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Midterm 2 – Solutions

**Problem 1**

| $x_1$ | $x_2$ | $-x_3$ | $-x_4$ | $x_5$ | -1 | |
|---|---|---|---|---|---|---|
| $10^{11}$ | 2 | 0 | 0 | 6 | -1 | $=- x_6$ |
| 0 | -1 | -2 | 0 | 3 | -2 | $= -x_7$ |
| -1 | 0 | 0 | 4 | -4 | 3 | $= -x_8$ |
| 2 | 0 | $3^{-300}$ | 0 | 1 | 2 | $= f$ -> min |

Multiply columns 3 and 4, and the last row by -1

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | -1 | |
|---|---|---|---|---|---|---|
| $10^{11}$ | 2 | 0 | 0 | 6 | -1 | $=- x_6$ |
| 0 | -1 | 2 | 0 | 3 | -2 | $= -x_7$ |
| -1 | 0 | 0 | -4 | -4 | 3 | $= -x_8$ |
| -2 | 0 | $3^{-300}$ | 0 | -1 | -2 | $= f$ -> max |

Row 1 is a bad row, so the LP is infeasible

**Problem 2**

$x_1, x_2 >= 0$
$2x_1 + 3x_2 <=6$
$3x_1 + 2x_2$ -> max

$x_1 = 3$
$x_2 = 0$
max = 9

**Problem 3**

| $x_1/3000$ | $x_1/3000002$ | $x_3/3000003$ | $x_4/3000005$ | -1 | |
|---|---|---|---|---|---|
| $10^{11}$ | -1/3000002 | 1/300004 | 0 | -2 | $= -x_6$ |
| -1/3000002 | -1/3000002 | 0 | 0 | 1 | $= -x_7$ |
| -1 | 0 | -1/3000002 | -4/3005 | 2 | $= -x_8$ |
| 2 | 0 | 0 | 1/305 | 2 | $= f$ -> min |

Multiply the last row by -1

Row 1 is a bad row, so the LP is infeasible

**Problem 4**

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 0 | 0 | 0 | $3-10^{-100}$ | 5 | 10 |
| 0 | 2 | 0 | 0 | 2 | 1 | 4 |
| 0 | 0 | 2 | 0 | 2 | 0 | $3-10^{-100}$ |
| 0 | 0 | 0 | 2 | 2 | $3-10^{-100}$ | 0 |
| 0 | 0 | 0 | $-10^{-100}$ | 0 | 0 | $3-10^{-100}$ |

Column 1 dominates columns 5, 6, 7
Row 4 dominates row 5

This leaves matrix:

| | | | |
|---|---|---|---|
| 2 | 0 | 0 | 0 |
| 0 | 2 | 0 | 0 |
| 0 | 0 | 2 | 0 |
| 0 | 0 | 0 | 2 |

His optimal strategy: $[¼, ¼, ¼, ¼, 0]^T$
Her optimal strategy: [¼, ¼, ¼, ¼, 0, 0, 0]
Value of game = ½

**Problem 5**

| | | | | | | |
|---|---|---|---|---|---|---|
| $-10^{100}$ | $3-10^{-100}$ | $3-10^{-100}$ | $3-10^{-100}$ | $3-10^{-100}$ | $3-10^{-100}$ | $3-10^{-100}$ |
| $0^*$ | 2 | 0 | -1 | $-2^{\Delta}$ | -1 | -1 |
| $0^*$ | $3^*$ | $3^*$ | 0 | $-2^{\Delta}$ | 0 | -1 |
| $-1^{\Delta}$ | 0 | $3^*$ | 0 | 2 | $3^*$ | 0 |
| $0^{*\Delta}$ | $3^*$ | 1 | $4^*$ | $3^*$ | $0^{\Delta}$ | $4^*$ |

$^*$ denote maximums
$^{\Delta}$ denote minimums

Saddle point on row 5, column 1

Value of game = 0