s conjecture is confirmed in the sequel in special cases (Theorem 6 and the results 4).

OREM 6. A normal matrix $A \in R_m$ is canonically determined if and only if all the Fi ariants of the matrix $(xE - A)$ are principal ideals.

of. In view of Corollary 2 to Theorem 5, it suffices to consider the situation when ormal matrix and not all the ideals $\mathcal{D}_s(xE-A)$ are principal, and to show that in e there exists a matrix $B \in R_m$ which is not similar to A but which has the same Fit ariants. In view of Theorem 2 it suffices to do this for the case that the polynomi primary, i.e., there exists an absolutely irreducible polynomial $G(x) \in R[x]$ such $x) = \bar{G}(x)^r$. Then all nonidentity ideals $\mathcal{D}_s(xE-A)$ are primary ideals, contained aximal ideal $P = (G(x), J(R))$; this will be assumed throughout in the sequel.

MA. Assume that $\mathcal{K} = \mathrm{Diag}(K_1(x), K_2(x))$ is a quasicanonical matrix and $\mathcal{D}_1(\mathcal{K})$ is a ipal primary ideal contained in P. Then there exists a quasicanonical matrix $\mathcal{K}'(x)$ orm $\mathcal{K}' = \begin{pmatrix} K_1(x) & L(x) \\ N(x) & K_2(x) \end{pmatrix}$, which has the same Fitting invariants as $\mathcal{K}$ and such that for , $V(x) \in R[x]_2$ the condition $U\mathcal{K}' = \mathcal{K}V$ implies $|U(x)| \in P$.

of. Divide $K_2(x)$ by $K_1(x)$ with remainder:

$$K_2(x) = Q_1(x)K_1(x) + L(x), \quad \deg L(x) < \deg K_1(x). \tag{14}$$

$L(x) \neq 0$ because $\mathcal{D}_1(\mathcal{K})$ is nonprincipal, but $\bar{L}(x) = \bar{0}$ because $\bar{K}_1(x) \mid \bar{K}_2(x)$. Since x] and $J^n = 0$ one can find an element $\pi \in J$ with the properties $\pi L(x) = N(x) \neq 0$, $= 0$. We will show that for given $L(x)$ and $N(x)$ the quasicanonical matrix $\mathcal{K}'(x)$ in the statement of the Lemma has the required properties.

s obvious that $L(x), N(x) \in \mathcal{D}_1(\mathcal{K})$, and thus $\mathcal{D}_1(\mathcal{K}) = \mathcal{D}_1(\mathcal{K}')$; $N(x)L(x) = 0$ implies $) = \mathcal{D}_2(\mathcal{K}') = (K_1(x) \cdot K_2(x))$. Assume that there exist matrices $U(x), V(x) \in R[x]_2$ such $\mathcal{K}'(x) = \mathcal{K}(x)V(x)$; this implies in particular the following equations:

$$U_{11}(x)K_1(x) + U_{12}(x)N(x) = K_1(x)V_{11}(x), \tag{15}$$
$$U_{11}(x)L(x) + U_{12}(x)K_2(x) = K_1(x)V_{12}(x). \tag{16}$$

we obtain that $K_1(x) \mid U_{12}(x) \cdot N(x)$, and since $\deg N(x) < \deg K_1(x)$, we have $(U_{12}(x)$, (e), i.e.,

$$\bar{G}(x) \mid \bar{U}_{12}(x). \tag{17}$$

together with (14) we obtain $(U_{11}(x) + U_{12}(x))L(x) = K_1(x)(V_{12}(x) - U_{12}(x)Q(x))$. $) \mid (U_{11}(x) + U_{12}(x)) \cdot L(x)$, and since $\deg L(x) < \deg K_1(x)$ we have $(U_{11}(x) + U_{12}(x)$, (e). Since the polynomial $K_1(x)$ is primary, it follows from (17) and the last re $\bar{G}(x) \mid \bar{U}_{11}(x)$. In conjunction with (17) it follows that $|U(x)| \in P$. This concludes of the Lemma.

me now that the matrix $xE - A$ is equivalent to a diagonal quasicanonical matrix $K_1(x), \ldots, K_m(x))$, where $|\mathcal{K}(x)|$ is a primary polynomial from P. Since $\bar{K}_i \mid \bar{K}_{i+1}$ ll the ideals $\mathcal{D}_s(\mathcal{K})$ are principal it follows from Theorem 5 that there exists an at $K_i(x) \nmid K_{i+1}(x)$. Then we have:

$$K_{i+1}(x) = Q(x)K_i(x) + L(x), \quad \deg L(x) < \deg K_i(x), \quad L \neq 0, \quad \bar{L} = \bar{0}. \tag{18}$$

polynomial $N(x) = \pi L(x)$ as in the Lemma and consider the matrix

$$\mathcal{K}' = \mathrm{Diag}\left(K_1, \ldots, K_{i-1}, \begin{pmatrix} K_i & L \\ N & K_{i+1} \end{pmatrix}, K_{i+2}, \ldots, K_m\right).$$

4 there exists a matrix $B \in R_m$ such that $xE - B \sim \mathcal{K}'(x)$. We will show that for matrices A and B Eqs. (13) hold, but $A \not\approx B$.

easy to see that the ideal $\mathcal{D}_s(xE-B) = \mathcal{D}_s(\mathcal{K}')$ is obtained from a system of gene he ideal $\mathcal{D}_s(xE-A) = \mathcal{D}_s(\mathcal{K})$ by multiplications of the form $K_{i_1} \cdot \ldots \cdot K_{i_{s-1}} \cdot L$, where $\{i_1, \ldots, i_{s-1}\}$. But in view of (18) all such products lie in $\mathcal{D}_s(\mathcal{K})$. Hence $_s(\mathcal{K})$ and Eqs. (13) hold.

e that $A \approx B$. Then it follows from Theorems 1 and 3 that there exists a module m $\mathfrak{M}(\mathcal{K}) \approx \mathfrak{M}(\mathcal{K}')$. In view of the decompositions