1. Restricted Nim. Last move wins. Players alternate and can take 1,5, or 6 stones in a move from a pile.
Initial position: 2 piles, 10 and 20 stones.

First we do one pile

stones	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
value take	L	W 1	L	W 1	L	W 1 or 5	W 6	W 5	W 6	W 5	W 6	L	W 1	L	W 1	L	W 1 or 5	W 6	W 5

Period is 11. A winning strategy:
reduce modulo 11 and then use the table.

Now we consider two piles:

	0	1	2	3	4	5	6	7	8	9
10										
9	W									L
8							L		L	1
7				L				L	1	6
6							L	1	L	1
5		L		L		L	1or 5	6	5	6
4	L		L		L	1	6	5 or 1s	6	5
3	1s	L		L	1 or 1s	L	1	L	1	6
2	L		L	1 or 1s	L	1,1s,or 5	6	1s	6	5
1	1	L	1 or 1s	L	1 or 1s	L	1	6	1 or 1s	6
0	L	1	L	1	L	1or 5	6	5	6	5

1s means that we take 1 from a smaller pile (going 1 step south).

A winning strateg at (10,20): we take 1 stone from pile of 10. Now we have (9, 20) which are congruent modulo 11.
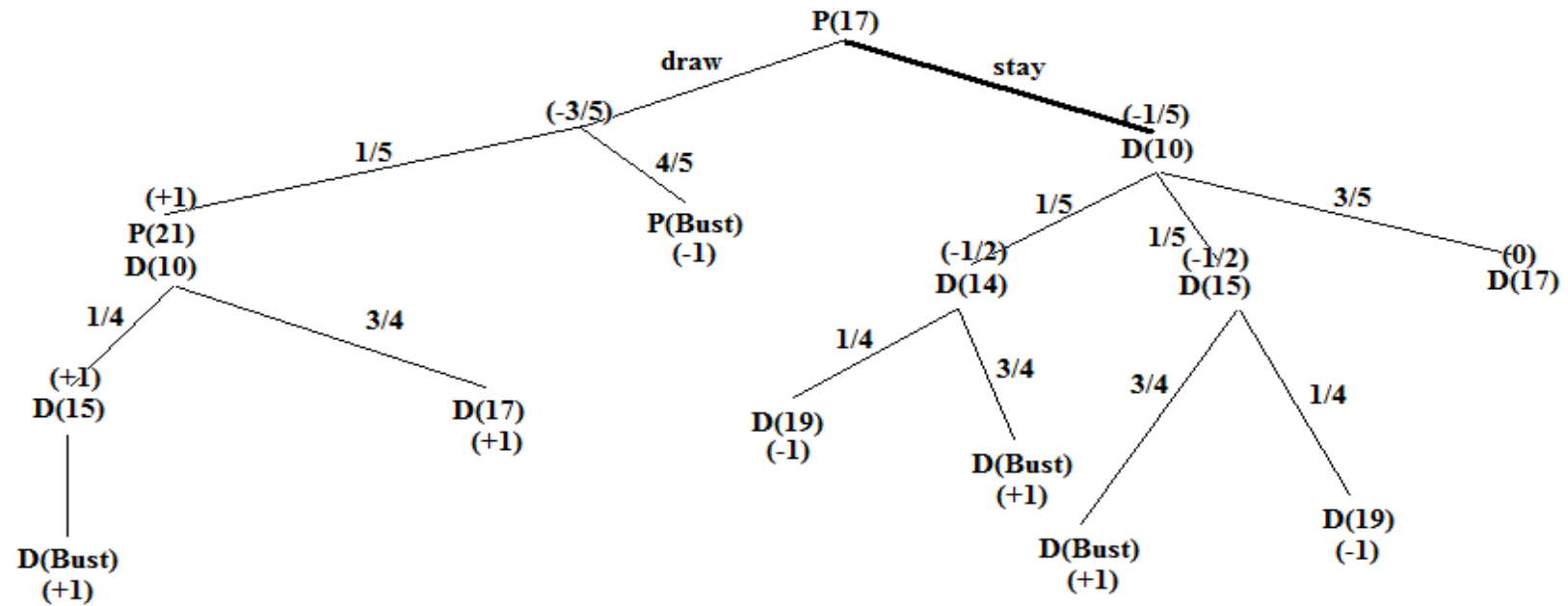
Then we keep the numbers congruent modulo 11 if possible.

Let in position (m,n) with m, n congruent modulo 11 the second player takes x from m. Then, if possible we either take x from n or 11- x from m.

