

1--2. Solve linear programs where all $x_i \geq 0$:

| x_1 | x_2 | -2 | $-x_5$ | Problem 1 |
|-------|-------|----|--------|--------------------|
| 1 | -2 | 3 | 4 | $= x_3$ |
| 1 | -2 | 0 | 3 | $= -x_2$ |
| 0 | 2 | -3 | 0 | $\rightarrow \max$ |

$$\Rightarrow \begin{array}{ccc|c} x_1 & x_2 & x_5 & 1 \\ \hline 1 & -2 & -4 & -6 \\ -1 & 2 & 3 & 0 \\ 0 & -2 & 0 & -6 \end{array} \begin{array}{l} = x_3 \\ = x_2 \\ \rightarrow \min \end{array}$$

$$\text{Pivot at } * \begin{array}{ccc|c} x_2 & x_5 & 1 \\ \hline -1 & 0 & -1 & -6 \\ -1 & 2 & 3 & 0 \\ 0 & -2 & 0 & -6 \end{array} \begin{array}{l} = x_3 \\ = x_1 \\ \rightarrow \min \end{array}$$

$$\Rightarrow \begin{array}{ccc|c} x_2 & x_5 & 1 \\ \hline -1 & -1 & -6 \\ 1 & 3 & 0 \\ -2 & 0 & 6 \end{array} \begin{array}{l} = x_3 \\ = x_1 \\ \rightarrow \min \end{array}$$

Row 1 is Bad so LP is infeasible

| $2x_1$ | $-x_2$ | 1 | $-x_4$ | Problem 2 |
|--------|--------|----|--------|--------------------|
| 1 | -2 | 3 | 4 | $= x_3$ |
| 1 | 2 | 0 | -1 | $= -x_4$ |
| 0 | -2 | -3 | -2 | $\rightarrow \min$ |

$$\Rightarrow \begin{array}{ccc|c} x_1 & x_2 & x_4 & 1 \\ \hline 2 & 2 & -4 & 3 \\ -2 & 2 & -1 & 0 \\ 0 & 2 & 2 & -3 \end{array} \begin{array}{l} = x_3 \\ = x_4 \\ \rightarrow \min \end{array}$$

$$\text{Pivot at } * \begin{array}{ccc|c} x_4 & x_2 & x_4 & 1 \\ \hline -1 & 3 & -5 & 3 \\ -1/2 & 1 & -1/2 & 0 \\ 0 & 2 & 2 & -3 \end{array} \begin{array}{l} = x_3 \\ = x_1 \\ \rightarrow \min \end{array}$$

$$\Rightarrow \begin{array}{ccc|c} x_2 & x_4 & 1 \\ \hline 3 & -6 & 3 \\ 1 & -1 & 0 \\ 2 & 2 & -3 \end{array} \begin{array}{l} = x_3 \\ = x_1 \\ \rightarrow \min \end{array}$$

so $\min = -3$
at $x_2 = x_4 = 0$
 $x_3 = 3$
 $x_1 = 0$

3. Solve transportation problem

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|-----------|---|---|---|
| 2 | 1 | 3 | 2 | 1 | 2 | 1 | 6 | 2 | 1 | 3 | 9 | | | |
| 1 | 6 | 2 | 3 | 1 | 1 | 1 | 1 | 3 | 1 | 2 | 8 | | | |
| 2 | 2 | 1 | 6 | 3 | 1 | 2 | 1 | 3 | 5 | 2 | 1 | 9 | | |
| 5 | 1 | 1 | 3 | 1 | 1 | 2 | 4 | 1 | 1 | 2 | 3 | 1 | 4 | 8 |
| 6 | 1 | 6 | 1 | 6 | 1 | 6 | 1 | 6 | 1 | 6 | dem \ sup | | | |

Solved in phase 1

$$\boxed{\min = 41}$$

4. Solve transportation problem

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|-----------|---|---|
| 3 | 1 | 1 | 3 | 2 | 1 | 2 | 2 | 2 | 2 | 1 | 6 | 9 | |
| 1 | 6 | 1 | 3 | 1 | 1 | 5 | 1 | 2 | 3 | 1 | 8 | | |
| 2 | 2 | 0 | 6 | 3 | 1 | 3 | 1 | 3 | 1 | 2 | 9 | | |
| 2 | 3 | 5 | 1 | 1 | 2 | 1 | 1 | 2 | 4 | 0 | 1 | 2 | 8 |
| 6 | 1 | 6 | 1 | 6 | 1 | 6 | 1 | 6 | 1 | 6 | dem \ sup | | |

Solved in phase 1

$$\boxed{\min = 32}$$

5. Solve job assignment problem

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 2 |
| 2 | 3 | 0 | 1 | 2 |
| 3 | 2 | 1 | 3 | 1 |
| 1 | 3 | 4 | 5 | 1 |
| 1 | 5 | 2 | 1 | 6 |

$$\begin{array}{ccccc} 0 & 0 & 3 & 3 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 4 & 0 \\ 0 & 3 & 2 & 0 & 5 \end{array}$$

$$1 \text{ worker at each } * \quad \min = 1 + 2 + 0 + 1 + 1 = 5 = \min$$