Thermal Linear Expansion of Wires

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Introduction

As a material changes in temperature, it can expand and contract. The extent to which a material does this can be characterized by a coefficient of expansion, α. The expansion of a material along any given direction can be expressed by

$$\alpha = \frac{1}{l}\frac{dl}{dT} \tag{1}$$

where l is the length of wire going through a temperature change and T is the temperature. Since only the expansion along one dimension is what is to be determined, measuring the expansion of a wire of material is an efficient way to determine its α. For this experiment, two assumptions are used to simplify this equation. First, the value of α is assumed to not change significantly between the temperatures used for this experiment. Secondly, the total expansion observed over the course of this experiment is so small that the total length of the wire measured does not change. That is, the total length of expansion of the wire is unable to be measured by the means used to measure the total length of wire itself.

Experimental Details

A hot water bath was prepared. The water heated up to around $60°C$. A copper tube was placed through a PVC pipe. The length of the copper tube was measured to be $l = 61.5$ An aquarium pump pumped water from the hot water bath, through the PVC pipe, and back out to the hot water bath. This kept the hot water circulating around the outside of the copper tube. The wire to be tested was strung through the copper tube. At the bottom of the tube, the wire was tied to a $1000g$ weight. The wire then went through the copper tube, around a pulley and tied onto a $200g$ weight as shown in fig. 1. The radius of the pulley was measured to be $r = 4.87cm$

When the temperature of the wire sample was to be raised, the aquarium pump was turned on and the circulating water heated up the copper pipe. When the temperature of the sample was to be lowered, the aquarium pump was turned off and the copper pipe was allowed top come to room temperature. To monitor the temperature inside the copper pips, a thermocouple thermometer was put about halfway down the pipe. since α is very small for these wires, an optical lever was used to magnify the amount the length of the wire, L, changed.As the wire expands or contracts it will rotate the pulley. A mirror was attached to this pulley. A meter stick with a telescope on the top of it was placed $d = 348cm$ away. Through the telescope, one could see the image of the ruler through the mirror mounted on the pulley. As the pulley rotated, different sections of the ruler could be seen. This setup is shown in fig. 2. The axis of rotation was marked on the mirror buy marking it on an index card that was overlaid. When the cross-hair of the telescope was lined up with the axis of rotation, the mark on the ruler observed at that point was taken to be the x value for a particular temperature

