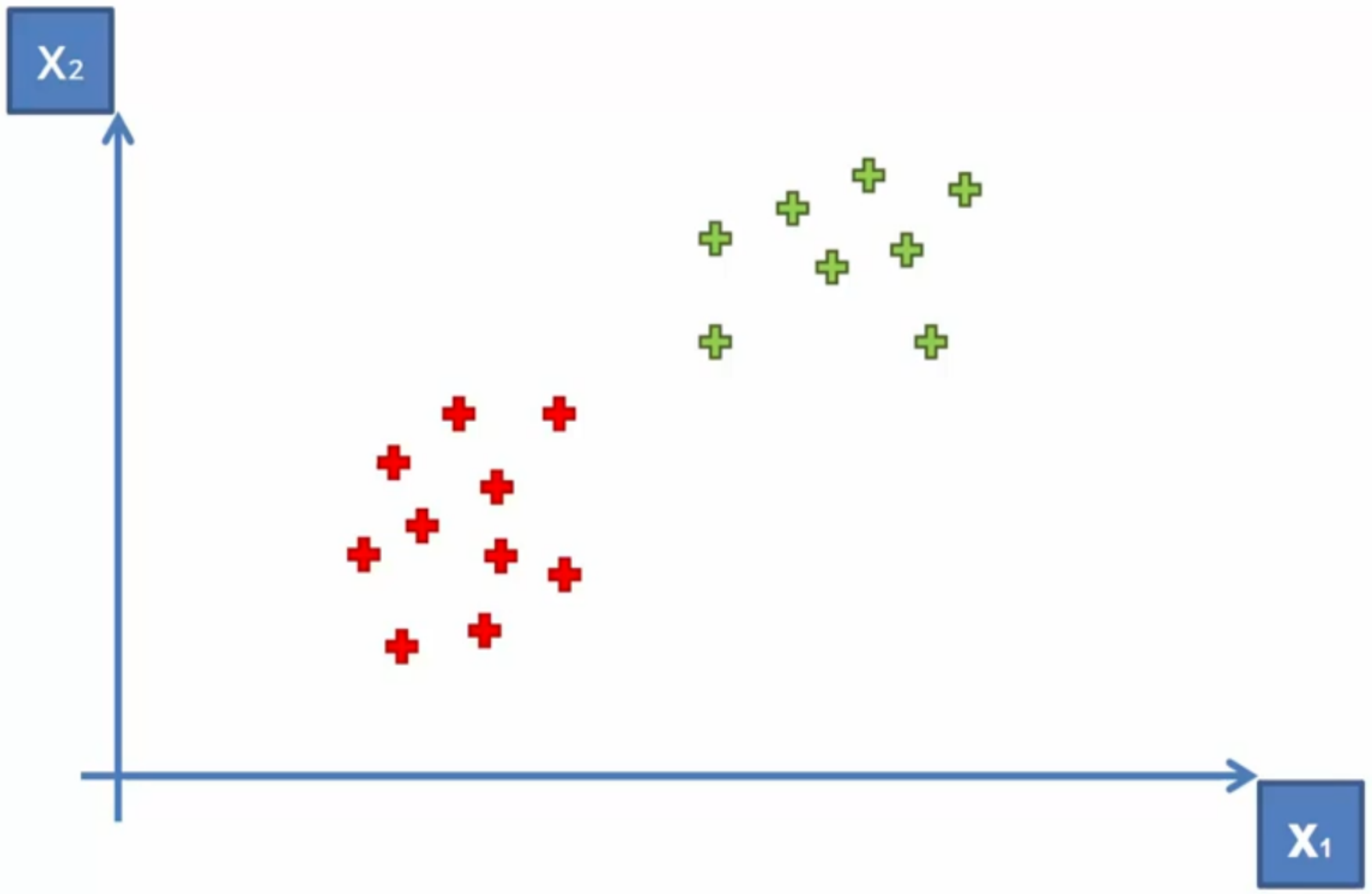
Support Vector Machine

Content

- Support Vector Machine
  - Linearly Separable
  - Non-Linearly Separable
  - Hyperplane / Decision Boundary
  - Margin
  - Marginal Distance
  - Support Vectors
  - Kernel SVM

How to separate these points ?



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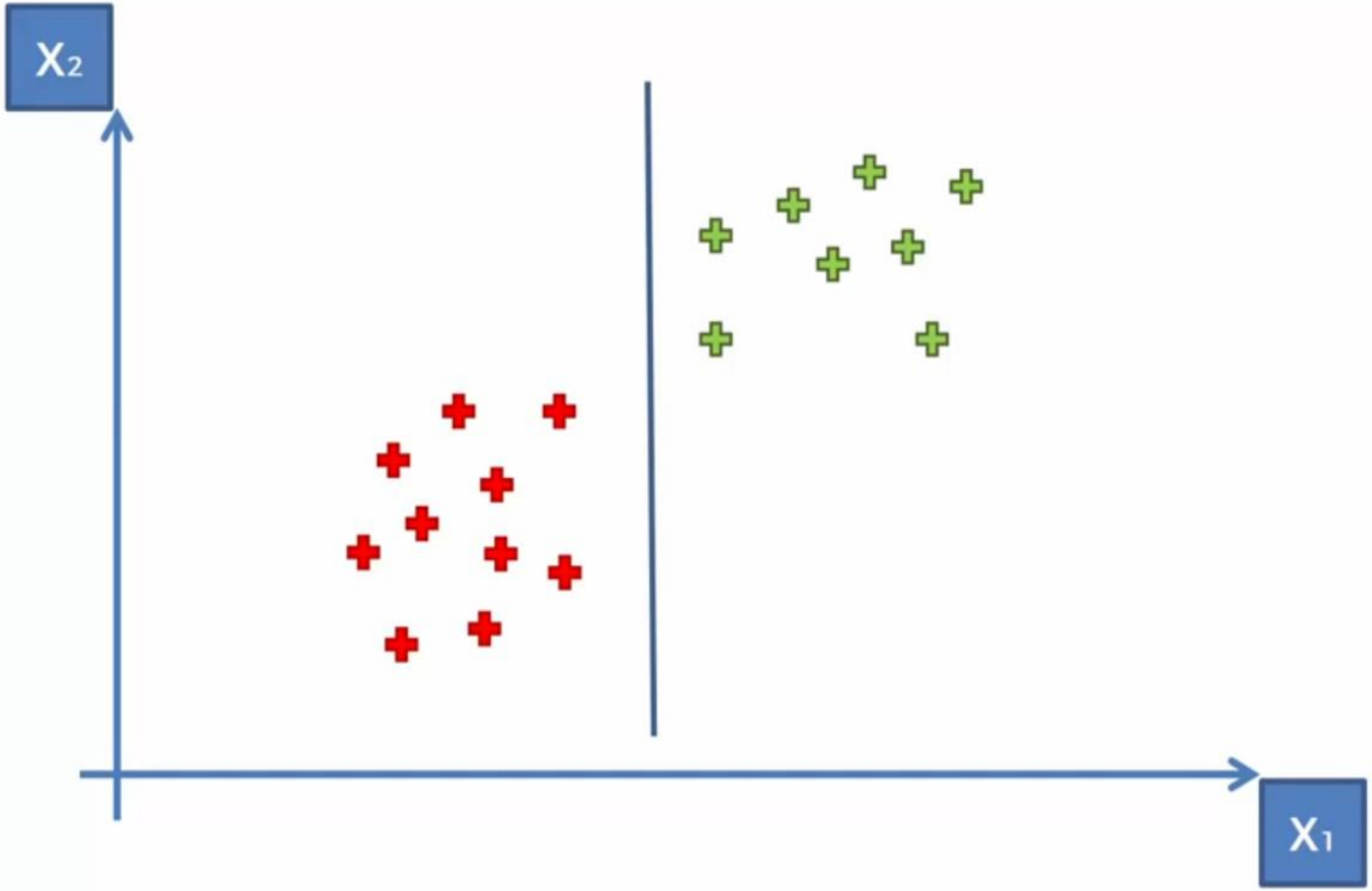

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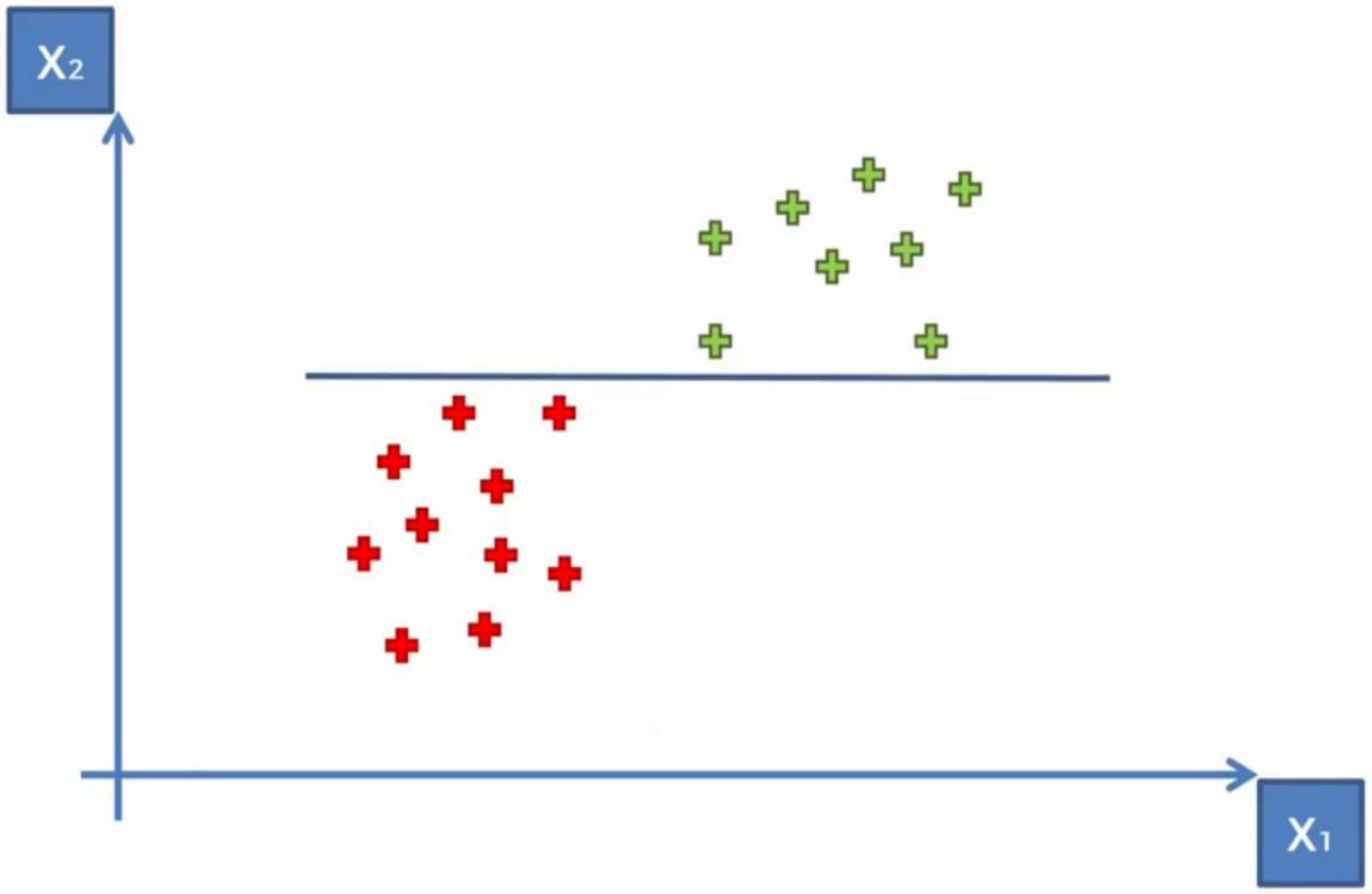
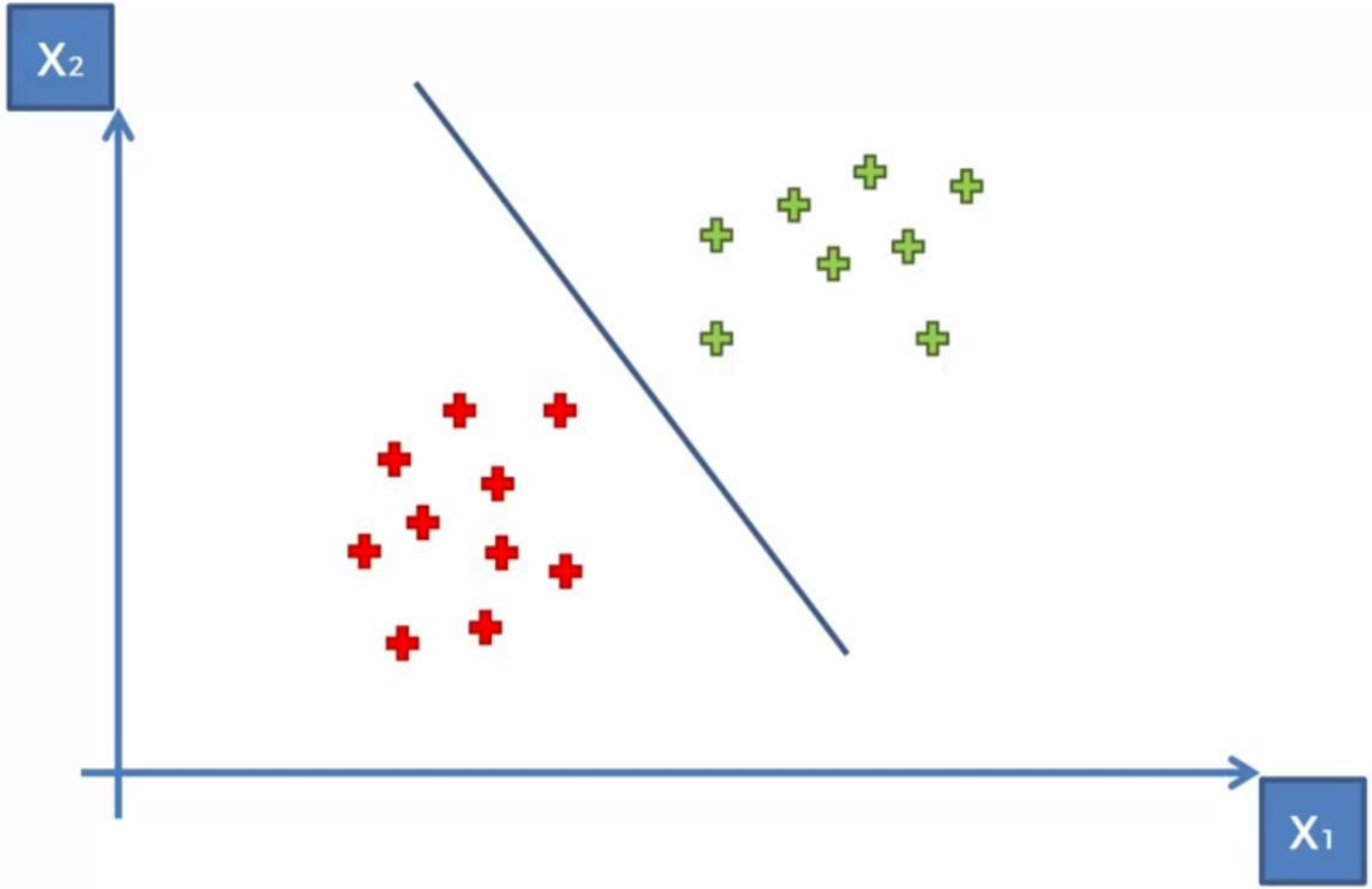
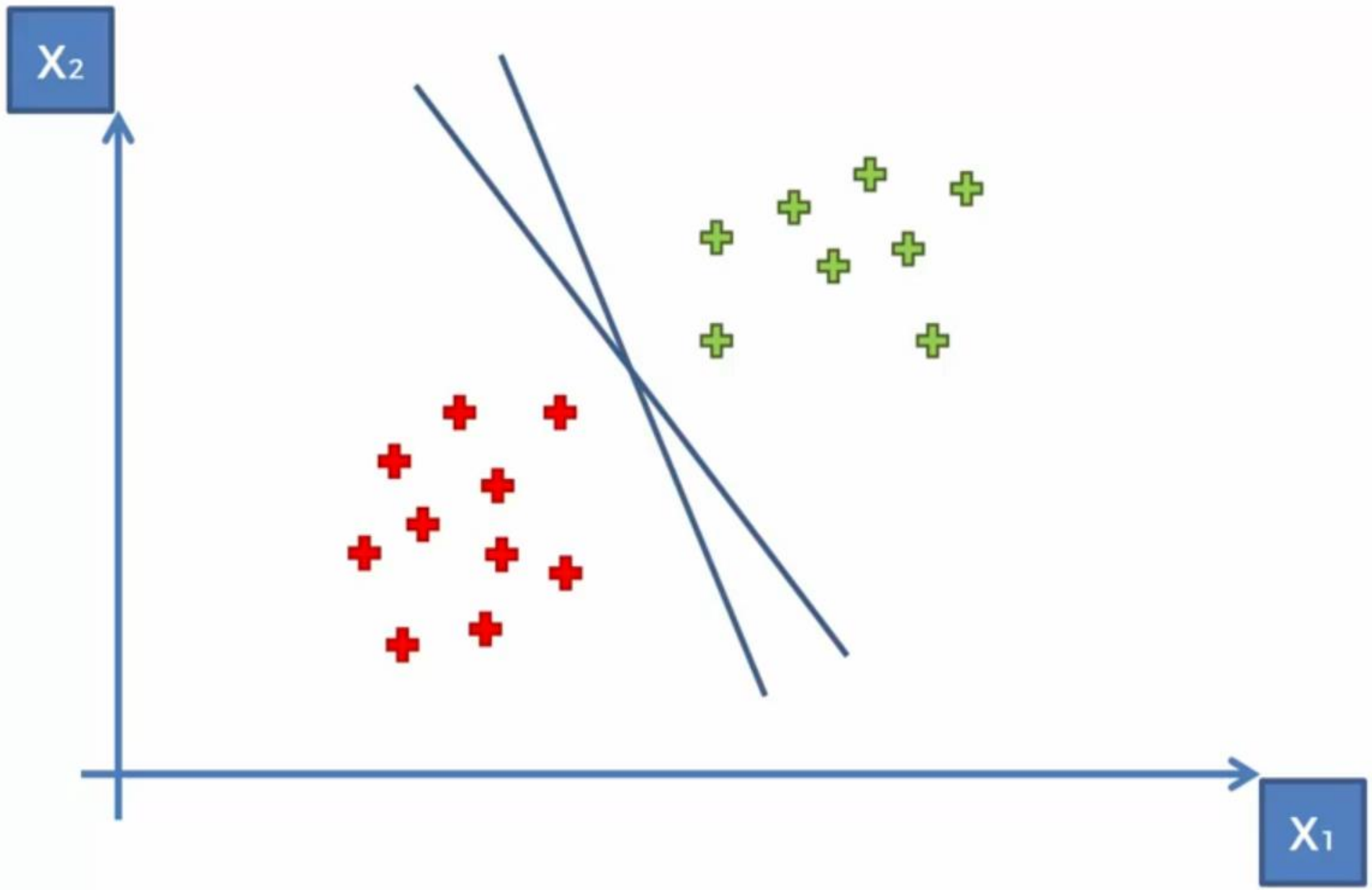


SVM



SVM



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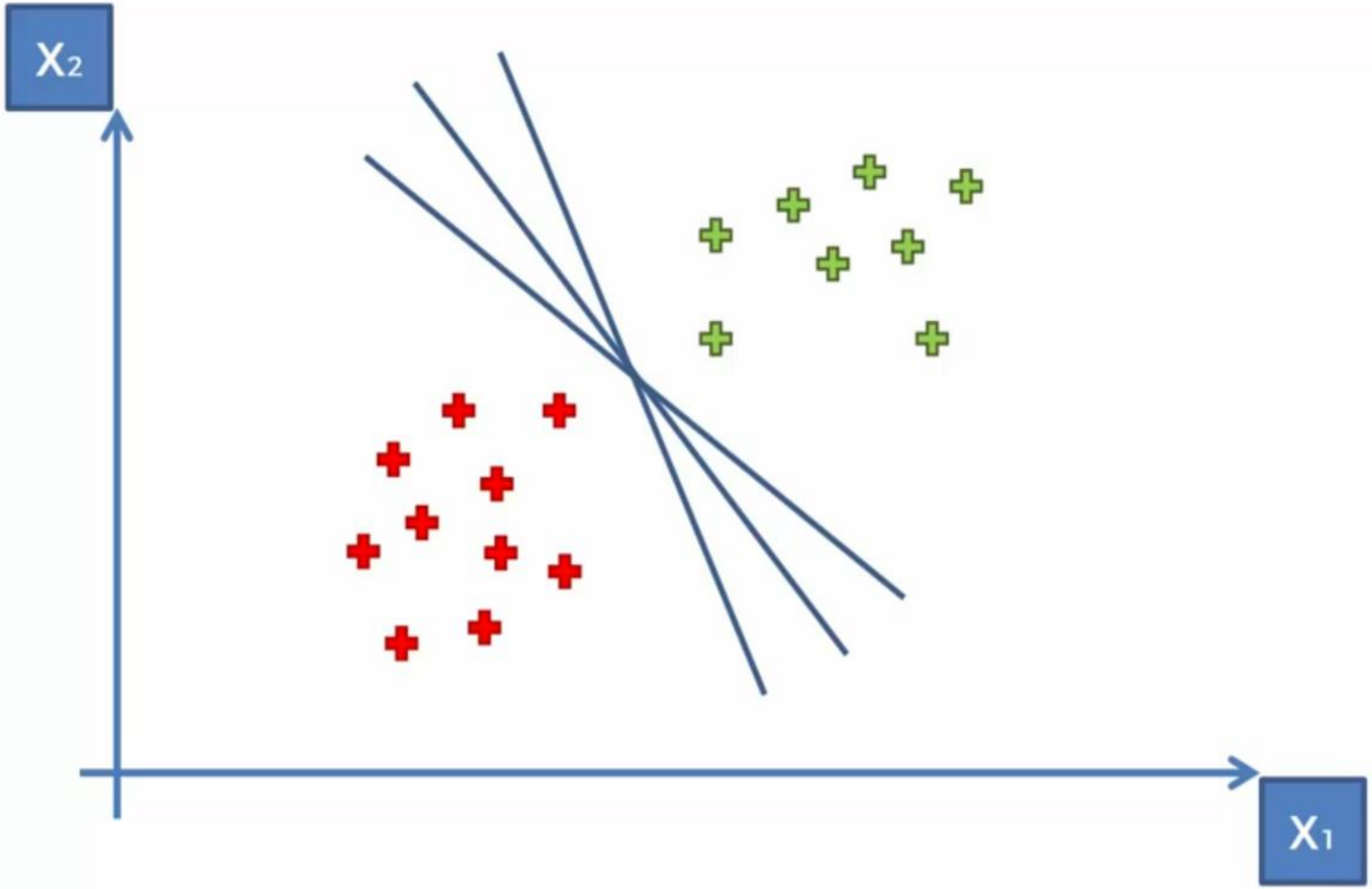
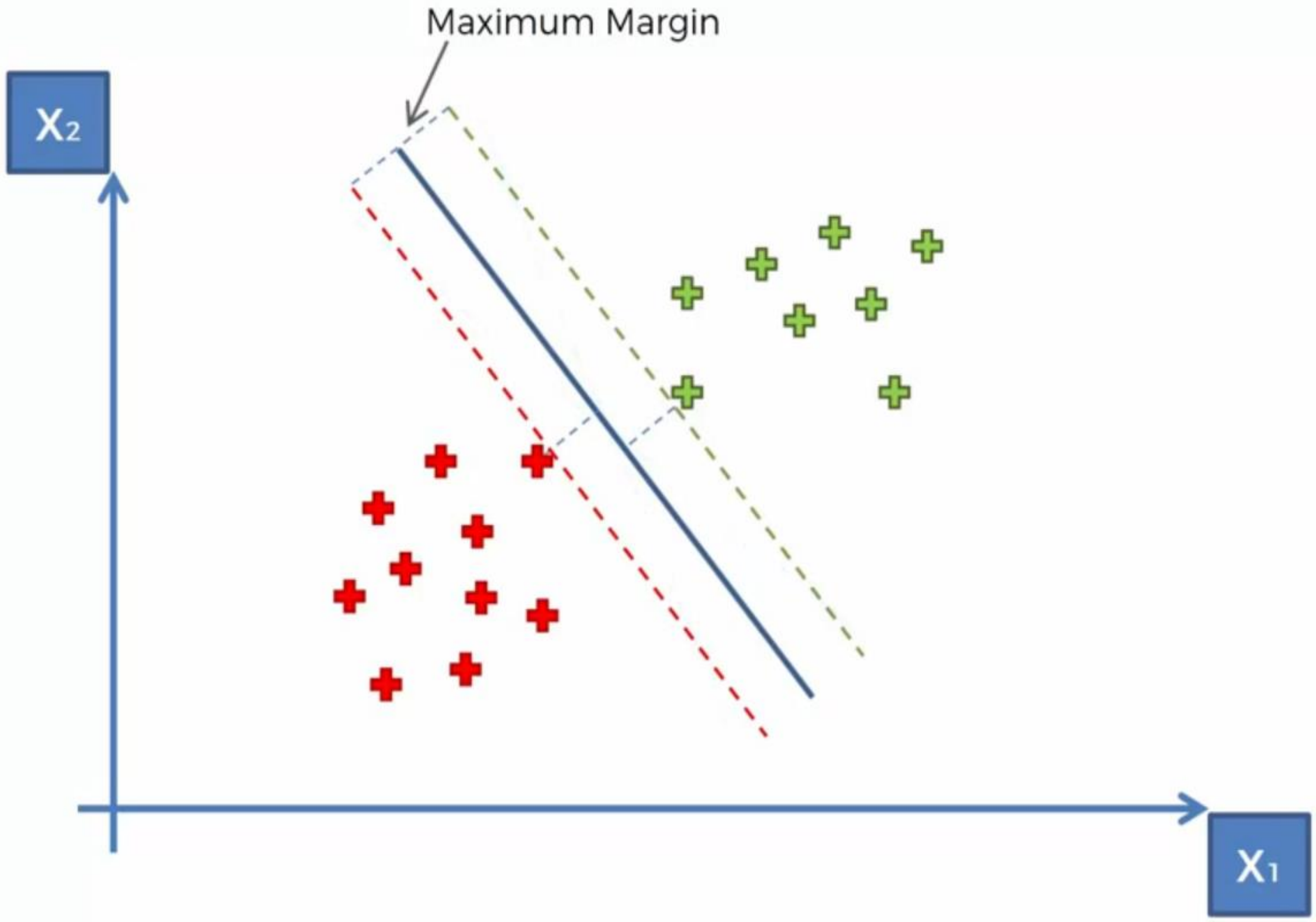
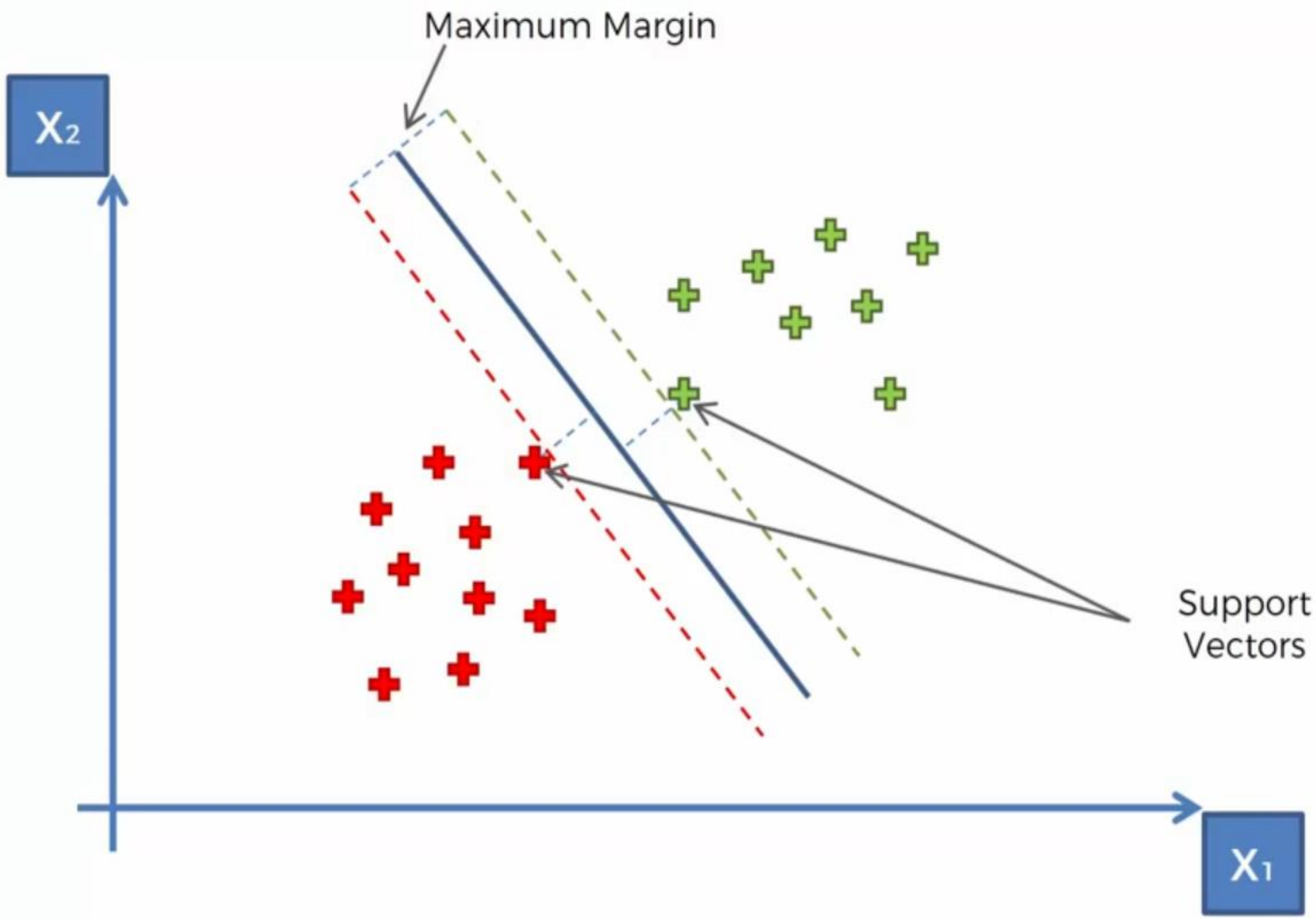


