is not square, then we will use the pseudo-inverse $(P^T P)^{-1} P^T$.

$$\hat{\Pi} = \Delta \Pi + (I - \Delta) P^{-1} Q \quad (13)$$

Only the shrinkage factor, Δ, still needs to be mapped back to the Black-Litterman model. If we consider formula (13) in relation to formula (10) from the Original Reference Model we can arrive at one possible way to compute Δ which is consistent with the usual form of Bayesian shrinkage and provides independent control over the shrinkage of each view.

$$\Delta = \frac{\Sigma_\mu^{-1}}{\Sigma_\mu^{-1} + P^T \Omega^{-1} P} \quad \text{and} \quad (I - \Delta) = \frac{P^T \Omega^{-1} P}{\Sigma_\mu^{-1} + P^T \Omega^{-1} P} \quad (14)$$

Here, Ω is a diagonal matrix containing a non-negative measure (ω_i) for each view which corresponds to uncertainty in the view. By varying the uncertainty, ω_i, from $(0,\infty)$ we can vary the value of δ_i over the interval (0,1). Note that ω_i is inversely proportional to δ_i. This is sufficient to parametrize our mixing model. One drawback is that specifying Ω is not an intuitive way to quantify the desired amount of shrinkage.

We can substitute (14) into (13) and arrive at the Alternative Reference Model.

$$E(r) \sim N([\Sigma_\mu^{-1} \Pi + P^T \Omega^{-1} Q][\Sigma_\mu^{-1} + P^T \Omega^{-1} P]^{-1}, \Sigma_\mu) \quad (15)$$

As we have seen τ does not enter the model, there is no need for another free parameter.

Idzorek (2005) takes a different approach to express the shrinkage factor δ. specifying a confidence in each view as a percentage, which maps onto the term (1- δ_i). Using this confidence level for each view, his method computes the values of ω_i . Specifying the confidence in this manner is a more intuitive way to specify the shrinkage.

Choosing a Reference Model

Now that the two reference models have been described we can provide some guidance on using one model or the other. The most common reference model in the literature is the Alternative Reference Model. In fact, only in the papers by Litterman have we seen the Original Reference Model described and used. We can see this by examining whether authors update the posterior precision, or use the covariance portion of formula (10), or whether they just use the prior covariance, formula (12). If they do not update the posterior precision and covariance, then they are using the Alternative Reference Model. In this case there is no need to consider τ, as it is not a part of the model.

The primary reason to use the Original Reference Model would be to pick up the additional information from the model via the updated posterior covariance matrix. This additional information from the model comes at the cost of needing to determine the additional factor τ. Investors willing to accept the simpler Alternative Reference Model can avoid the need to consider τ.

Worked Example with Both Reference Models

When we use the Original Reference Model, the posterior covariance may be smaller than the prior covariance if the views improve the precision of the estimate. Absolute views generally make larger improvements in the precision of the estimate. Relative views make weak or no improvements in the precision of the estimate. Intuitively this makes sense as relative views do not provide an improved estimate of the mean, just extra information on the relationship between the estimates. We can measure the precision of the estimates by summing the unconstrained weights. We can compute an effective posterior measure of uncertainty/precision as shown below

(16) $$\eta = \frac{(1 - \sum_{1}^{n} w_i)}{\sum_{1}^{n} w_i}$$

The value η in formula (16) can be compared to τ and can be used to measure the uncertainty in the prior or posterior estimates of the mean. When viewing the prior estimates, $\eta = \tau$. We can compare η between the prior and the posterior to determine the relative improvement in the precision of the estimates.

One of the interestingly artifacts of the Black-Litterman model is that while the estimated return of an asset without views can change, the unconstrained weight of the asset in the portfolio does not change at all. We can prove this point by examining the formula 17 in He and Litterman (1999). Λ is a k x 1 matrix with one row per view. The $P^T\Lambda$ term will always be zero for an asset in no views. This is true regardless of which reference model we use. Under the Original Reference model, our investor with less than 100% confidence in their prior estimates is not 100% invested, but is only invested in the fraction $1/(1 + \tau)$. This is because of formula (6) which shows the prior dispersion of realized returns about the estimated mean is $(1 + \tau)\Sigma_{\mu}$. The asset weights in an unconstrained portfolio based only on the prior will be $w_{eq}/(1 + \tau)$. Because the posterior precision of the estimated mean will be equal or higher, the investor will invest an equal or larger fraction of their wealth in the portfolio and the asset allocation will experience some change solely because of this change. In the Alternative Reference Model the unconstrained weights will generally sum to 1, and we do not consider precision of the posterior estimates. Of course, since most portfolio optimization is constrained, the final asset weights will most likely change even when an investor has no view on a specific asset.

Now we will work an example using both the Original Reference model and the Alternative Reference model. The details of the example can be found in Appendix A. We examine two scenarios

- Relative Views – Investor has two relative views
- Absolute View – Investor adds an absolute view on Germany with return = return from the first scenario.