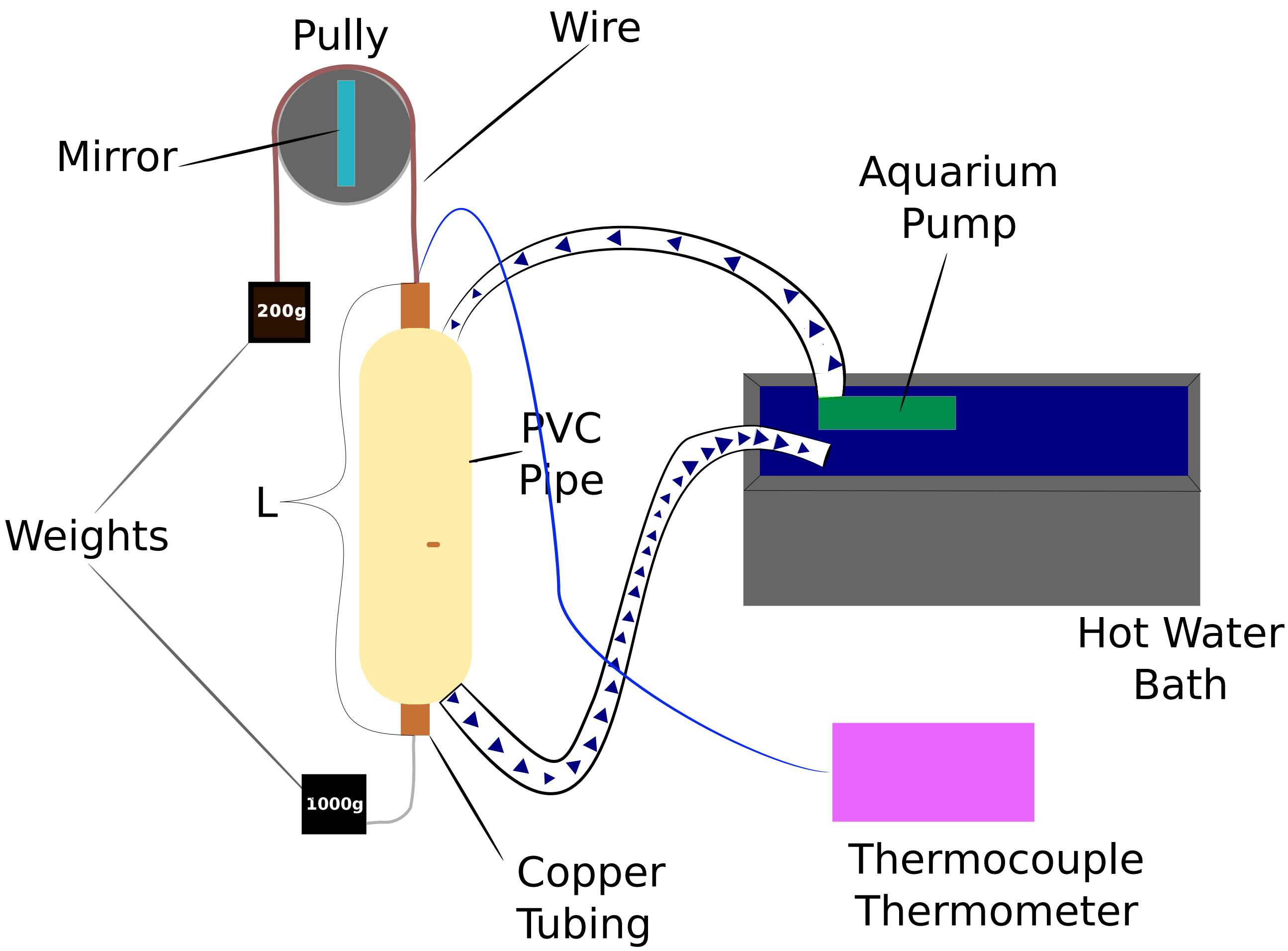

Figure 1: A diagram showing the method by which the wires were heated.

The view through the telescope can be seen in fig. 3. Because the distance, d, between the pulley and the telescope-ruler system was so much longer than the telescope-ruler height, the angle θ formed, as seen in fig. 2, can be approximated by $sin(\theta) \approx \theta$. This allows the following relation:

$$\Delta\theta = \frac{\Delta x}{d} \implies (d\theta) = \frac{1}{d}(dx) \tag{2}$$

where d represents the distance between the mirror and the telescope. The amount θ changes can be related to the angle that the pulley rotatesϕ by

$$\Delta\phi = \frac{\Delta\theta}{2} \implies (d\phi) = \frac{1}{2}(d\theta). \tag{3}$$

Putting equations 2 and 3 together yield

$$(d\phi) = \frac{1}{2d}(dx). \tag{4}$$

Since the rotation of the pulley is cased by the expansion of the wire on it, the change in wire length dl can be related to the rotation of the pulley by