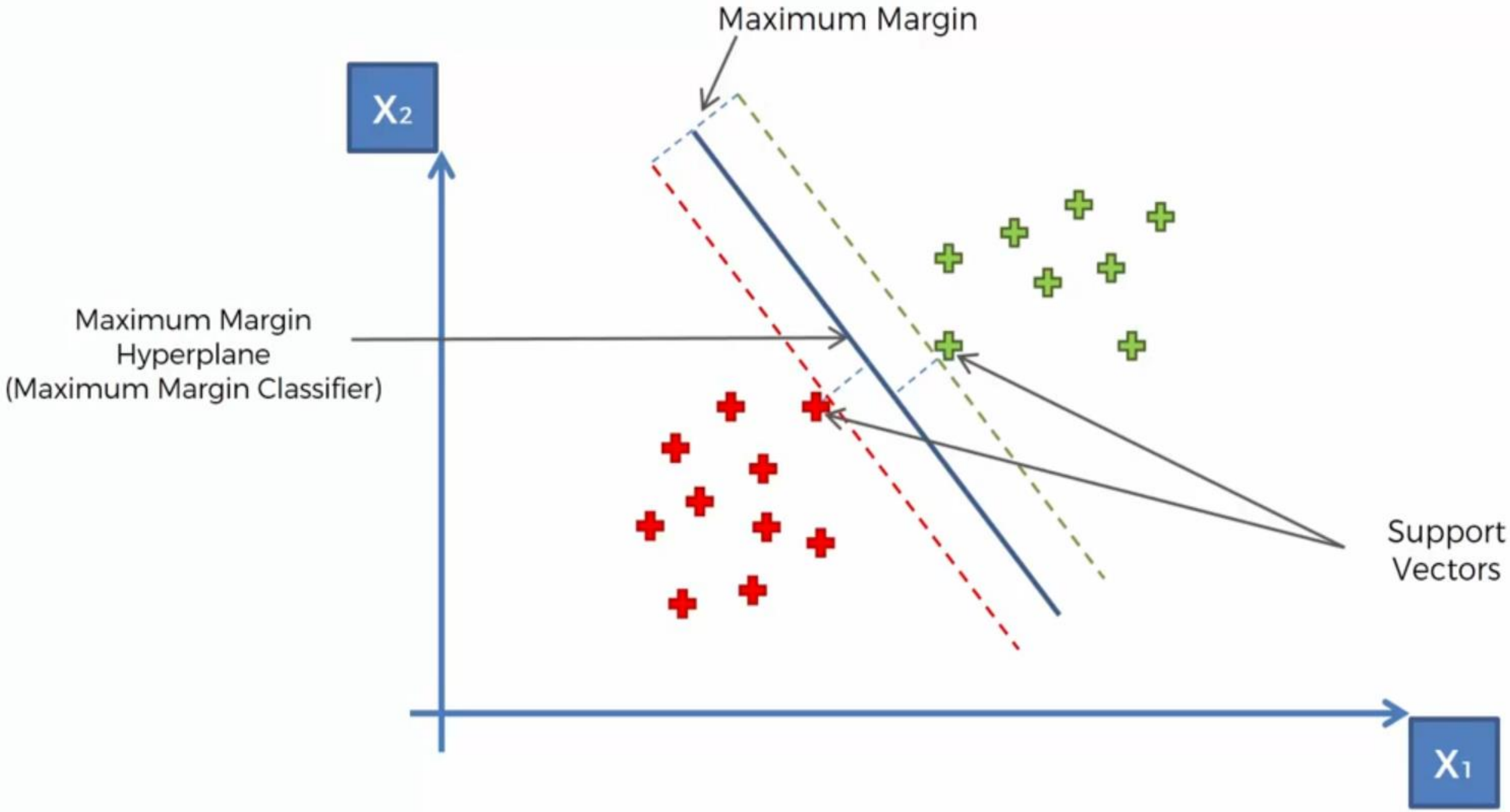
Maximum Margin



Support Vectors



Hyperplanes



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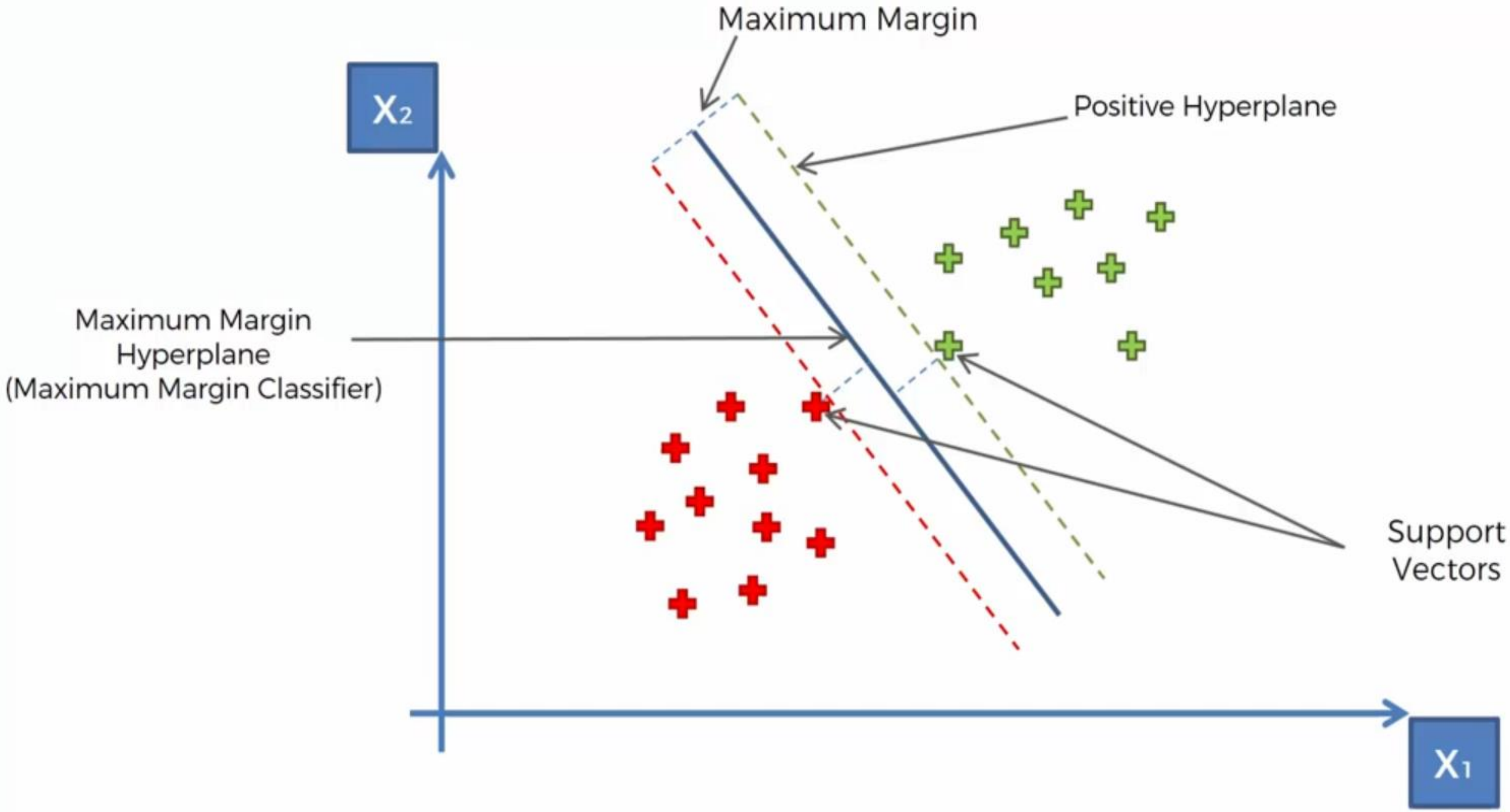

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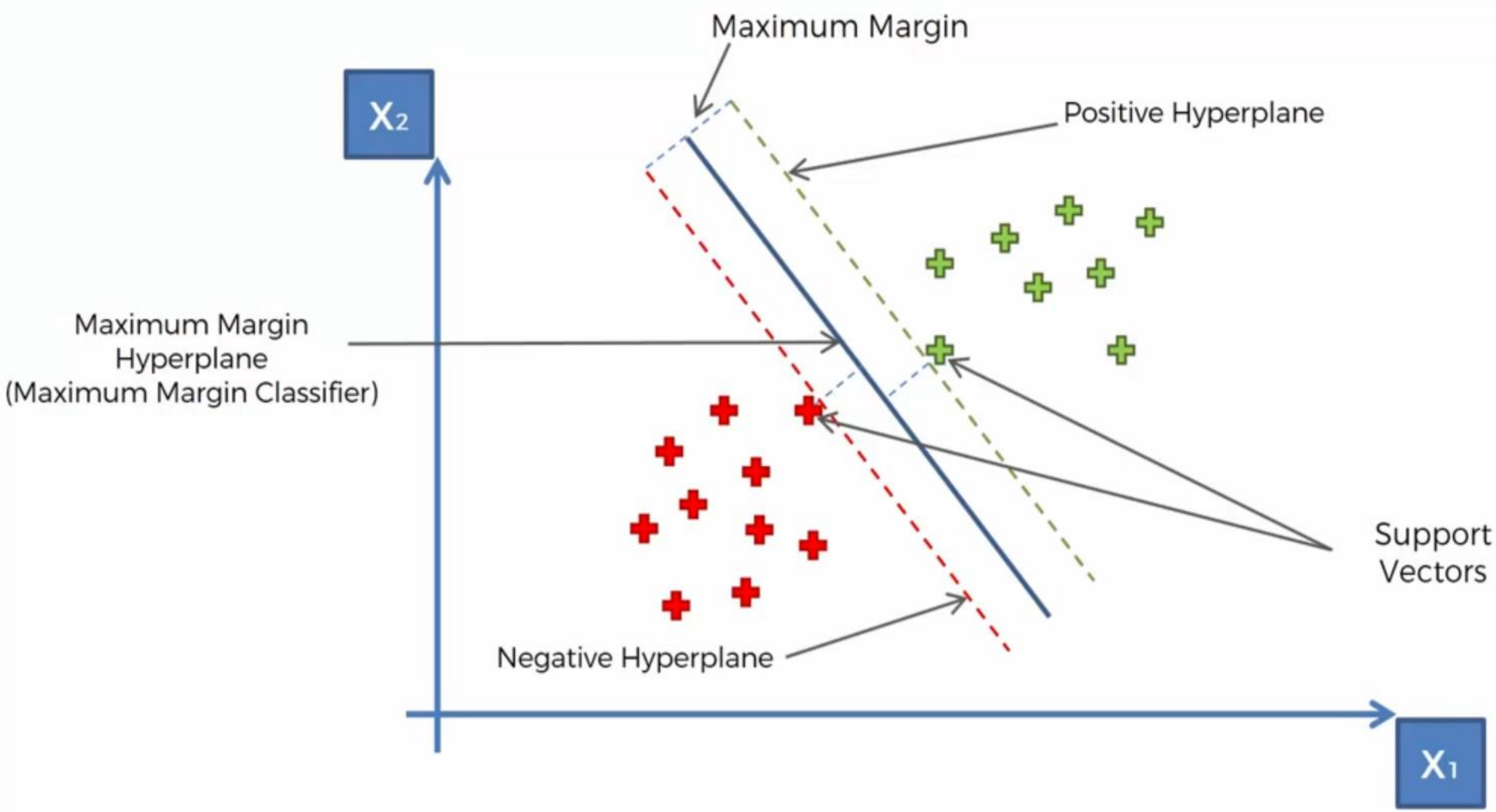

What’s So Special About SVMs?

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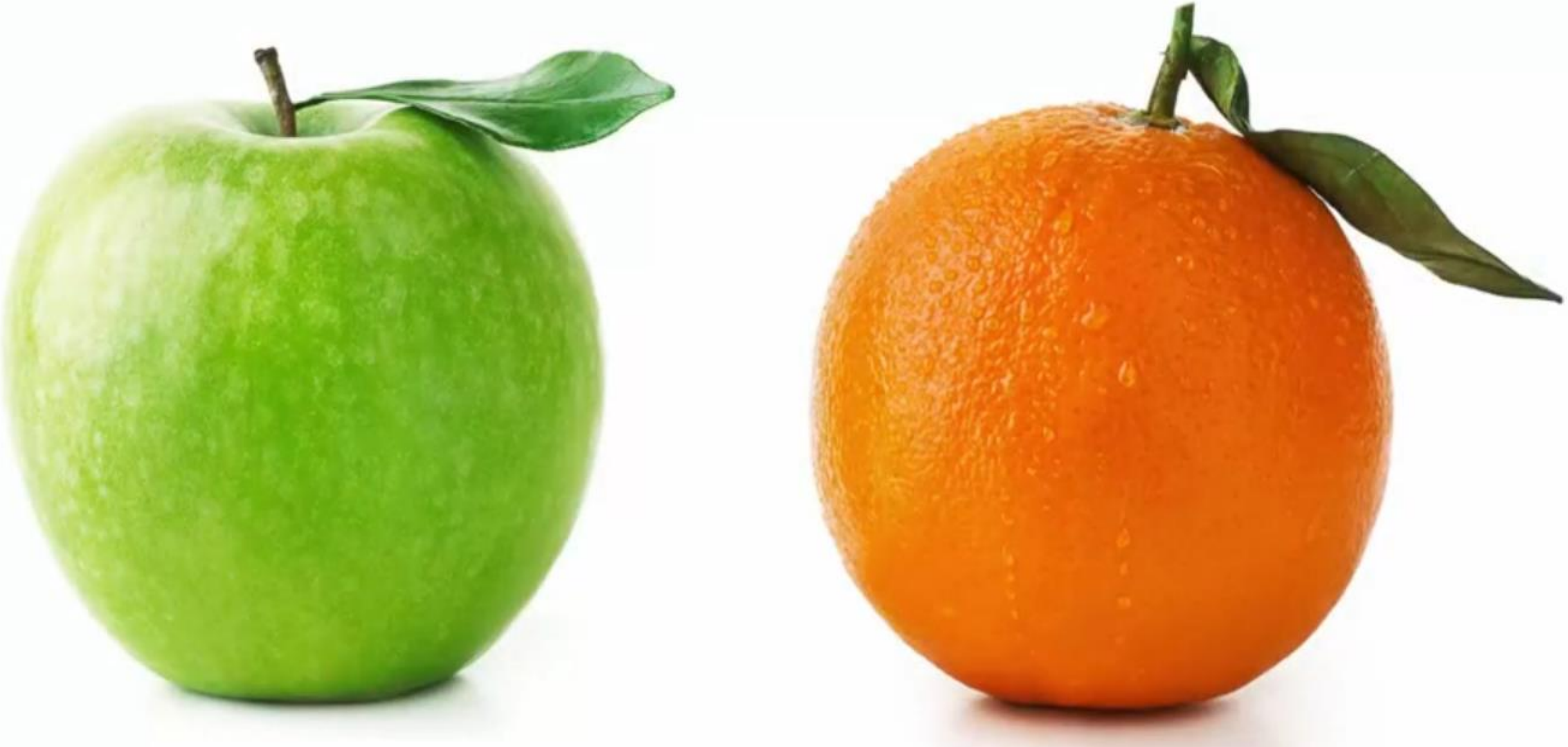

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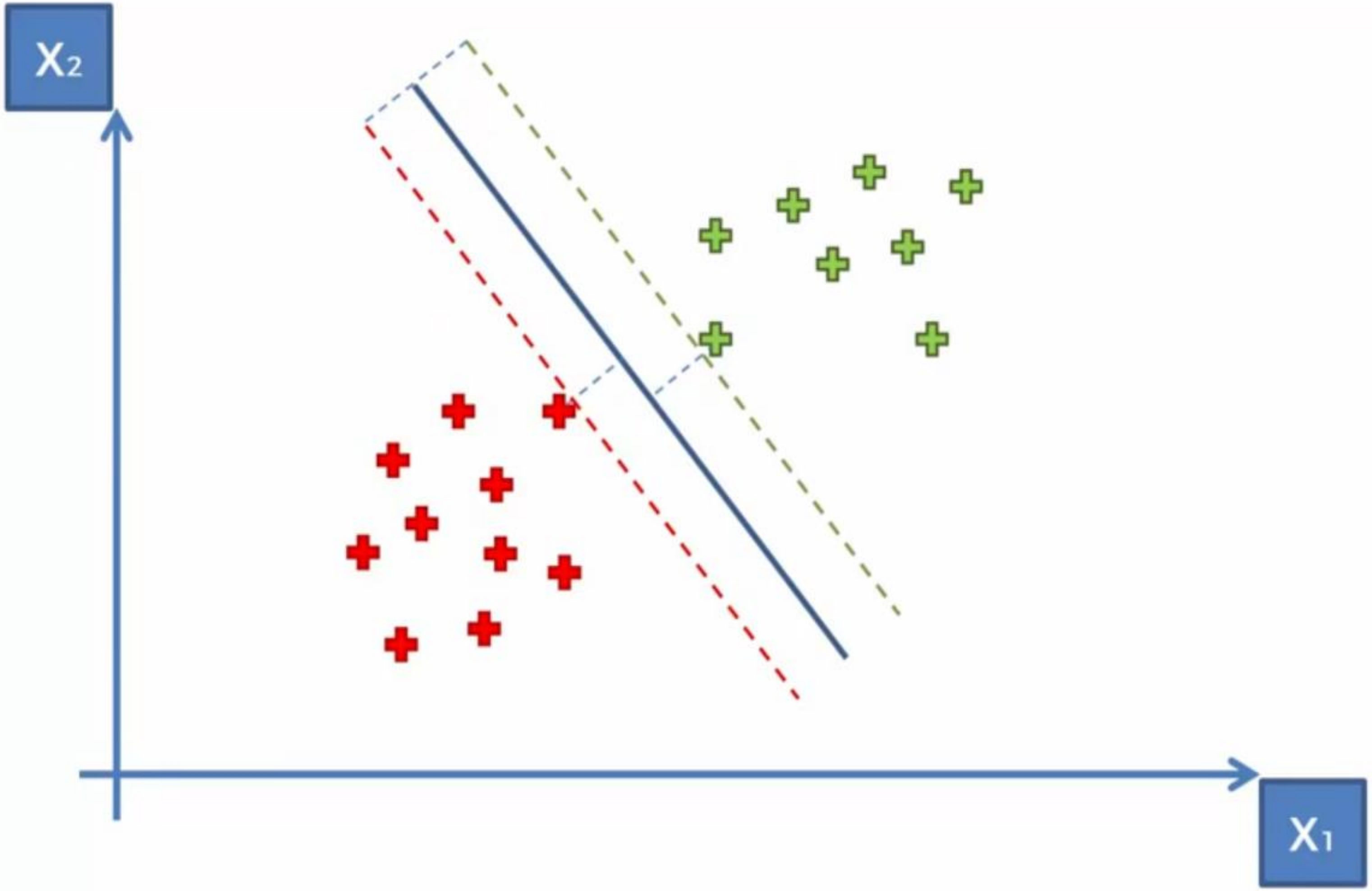

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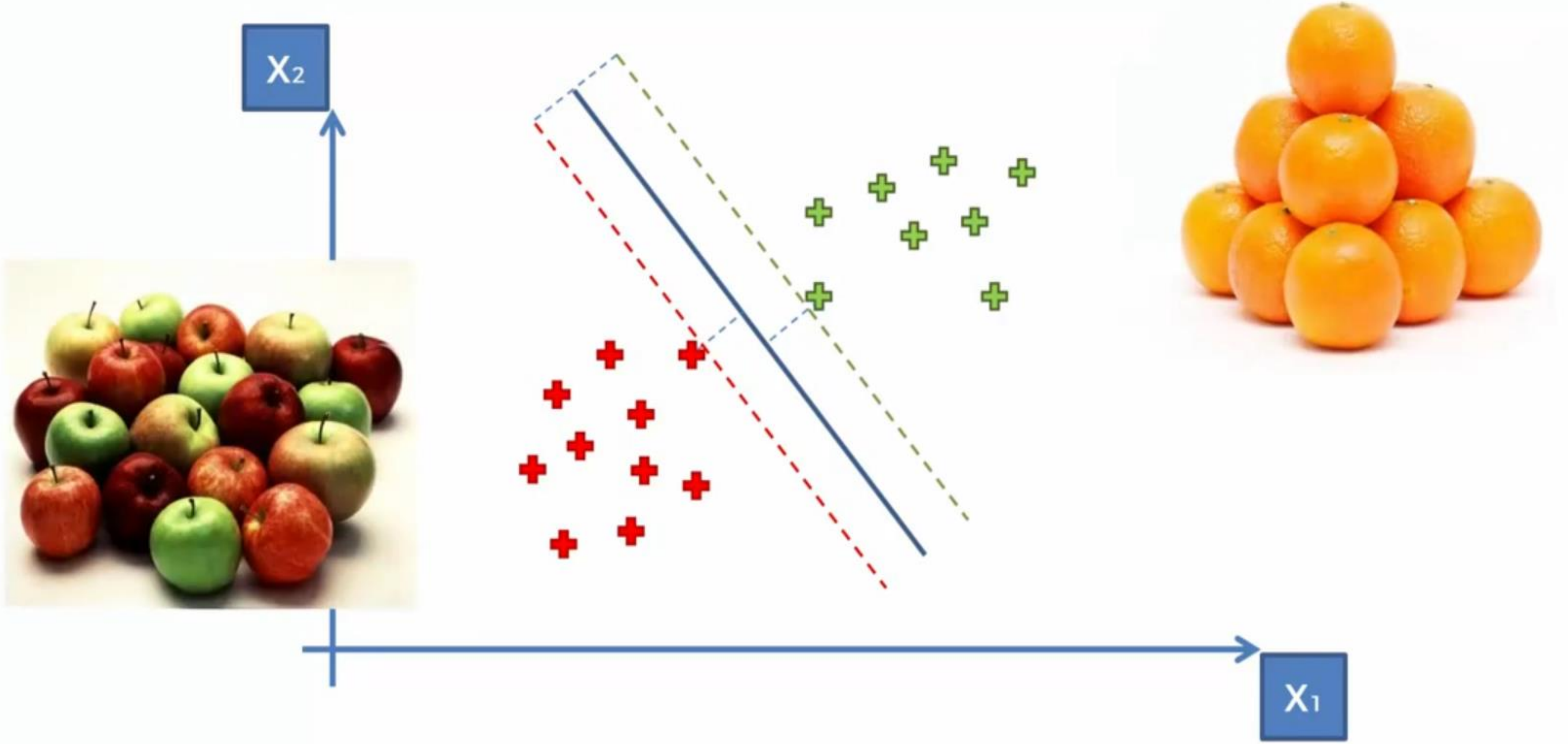

