Assignment 8

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16. (a) Problem 29.2-7 from the CLRS text.

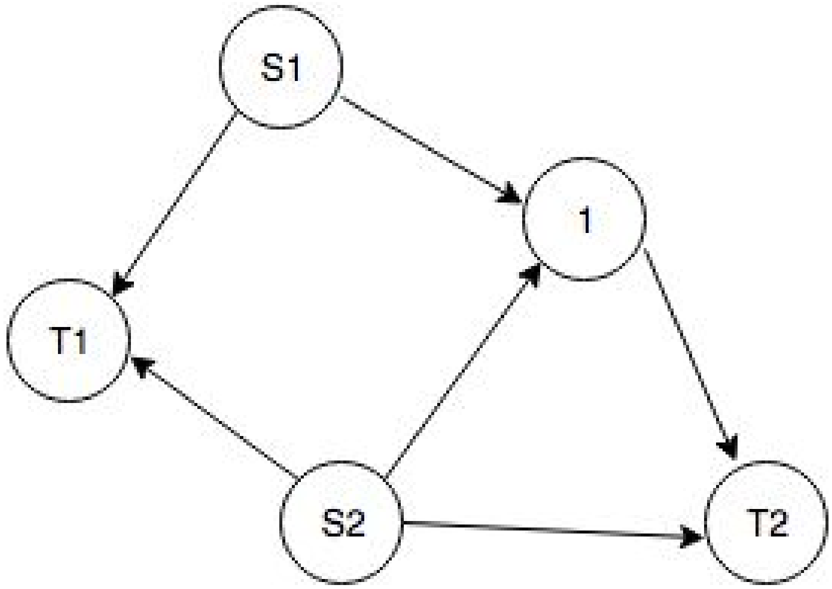
Answer: The goal of this problem is to minimize $\sum_{u,v \in V} a(u,v) f_{uv}$. The constraints of the problem are:
(1) flow-conservation: $\sum_{v \in V} f_{iuv} = \sum_{v \in V} f_{ivu}$, for each $u \in V - (s,t)$ and for each $i = 1, 2, .., k$
(2) capacity-conservation: $\sum_{i=1}^{k} f_{iuv} \leq c(u,v)$, for each $u, v \in V$
(3) $\sum_{v \in V} f_{isv} - \sum_{v \in V} f_{ivs} = d_i$, for each $i = 1, 2, .., k$
(4) $f_{iuv} \geq 0$, for each $u, v \in V$ and for each $i = 1, 2, .., k$

16. (b) Give an example of an instance where the optimal objective value to the (rational) linear program is less than the optimal objective value for the integer linear program.

The instance is a graph like the one shown below. The demand from S_1 to T_1 is 1 and the demand from S_2 to T_2 is also 1. The capacity of all arcs are 1. The cost from S_1 to T_1 is 2. The cost from S_2 to T_1 is 0. The cost from S_1 to 1 is 1. The cost from 1 to T_2 is 2. The cost from S_2 to 1 is 3. The cost from S_2 to T_2 is 0.

We represent arc $S_1 - T_1$ as a, arc $S_2 - T_1$ as b, arc $S_1 - 1 - T_2$ as c, arc $S_2 - T_2$ as d, and arc $S_2 - 1 - T_2$ as e.

So we want to minimize $2a + 3c + 3e$. The optimal integer solution is one flow of b and one flow of c, which costs 3. The optimal rational solution is 1/2 of b, 1/2 of a, 1/2 of d, and 1/2 of c, which costs 5/2. The optimal rational solution has less cost than the optimal integer solution.


S1
1
T1
S2
T2