$$(dl) = r(d\phi) = \frac{r(dx)}{2d} \tag{5}$$

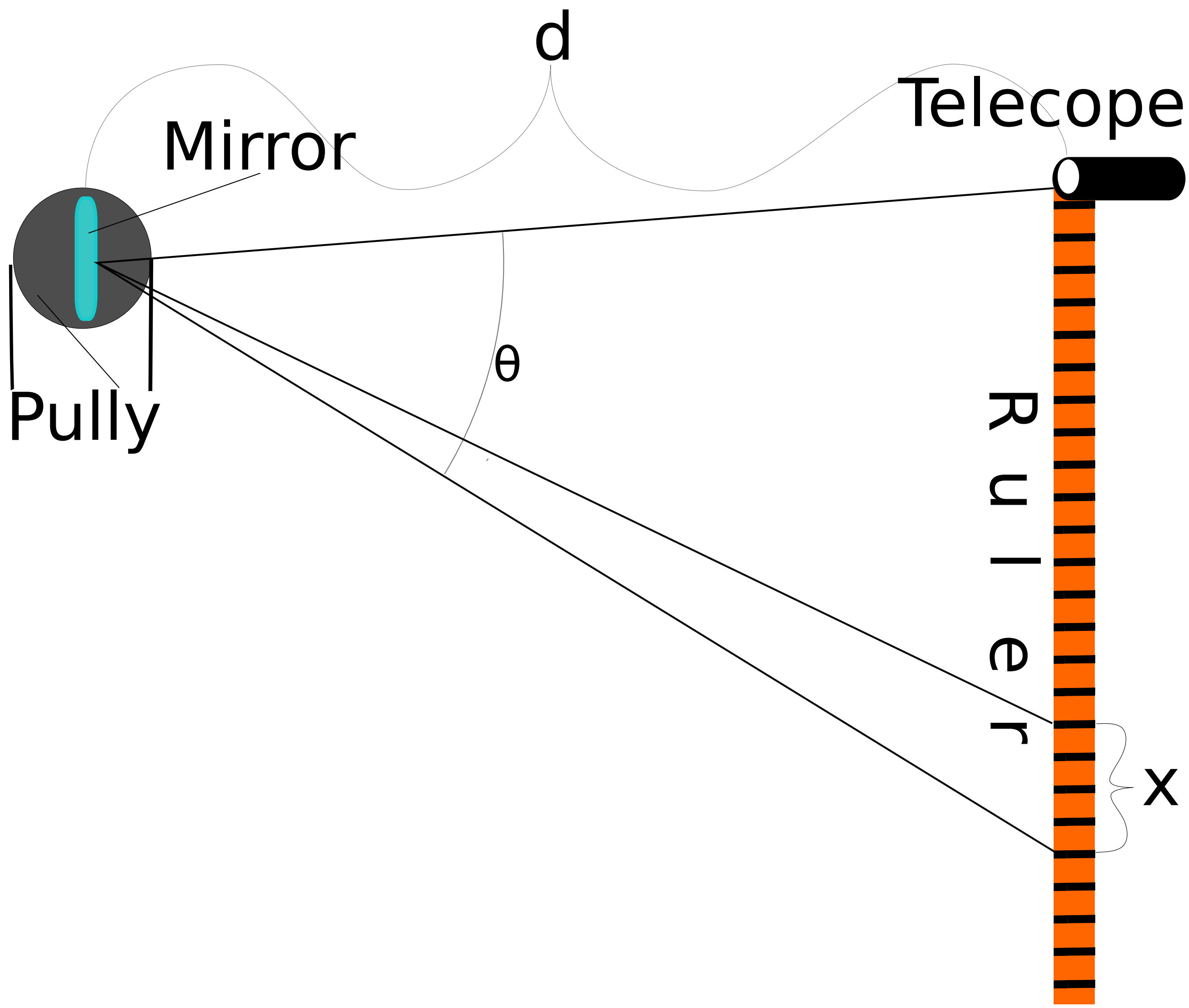

Figure 2: Diagram showing the setup used to measure the rotation of the pulley

where r is the radius of the pully. Combining equations 1 and 5 yields

$$\alpha = \frac{r}{2ld}\frac{dx}{dT} \tag{6}$$

First, a copper wire was examined. Six x values were recorded at various temperatures as the sample heated up from $21.9°C$ to $59.0°C$. The the resulting data can be seen in table 1.