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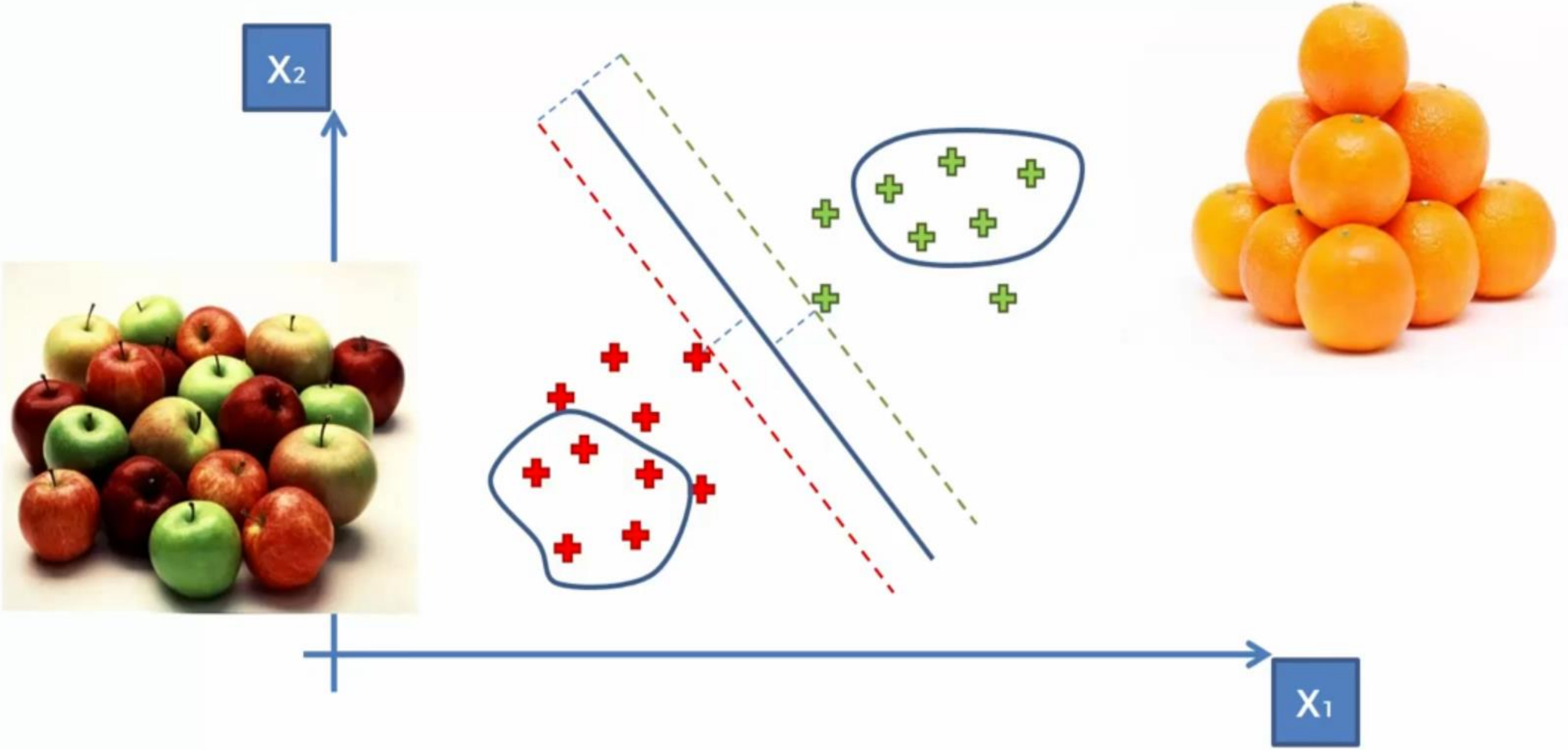

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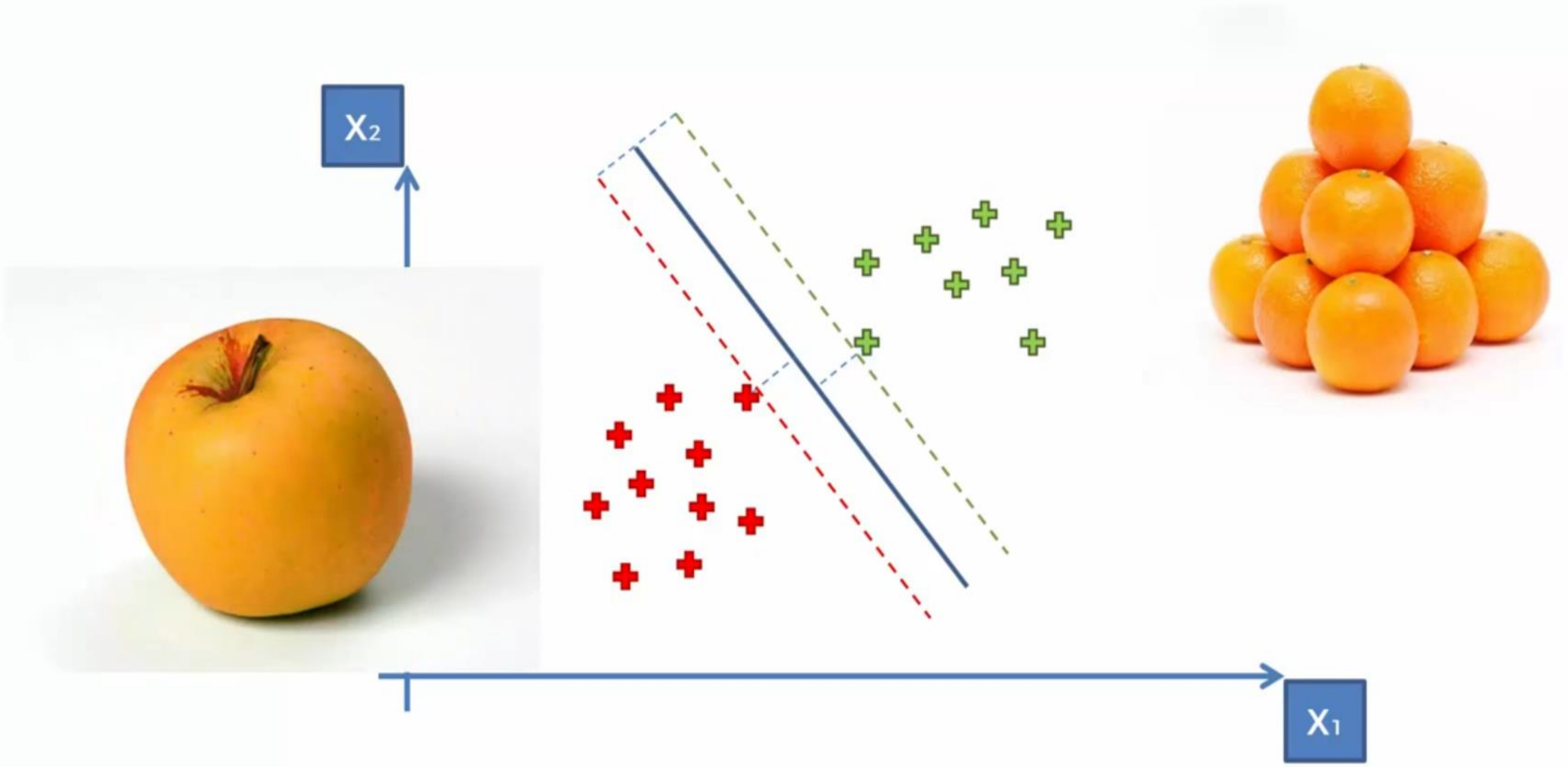

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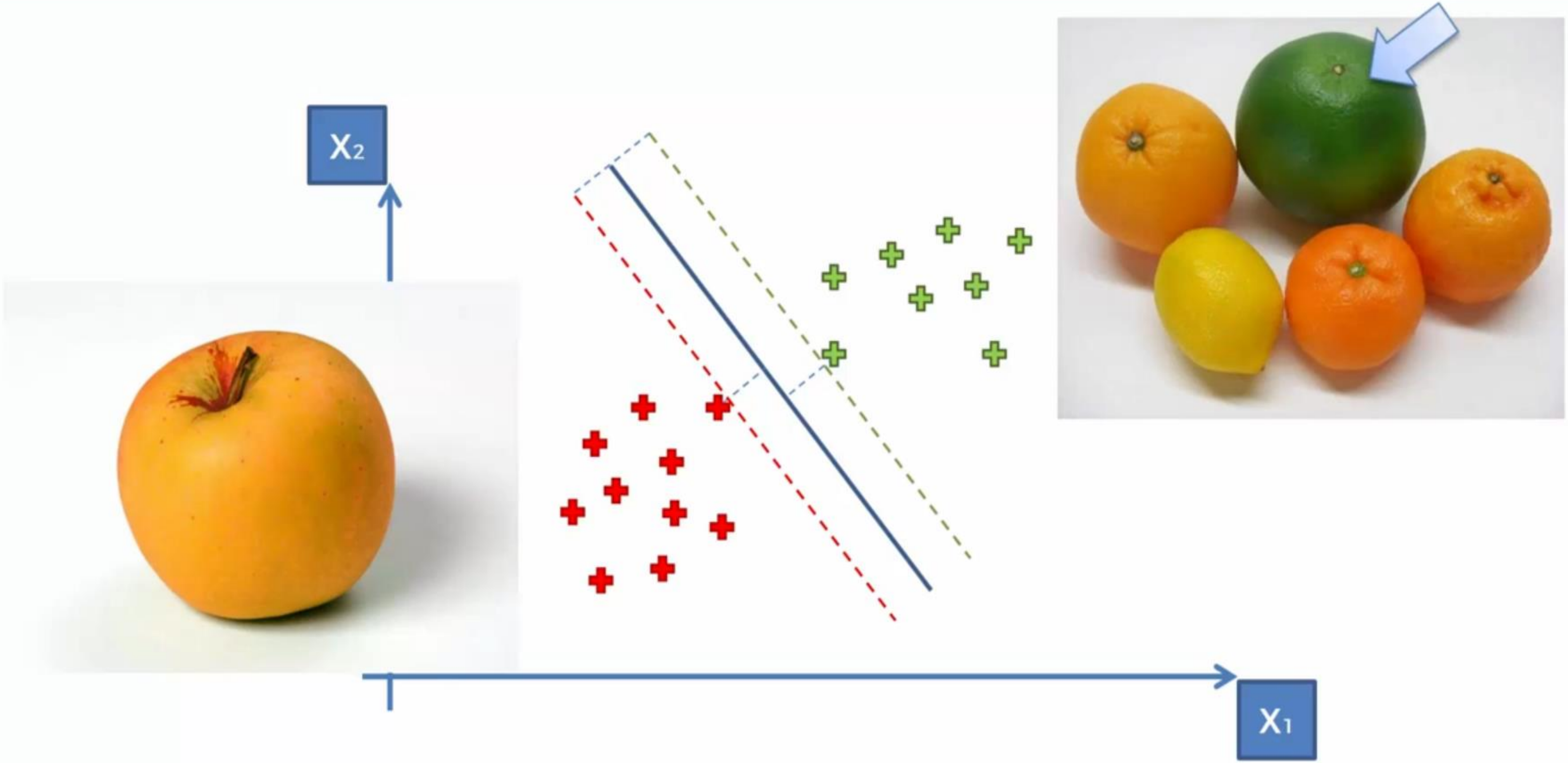
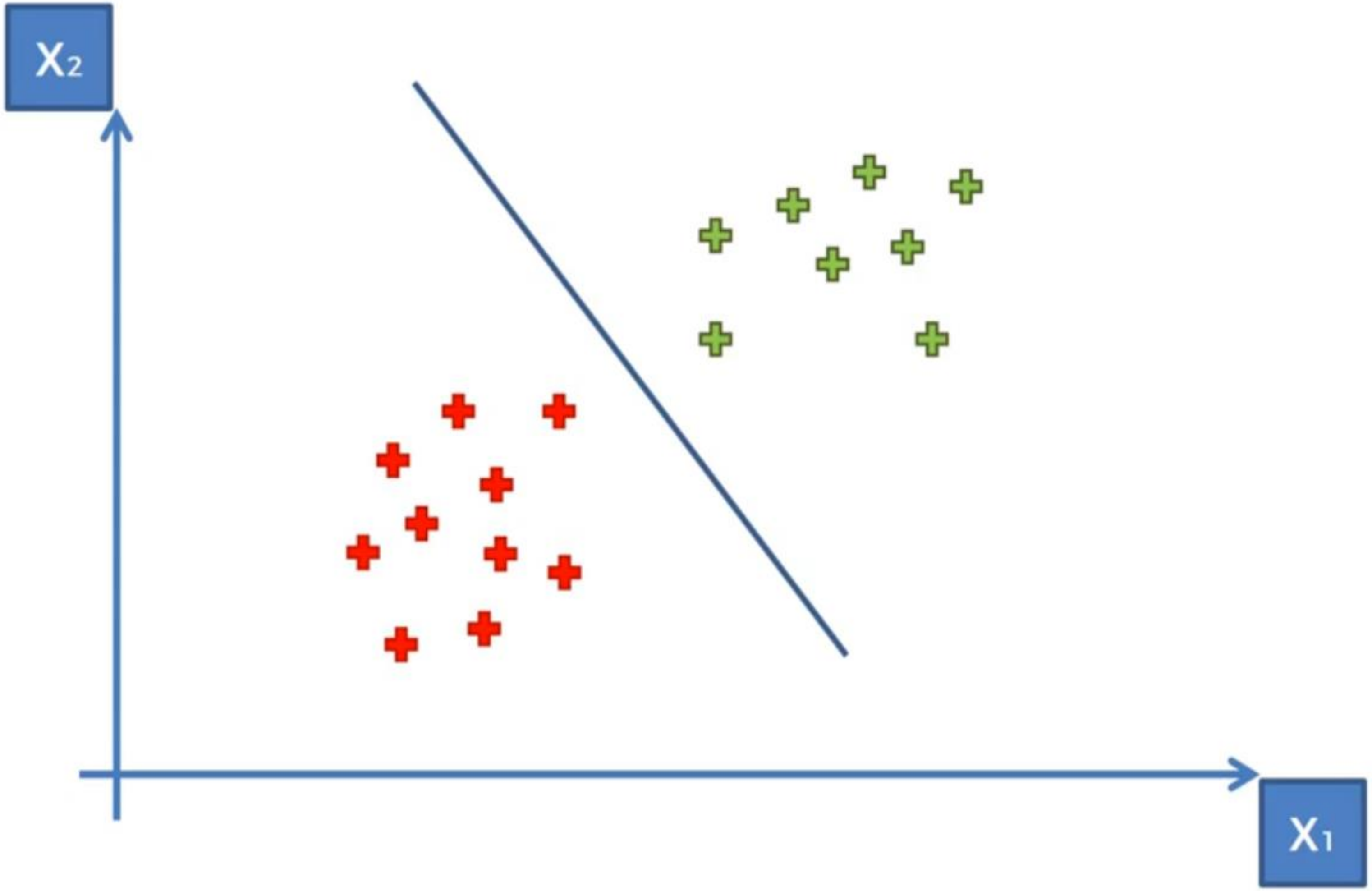


SVM separates well these points



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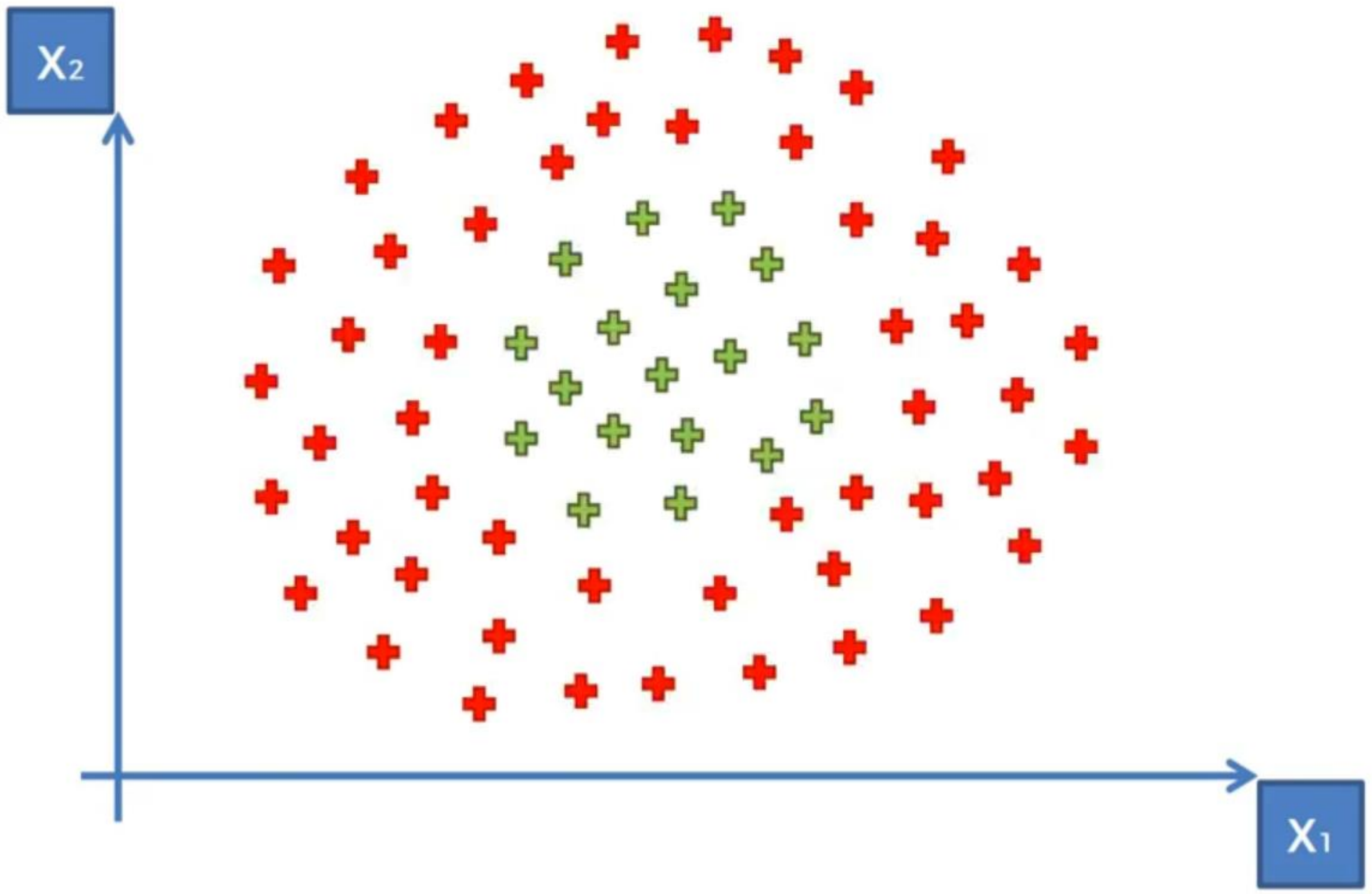

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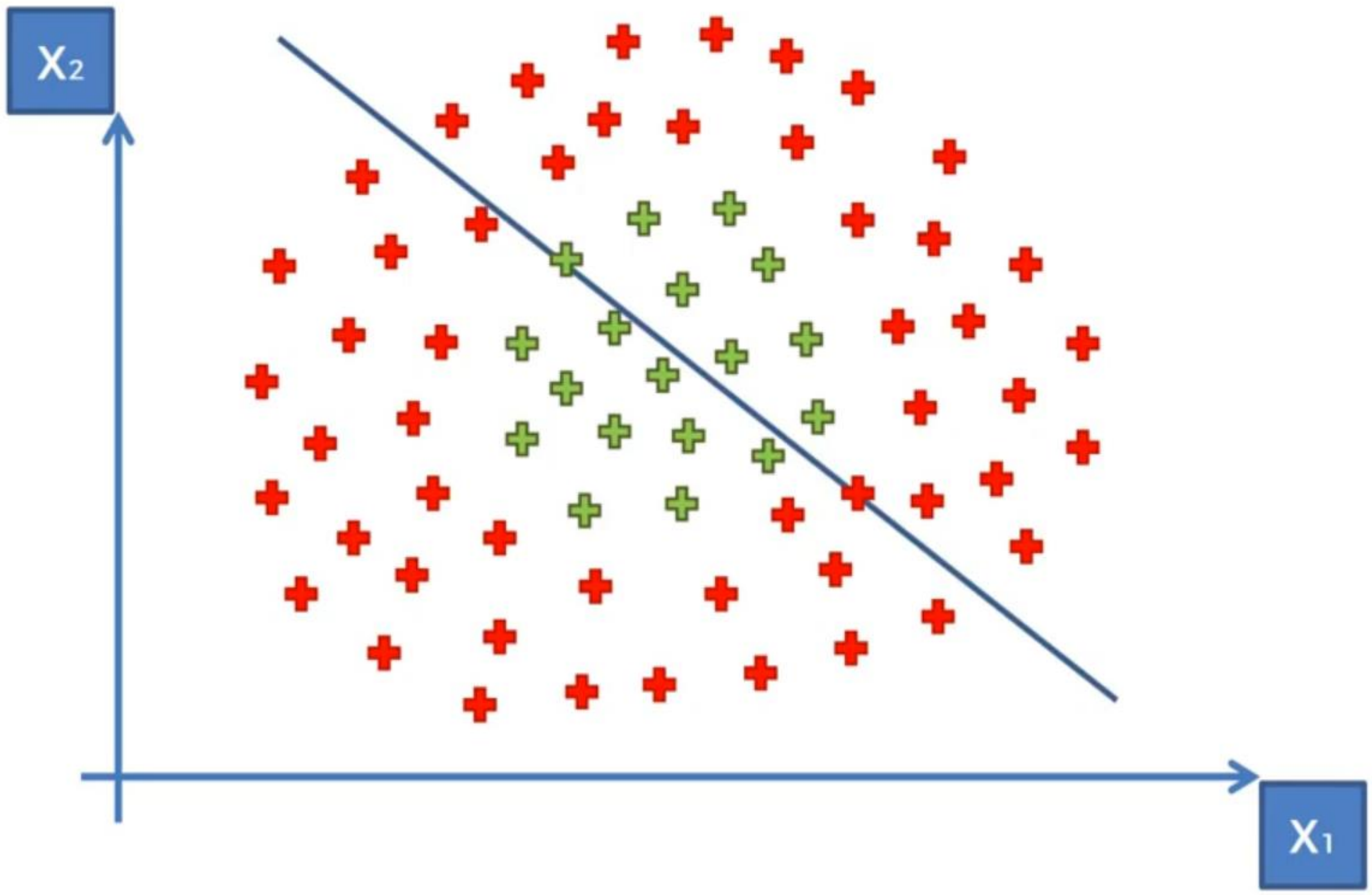

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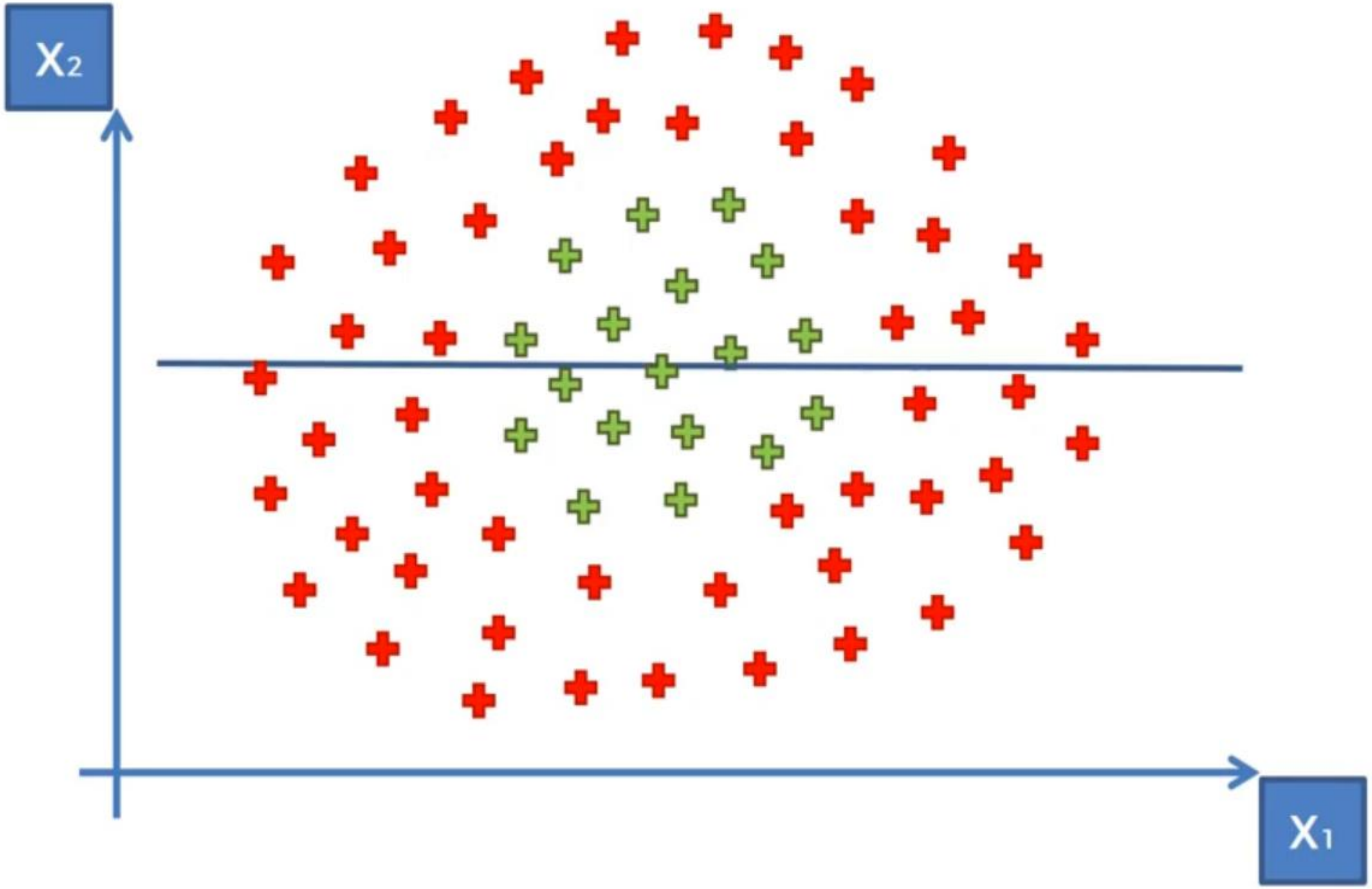

