that for the matrix A one of the conditions (a), (b) fails; we will show that ts a matrix $B \in R_m$ which is not similar to A and such that $\mathrm{Ann}(B) = \mathrm{Ann}(A)$,) $\in R[x]$ be a monic absolutely irreducible polynomial, $\deg G(x) = g$, $\chi_{\bar{A}}(x) =$ Denote by $N_k(G)$ the $gk \times gk$ matrix of the form

$$N_k(G) = \begin{pmatrix} S(G) & E & 0 & \dots & 0 \\ & & & & \dots \\ 0 & S(G) & E & \dots & E \\ \dots & & \dots & & \\ 0 & \dots & & & S(G) \end{pmatrix}, \quad E = E_{g\times g},$$

a generalized Jordan block. Then there exist $n_1, \dots, n_t$ such that

$$\bar{A} \approx \mathrm{Diag}(N_{n_1}(\bar{G}), \dots, N_{n_t}(\bar{G})),$$

s of generality we may assume that

$$\bar{A} = \mathrm{Diag}(N_{n_1}(\bar{G}), \dots, N_{n_t}(\bar{G})), \text{ where } n_1 \leqslant n_2 \leqslant \dots \leqslant n_t. \tag{26}$$

$$N = \mathrm{Diag}(N_{n_1}(G), \dots, N_{n_t}(G)) \in R_m. \tag{27}$$

have

$$A = N + V, \text{ where } V \in J_m. \tag{28}$$

matrix C with elements from the ideal (0:J) and put

$$D = \mathrm{Diag}(C, 0, \dots, 0)_{m\times m}, \quad B = A + D. \tag{29}$$

sume that $F(x) \in R[x]$ is a monic polynomial such that $\bar{F}(x) = \bar{G}(x)^q$, $q > n_1$, $F(B) = 0$.

e polynomial $F(x)$ is of the form $F(x) = G(x)^q + H(x)$, where $H(x) \in J[x]$, there- e equations $H(B) = H(A)$ and $F(B) = G(B)^q + H(A)$. It remains to show that

definition of the matrix D and Eqs. (27)-(29) it is easy to show that for every

$$B^i = (A+D)^i = A^i + \sum_{s=0}^{i-1} A^s D A^{i-1-s} = A^i + \sum_{s=0}^{i-1} N^s D N^{i-1-s} = A^i + D_i. \tag{30}$$

$_i$ the entries on the intersection of the first g rows with the first $n_1 g$ to the ideal (0:J), and the remaining entries are zero. On the other hand ee that in the matrix $G(B)$ the entries in the first g columns lie in the ideal e have $G(B)(A + D)^i = G(B)A^i$ for $i \geq 1$. It is clear, therefore, that

$$G(B)^q = G(B)\,G(A)^{q-1}. \tag{31}$$

$\geq n_1$ all elements in the first $n_1 g$ columns of the matrix $G(A)^{q-1}$ lie in the) and by the above mentioned properties of the matrices D_i it follows that $(A)^{q-1}$ for $i \geq 1$. Therefore $G(B)G(A)^{q-1} = G(A)^q$ and it now follows from (31) $A)^q$. This concludes the proof of the Lemma.

ose the matrix C in (29) so that B is not similar to A. For example, we might with nonzero trace. Then the trace of B is different from the trace of A is clear that we still have $\bar{B} \approx \bar{A}$.

to show that $\mathrm{Ann}(B) = \mathrm{Ann}(A)$. Since $\mathrm{Ann}(B) \cap J[x] = \mathrm{Ann}(A) \cap J[x]$ it suf- hat if (24) holds then $F(B) = 0$. This follows from the preceding Lemma. In- s from (23) that if $\chi_{\bar{A}}(x) = \bar{G}(x)^n$ the minimal polynomial $F(x)$ of A satisfies r some $q \in \mathbb{N}$. If A does not satisfy condition (a) the degree of $F(x)$ is rger than the degree of a minimal polynomial of the matrix A, and thus by (26) oes not satisfy condition (b) then $n_1 < n_t$ and again $q > n_1$ since $\bar{F}(x)$ is e minimal polynomial of $\bar{A}$ which equals $\bar{G}(x)^{n_t}$. Thus the conditions of the fied for $F(x)$, and $F(B) = 0$. This concludes the proof of Theorem 8.

t in the introduction we called a polynomial $F(x) \in R[x]$ a strong invariant matrices A, B in the class $\mathscr{V}(F, R_m)$ of all matrices $A \in R_m$ satisfying $F(A) =$ $A \approx B$ is equivalent to $\bar{A} \approx \bar{B}$.