The same was then done for a sample of nickle chrome wire as it cooled from $62.3°C$ to $29.7°C$. The results of that can be seen in table 2

Analysis & Discussion

For this experiment, it was assumed that α does not vary considerably based on the temperatures used. It can then be said that α is constant. Equation 6 then implies that $\frac{dx}{dt}$ must be constant since all the other variables did not change considerably. By plotting the x variable against T, $\frac{dx}{dt}$ can be determined graphically. The data collected on the copper wire is put in a graph and a best fit line

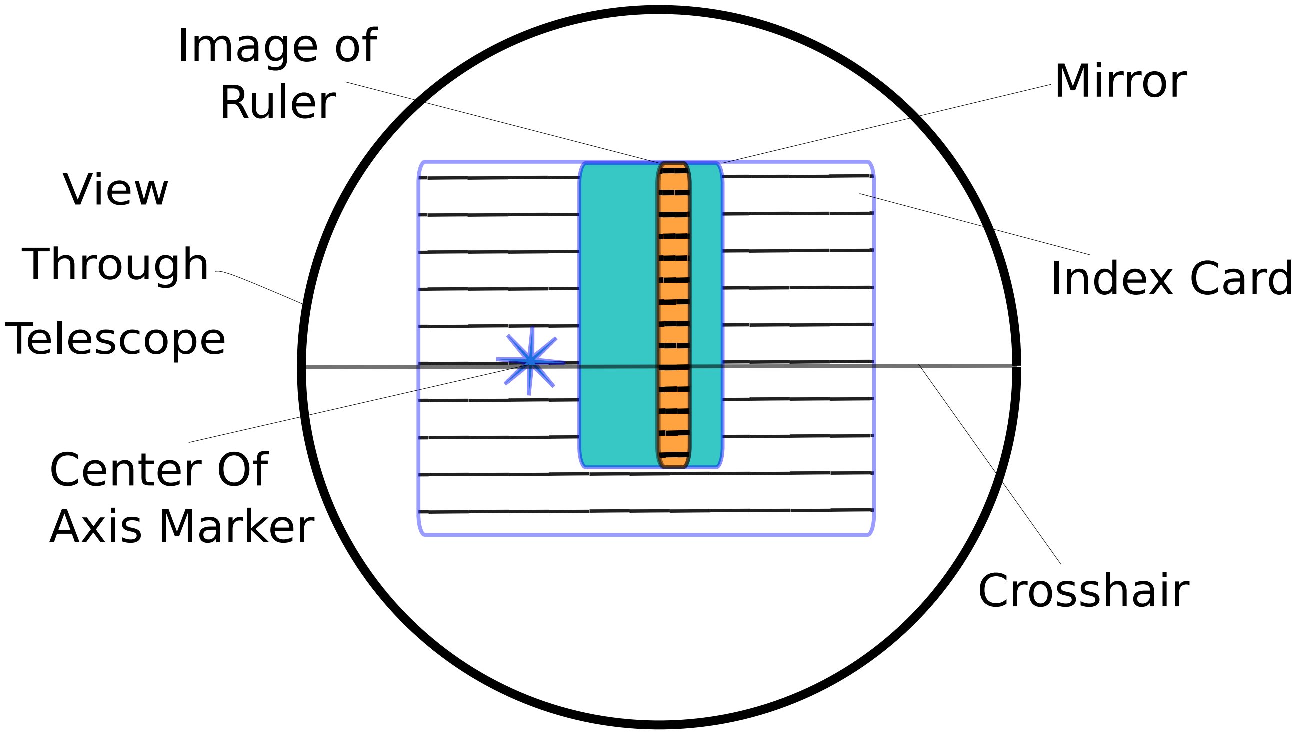

Figure 3: A view of the ruler image as seen through the view of the telescope.

Table 1: "x" Values for the Copper Sample

Temperature	"x" value
21.9	78.
44.0	80.
49.2	82.
53.6	83.
56.4	84.
59.0	85.

is generated 4. The slope of this line suggests $\frac{dx}{dT} = 0.1826 \frac{cm}{°C}$. Using this along with equation 6, the value of α for copper is calculated to be $2.08 \times 10^{-5} \frac{1}{°C}$

The data for the nickel chrome sample is put into a graph and a best fit line is generated 5. The slope of this line suggests $\frac{dx}{dT} = 0.15990401 \frac{cm}{°C}$. Using this along with equation 6, the value of α for nickel chrome is calculated to be $1.82 \times 10^{-5} \frac{1}{°C}$.