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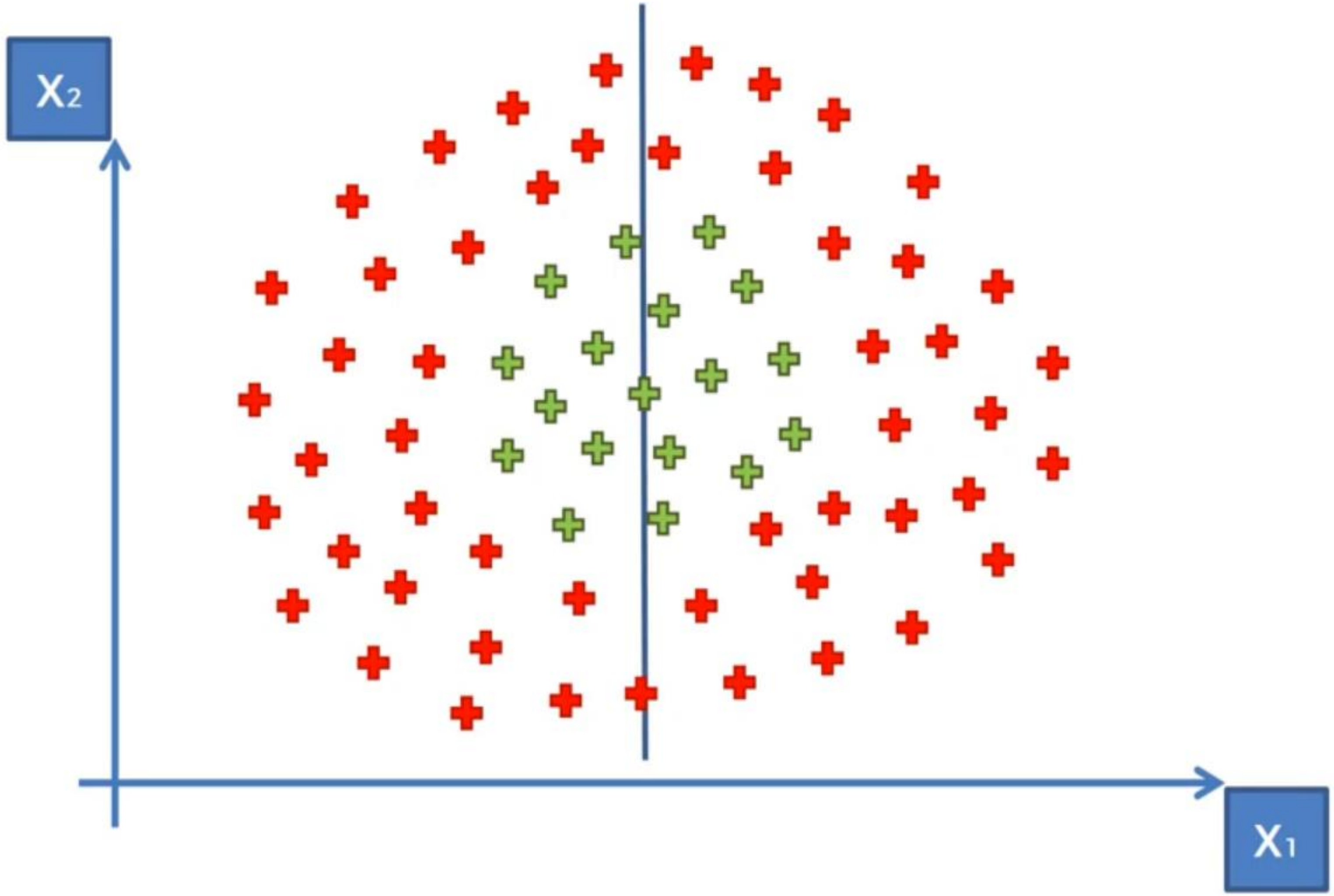

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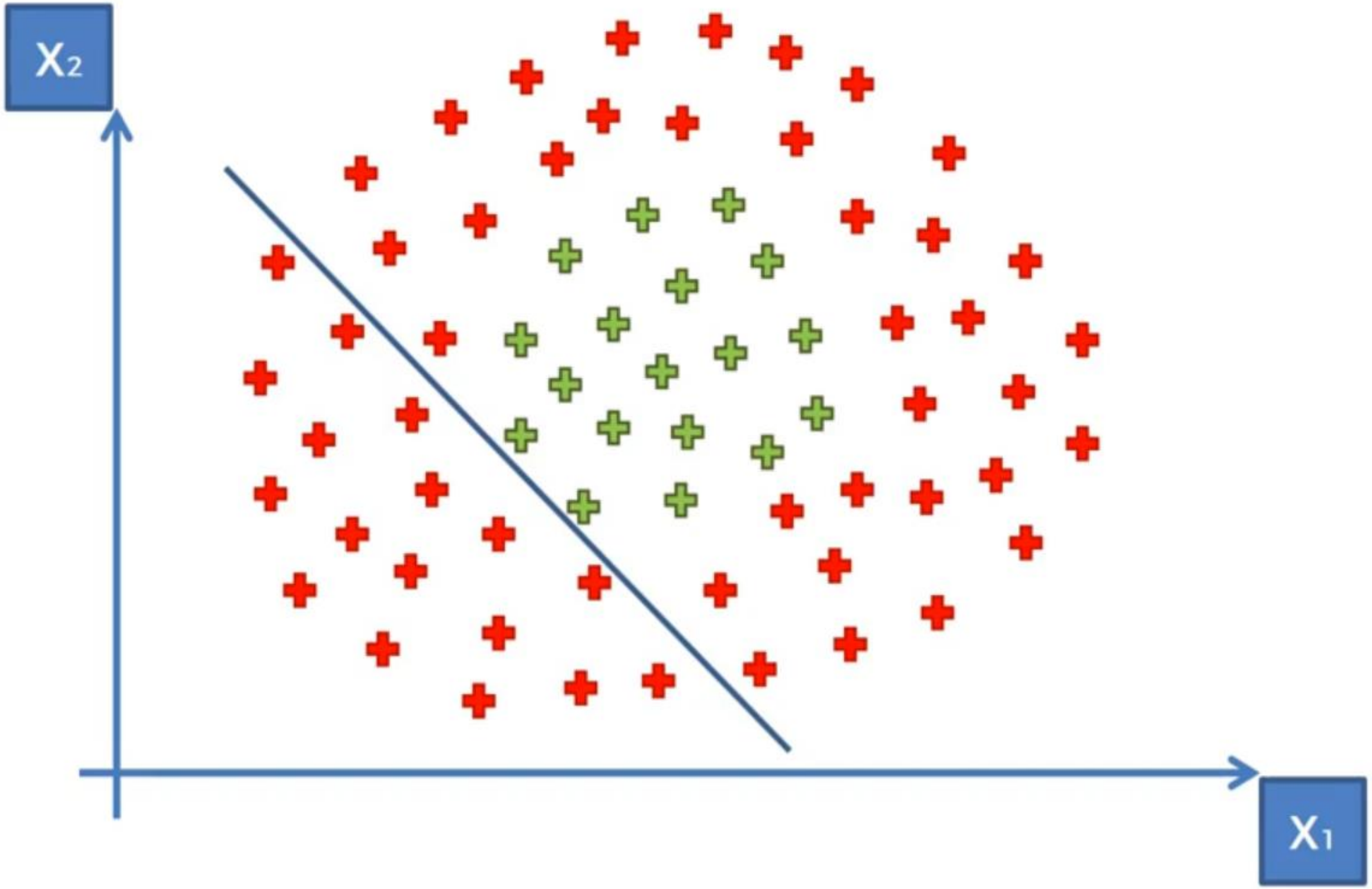

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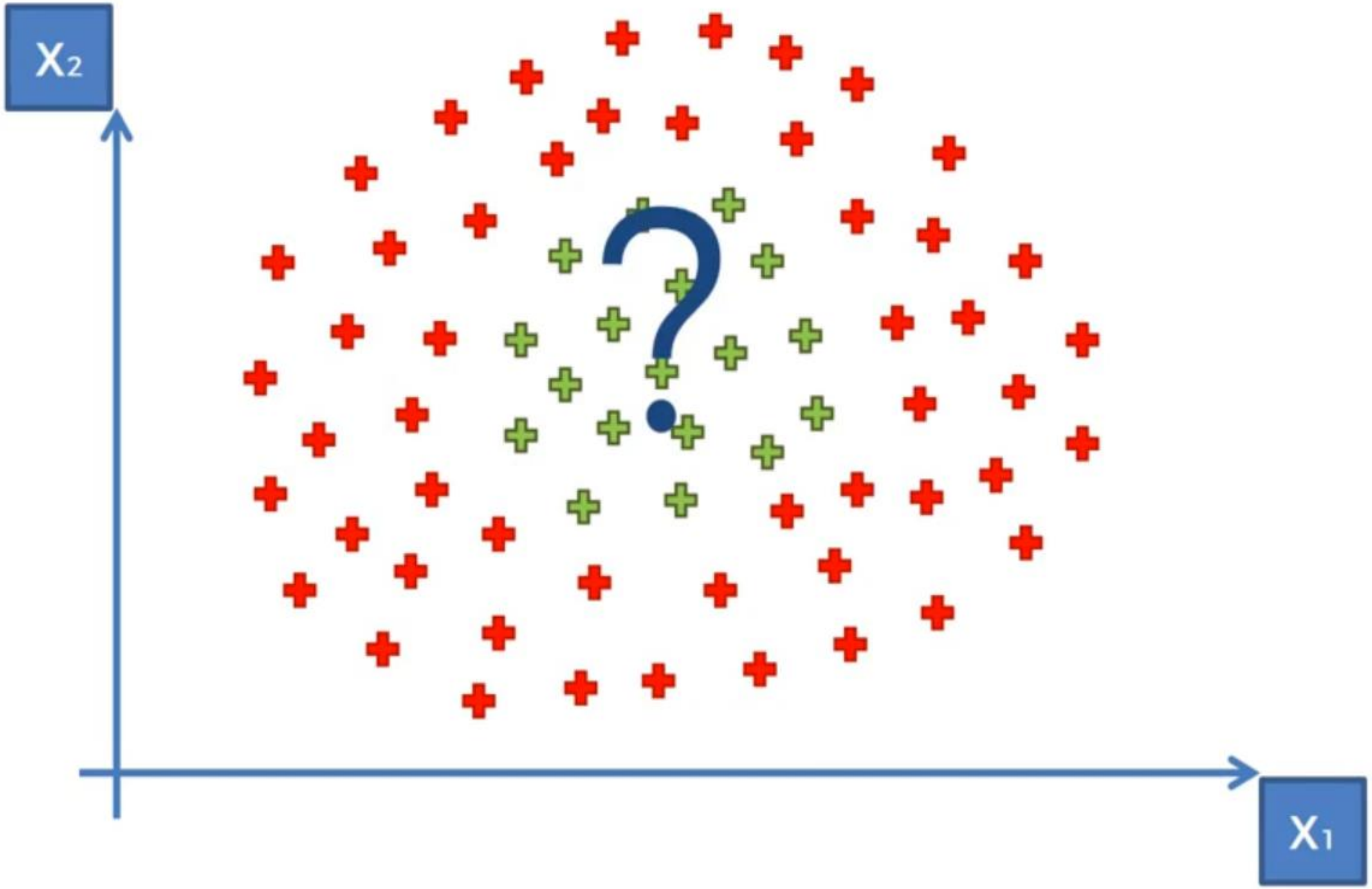
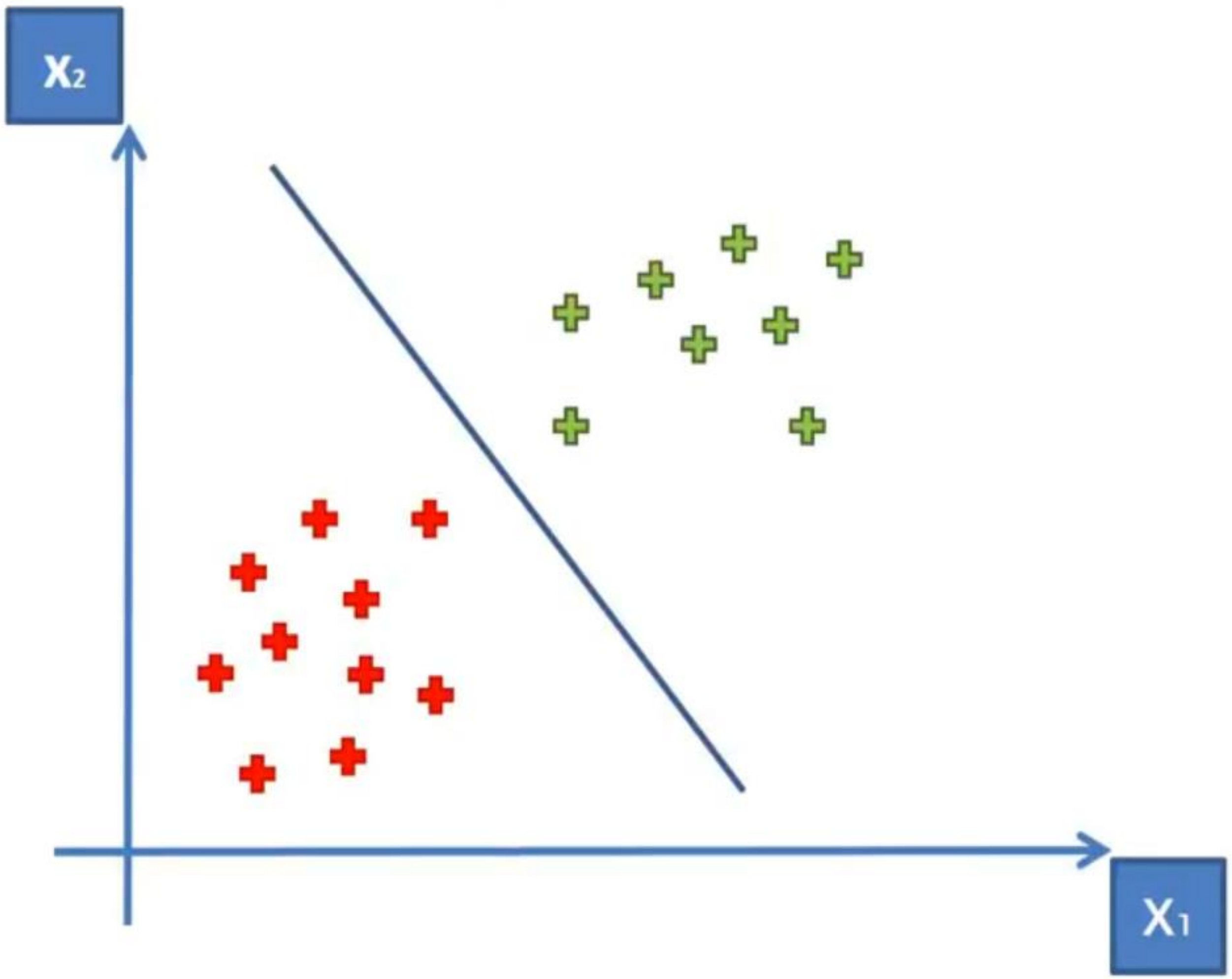


Why ?

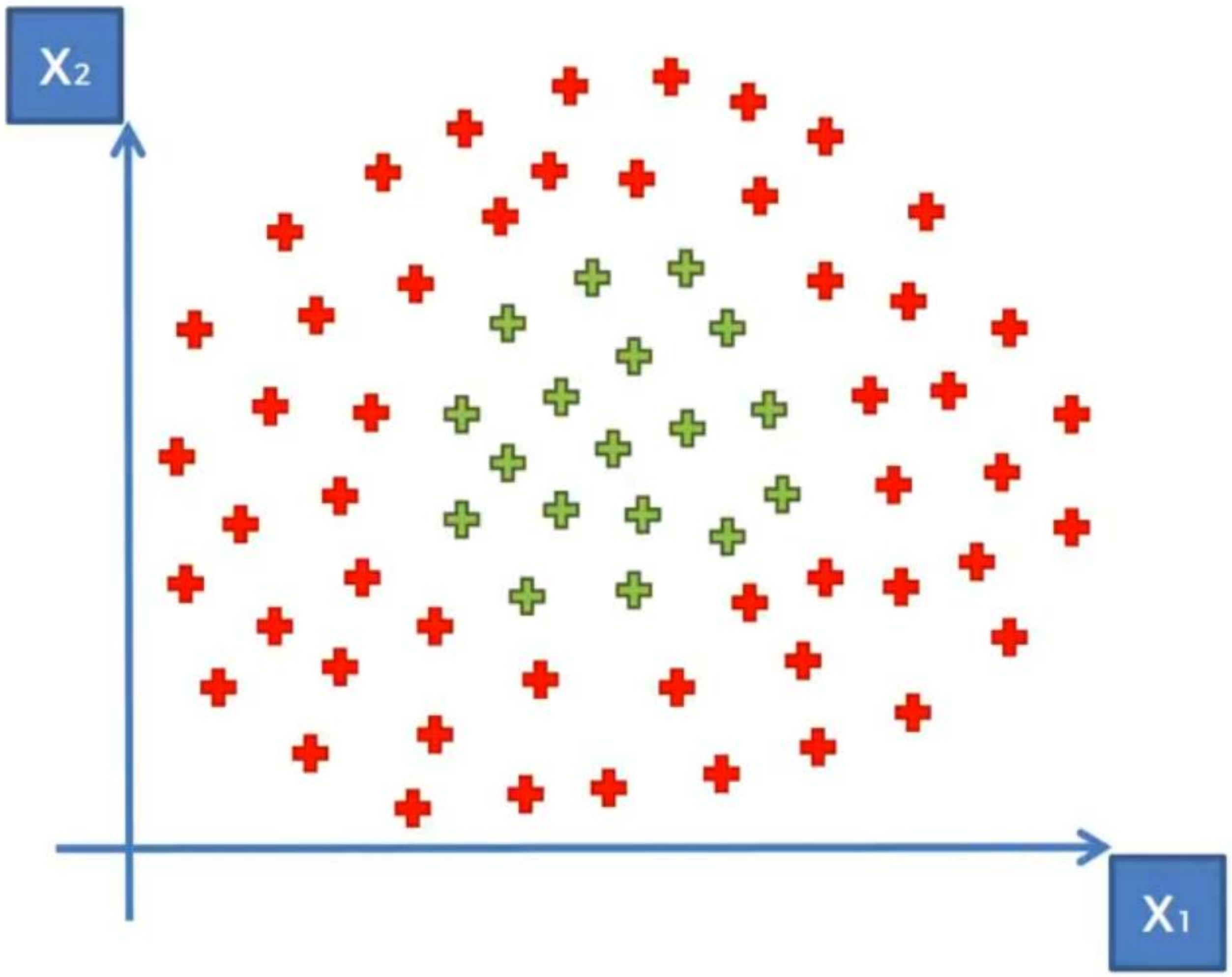
Because the data points are
not LINEARLY SEPARABLE

Linear Separability

Linearly Separable

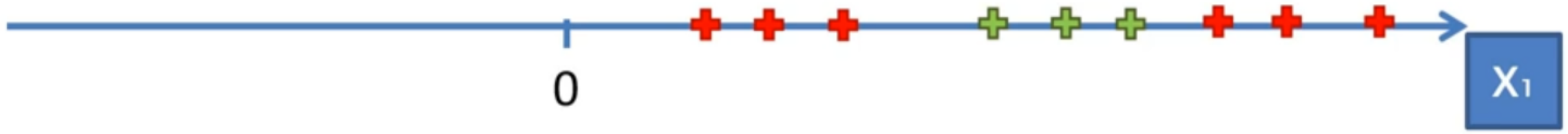


Not Linearly Separable



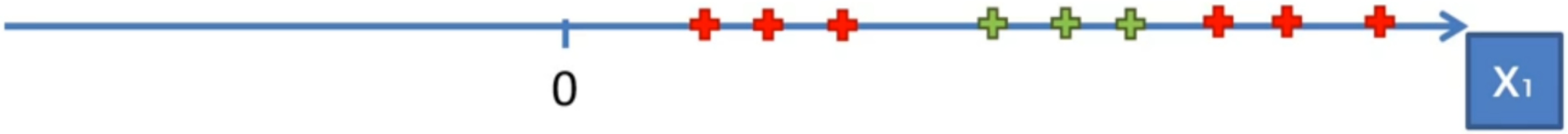
A Higher-Dimensional Space

Mapping to a Higher Dimension



Mapping to a Higher Dimension

$f = x - 5$



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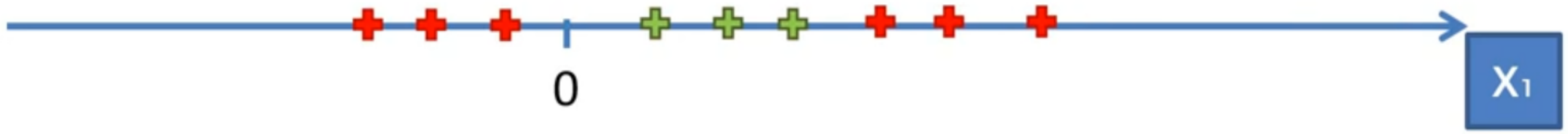
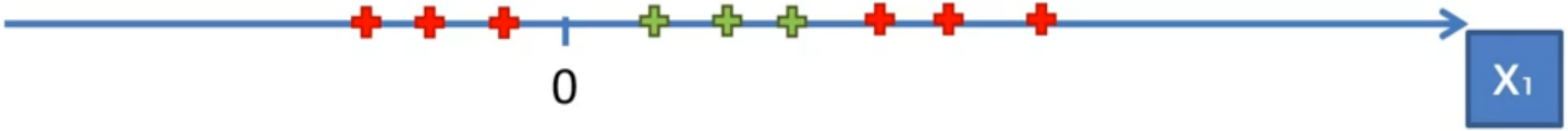


Mapping to a Higher Dimension

$f = x - 5$

$f = (x - 5)^2$



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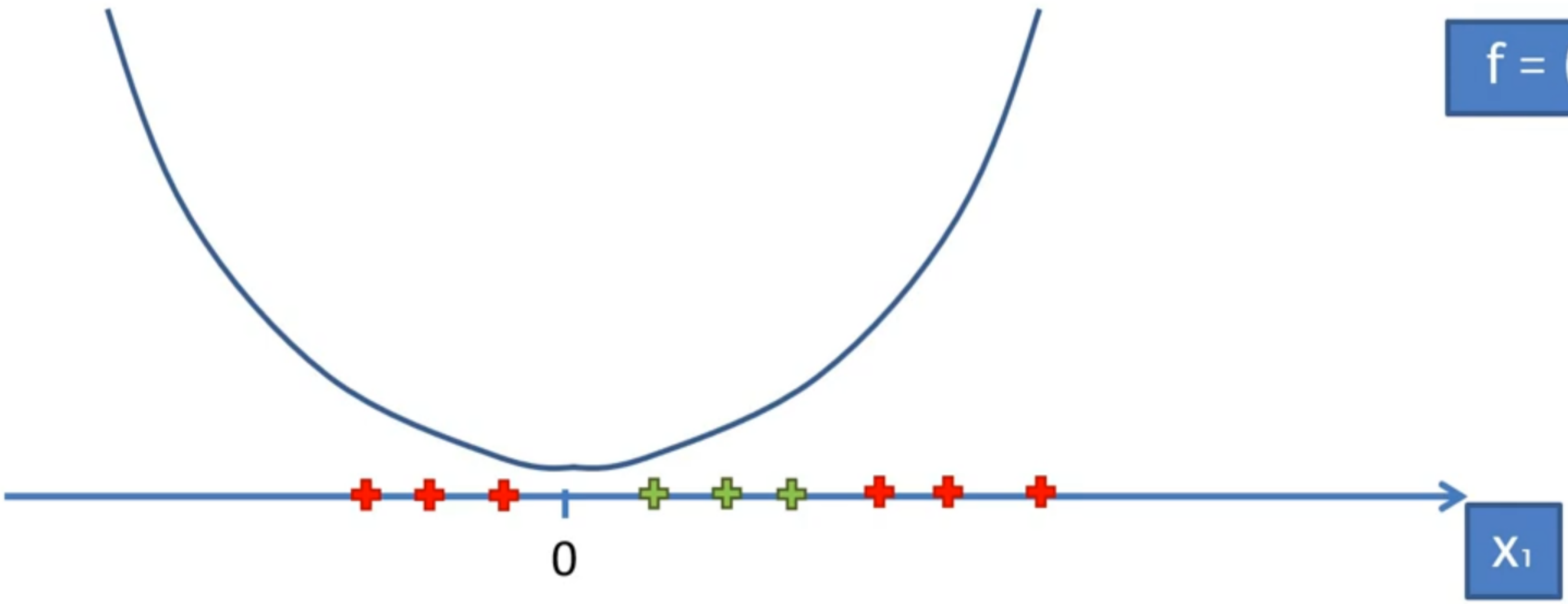

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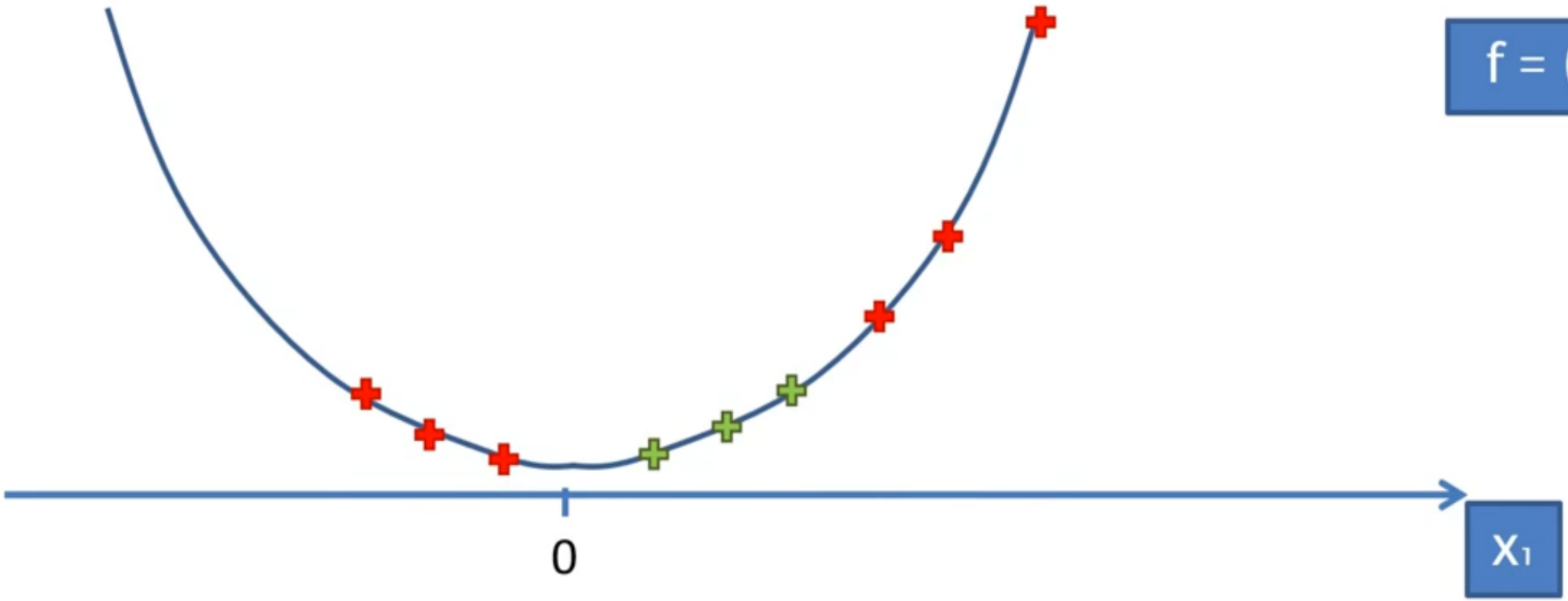

