olynomial $F(x) \in R[x]$ is called a generalized Eisenstein polynomial if there exist
utely irreducible monic polynomial $G(x) \in R[x]$ such that

$$F(x)=G(x)^q+F_{q-1}(x)G(x)^{q-1}+\dots+F_0(x), \qquad (3$$

g $F_i(x) < \deg G(x)$, $F_i(x) \in J[x]$ for $i=\overline{0,\ q-1}$ and $F_0(x) \notin J^2[x]$ if $J \neq 0$.

OREM 9. If the polynomial $F(x) \in R[x]$ is a product of pairwise relatively prime zed Eisenstein polynomials: $F(x) = H_1(x)\cdot\ldots\cdot H_t(x)$, then it is a strong invariant. , any matrix $A\in\mathscr{V}(F, R_m)$ is similar to some matrix $\mathrm{Diag}(S(H_{i_1}), \dots, S(H_{i_s}))$.

of. By theorem 2 we may assume that the polynomial $F(x)$ is of the form (32). By 3 it suffices to prove that in this case the following relations hold for any $A\in\mathscr{V}($

$$\mathrm{Ann}(A)=(F(x)), \qquad (3$$

$$\bar{A}\approx\mathrm{Diag}(N_q(\bar{G}), \dots, N_q(\bar{G})). \qquad (3$$

, it is enough to prove (#4), because this implies that $\bar{F}(x)$ is the minimal polyor $\bar{A}$ and since $F(A) = 0$ it follows from Theorem 7 that (33) holds. Without loss ty we may assume that conditions (26), (28) hold for A. In this case we have n_i emains to prove that $n_1 = q$. From now on we assume that $J(R) \neq 0$.

ume that $n_1 < q$ and $\deg G(X) = g$. On the set of matrices R_m we define two reduct s Δ_1 and Δ_2 as follows. For arbitrary matrices $C = (c_{ij}) \in R_m$ we put

$$\Delta_1(C)=\begin{pmatrix} c_{11} & \dots & c_{1,n_1\cdot g} \\ \dots & & \\ c_{n_1\cdot g,1} & \dots & c_{n_1\cdot g,n_1\cdot g}\end{pmatrix}, \quad \Delta_2(C)=\begin{pmatrix} c_{11} & \dots & c_{1g} \\ \dots & & \dots \\ c_{g1} & \dots & c_{gg}\end{pmatrix}.$$

show that under the given assumptions $\Delta_2(F(A)) \neq 0$, i.e., $F(A) \neq 0$. From this th will obviously follow.

(28) holds then $G(A) = G(A) + W$, where $W \in J_m$, and for each $r \geq 1$ we have

$$G(A)^r\equiv G(N)^r+\sum_{s=0}^{r-1}G(N)^sWG(N)^{r-1-s}\ (\mathrm{mod}\ J^2).$$

, if we use (32) we obtain $F(A)\equiv G(N)^q+\sum_{r=0}^{q-1}F_r(N)G(N)^r+\sum_{s=0}^{q-1}G(N)^sWG(N)^{q-1-s}\ (\mathrm{mod}\ J^2)$.

e it follows from (27) and the condition $n_t \leq q$ that $G(N)^q = 0$ we get

$$F(A)\equiv\left(\sum_{r=1}^{q-1}F_r(N)G(N)^r\right)+\left(\sum_{s=0}^{q-1}G(N)^sWG(N)^{q-1-s}\right)+\Big(F_0(N)\Big)\ (\mathrm{mod}\ J^2). \qquad ($$

e terms on the right-hand side of (35) by W_1, W_2, W_3, respectively. Note that x $G(N)$ the first g columns equal zero, hence

$$\Delta_2(W_1)=0.$$

the matrix $W_2=\sum_{s=0}^{q-1}G(N)^sWG(N)^{q-1-s}$. By the block-diagonal structure of the ma e

$$\Delta_1(W_2)=\sum_{s=1}^{q-1}G(N_{n_1}(G))^s\,\Delta_1(W)\,G(N_{n_1}(G))^{q-1-s}.$$

we have $G(N_{n_1}(G))^{q-1} = 0$ and hence

$$\Delta_1(W_2)=\sum_{s=1}^{q-1}G(N_{n_1}(G))^s\,\Delta_1(W)\,G(N_{n_1}(G))^{q-1-s}.$$

s > 0 the first g columns of the matrix $F(N_{n_1}(G))^s$ are zero it follows from

$$\Delta_2(W_2)=0.$$