

$$\mathfrak{M}(\mathcal{K}) = \sum_{i=1}^m \mathfrak{M}(K_i),$$

$$K_i) + \mathfrak{M}\left(\begin{pmatrix} K_i & L \\ N & K_{i+1} \end{pmatrix}\right) + \mathfrak{M}(K_{i+2}) + \dots + \mathfrak{M}(K_m)$$

obtain therefore an isomorphism

$$\mathfrak{M}\left(\begin{pmatrix} K_i & L \\ N & K_{i+1} \end{pmatrix}\right) \cong \mathfrak{M}\left(\begin{pmatrix} K_i & 0 \\ 0 & K_{i+1} \end{pmatrix}\right). \quad (19)$$

and consider the  $k \times k$  matrices

$$\begin{pmatrix} E & 0 \\ 0 & \mathfrak{U} \end{pmatrix}, \text{ where } \mathfrak{U} = \begin{pmatrix} K_i(x) & 0 \\ 0 & K_{i+1}(x) \end{pmatrix};$$

$$\begin{pmatrix} E & 0 \\ 0 & \mathfrak{U}' \end{pmatrix}, \text{ where } \mathfrak{U}' = \begin{pmatrix} K_i(x) & L(x) \\ N(x) & K_{i+1}(x) \end{pmatrix}.$$

isomorphism (19) we have an isomorphism

$$\mathfrak{M}(\mathcal{K}_1) \cong \mathfrak{M}(\mathcal{K}_1'). \quad (20)$$

are equivalent to some characteristic matrices  $xE - \Lambda_1$  and  $xE - \Lambda_1'$ . By Theorem 3 and (20) that the matrices  $\mathcal{K}_1$  and  $\mathcal{K}_1'$  are not equivalent. This is impossible.

$\mathcal{K}_k$  are such that

$$U(x)\mathcal{K}_1' = \mathcal{K}_1 V(x).$$

Divide  $V$  into blocks  $U_{ij}$ ,  $V_{ij}$  corresponding to the blocks of the matrices  $\mathcal{K}_1$  and  $\mathcal{K}_1'$ . The equations

$$U_{21} = \mathfrak{U} V_{21}, \quad (21)$$

$$U_{22} \mathfrak{U}' = \mathfrak{U} V_{22}. \quad (22)$$

Elements of the matrix  $U_{21}$  belong to the ideal  $P$ . In view of (21)  $|U_{22}| \in P$ . Therefore,  $|U| \in P$  and the matrix  $U(x)$  is non-invertible. This is a contradiction. The proof of Theorem 6.

## Strong Invariants

were announced in [7]. If  $A \in R_m$  we denote by  $\text{Ann}(A)$  the ideal of  $R[x]$  with  $F(A) = 0$ . As was shown in [14] the ideal  $\text{Ann}(A)$  is determined by the two leading Fitting invariants:

$$F_1(A) = \mathcal{D}_{m-1}(xE - A) = (\chi_A(x) : \mathcal{D}_{m-1}(xE - A)). \quad (23)$$

are determined if for any matrix  $B \in R_m$  the condition  $A \sim B$  is satisfied. Conditions:  $\text{Ann}(A) = \text{Ann}(B)$  and  $A \sim B$ . By (23) it is clear that the leading Fitting invariants are canonically determined. Before we describe polynomials we derive some more specific results about the ideal  $\text{Ann}(A)$ .

A matrix  $A \in R_m$  is a monic polynomial  $F(x) \in R[x]$  of smallest degree. The minimal polynomial of a matrix is not in general unique. For

example  $\mathbb{Z}/4$  has minimal polynomials  $x^2$  and  $x(x+2)$ . The proper-

ties of polynomials over  $R$  are not well studied. For example it is unknown whether the minimal polynomial of a matrix which divides the characteris-

tic polynomial for  $A \in R_m$  and  $H(x) \in \text{Ann}(A)$ . Then  $H(x) = Q(x)F(x) + R(x)$  and  $L(A) = 0$ . Here we have  $L(x) \in J[x]$  since otherwise by a theorem of McCoy there is a monic polynomial  $L_0(x)$  which is an associate of  $L(x)$  with  $L_0(A) = 0$ . This is impossible since  $L_0(A) = 0$ . We have therefore

$$\text{Ann}(A) = (F(x)) + \text{Ann}(A) \cap J[x]. \quad (24)$$