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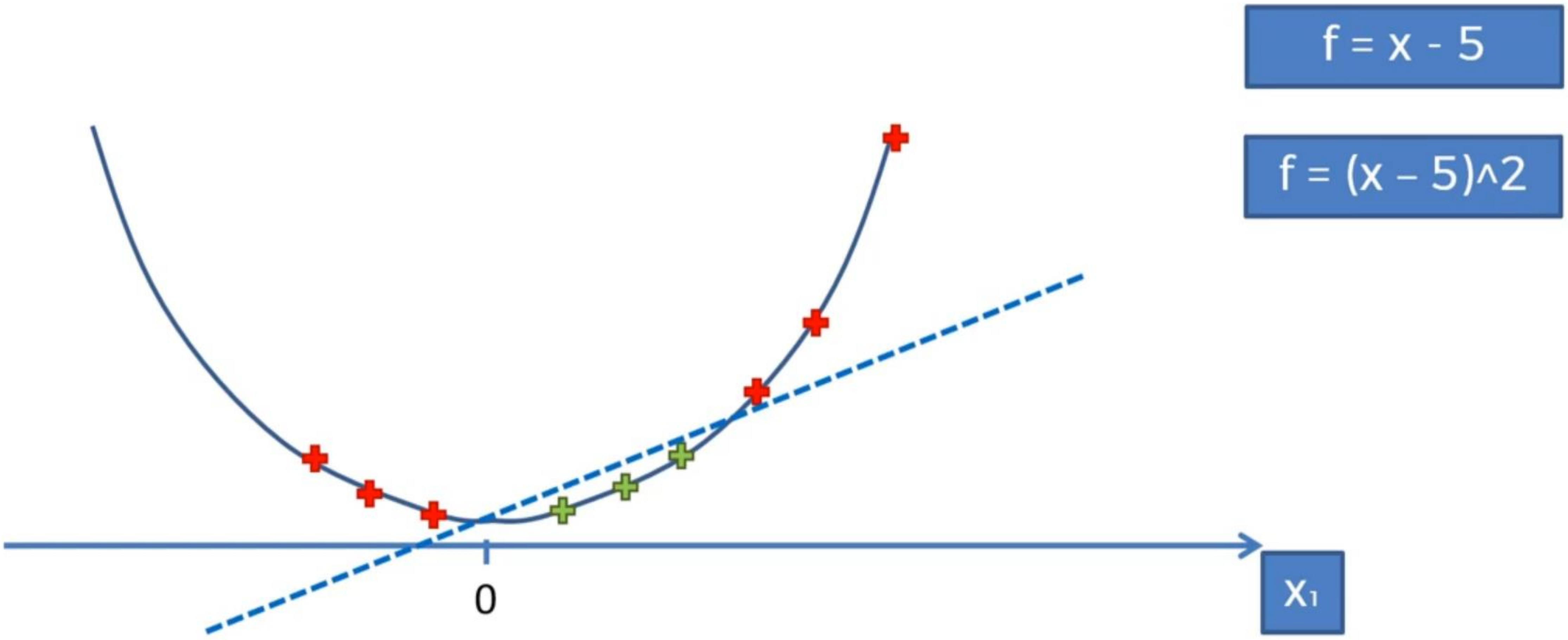

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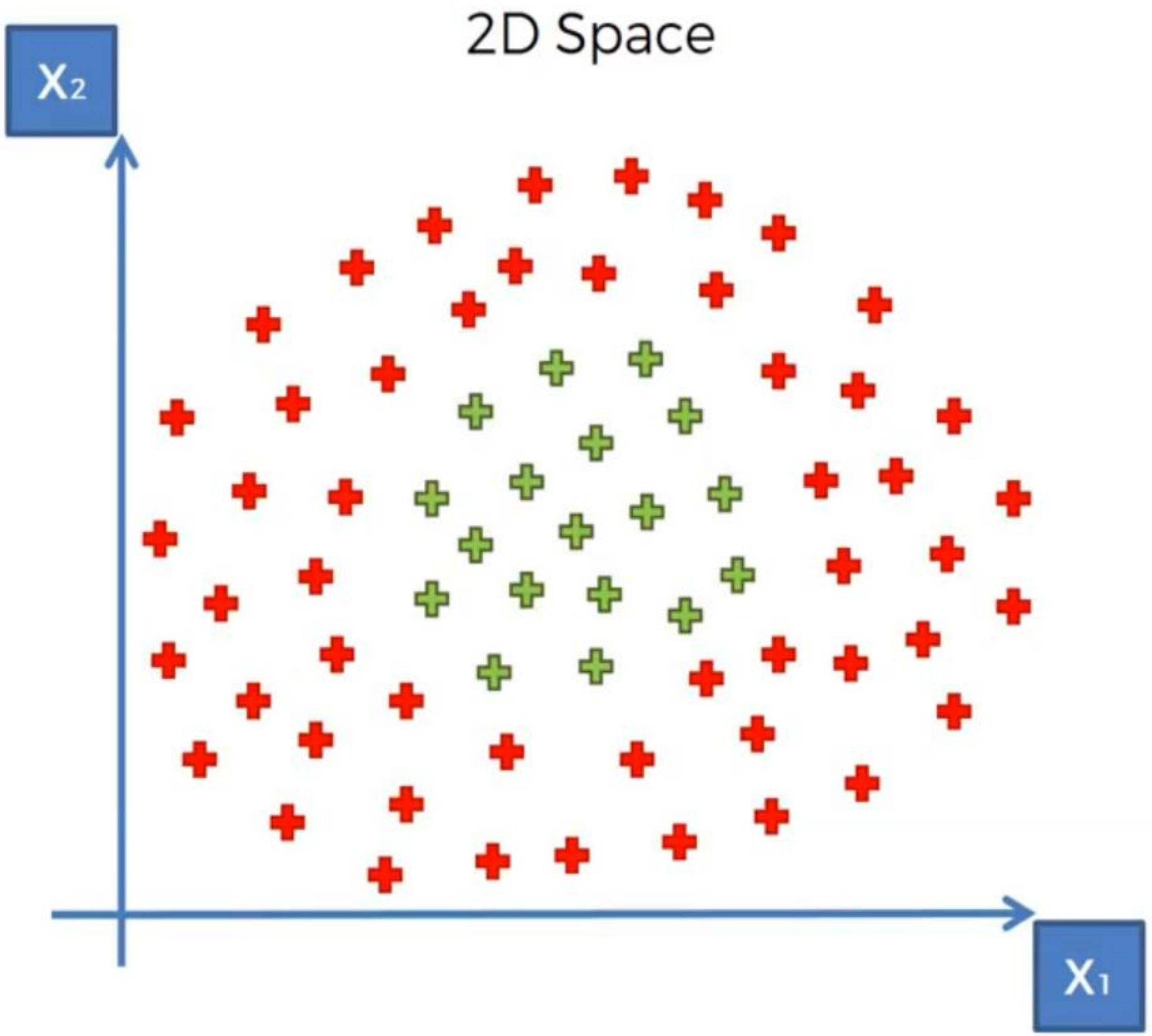

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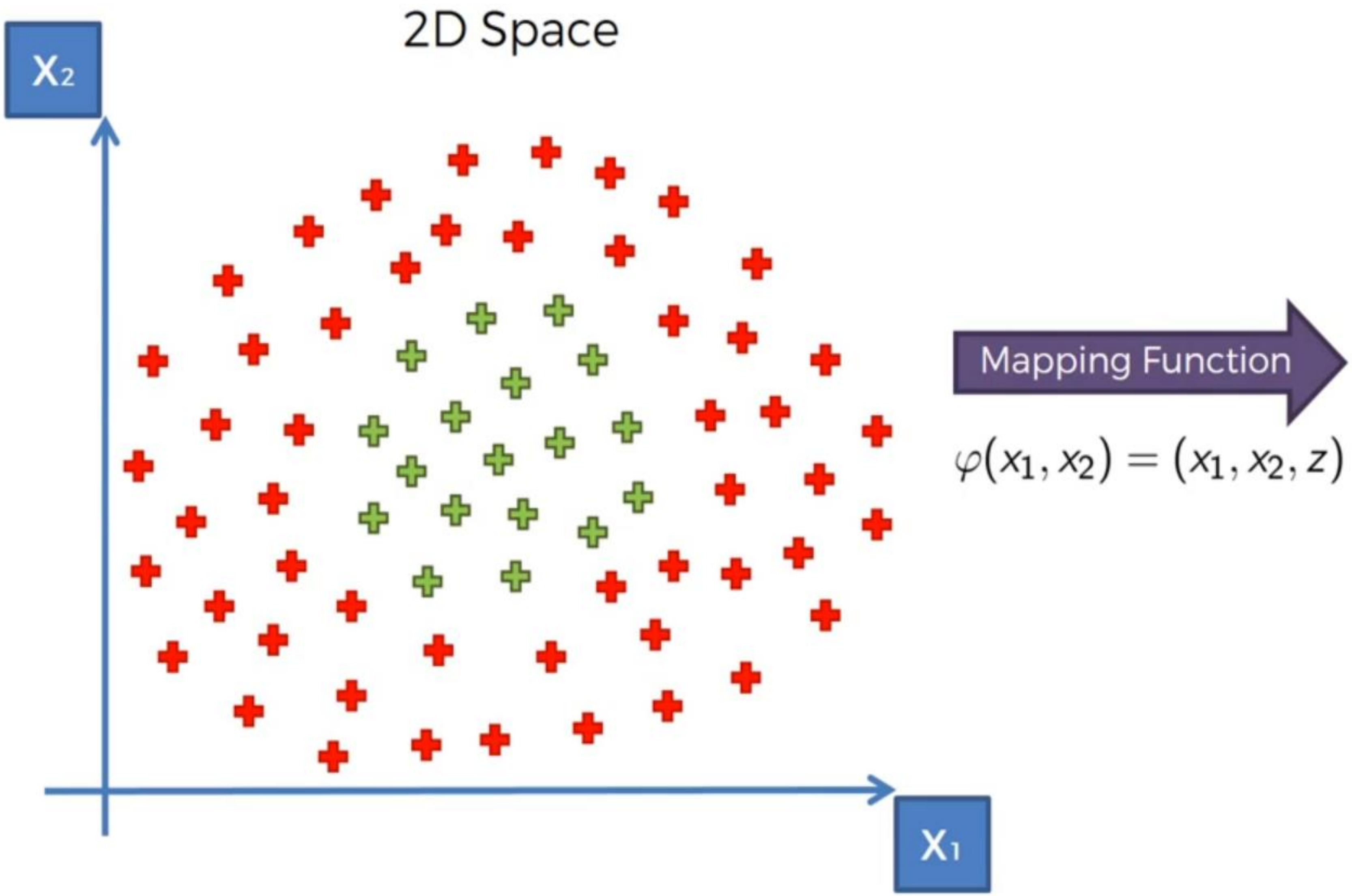
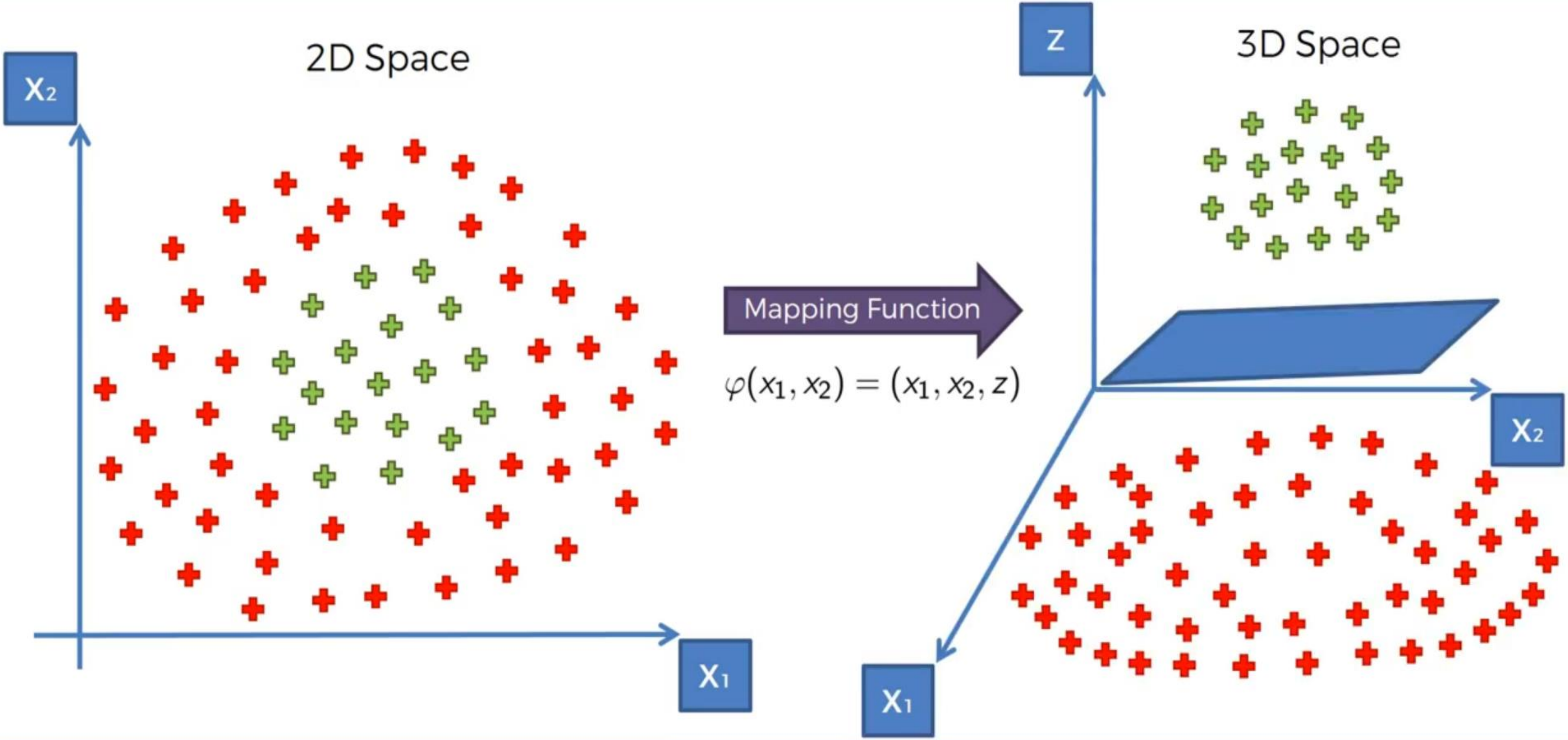


$$\varphi(x_1, x_2) = (x_1, x_2, z)$$

Mapping to a Higher Dimension



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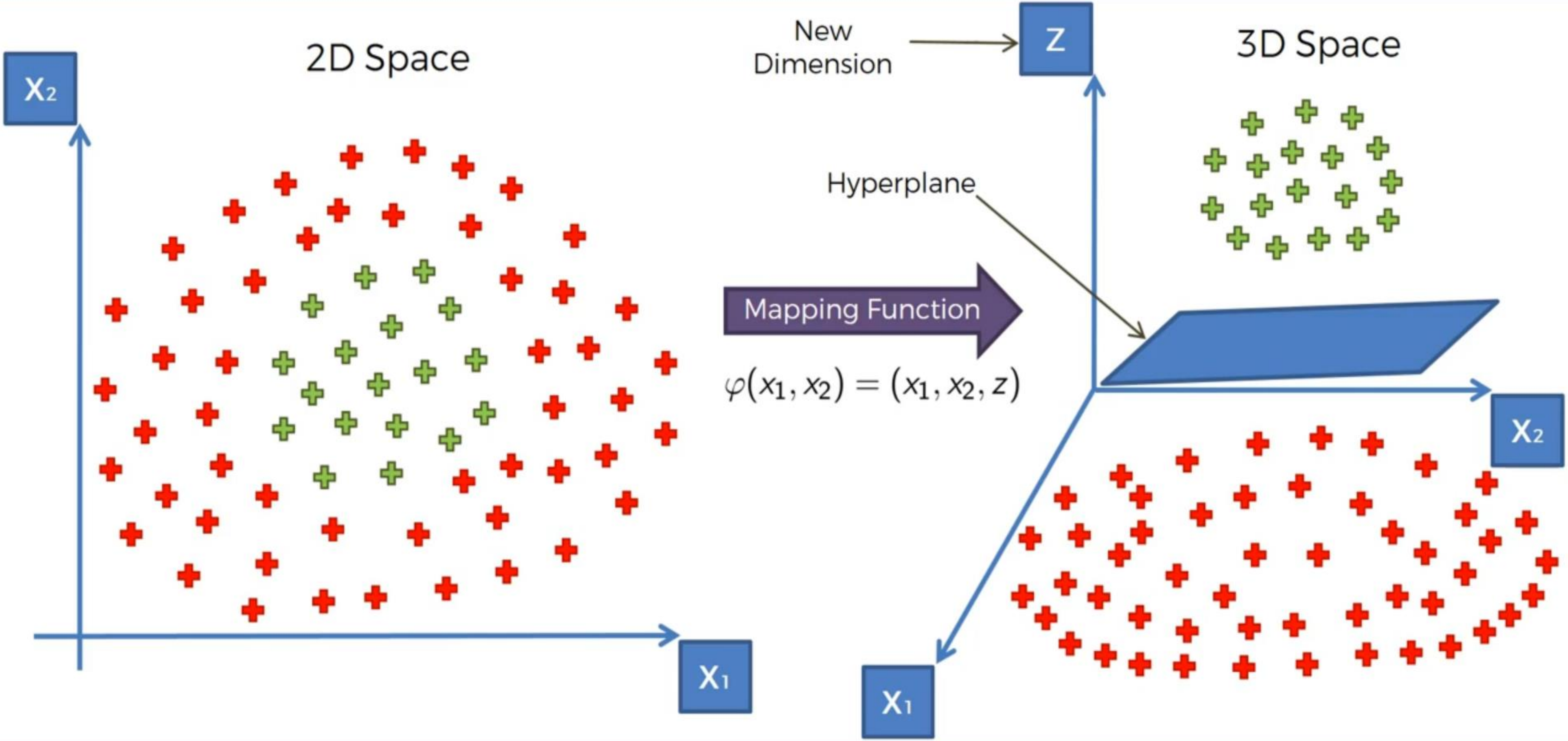
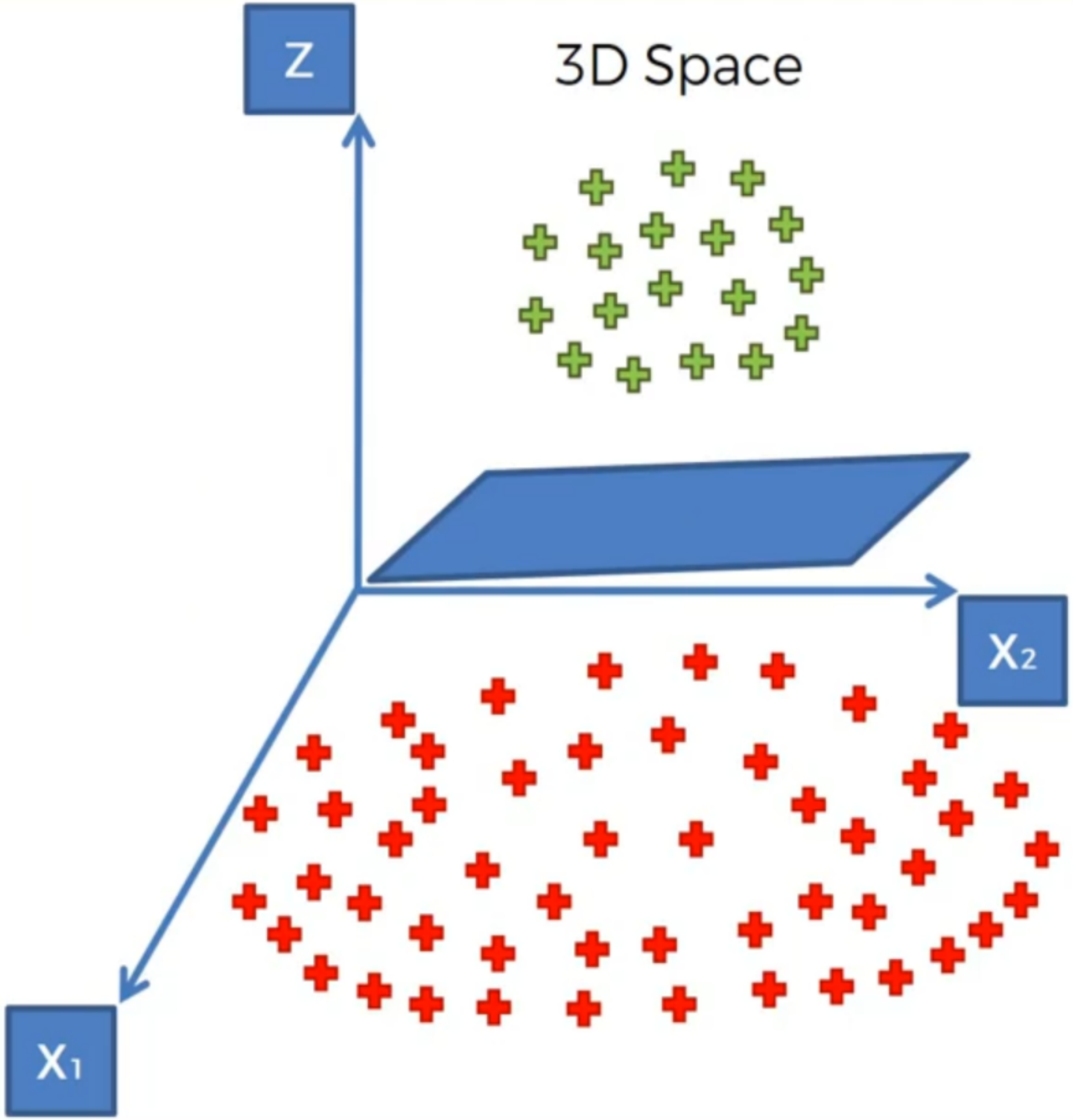


Projecting back to 2D Space



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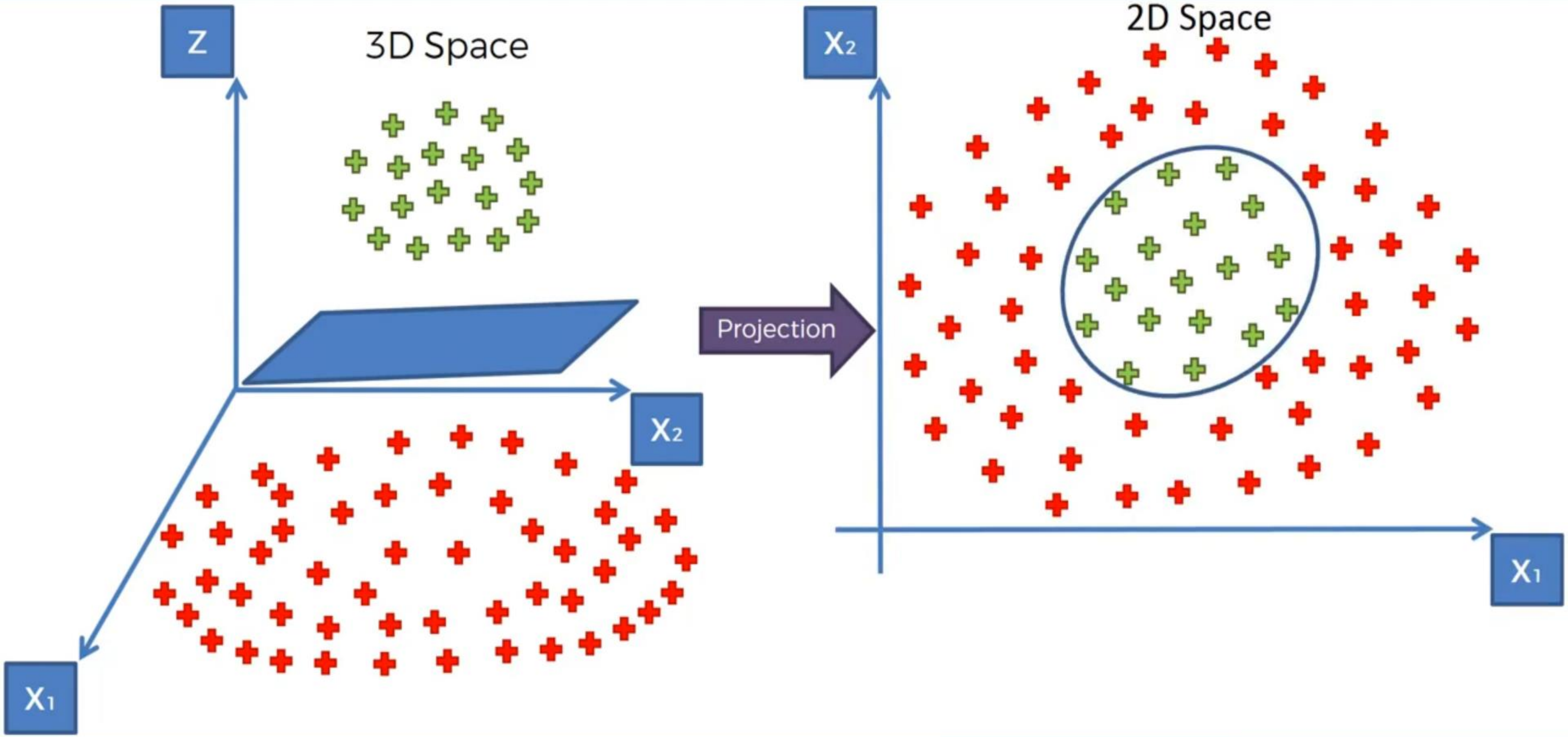

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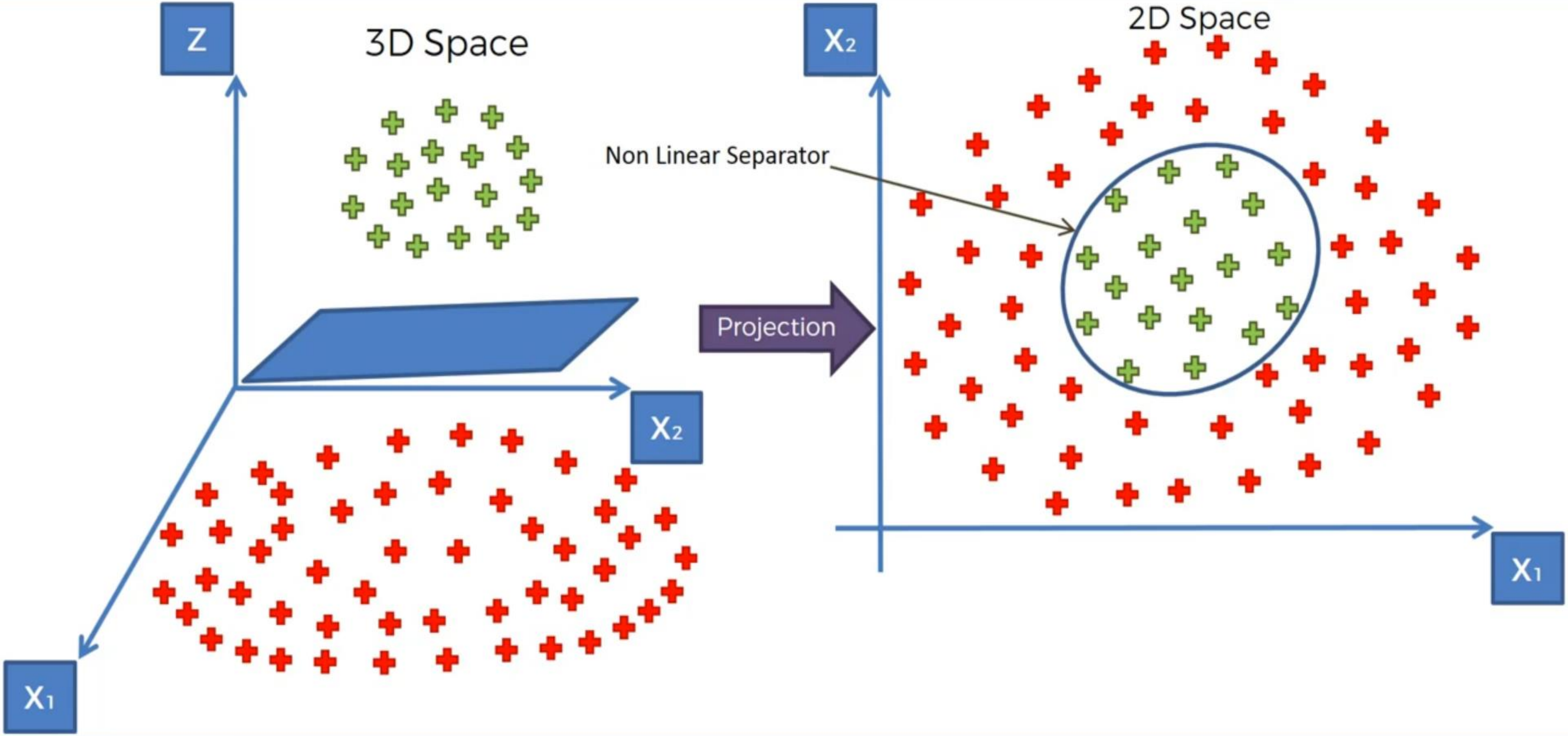

But there is a catch...