It remains to consider the the case when we cannot take x from the other pile nor we can take 11-x from the same pile.

Then n < 6 and m <11, hence m = n. therefore we can take x from the other pile.

2. Blackjack. Player (P) has hard 17. Dealer (D) shows 10. Cards remaining including D's face-down are 4, 5, 7, 7,7.
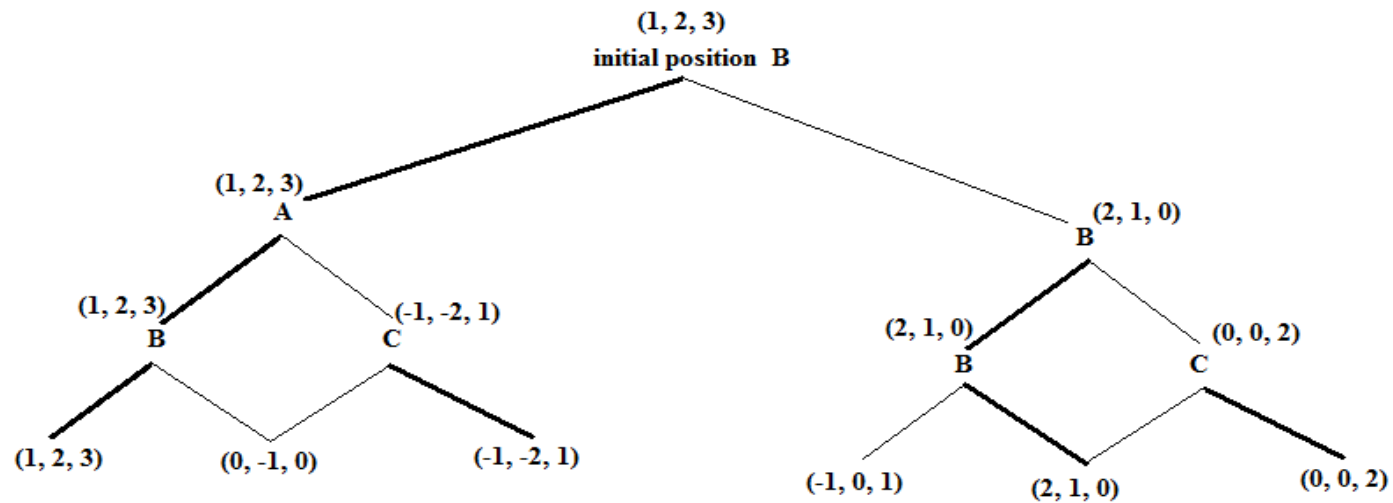
P(17)

draw — stay

(-3/5)   (-1/5) D(10)

1/5   4/5   1/5   3/5

(+1) P(21) D(10)   P(Bust) (-1)   (-1/2) D(14)   1/5 (-1/2) D(15)   (0) D(17)

1/4   3/4

(+1) D(15)   D(17) (+1)   1/4   3/4   3/4   1/4

D(19) (-1)   D(Bust) (+1)

D(Bust) (+1)   D(Bust) (+1)   D(19) (-1)

Player stays, expected payoff is -$1/5

3. 2 player game in normal form.

-7, 1	4, 0	-1, 3	0, 0	3*, 3	-3, 4*
5*, -1	5*, 0	-2, 5*	1, 4	3*, 1	0, 0
0, 5*	4, -1	-2, 4	6*, 0	0, 3	1, 1
1, 6*	5*, 0	0*, 3	6*, 0	3*, 5	3*, 3

No equilibrium.

4. Extensive form, 3 players, A B, C.

(1, 2, 3)
initial position B

(1, 2, 3)
A

(2, 1, 0)
B

(1, 2, 3)
B

(-1, -2, 1)
C

(2, 1, 0)
B

(0, 0, 2)
C

(1, 2, 3)   (0, -1, 0)   (-1, -2, 1)

(-1, 0, 1)   (2, 1, 0)   (0, 0, 2)

5. Game with 3 players, A, B, C. in normal form.

Strategy			Payoff		
A	B	C	A	B	C
1	1	1	0*	-1	1*
1	1	2	1*	1*	-2
1	2	1	1*	0*	0
1	2	2	0*	0	1*
2	1	1	0*	-1	1*
2	1	2	1*	1*	-2
2	2	1	1*	0*	-1
2	2	2	-1	1*	0*
3	1	1	0*	-1	1*
3	1	2	-1	1*	-2
3	2	1	1*	0*	0
3	2	2	-1	0	1*

No Equilibrium.