Proof. Assume that (a) holds and put $\mathscr{K}_0(x)=\mathrm{Diag}(K_1(x), \dots, K_m(x))$. Let $\mathscr{K}(x)$ be any canonical matrix which is equivalent to xE − A and has properties (4)-(8). Then $\sim\mathscr{K}_0(x)$ and we have $\overline{\mathscr{K}}(x)=\overline{\mathscr{K}}_0(x)$ and $\mathscr{D}_s(\mathscr{K})=\mathscr{D}_s(\mathscr{K}_0)=(K_1(x)\cdot \dots \cdot K_s(x))$ for $s=\overline{1, m}$. we obtain successively for $s=\overline{1, m}$ the equations $K_{ss}(x) = K_s(x)$, $K_{si}(x) = K_{is}(x) = 0$ $\neq s$, i.e., $\mathscr{K}=\mathscr{K}_0$. This proves the implication (a) ⇒ (b).

Assume that (b) holds and assume that the quasicanonical matrix which is equivalent to A has the form (4). We show first of all that it is a diagonal matrix. Assume the con-. Then we have for some $i \geq 1$

$$\mathscr{K}=\mathrm{Diag}\left(K_{11}(x), \dots, K_{i-1\, i-1}(x), \begin{pmatrix} K_{ii}(x) & \dots & K_{im}(x) \\ \dots & & \dots \\ K_{mi}(x) & \dots & K_{mm}(x)\end{pmatrix}\right),$$

or some $j > i$ either $K_{ij}(x) \neq 0$ or $K_{ji}(x) \neq 0$. Assume that $K_{ij}(x) \neq 0$. Then it follows (7) and (8) that $K_{ii}(x) + K_{ij}(x) = K'_{ii}(x)$, and if we add column j of $\mathscr{K}$ to column i we n

$$\mathscr{K}\sim\mathscr{K}'=\mathrm{Diag}\left(K_{11}, \dots, K_{i-1\, i-1}, \begin{pmatrix} K'_{ii} & K_{ii+1} & \dots & K_{im} \\ \dots & \dots & & \dots \\ K'_{mi} & K_{mi+1} & & K_{mm}\end{pmatrix}\right),$$

the matrix $\mathscr{K}'$ has properties (4)-(7). Using the remark following the proof of the part of Theorem 4, we can say that the matrix $\mathscr{K}'$ is equivalent to a quasicanonical x $\mathscr{K}''$ of the form

$$\mathscr{K}''=\mathrm{Diag}\left(K_{11}, \dots, K_{i-1\, i-1}, \begin{pmatrix} K'_{ii} & K''_{ii+1} & \dots & K''_{im} \\ \dots & & & \\ K''_{mi} & & \dots & K''_{mm}\end{pmatrix}\right),$$

is not equal to $\mathscr{K}$, since $K'_{ii} \neq K_{ii}$. Consequently, $\mathscr{K}$ is a diagonal matrix: $\mathscr{K}=\mathrm{Diag}$ $\dots, K_m)$. If then there exists an $i < m$ such that $K_{i+1}(x) = K_i(x)Q(x) + K_{ii+1}(x)$, where $_{ii+1}(x) <$ deg $K_i(x)$ and $K_{ii+1}(x) \neq 0$, one sees without difficulty that in view of (5) $(x) = \overline{0}$ and the matrix $\mathscr{K}$ is equivalent to a quasicanonical matrix different from $\mathscr{K}$,

$$\mathscr{K}'=\mathrm{Diag}\left(K_1, \dots, K_{i-1}, \begin{pmatrix} K_i & K_{ii+1} \\ 0 & K_{i+1}\end{pmatrix}, K_{i+2}, \dots, K_m\right).$$

efore, each element on the main diagonal of $\mathscr{K}$ is divisible by the preceding one and $\mathscr{D}_s(xE$ $=\mathscr{D}_s(\mathscr{K})=(K_1\cdot\ldots\cdot K_s)$ is a principal ideal for s = 1, m. This proves the implication ⇒ (c).

Finally, assume that (c) holds and let (4) by any quasicanonical matrix for xE − A. we have $\overline{\mathscr{D}_1(xE-A)}=(\bar{K}_{11}(x))$ and $K_{11}(x)\in\mathscr{D}_1(xE-A)$, hence $\mathscr{D}_1(xE-A)=(K_{11}(x))$. Conse-ly all elements of the matrix $\mathscr{K}(x)$ are divisible by $K_{11}(x)$ and $K_{1i}(x) = K_{i1}(x) = 0$ for , m. In this case $K_{11}(x)\cdot K_{22}(x)\in\mathscr{D}_2(xE-A)$ holds and from $\overline{\mathscr{D}_2(xE-A)}=(\bar{K}_{11}\cdot\bar{K}_{22})$ we deduce $E-A)=(K_{11}\cdot K_{22})$. It follows that all elements $K_{11}(x)K_{ij}(x)$ for i, j ≥ 2 are divisible $_{1}(x)K_{22}(x)$, i.e., $K_{22}|K_{ij}$ and $K_{2i} = K_{i2} = 0$ for $i = \overline{3, m}$. Continuing in this fashion we n $\mathscr{K}=\mathrm{Diag}(K_{11}, \dots, K_{mm})$ and $K_{ii}|K_{i+1\, i+1}$ for $i = \overline{1, m-1}$, i.e., (c) ⇒ (a). This con-s the proof of Theorem 5.

We shall say that a matrix $A \in R_m$ is canonically determined if whenever $B \in R_m$ satisfies then $B \approx A$.

COROLLARY 1. If $A \in R_m$ and the minimal polynomial of the matrix $\bar{A} \in \bar{R}_m$ coincides with haracteristic polynomial then $A \approx S(\chi_A(x))$.

The proof follows from the equations $\mathscr{D}_m(xE-A)=(\chi_A(x))$, $\mathscr{D}_{m-1}(xE-A)=\dots=\mathscr{D}_1(xE-A)=(e)$.

COROLLARY 2. If $A \in R_m$ and all Fitting invariants of the matrix (xE − A) are principal ls then A is canonically determined.

All examples studied by the author suggest that the converse of Corollary 2 holds also,

Conjecture. The matrix $A \in R_m$ is canonically determined if and only if all the Fitting riants of the matrix xE − A are principal ideals.