Mapping to a Higher Dimensional Space
can be highly compute-intensive

The Kernel Trick

The Gaussian RBF Kernel

$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x} - \vec{l}^i\|^2}{2\sigma^2}}$$

The Gaussian RBF Kernel

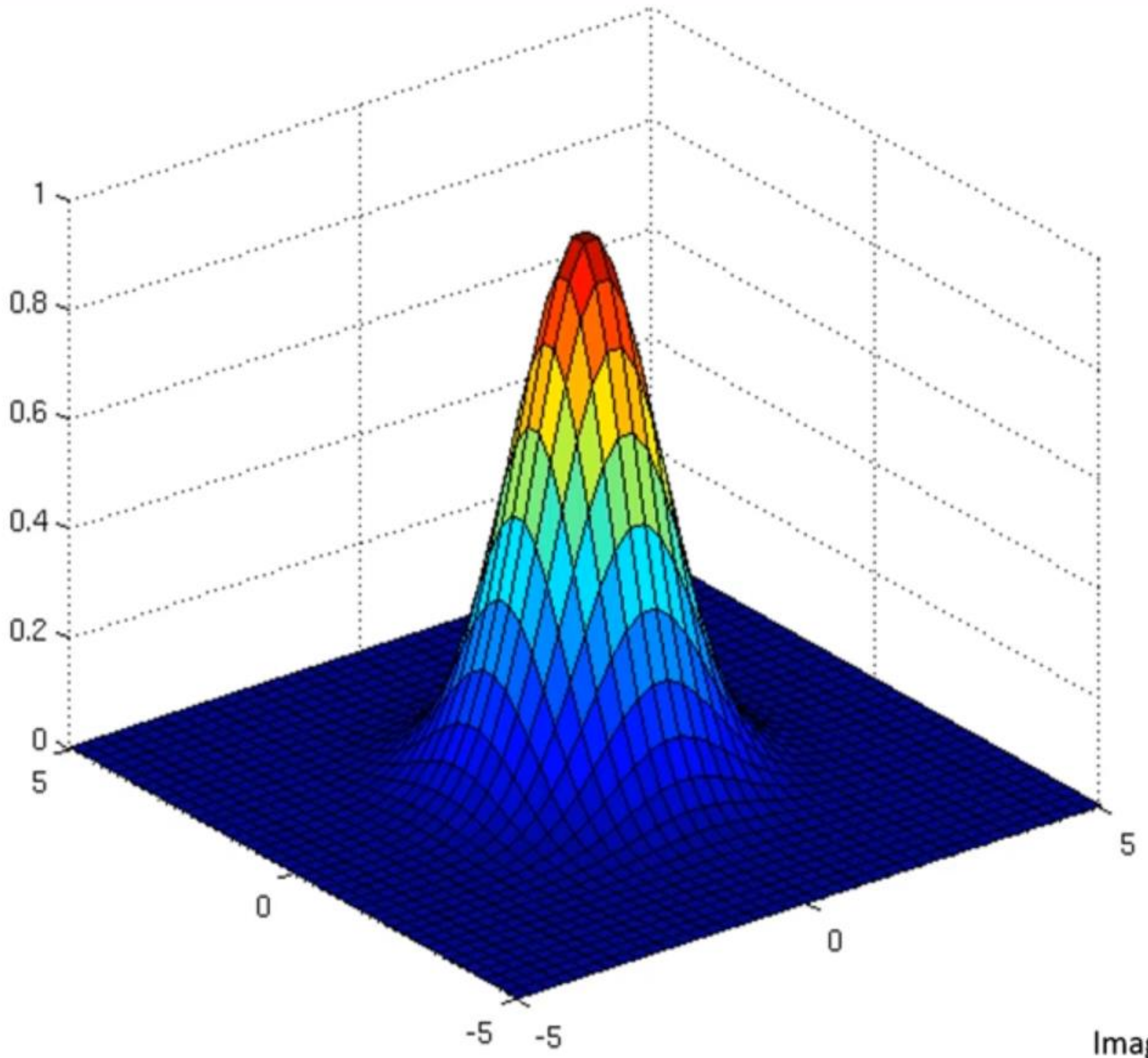

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Image source: http://www.cs.toronto.edu/~duvenaud/cookbook/index.html

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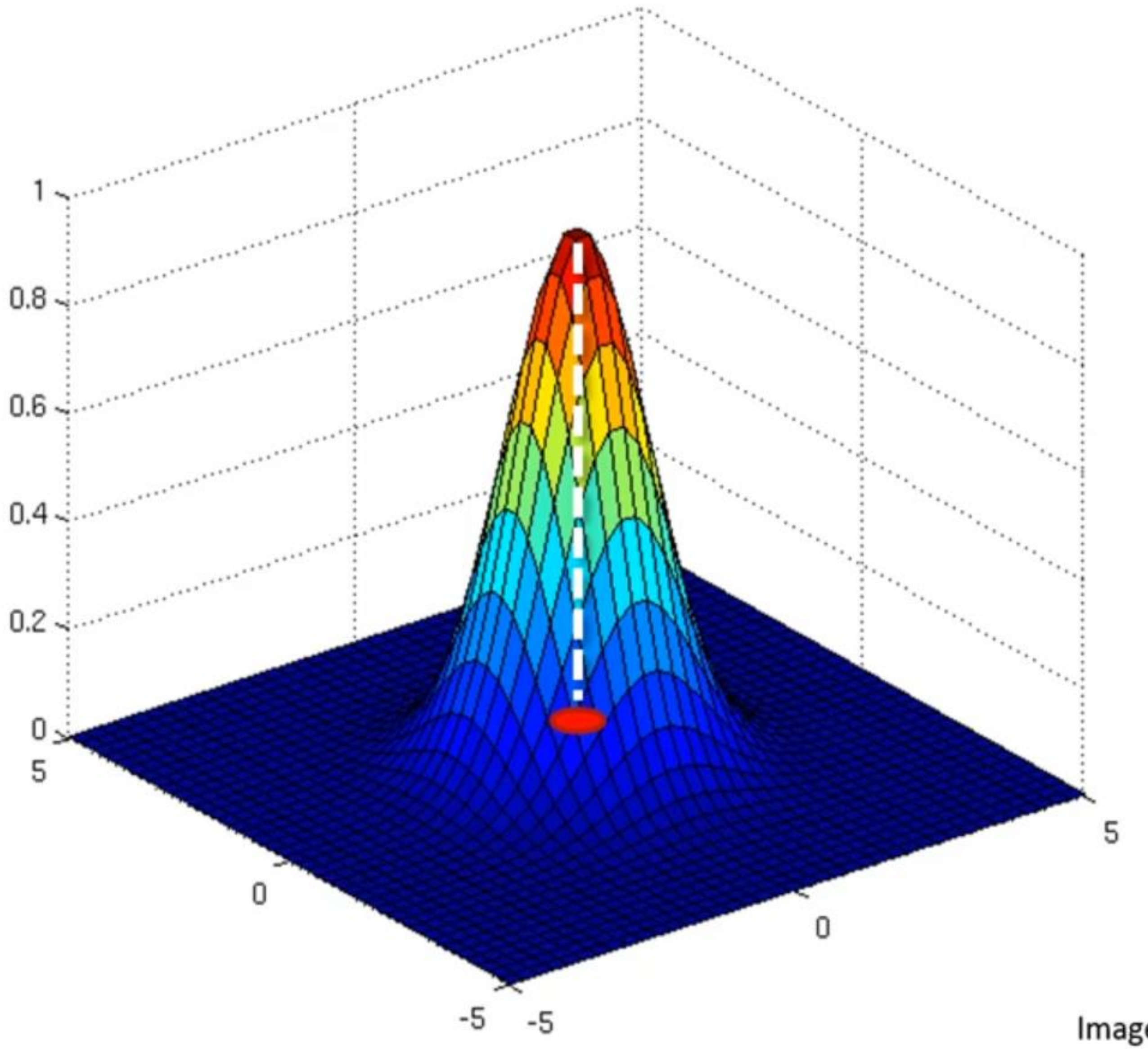

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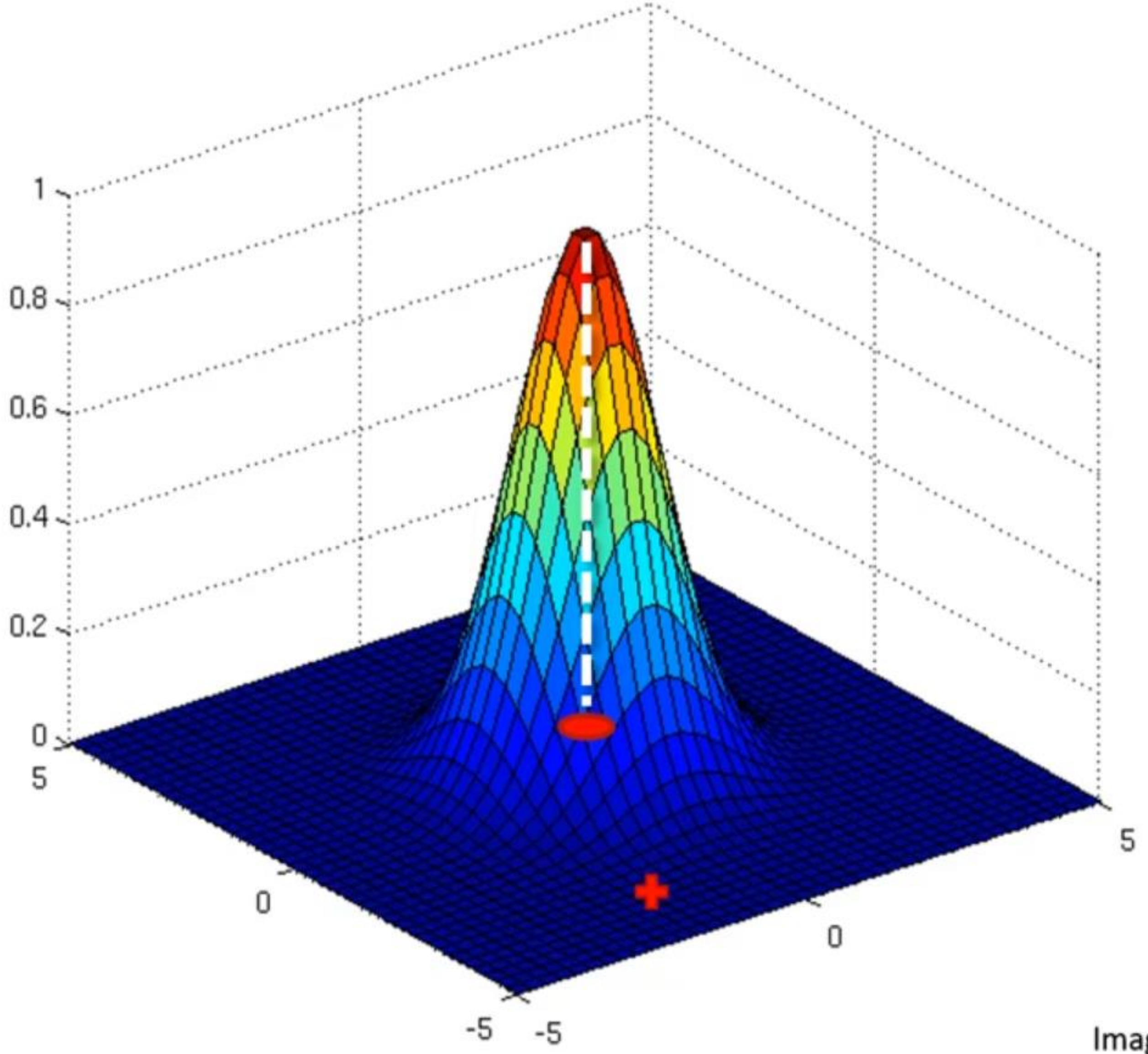

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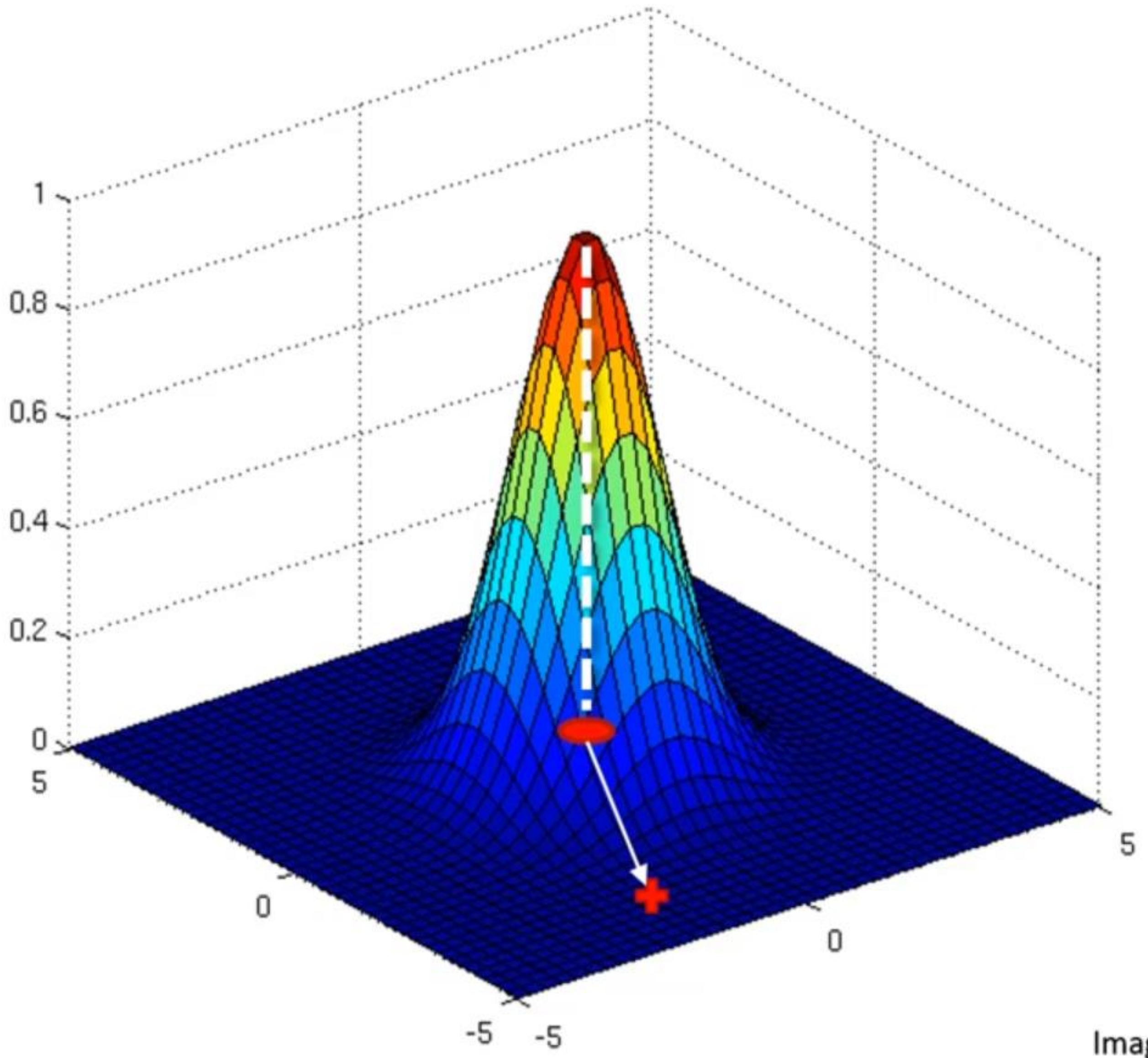

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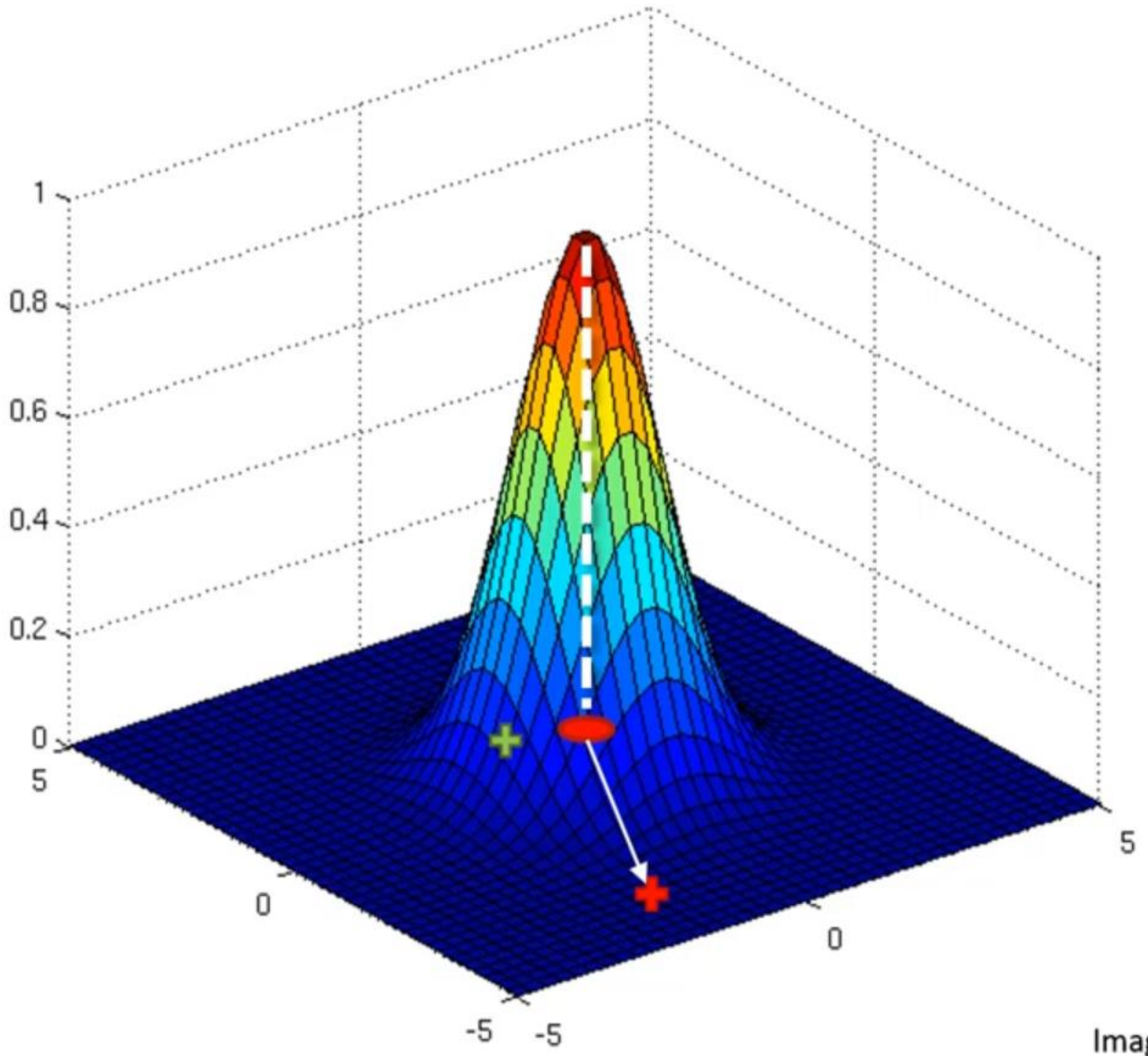

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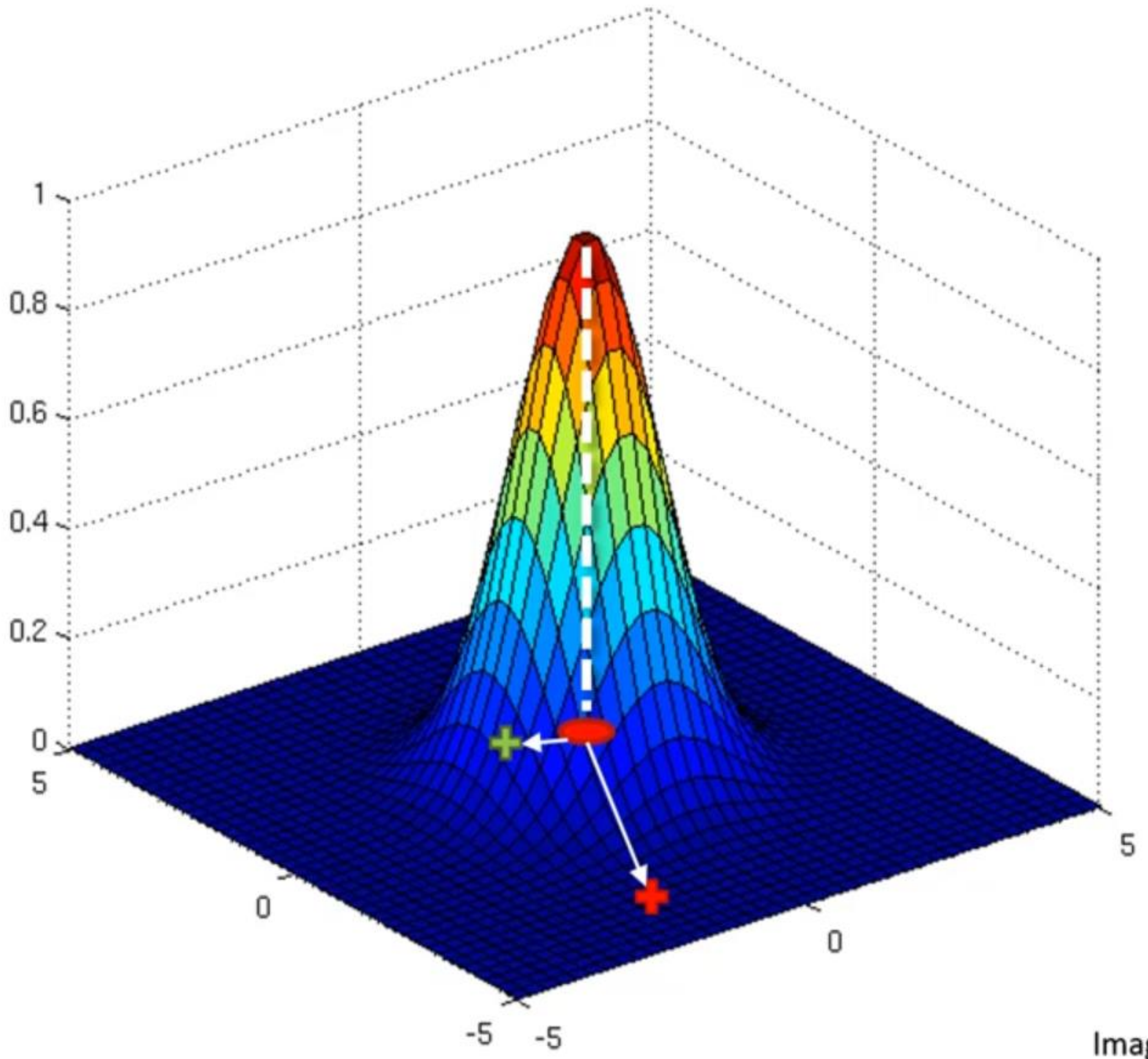