This construction should allow scenario two to illustrate the difference in the unconstrained weights caused solely by the updated posterior covariance matrix.

Table 1 shows that when using the Original Reference Model the unconstrained weights of the assets without views (Japan) do not change from the prior unconstrained portfolio. Note that because the investors confidence in the prior is less than 100%, the prior asset weights differ from the equilibrium by a factor of $1/(1 + \tau)$. We can also see that an absolute view which only changes the posterior covariance will cause changes to the unconstrained asset allocation, but only to the assets with views. In this case an absolute view of moderate confidence caused an additional 140bps shift in the unconstrained weights of the asset in the view. Other assets with views had changes in their allocation from 4 bps for an asset in an unrelated view, to 19bps and 44bps for assets coupled to Germany through a relative view.

Table 1 – Original Reference Model Results

Asset	Equilibrium Weights	Orig Ref Model Prior Weights	Orig Ref Model Relative Views	Orig Ref Model Absolute View
Brazil	3.39%	3.23%	11.89%	11.85%

China	6.32%	6.02%	22.16%	22.10%
France	8.23%	7.84%	21.60%	21.41%
Germany	9.96%	9.49%	26.14%	27.54%
Japan	13.56%	12.91%	12.91%	12.91%
UK	11.81%	11.25%	-19.18%	-18.74%
US	46.73%	44.50%	19.70%	19.80%
Sum	100.00%	95.24%	95.24%	96.88%
η		0.050	0.050	0.032

Table 2 shows that in the Alternative Reference Model the prior weights match the equilibrium weights as uncertainty in the estimated covariance is not a factor. It also shows that the posterior unconstrained weight of an asset included in no views does not change from the equilibrium weight. Further it shows no impact from the absolute view on Germany, the expected return is already taken into account by the results from scenario 1.

Table 2 – Alternative Reference Model Results

Assets	Equilibrium Weights	Alt Reg Model Prior Weights	Alt Ref Model Relative Views	Alt Ref Model Absolute View
Brazil	3.39%	3.39%	12.43%	12.43%
China	6.32%	6.32%	23.17%	23.17%
France	8.23%	8.23%	22.25%	22.25%
Germany	9.96%	9.96%	26.93%	26.93%
Japan	13.56%	13.56%	13.56%	13.56%
UK	11.81%	11.81%	-19.18%	-19.18%
US	46.73%	46.73%	20.84%	20.84%
Sum	100.00%	100.00%	100.00%	100.00%

With both reference models the unconstrained weights for an asset with no views, e.g. Japan, do not change. This matches our assertion earlier that this is a general property of the Black-Litterman model.

Clearly the Original Reference Model leads to larger movements from the equilibrium. This is because we are simultaneously updating our estimate of the mean, and increasing the precision of that estimate. This causes the asset allocation to tilt subtly more toward the assets included in the views with higher precision. The impact of the updated posterior covariance is a second order effect, as we can see from Table 1 it is less than 10% of the total change from the prior unconstrained weight.

Setting τ in the Black-Litterman Model

Now we will consider how to select a specific value of τ in more detail. A brief survey of the literature will be helpful. Given the previous discussion, we will focus on authors who use the Original Reference Model in this section.

From Black and Litterman (1991)

> *Because the uncertainty in the mean is much smaller than the uncertainty in the return itself, τ will be close to zero. The equilibrium risk premiums together with tS determine the equilibrium distribution for expected excess returns. We assume this information is known to all; it is not a function of the circumstances of any individual investor.*

He and Litterman (1999) propose considering τ as the ratio of the sampling variance to the distribution variance, and thus it is 1/t. They use a value of τ of 0.05 which they describe as

> *"...corresponds to using 20 years of data to estimate the CAPM equilibrium returns."*

As described previously, τ is the constant of proportionality between Σ_π and Σ. We will examine three ways in which we might select the value for τ. It is important to remember that τ is a measure of the investor's confidence in the prior estimates, and as such it is largely a subjective factor.

We will consider three methods to select a value for τ

- Estimate τ from the standard error of the equilibrium covariance matrix
- Use confidence intervals
- Examine the investor's uncertainty as expressed in their prior portfolio

First, we will approach the problem from the point of view of He and Litterman (1999). If we were using regression techniques to find π using formula (3), then Σ_π would be the sampling variance or square of the standard error of the regression. Formula (14) is the expression for the standard error where the residual is normally distributed which is assumed in the model.

(17) $$Std\ Err = \frac{\sigma}{\sqrt{n}}$$

In the Black-Litterman model we do not use a regression approach to find the prior estimate of the mean return, we solve for it using equilibrium techniques that have no clear standard error term. We do however generate the prior covariance matrix using a sample of returns, and we have a consistent n to use with our Σ_μ which we could use to estimate a standard error for the estimate of the mean.

