Assignment 7

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14. Prove Lemma 5.4 in http://people.cs.pitt.edu/ kirk/cs2150/matroid-intersect-notes.pdf

Answer: We can use induction and strong exchange property to prove this lemma. Let $M' = (E, S' \in I : |S'| <= |S|)$. So S and T are bases of M'. If we take $x \in T\backslash S$, based on strong exchange property, there exists $y \in S\backslash T$ such that $S-y+x \in B$ and $T-x+y \in B$, B is the set of bases of M'. It's easy to tell that $S-y+x$ and $T-x+y$ are both independent in M. Based on the definition of exchange graph, we have (y, x) as an edge in $D_M(S)$. Then we can keep using induction by replacing T using $T-x+y$, $|S\backslash T| = |S\backslash(T-x+y)|+1$. Since $|S| = |T|$, in the end we can find all matching pairs in $D_M(S)$, which means we find a perfect matching. So we can conclude that if S and T are independent sets in M with $|S| = |T|$, then there exists a perfect matching between $S\backslash T$ and $T\backslash S$ in $D_M(S)$.

15. (a) Problem 29.2-6 from the CLRS text. So you want to given an integer linear programming formulation that models the bipartite matching problem.

Answer: For a given graph $G(V, E)$, in order to write a linear program to solve maximum matching problem we need a variable x_e for each edge $e \in E$. The intended meaning of x_e is whether edge e is in the matching or not. If e is in the matching $x_e = 1$ and if it is not, $x_e = 0$. We would also need a constraint for each vertex $v \in V$ since a vertex in a graph can be in at most one matching. To write these as a linear programming formula we have:

$$\max \sum_{e \in E} x_e \qquad \text{s.t.}$$

$$\forall v \in V \quad \sum_{e \text{ adjacent to } v} x_e \leq 1$$

$$\forall e \in E,\ x_e \in \{0, 1\}$$

15. (b) Consider the relaxed linear program where the integrality requirements are dropped. Explain how to find an integer optimal solution from any rational optimal solution to this relaxed linear program

Answer: The problem is meaningful only if for each edge of e is assoiated with a weight w_e. Same as (a),we need a variable x_e for each edge $e \in E$. The intended meaning of x_e is the percentage of edge e that is used in the matching. Similarly, the linear programming formula is:

$$\max \sum_{e \in E} x_e * w_e \qquad \text{s.t.}$$
$$\forall v \in V \quad \sum_{e \text{ adjacent to } v} x_e \leq 1$$
$$\forall e \in E,\ x_e \geq 0$$