Name:

Instructions: You may have a calculator (no TI-89 or equivalent) and a handwritten $3'' \times 5''$ note card during the exam. You may not have any devices that are connected to the internet. I will provide scratch paper if you need it. There are 70 points on this exam. The other 30 are from your take-home final.

True/False (4 points each)

If you answer "True," provide a brief justification. If you answer "False," provide a counterexample.

1. ________ Let n be an odd integer. Then $n^2 - 1$ is even.

2. ________ If $\sqrt{2}$ is rational, then chocolate milk comes from brown cows.

3. ________ Let A and B be subsets of a universal set. Then $|A - B| = |A| - |B|$.

4. ________ Every surjection $f\colon \mathbb{R} \to \mathbb{R}$ is invertible.

5. ________ $<$ is an equivalence relation, where $<$ is the usual 'less than' relation on $\mathbb{R}$.

Short Answer (4 points each)

Your answers should be a sentence or two, or perhaps some mathematical symbols.

6. Write the negation of the following sentence in plain English:

   If I had a hammer, I'd hammer in the morning.

7. Write the following set in set-builder notation:

$$\left\{\ldots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27 \ldots\right\}$$

8. Let P, Q, R be statements. Write the contrapositive of

$$[(P \wedge \neg Q)] \rightarrow (\neg P \vee R)$$

   in symbolic notation and simplify using DeMorgan's Laws.

9. Let $A = \{x \in \mathbb{Z} \mid x^2 = 1\}$, $B = \{y \in \mathbb{R} \mid y^2 = 2\}$, and $C = \{-2, -1, 0, 1, 2\}$. Write $(C - A) \times B$ in roster notation.

10. Give an example of a relation that is not symmetric, but is reflexive and transitive. You may do this either by specifying a set and a relation on it using mathematical symbols, or you may draw a digraph representing such a relation.

Proofs (10 points each)

11. Choose **three** of the following four propositions and provide a full proof for each of them. Attach additional pages as necessary. (If you attempt all four, please make it clear which ones you would like me to grade.)

    (a) Prove that $3 \mid 5^n - 2^n$ for all $n \in \mathbb{N}$.

    (b) Let $x \in \mathbb{R}$ such that $x > 0$. Prove that $(1+x)^n > 1 + xn$ for all natural numbers $n \geq 2$.

    (c) Let U be a universal set, and let A and B be nonempty subsets of U. Prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

    (d) Define a relation $\sim$ on $\mathbb{Z}$ as follows: $a \sim b$ if and only if $3a + b \equiv 0 \pmod 4$ for all $a, b \in \mathbb{Z}$. Prove that $\sim$ is an equivalence relation and determine all of the equivalence classes for $\sim$.