The elements of the resulting matrix B' are already independent of the choice of the elemen u, v, its nonzero parameters $b_{32}' = b_{32} = \beta$, $b_{11}' = b_{22} = \gamma$, $b_{33}' = \delta$ can be varied independently with only one restriction: $\beta \neq 0$, and they are uniquely determined by the original mat B. This is described in row III of Table 1.

c) $\bar{b}_{32} = 0$, $\bar{b}_{31} \neq \bar{0}$. By choosing $u, v \in R^*$ we can in this case get $b_{31}' = uv^{-1}b_{31} = e$ and if we then choose $y, z \in R$ we can obtain that $\varphi = u^{-1}b_{12}z - v^{-1}b_{31}y = -b_{11}$. In this case $b_{11}' = b_{22}' = 0$ and the elements $b_{12}' = u^{-1}vb_{12} = b_{31}b_{12} = \beta$, $b_{33}' = b_{33} + b_{22} + b_{11} = \alpha$ are uniquely determined by the matrix B and can take arbitrary values from R (row IV in Table 1)

d) $\bar{b}_{32} = \bar{b}_{31} = \bar{0}$, $\bar{b}_{12} \neq \bar{0}$. Similar reasoning as in part c) shows that the matrix B' can be written in the form

$$B' = \begin{pmatrix} 0 & e & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha \end{pmatrix}$$

(row V in Table 1).

It remains to note that by (42), (44) it is impossible to transform any matrix $\pi \cdot B$ of the form (45) which satisfies one of the conditions a)-d) by means of a transformation (43) into another matrix $\pi \cdot B'$ of the same form (45) which satisfies a different one of these conditions.

(iii) deg $F_1(x) = 1$. Then there exists $r \in R$ such that

$$\bar{F}_1(x) = x - \bar{r}, \quad A = rE + \pi \cdot B \text{ and } \bar{F}_2(x) = (x - \bar{r})^2.$$

It is clear that the matrix $A' = rE + \pi B'$ is similar to A if and only if the matrices $\bar{B}$ and $\bar{B}'$ are similar over the field $\bar{R}$. In this case then the class of matrices which are conjugate to A is completely determined by the minimal polynomial $f(x) \in \bar{R}[x]$ of the matrix $\bar{B}$.

a) If deg $f(x) = 1$, then $f(x) = x - \bar{b}$, $b \in R$, and $\pi \cdot B = \pi \cdot b \cdot E$, $A = \rho E$, where $\rho = r + \pi \cdot b$ (row VI of Table 1).

b) If $f(x) = (x - \bar{\beta}) \cdot (x - \bar{\gamma})$, $\bar{\beta} \neq \bar{\gamma}$, then

$$\bar{B} \approx \mathrm{Diag}(\bar{\beta}, \bar{\beta}, \bar{\gamma}), \; A \approx \mathrm{Diag}(\rho + \pi\beta, \rho + \pi\beta, \rho + \pi\gamma),$$

where $\rho = r$ (row VII of Table 1).

c) If $f(x) = (x - \bar{\beta})^2$, then $\bar{B} \approx \mathrm{Diag}\left(\bar{\beta}, \begin{pmatrix} \bar{\beta} & \bar{e} \\ 0 & \bar{\beta} \end{pmatrix}\right)$,

$$A \approx \mathrm{Diag}\left(\rho + \pi\beta, \begin{pmatrix} \rho + \pi\beta & \pi \\ 0 & \rho + \pi\beta \end{pmatrix}\right),$$

$\rho = r$ (row VIII of Table 1).

d) If $f(x) = x^3 - \bar{\alpha}x^2 - \bar{\beta}x - \bar{\gamma} = \chi_{\bar{B}}(x)$, then $A \approx \rho E + \pi S(\chi_B)$, $\rho = r$ (row IX of Table 1) The second statement of Theorem 11 is established, and the first statement follows immediate from an inspection of Table 1.

Remark I. This Theorem shows that sometimes one can decide whether a matrix $A \in R_m$ is normal from its Fitting invariants, even in the case when they are not principal, although in this case Theorem 6 shows that A is not canonically determined. Indeed, one can say that if under the assumptions of Theorem 11 the Fitting invariants of a matrix are of the form $\mathcal{D}_1(xE - A) = (e)$, $\mathcal{D}_2(xE - A) = (x - \rho, \pi)$, $\chi_A(x) = (x - \rho)^3 - \pi\alpha(x - \rho)^2 - \pi\beta(x - \rho)$ and $\bar{\beta} \neq \bar{0}$ then the matrix A is normal and $A \approx \mathrm{Diag}(\rho + \pi\gamma, S(G(x)))$, where $G(x) = (x - \rho)^2 - \pi(\gamma + \delta)(x - \rho)$ $\pi\beta$, $2\gamma + \delta = \alpha$ (row III of Table 1). It would be interesting to obtain descriptions of such situations for wider classes of matrices.

Remark II. If one considers all the possibilities listed in the table accompanying Theorem 11 one sees easily that for matrices $A \in R_3$ (assuming that $\pi^2 = 0$) the following hold:

$$(\mathcal{D}_{s+1}(xE - A) : \mathcal{D}_s(xE - A)) \supseteq (\mathcal{D}_{s+2}(xE - A) : \mathcal{D}_{s+1}(xE - A)).$$

These are analogs of the well-known relations between invariant factors of a matrix over a principal ideal ring. The following question remains open: do these relations hold for $A \in R_m$ in the case $m \geq 2$ and for an arbitrary commutative ring R?