

matrix $W_3 = F_0(N)$. Clearly, $\Delta_2(W_3) = F_0(S(G))$. Since $G(x)$ is a minimal polynomial of the matrix $S(G)$ and $\bar{G}(x)$ is the minimal polynomial of the matrix $S(G)$ it follows for the matrix $S(G)$ there are no nonzero polynomials modulo J^2 of degree ≤ 1 which annihilate it modulo J^2 . Consequently,

$$\Delta_2(W_3) = F_0(S(G)) \not\equiv 0 \pmod{J^2}.$$

from (35), (36) and (38) that $\Delta_2(F(A)) \not\equiv 0 \pmod{J^2}$, i.e., $F(A) \neq 0$. Proof of Theorem 9.

As observed in the introduction, Theorem 9 includes as special cases the results for commutative rings. Moreover, Theorem 9 significantly extends the results obtained in the previous papers. For example, over $\mathbb{Z}/2^n$, the polynomial $x^2 + e$ which could be considered comparable to the latter a strong invariant: $x^2 + e = (x + e)^2 - 2(x + e) + 2e$. In general, over \mathbb{Z}/p^n every polynomial which is congruent modulo p^2 to $x^p + (p - 1)x$ is a strong invariant.

Second and Third Degree over a Principal Ideal Ring

Let R denote a commutative artinian principal ideal ring, $J = \pi R \neq 0$. Then there exists a number n such that $J^n = 0$ and all the proper ideals of R are $J = \pi R, \pi^2 R, \dots, \pi^{n-1} R$ (cf. [15, Chap. 4]). In this case the similarity problem for matrices $A \in R_m$ with $A \neq 0$ since if $A \sim B$ if and only if the images of the matrices A_1 and B_1 over the residue field \bar{R} are similar.

Assume that $A \in R_2$ and $\bar{A} \neq \bar{0}$. Then exactly one of the following cases occurs:

1) $A = (e)$ and $A \sim S(\chi_A(x))$, $\text{Ann}(A) = (\chi_A(x))$.

2) $A = (x - r)$, where $r \in R$, and $A = rE$, $\text{Ann}(A) = (x - r)$.

3) $A = (x - r, \pi^k)$, where $r \in R$, $0 < k < n$, and $A \sim rE + \pi^k S(G(x))$, where $G(x)$ is a polynomial of degree k and the polynomial $\pi^k G(x)$ is uniquely determined by the matrix $\chi_A(x) = \begin{pmatrix} x - r & \pi^k \\ 0 & x - r \end{pmatrix}$, $\text{Ann}(A) = (\chi_A(x), \pi^{n-k}(x - r))$.

The matrix A is canonically determined if and only if all its Fitting invariants are known.

Let $\mathcal{D}_s(xE - A) = \mathcal{D}_s(xE - \bar{A})$, $s = 1, 2$ it follows that if $\mathcal{D}_1(xE - A) = (F(x)) + \mathcal{D}_1$ and $F(x)$ is a monic polynomial, then $\deg F(x) \leq 1$. Therefore one of the following cases must apply to the ideal $\mathcal{D}_1(xE - A)$.

1) If the statements of the theorem are consequences of the fact that all the principal ideals of R are $\pi^k R$ and that therefore A is a normal matrix.

It is clear that the matrix A is of the form $A = rE + \pi^k B$, where the minimal polynomial of B coincides with its characteristic polynomial. Indeed, if this is not the case, for suitable $b \in R$ and $A = r_1 E + \pi^{k+1} B_1$, i.e., the ideal $\mathcal{D}_1(xE - A)$ is (π^{k+1}) , contrary to the assumption. Consequently, if $\chi_B(x) = G(x)$ then $B \sim S(G)$ then $T^{-1}AT = rE + \pi^k S(G)$. Clearly, if also $A \sim rE + \pi^k S(G_1)$, then $G_1(x) \equiv G(x) \pmod{J^{n-k}}$, i.e., $\pi^k G_1(x) = \pi^k G(x)$.

Let $G(x) = x^2 - ax - b$ and that $b_1 \in R$ is an element with the properties $\pi^{2k} b_1 \equiv b \pmod{J^{n-2k}}$ (the existence of such an element is guaranteed by the conditions of the theorem). It is easily seen that the matrix $A_1 = rE + \pi^k S(G_1)$, where $G_1(x) = x^2 - ax - b_1$ is similar to A but $\mathcal{D}_s(xE - A) = \mathcal{D}_s(xE - A_1)$ for $s = 1, 2$. Consequently, if A is a canonically determined matrix. The last statement of Theorem 10 follows.

[16] representatives of classes of conjugate elements in the group R_2^* are given.

For degree three over R the situation becomes significantly more complex, it is only possible to describe possible canonical forms for $A \in R_3$ in the