Conclusion

The accepted value of α for copper is 1.67×10^{-5}[1]. This means this experiment produced a 25% error. The most significant cause of this error is most likly inaccuracies in reading the "x" value through the telescope. Due to the distances involved, even with the telescope, only the large "10cm" marks of the ruler were able to be read. The in between notches had to be counted and estimated which value was the one in the cross-hairs. In future iterations of this experiment, it may be helpful to have some slide able marker to attach to the meter stick to be able to line up with the cross-hairs and read the value of "x" afterwards. During the final measurements of this experiment, a pen was used somewhat

Table 2: "x" Values for the Nickel Chrome Sample

Temperature	"x" value
62.3	36.5
63.6	37.0
49.0	34.5
45.1	34.0
39.5	33.5
33.4	33.0
30.8	32.1
29.7	30.0

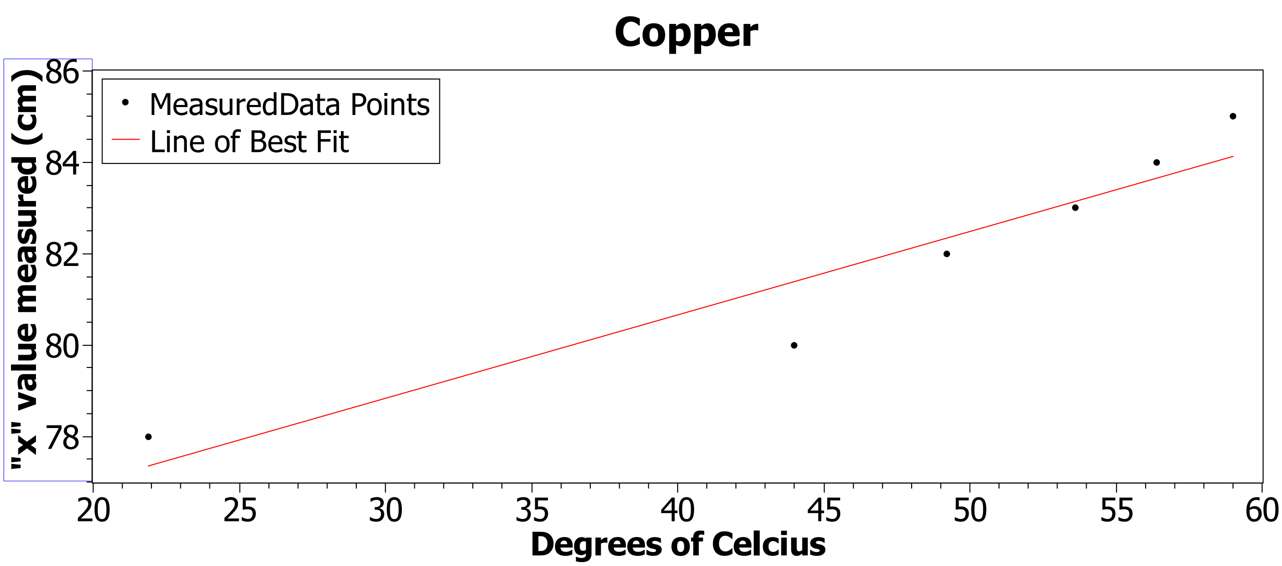

Figure 4: Graph showing the relation between the measured "x" value and the temperature for the copper sample

to this effect. If one was to make use of a laser as a means for the optical lever, a greater value of d may be obtained and would most likely result in a more accurate measurement. Another issue was the inability to get the copper wire completely straight. It is unclear what effect this may have had on the results.

References

References

[1] Fishbane,P.M., Gasiorowicz, S. and S. Thornton Physics for Scientists and Engineers, Prentice Hall, 1993.

