it is shown that if R is a finite principal ideal ring then every separable polynomial over R is a strong invariant. Exactly the same reasoning is valid also for arbitrary infinite rings R. It follows further that if $F(x) \in R[x]$ is a separable polynomial then every matrix $A \in \mathscr{V}(F(x), R_m)$ is normal.

These results of [3] imply as particular cases the results of the papers [4] for the class $\mathscr{V}(x^t - e, (\mathbf{Z}/p^n)_m)$, $(t, p) = 1$; [5] for the class $\mathscr{V}(x^2 - x, (\mathbf{Z}/p^n)_m)$; [6] for the class $\mathscr{V}(x^t - e, \bar{R}_m)$, where R is finite, $(t, \operatorname{char} \bar{R}) = 1$. We note that [6] considers the more general case when R is a finite local noncommutative ring; there it is also shown that $x^t - e$ is a strong invariant using Sylow's theorems.

In [7] a result was announced which described a wider class of strong invariants than separable polynomials. In that paper we also described the matrices $A \in R_m$ for which the condition $B \approx A$ is equivalent to the condition $\bar{B} \approx \bar{A}$ together with $\operatorname{Ann}(B) = \operatorname{Ann}(A)$, where $\operatorname{Ann}(A) = \{F(x) \in R[x], F(A) = 0\}$. We give the proofs in Sec. 3 of the present paper.

In [8] the similarity problem for matrices over R is reduced to the isomorphism problem for modules over R[x] (considering only the case when R is a principal ideal ring). The main result of that paper (Theorem III.2) states that if the ideal Ann(A) is principal then the matrix A is normal. This is incorrect, the simplest counterexample being the matrix $A = \operatorname{Diag}\left(\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right)$ over the ring $\mathbf{Z}/4$. For this matrix we have that $\operatorname{Ann}(A) = (x^2)$ is a principal ideal, but it is not similar to any matrix of the form $\operatorname{Diag}\left(2\alpha, 2\beta \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right)$, as can be verified directly. There are also errors in the proof of a correct result in that paper: Theorem II.3 (cf. our Sec. 3, Remark to Theorem 7).

In this paper we describe two methods which often allow to solve the problems connected with the similarity problem (section 1). In section 2 we consider conditions under which the class of matrices similar to A is uniquely determined by the Fitting invariants $\mathscr{D}_1(xE-A), \ldots, \mathscr{D}_m(xE-A)$ of its characteristic matrix $xE - A$ ($\mathscr{D}_k(xE-A)$ is the ideal in R[x] generated by the minors of order k of the matrix $xE - A$). Such matrices are called canonically determined. We conjecture that the canonically determined matrices are exactly those matrices A for which all the ideals $\mathscr{D}_k(xE-A)$ are principal. This conjecture holds in the case of normal matrices, and in section 4 it is verified in the case where R is a principal ideal ring for all matrices in R_2 and for all matrices in R_3 under the condition that $J(R)^2 = 0$. In these rings we give a method to construct canonical representatives for the similarity classes of matrices. It turns out that sometimes the normality of a matrix A can be established from its Fitting invariants, even in the case when they are not principal ideals, i.e., when the matrix A is not canonically determined.

1. Two Approaches to the Similarity Problem. Normal Matrices

With the matrix $A \in R_m$ we associate the left R[x]-module M(R) which is obtained from the R-module $R^{(m)}$ of column vectors of length m over R by defining the product of $F(x) \in R[x]$ with $\alpha \in R^{(m)}$ by $F(x) \circ \alpha = F(A) \cdot \alpha$. This module will be called the module associated with the matrix A (cf. [9, Chap. VII]).

THEOREM 1. Let $A, B \in R_m$. Then the following statements hold:

1. The matrices A and B are similar if and only if the R[x]-modules M(A) and M(B) are isomorphic.

2. The matrix A is similar to a reducible matrix if and only if the module M(A) is reducible. The similarity $\operatorname{Diag}(A_1, \ldots, A_t) \approx A$ is equivalent to the isomorphism $M(A) \cong M(A_0) \dotplus \ldots \dotplus M(A_t)$.

3. The module M(A) is cyclic if and only if $A \approx S(\chi_A(x))$ where $\chi_A(x) = |xE - A|$. The matrix A is normal if and only if the module M(A) decomposes into a direct sum of cyclic modules.

The proof of part 1 is given in [9, appendix to Chap. VII]. The proofs of parts 2 and 3 can be given easily using the same ideas as used in [9, Chap. VII] and [10, Chap. XV] in the study of modules associated with endomorphisms of vector spaces, together with the fact that every direct summand of the module M(A) is a free R-submodule of $R^{(m)}$ which is invariant under multiplication by A. More details on this are given in [8].