Formula (8) shows the basic relationship between τ and n from our covariance matrix calculations. Because we are using non-statistical methods to estimate the mean return, this is not a quantitative answer as to what value we should use for τ, it is just one way to provide some intuition around the scale of τ.

A second approach to establishing a reasonable value for τ is to use confidence intervals. This has a more direct connection with the model and our estimate of the model. Formula (4) illustrates the distribution of the estimate of the mean, about the mean return. From this distribution we can assert a confidence interval for our estimate using basic probability.

A plausible scenario we might encounter would be yearly equity like returns with μ = 8% and σ = 15%. Table 3 below shows the 95% and 99% confidence intervals for this scenario and various values of τ.

Table 3

τ	**95% Confidence**	**99% Confidence**
0.0167	$\mu \in$ (4.13%, 11.87%)	$\mu \in$ (2.19%, 13.81%)

τ	95% Confidence	99% Confidence
0.0250	$\mu \in$ (3.26%, 12.74%)	$\mu \in$ (0.88%, 15.12%)
0.0500	$\mu \in$ (1.30%, 14.71%)	$\mu \in$ (-2.06%, 18.06%)
0.2000	$\mu \in$ (-5.42%, 21.42%)	$\mu \in$ (-12.12%, 28.13%)
1.0000	$\mu \in$ (-22.00%, 38.00%)	$\mu \in$ (-37.00%, 53.00%)

We can see setting τ too high makes a very weak statement for our prior estimate of the mean. For example, where we select $\tau = 0.20$ then our estimate at the 99% confidence level is about 8% ± 20% which is not a very precise estimate for the mean of a distribution. In fact, there is almost a 30% chance that the actual mean expected return is less than 0%.

Third, we can consider τ from the point of view of a Bayesian investor. Given the uncertainty in the estimates, our Bayesian investor begins the process not fully invested. In fact the fraction of their wealth invested is $1/(1+\tau)$. This means they are holding the fraction $\tau/(1+\tau)$ of their assets in the risk free asset solely due to their uncertainty in their estimates. A value of $\tau = 1$ would lead to the investor being only 50% invested , and a value of $\tau = 0.25$ would lead to the investor being 80% invested based on the prior estimates. The value η which we defined in formula (16) is the measure of wealth invested given the posterior estimates. A plausible example of this case would be a prior asset allocation of 90-95% which results in a value of τ of between .053 and .11. In reality it would be very difficult to distinguish a real investor's allocation to the risk free asset due to uncertainty in their estimate from similar allocations for other reasons.

Summary

We have seen the two major reference models used with the Black-Litterman model. These reference models arise from very different ways of viewing the problem of estimating asset returns.

The Original Reference Model is developed from Theil's Mixed-Estimation Model and includes investor uncertainty in the estimates. Their posterior estimates include an updated covariance matrix. This model requires the investor to estimate an additional parameter τ which impacts the posterior Covariance matrix as well as the estimated returns.

The Alternative Reference Model is developed from Shrinkage Theory and is simpler than the Original Reference Model. It does not require the parameter τ, and requires the investor to only estimate one parameter to adjust the shrinkage.

Most of the Black-Litterman literature makes use of the Alternative Reference model explicitly or implicitly, and most investors would be well served to explicitly use the Alternative Reference Model rather than struggling with τ.

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Appendix A

Appendix A contains the specification of the example scenario. Note that this is essentially random data and any correspondence with a real state of the market is purely coincidental. It is for example purposes only.

MATLAB code implementing the example is available at http://www.blacklitterman.org.

We have the following correlation matrix, equilibrium weights and standard deviations of excess returns.

Correlation	Brazil	China	France	Germany	Japan	UK	US
Brazil	1.0000	0.4118	0.2830	0.4192	0.4227	0.2771	0.4022
China	0.4118	1.0000	0.6994	0.7044	0.3220	0.7203	0.6665
France	0.2830	0.6994	1.0000	0.7231	0.2868	0.7124	0.5032
Germany	0.4192	0.7044	0.7231	1.0000	0.2933	0.7126	0.5164
Japan	0.4227	0.3220	0.2868	0.2933	1.0000	0.3400	0.2607
UK	0.2771	0.7203	0.7124	0.7126	0.3400	1.0000	0.6011
US	0.4022	0.6665	0.5032	0.5164	0.2607	0.6011	1.0000
Weight	3.39%	6.32%	8.23%	9.96%	13.56%	11.81%	46.73%
Std Dev	24.20%	18.50%	17.00%	16.81%	19.61%	15.40%	19.11%

We use $\delta = 2.5$ for the risk aversion of the market and $\tau = 0.05$.

The first view is that emerging markets returns will exceed US returns by 2% with uncertainty equal to the diagonal of $P\tau\Sigma P'$ (precision of the view = precision of the prior).

The second view is that returns from France and Germany will exceed UK returns by 2% with the same precision as in the first view.

The third view is that German returns will be 5.158% (posterior when views 1 & 2 are applied) with precision as in the first and second view.