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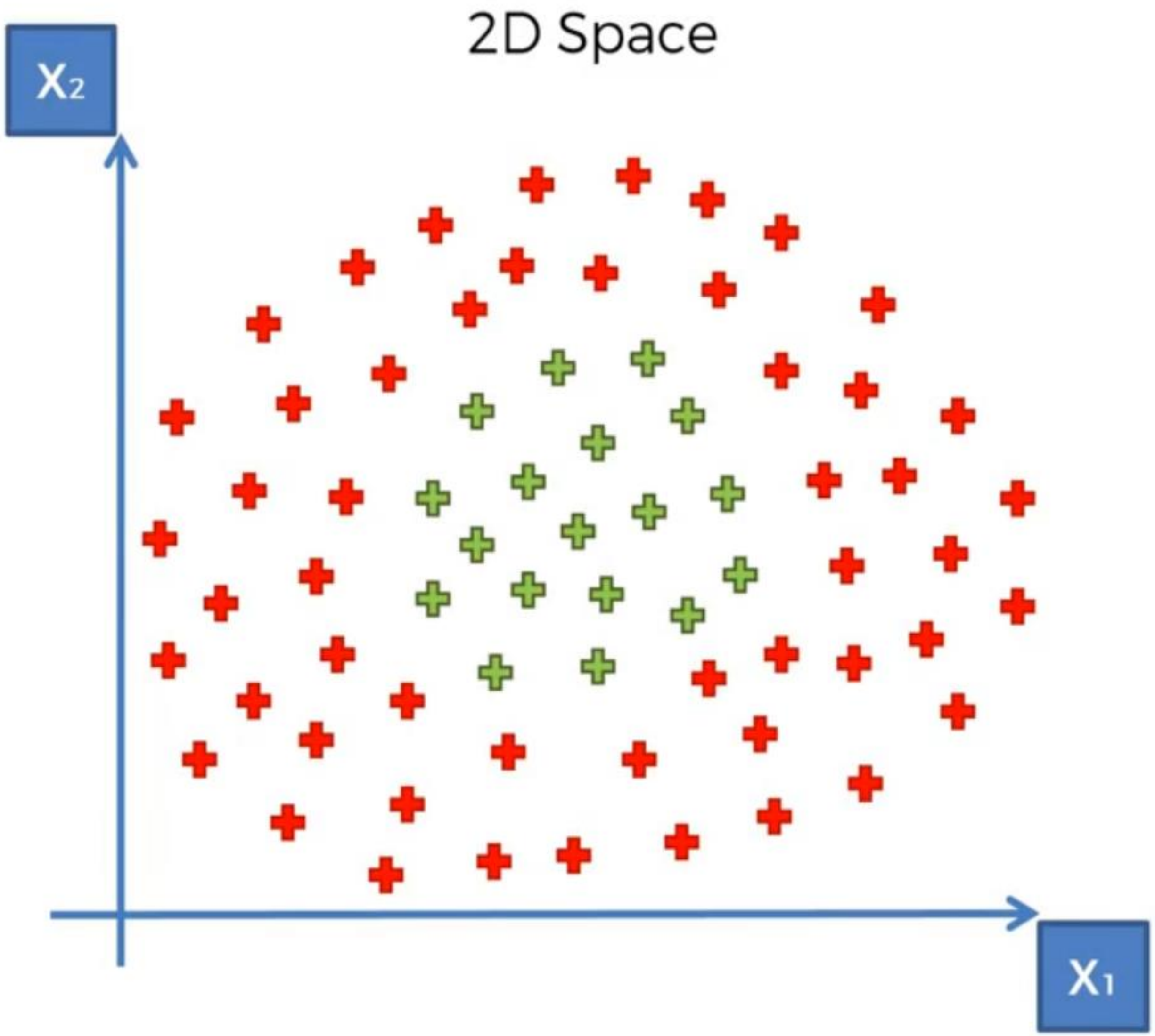

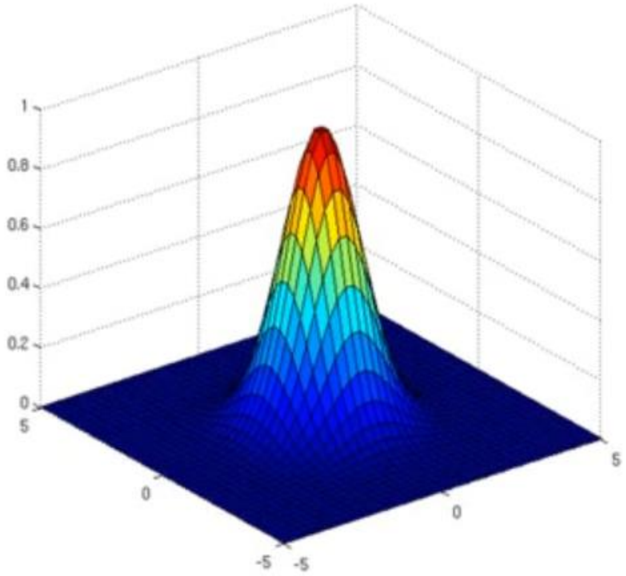

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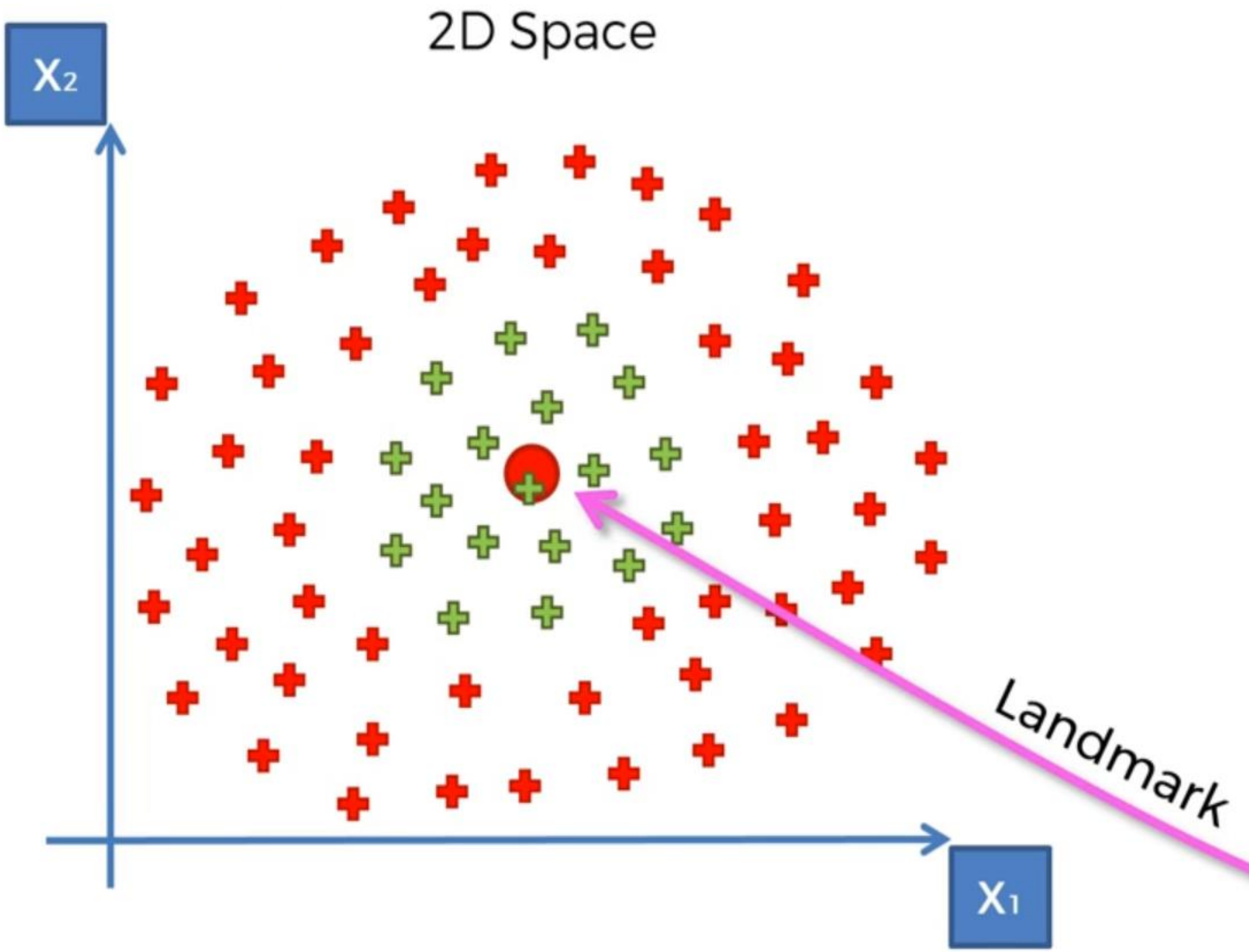

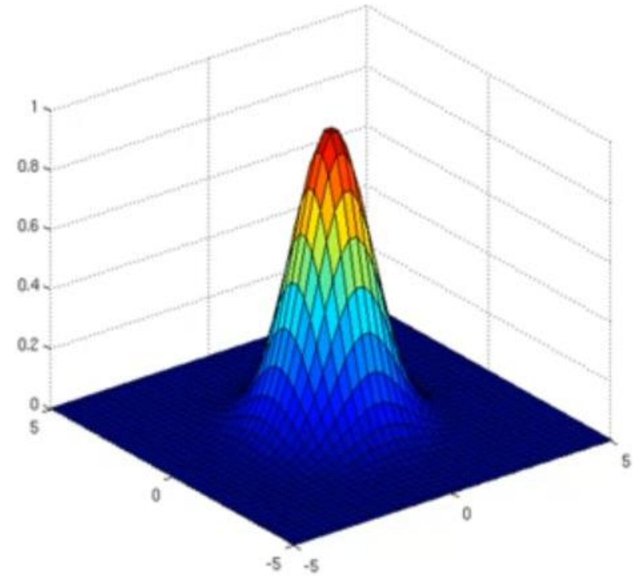

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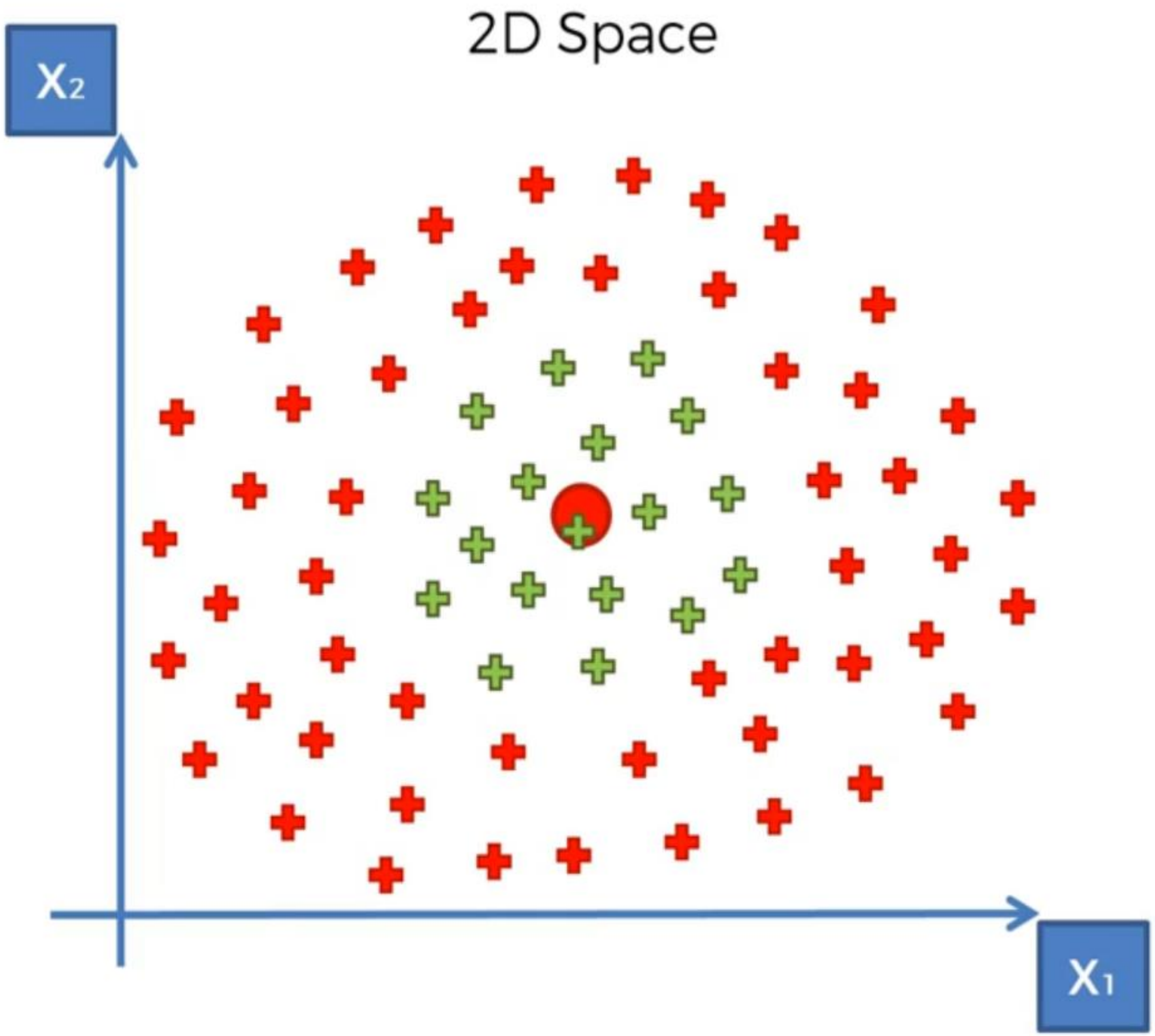

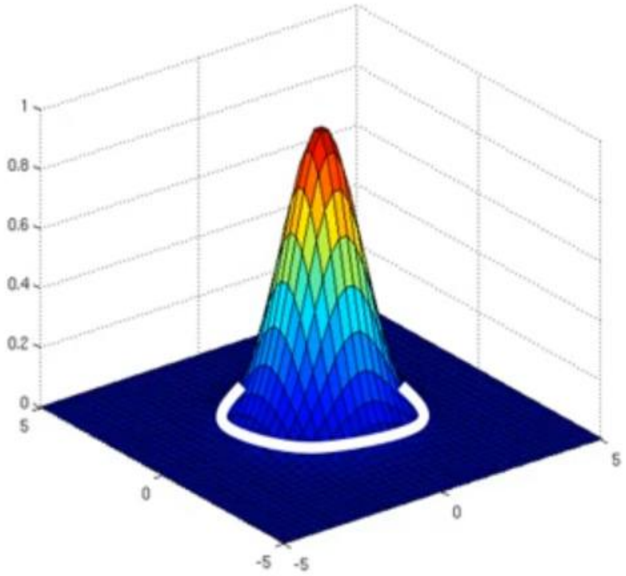

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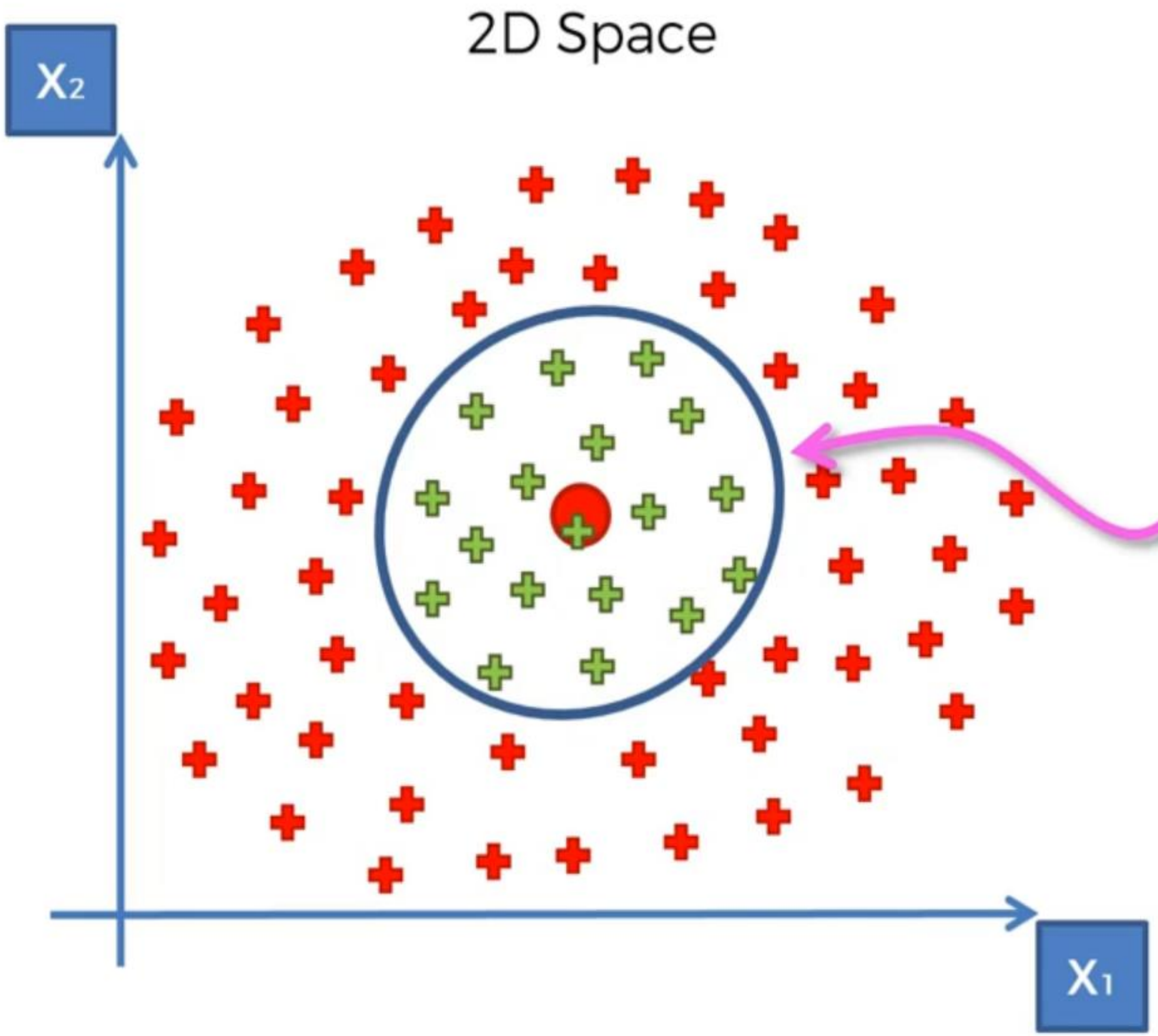

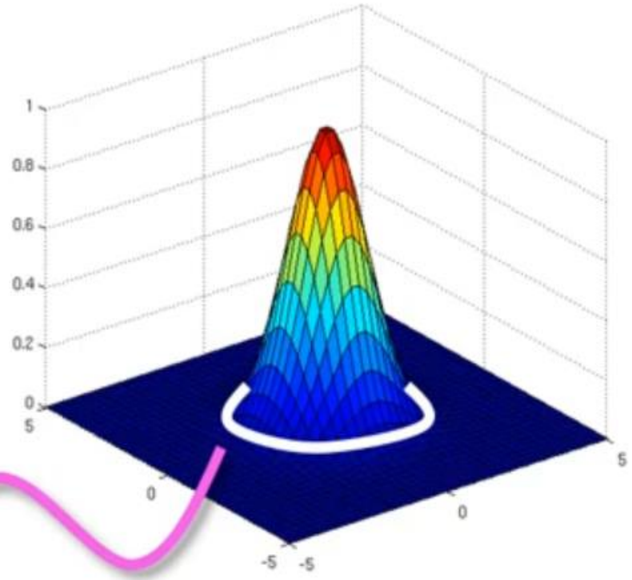

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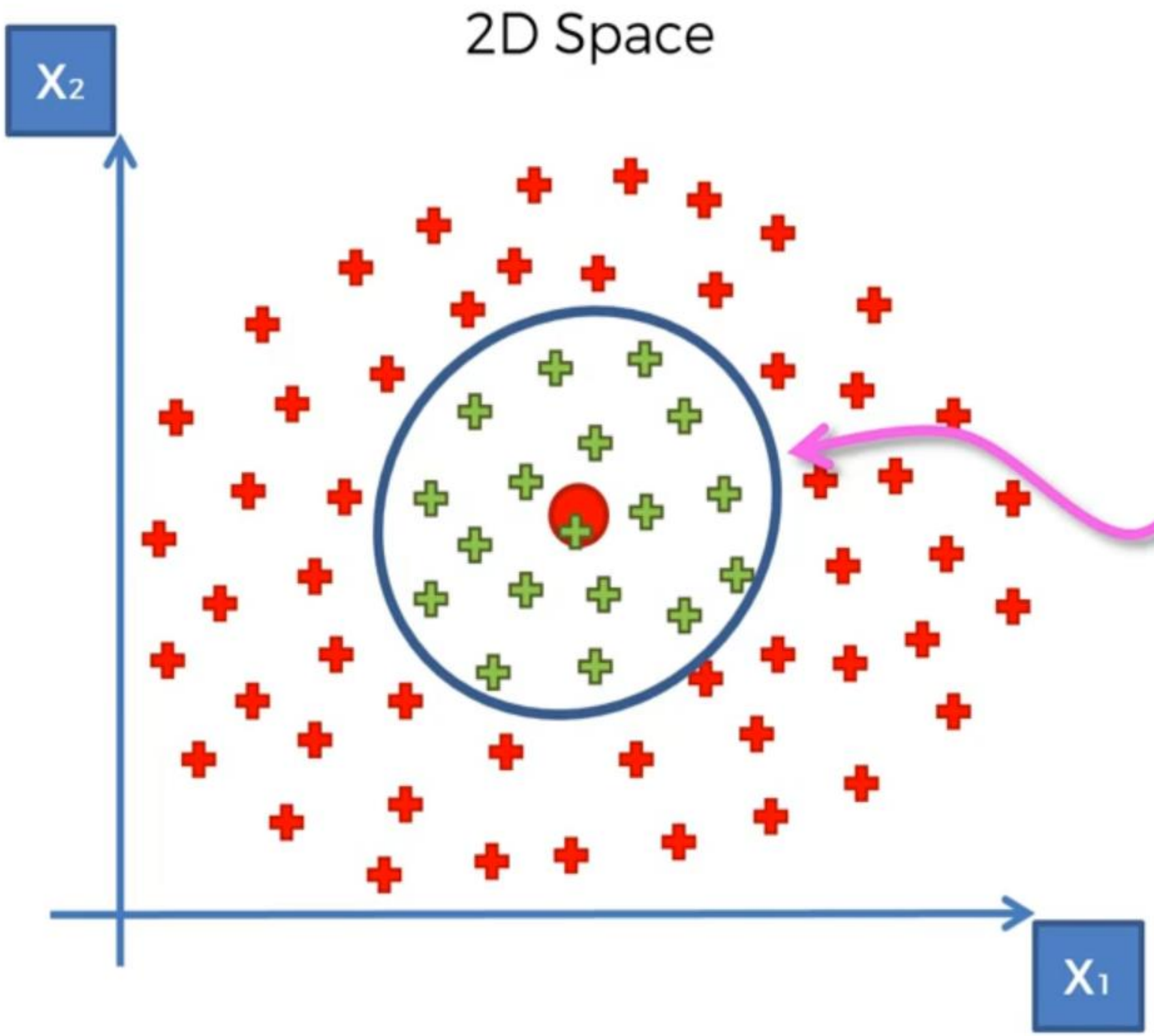

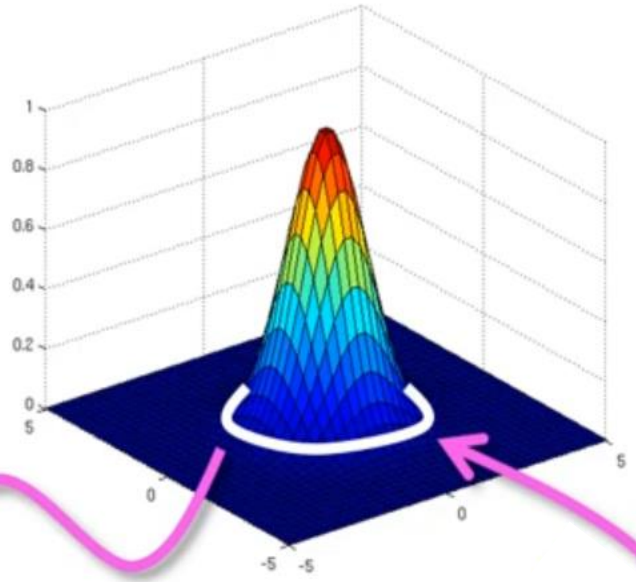

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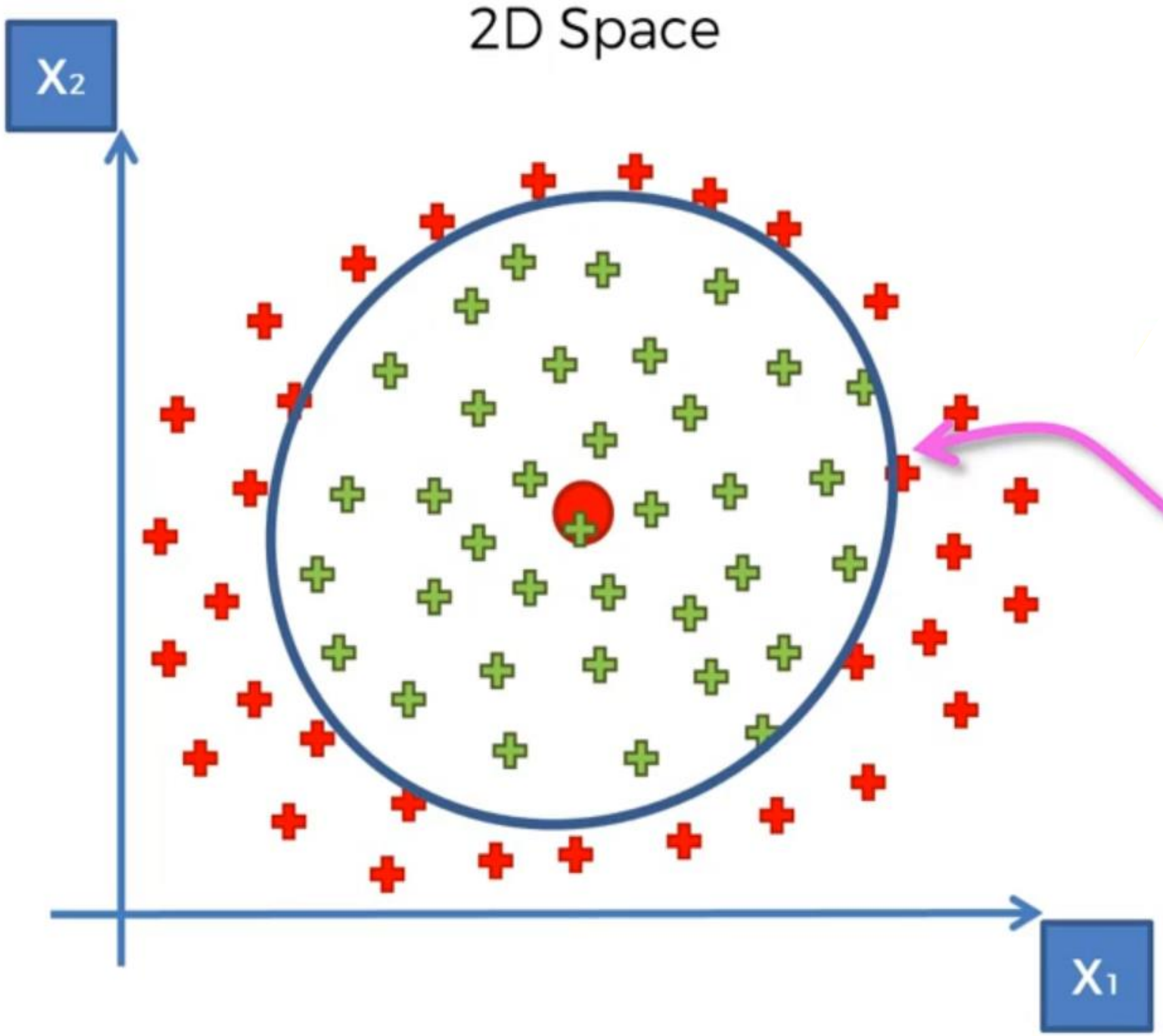
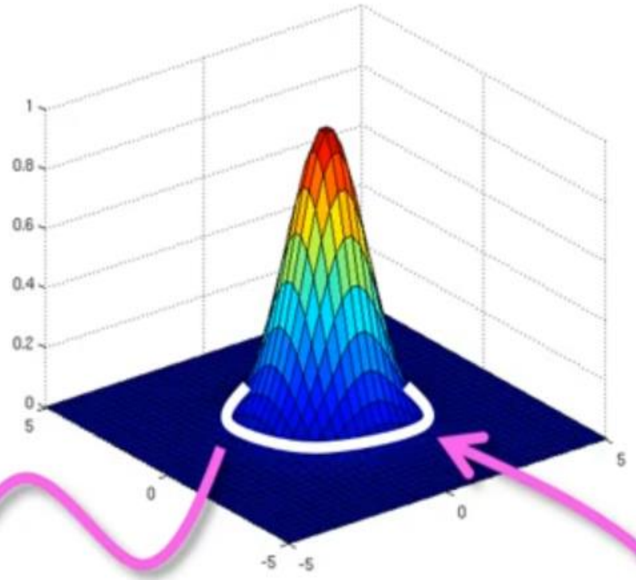


