1. Let $A \in R_3$, $J(R) = \pi R$, $\pi^2 = 0$. Then the matrix A is canonically deter-
nly if all Fitting invariants of the matrix $xE - A$ are principal ideals. The
for invariants of the matrix $xE - A$ and their corresponding canonical matrices
re listed in the accompanying Table 1.

e have

$$\mathscr{D}_s(xE-A) = F_s(x) + \mathscr{D}_s(xE-A) \cap J[x], \quad s=1, 2, 3,$$

a monic polynomial. Then $\mathscr{D}_S(x\bar{E} - \bar{A}) = (\bar{F}_S(x))$ and $F_3(x) = \chi_A(x)$, $\deg F_S(x) \leq$
. We consider all possibilities.

$_1(x) = 0$, $\deg F_2(x) = 0$. Then $\mathscr{D}_1(xE-A) = \mathscr{D}_2(xE-A) = (e)$ and by Corollary 1 of
$S(\chi_A)$ (row I of Table 1).

$F_1(x) = 0$, $\deg F_2(x) = 1$. Then $F_2(x) = x - r$ for suitable $r \in R$ and the matrix
o $\mathrm{Diag}\left(\bar{r}, \begin{pmatrix} \bar{r} & \bar{e} \\ \bar{0} & \bar{r} \end{pmatrix}\right)$. Put $C(r) = \mathrm{Diag}\left(r, \begin{pmatrix} r & e \\ 0 & r \end{pmatrix}\right)$. Without loss of generality, we
e that

$$A = C(r) + \pi B, \quad B = (b_{ij}). \tag{39}$$

select from among the matrices similar to (39) of the form

$$A' = C(r) + \pi B', \quad B' = (b'_{ij}), \tag{40}$$

nich B' contains the smallest possible number of distinct elements.
y to verify that if the matrices (39) and (4) satisfy

$$A' = T^{-1}AT, \text{ where } T \in R_3^*, \tag{41}$$

formation matrix T is of the form

$$T = U \cdot (E + \pi K), \text{ where } U = \begin{pmatrix} u & 0 & y \\ z & u & t \\ 0 & 0 & v \end{pmatrix}, K = (k_{ij}) \in R_3, \tag{42}$$

, $t \in R$. Here $U^{-1}C(r)U = C(r)$ and the matrix $\pi B'$ satisfies

$$\pi B' = \pi(U^{-1}BU + C(r)K - KC(r)). \tag{43}$$

ave

$$C(r)K - KC(r) = \begin{pmatrix} 0 & 0 & -k_{12} \\ k_{31} & k_{32} & k_{33} - k_{22} \\ 0 & 0 & -k_{32} \end{pmatrix}, \tag{44}$$

elements k_{ij} in the matrix K can be chosen arbitrarily, we can choose the ele-
, $k_{33} - k_{22}$, k_{32} in accordance with U and B and thus can obtain that $b'_{13} =$
0, $b'_{11} = b'_{22}$. By (44) the remaining elements of the matrix K do not in-
rm of the matrix B'. If we assume that the elements of the matrix K are
way and making use of (39)-(43) it is easy to verify that

$$\pi B' = \pi \begin{pmatrix} b_{11} + \varphi & u^{-1}vb_{12} - u^{-1}b_{32}y & 0 \\ 0 & b_{11} + \varphi & 0 \\ uv^{-1}b_{31} + v^{-1}b_{32}z & b_{32} & b_{33} + b_{22} - b_{11} - 2\varphi \end{pmatrix}, \tag{45}$$

$b_{31}y + u^{-1}b_{12}z - u^{-1}v^{-1}b_{32}yz$. Now we consider different particular cases ac-
values of the parameters in the original matrix B. We note at once that the
do not depend on t.

$\bar{b}_{31} = \bar{b}_{12} = \bar{0}$. In this case we have $\pi B' = \pi\,\mathrm{Diag}(b_{11}, b_{11}, b_{33} + b_{22} - b_{11})$,
oes not depend on the choice of the matrix U, and if we put $\rho = r + \pi b_{11}$, $\alpha =$
$_1$ we have the situation described in row II of Table 1.

$\bar{0}$. In this case $b_{32} \in R^*$ and if we put $y = vb_{12}b_{32}^{-1}$, $z = -ub_{31}b_{32}^{-1}$ we find

$$b_{12}' = b_{31}' = 0, \quad b_{11}' = b_{22}' = b_{11} - b_{31}b_{12}b_{32}^{-1},$$
$$b_{33}' = b_{33} + b_{22} - b_{11} + 2b_{31}b_{12}b_{32}^{-1}.$$