Assignment 11

Xiaoting Li (xil139)
Ziyu Zhang (ziz41)
Deniz Unal (des2014)

February 13 2019

19. Consider the problem of constructing maximum cardinality bipartite matching (See section 26.3 in the CLR text). Consider the natural integer linear program that you derived in a prior homework problem.

(a) Construct the dual linear program.

Answer: We construct the natrual integer linear program for maximum cardinality bipartite matching as

$$\max \sum_{e \in E} x_e \qquad \text{s.t.}$$
$$\forall v \in V, \sum_{e \text{ adjacent to } v} x_e \leq 1$$
$$\forall e \in E,\ x_e \in \{0, 1\}$$

To get the dual of the problem, we need to construct constraints from the objective in the primal and construct objective from the constraints in the primal. Let's have a variable α_v for the second constraint in the primal linear program. We can get the dual of this problem as below:

$$\min \sum_{v \in V} \alpha_v \qquad \text{s.t.}$$
$$\forall (u, v) \in E, \quad \alpha_u + \alpha_v \geq 1$$
$$\forall v \in V,\ \alpha_v \in \{0, 1\}$$

(b) Give a natural English interpretation of the dual problem (e.g. similar to how we interpreted the dual of diet problem as the pill problem)

The dual problem is to minimize the number of vertices which are the endpoints of more than one edge in the bipartite graph.

(c) We previously showed that there was always an integer optimal solution (so all values are 0 or 1) to the primal linear program. Does the dual problem always have an integer optimal solution? Justify your answer.

Based on strong duality, we learn that the optimal solution to a max problem in the primal linear program equals to the optimal solution to a min problem in the dual linear program. Previously we showed that there was always an integer optimal solution to the primal program. This optimal solution to the primal program is actually the optimal solution to the dual program. If this optimal solution to the primal program is an integer optimal solution, it means that this integer optimal solution is also the optimal solution to the dual program. So we can say the dual problem always has an integer optimal solution.

(d) Explain how to give a simple proof that a graph doesn't have a matching of a particular size based on the dual linear program. You should be able to come up with a method that would convince someone who knows nothing about linear programming.

Finding a matching of particular size is equivalent to finding certain number of vertices which are the endpoints of more than one edge in the graph. To get a matching of particular size, we have to get certain number of vertices which are the endpoints of more than one edge in the graph. Then we need to check whether the rest of the vertices in the graph (those which are the endpoints of one edge or zero edge) can construct edges in the graph. If the rest of the vertices (those which are the endpoints of one edge or zero edge) cannot construct that particular number of edges, it means that we cannot have a matching of a particular size.