σ



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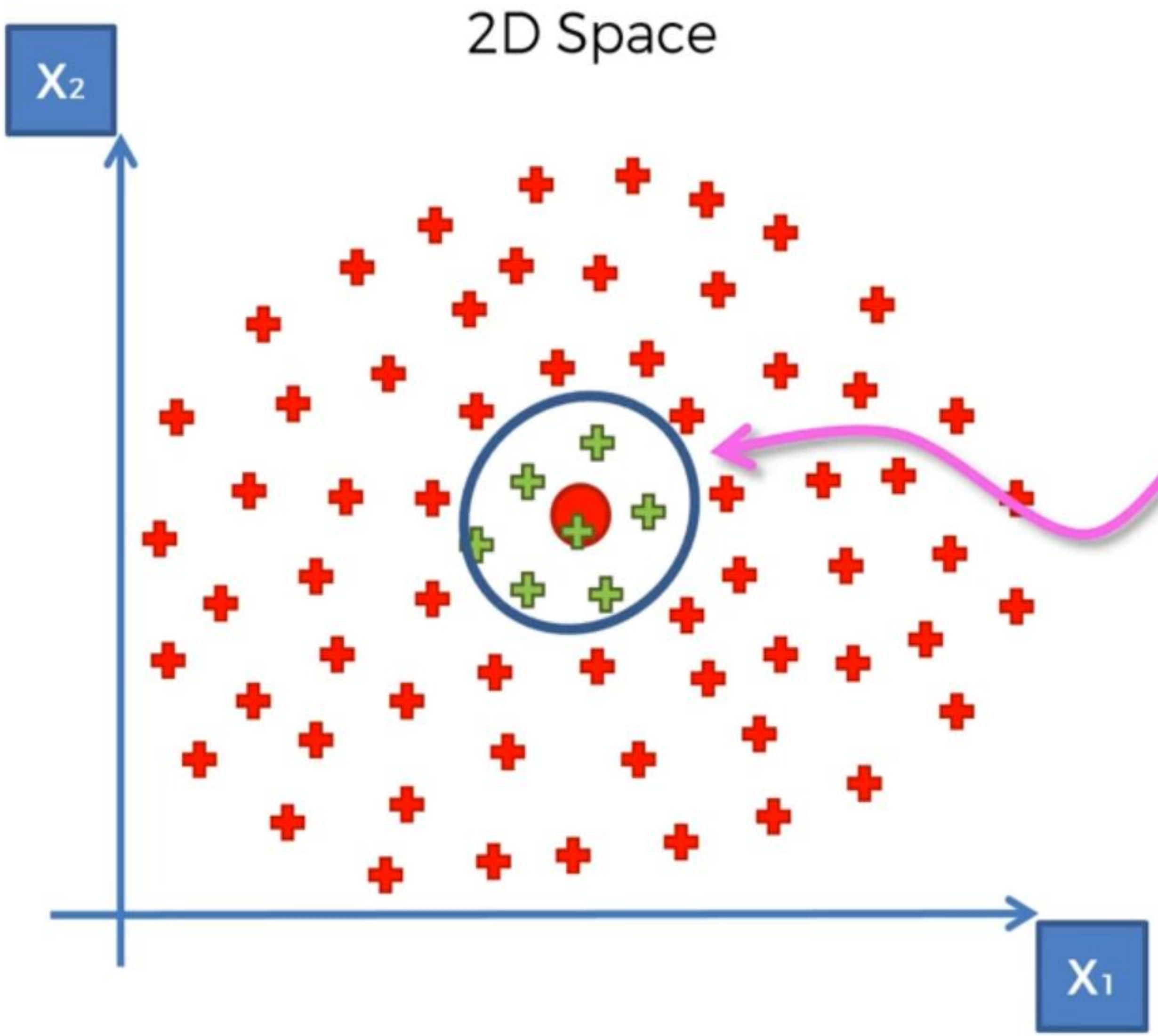
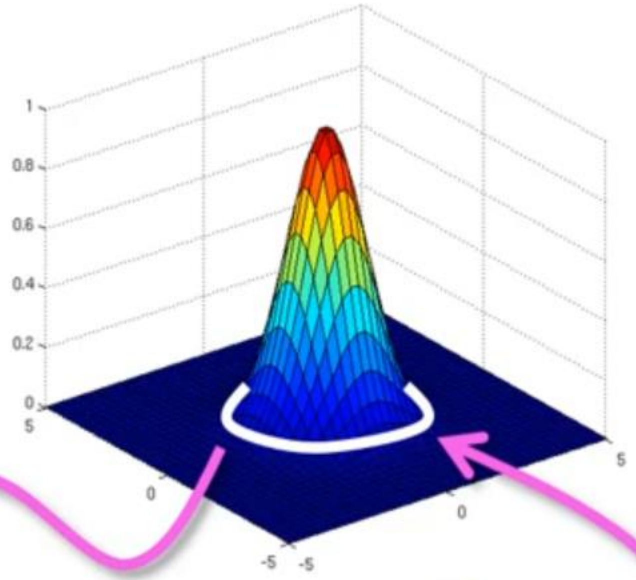


$\sigma \downarrow$



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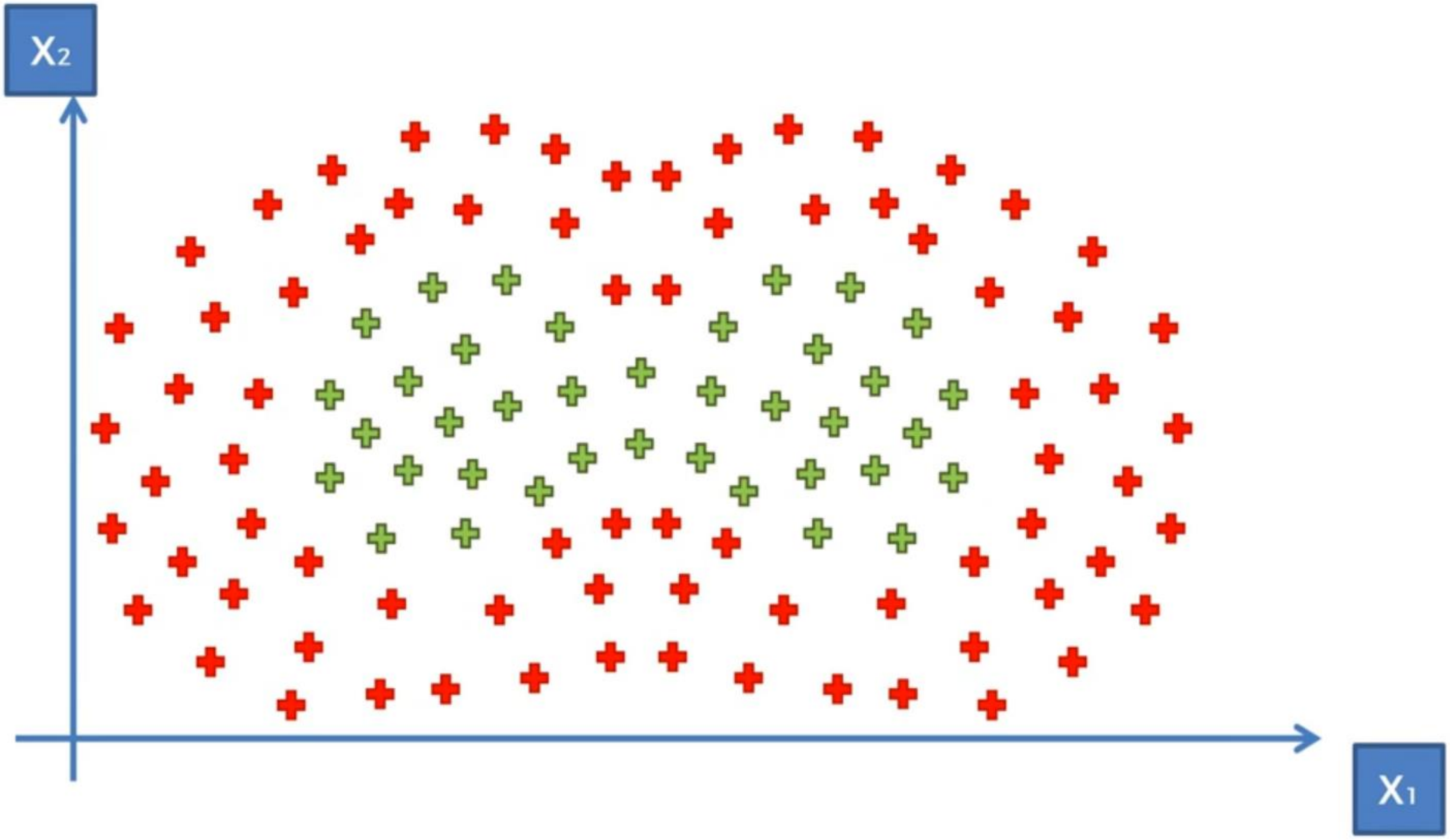

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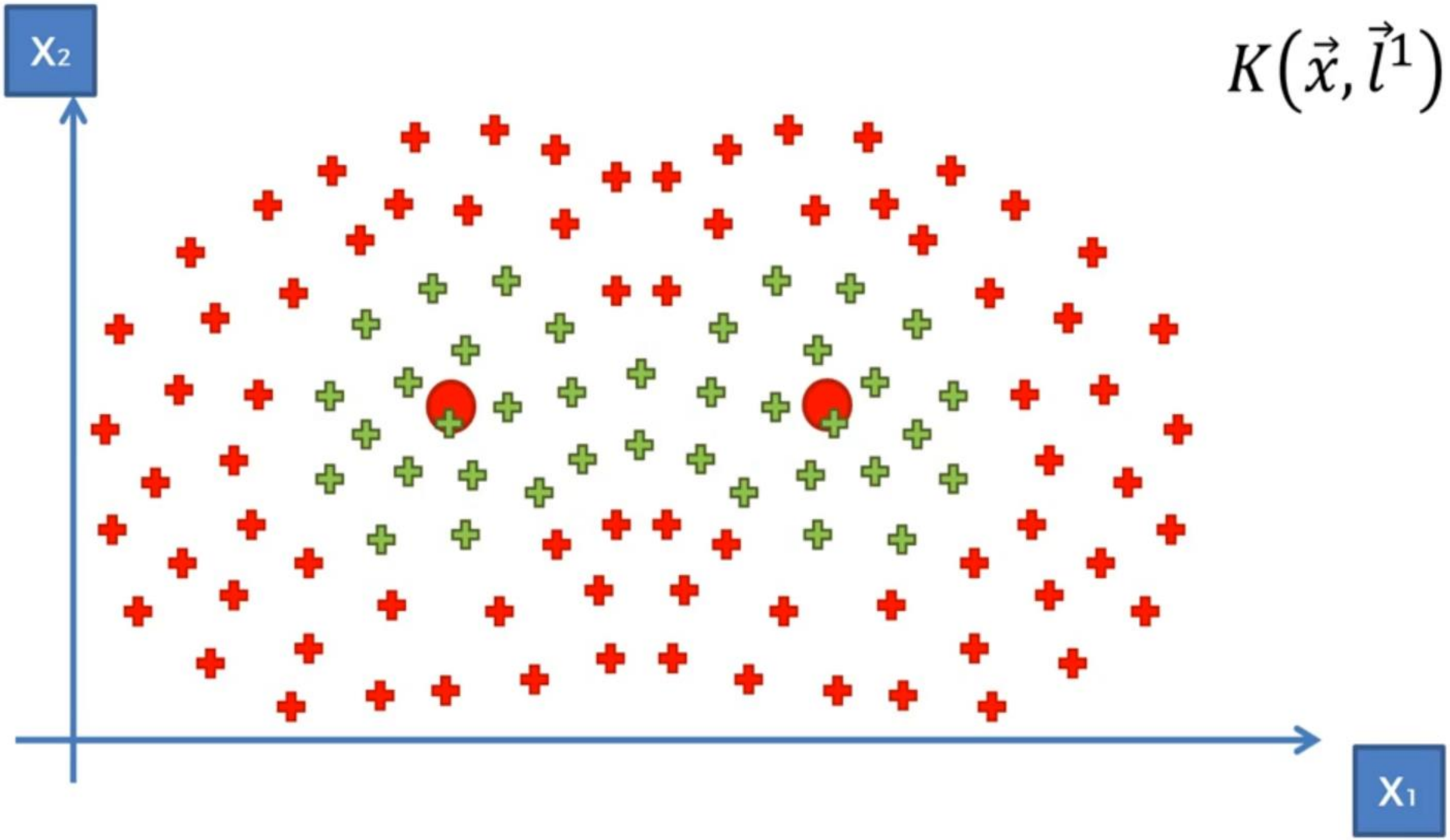
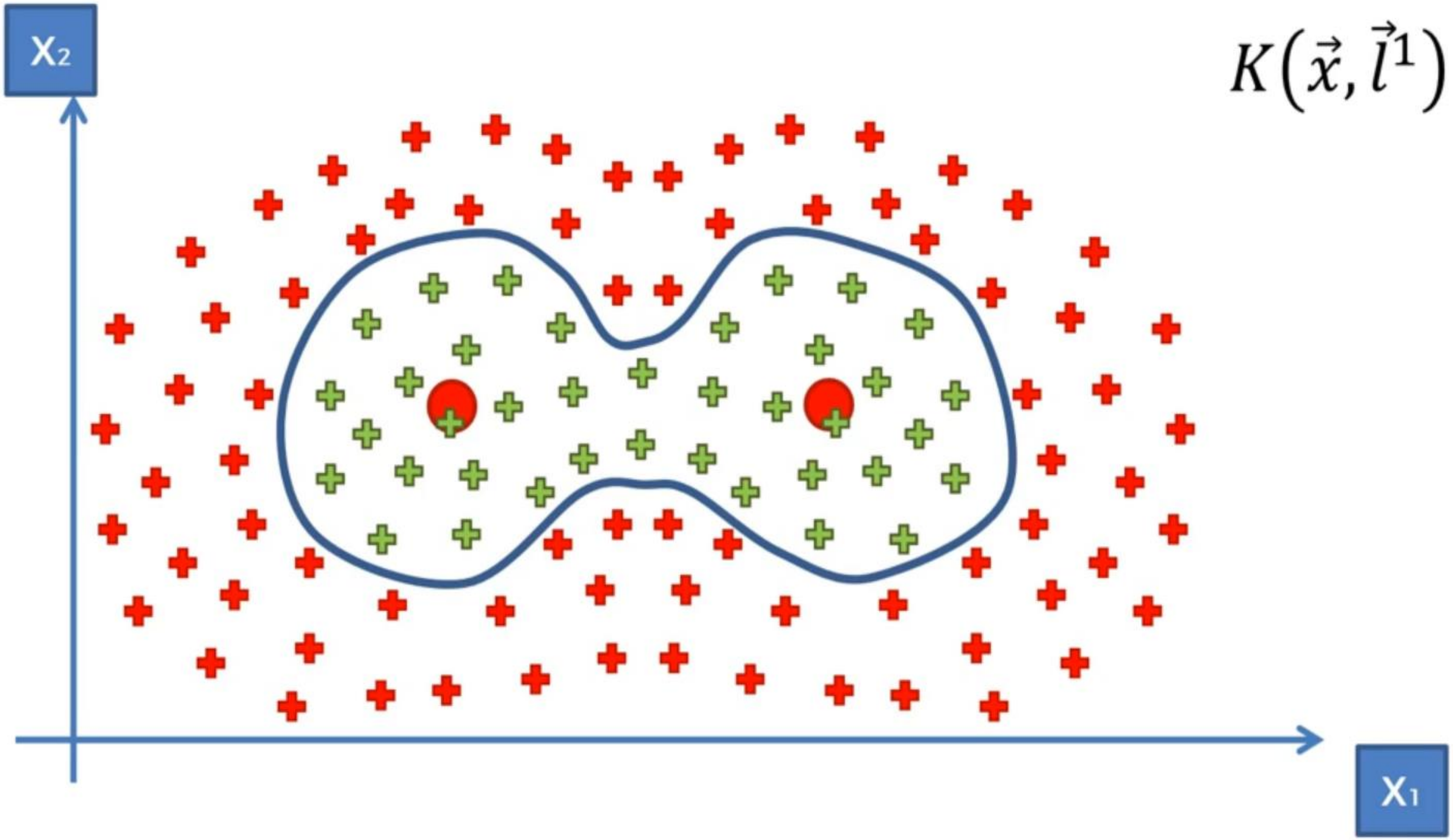


$$K(\vec{x}, \vec{l}^1) + K(\vec{x}, \vec{l}^2)$$

(Simplified Formula)

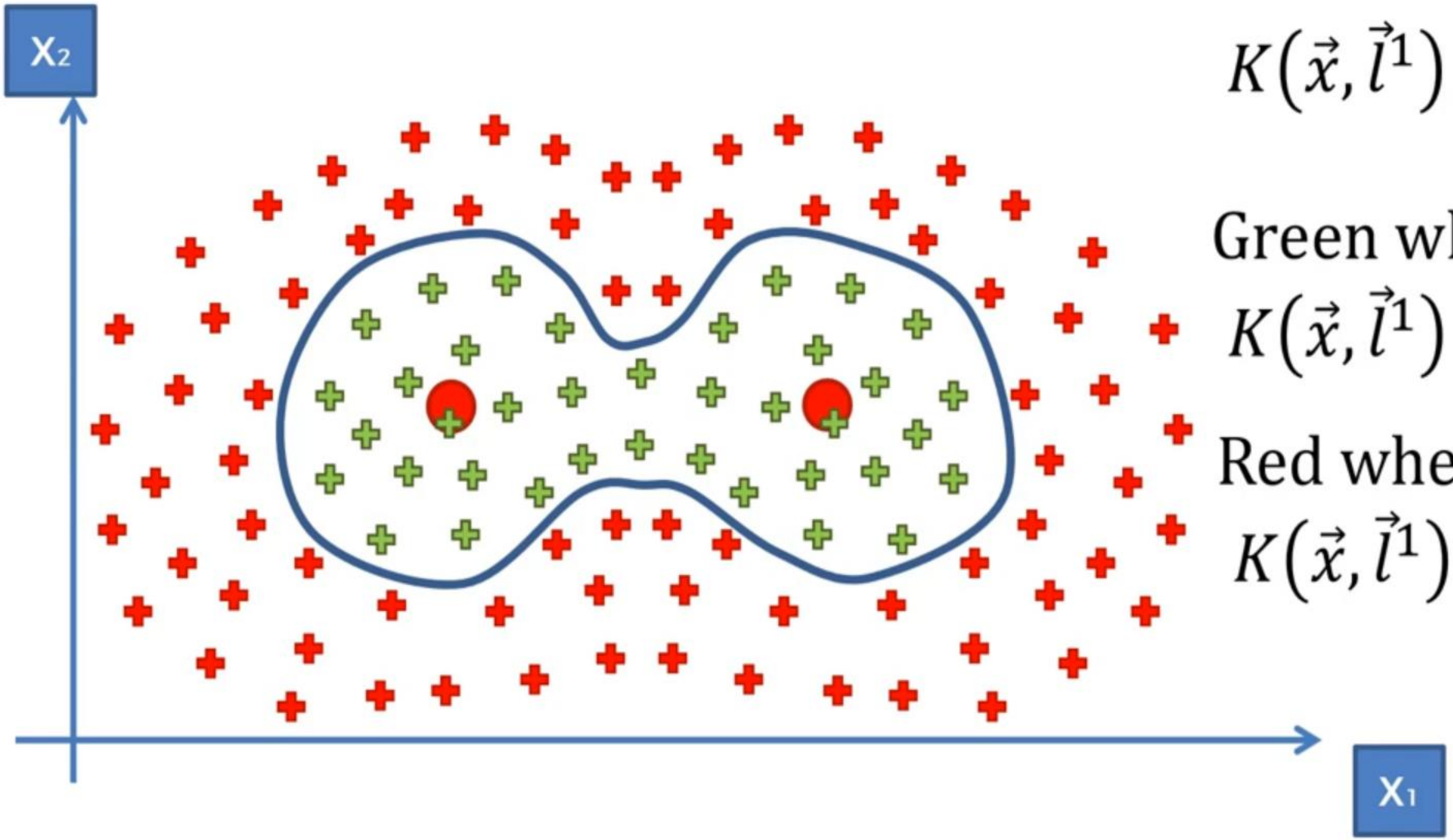
The Gaussian RBF Kernel



$$K\left(\vec{x}, \vec{l}^{1}\right) + K\left(\vec{x}, \vec{l}^{2}\right)$$

(Simplified Formula)

The Gaussian RBF Kernel



$$K\left(\vec{x},\vec{l}^{1}\right)+K\left(\vec{x},\vec{l}^{2}\right)$$

(Simplified Formula)

Green when:

$$K\left(\vec{x},\vec{l}^{1}\right)+K\left(\vec{x},\vec{l}^{2}\right)>0$$

Red when:

$$K\left(\vec{x},\vec{l}^{1}\right)+K\left(\vec{x},\vec{l}^{2}\right)=0$$

Types of Kernel Functions

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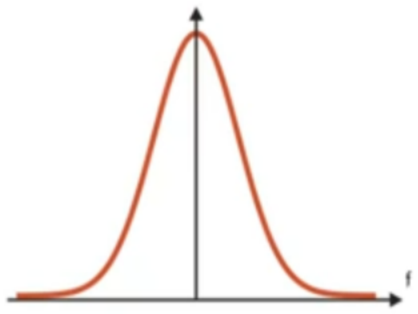
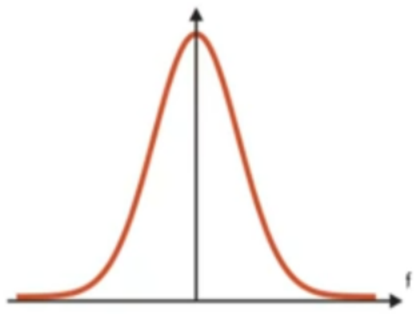


Gaussian RBF Kernel

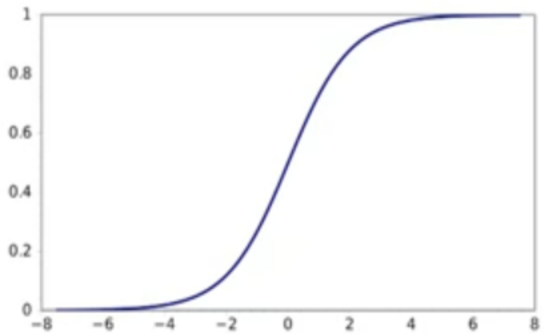
$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\left\|\vec{x}-\vec{l}^i\right\|^2}{2\sigma^2}}$$

Types of Kernel Functions



Gaussian RBF Kernel

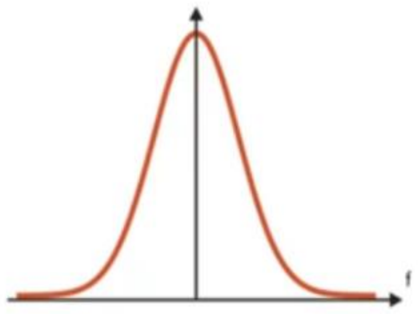
$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$



Sigmoid Kernel

$$K(X, Y) = \tanh(\gamma \cdot X^T Y + r)$$

Types of Kernel Functions



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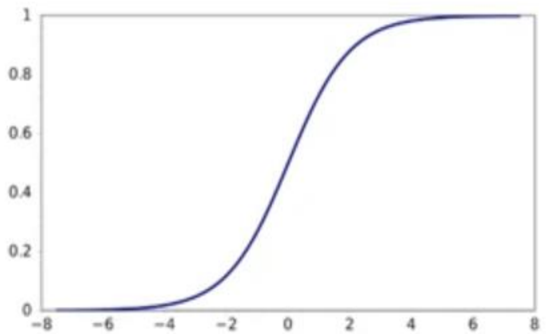

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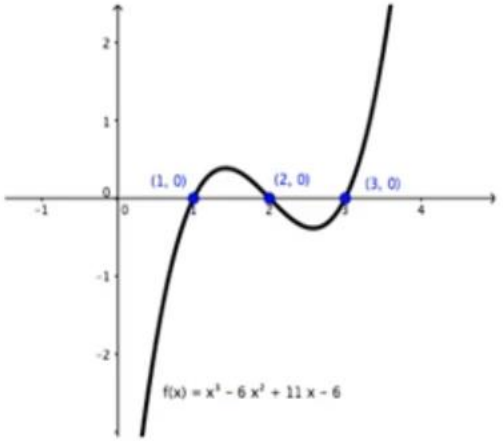

Polynomial Kernel

$$K(X,Y) = (\gamma \cdot X^T Y + r)^d, \gamma > 0$$