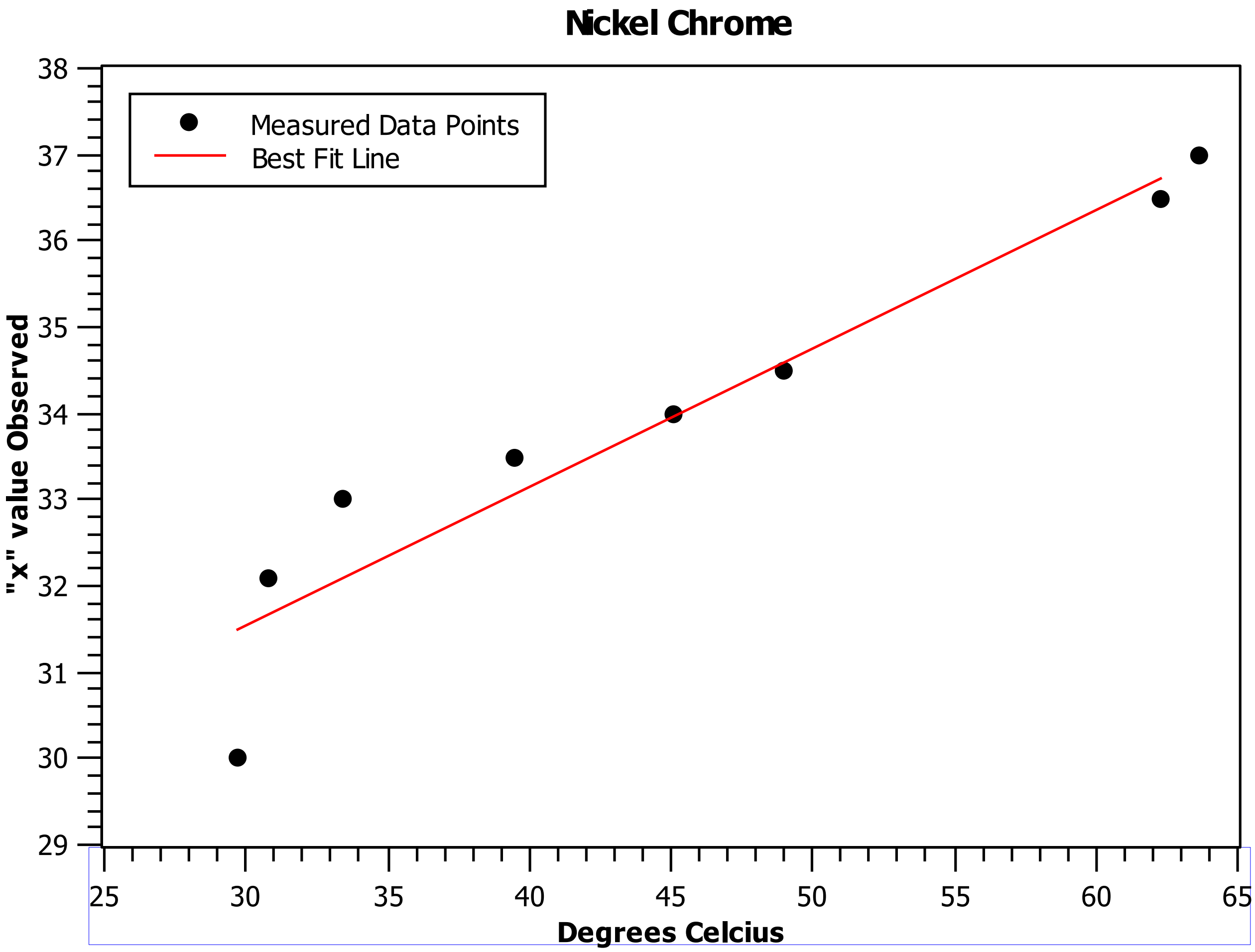

Figure 5: Graph showing the relation between the measured "x" value and the temperature for the nickel chrome sample