ultiplied by x. Then we perform similar operations on columns m, m − 1, ..., $m_1 + 1$. In he end we have

$$(xE-A)\sim\left(\begin{array}{cccc|cccc}
0 & -e & \dots & 0 & 0 & & \dots & 0\\
\dots & & & \dots & \dots & & & \dots\\
0 & \dots & & -e & 0 & & \dots & 0\\
K_{11}(x) & * & \dots & * & K_{12}(x) & * & & *\\
\hline
0 & \dots & & 0 & 0 & -e & \dots & 0\\
\dots & & & \dots & \dots & & & \dots\\
0 & \dots & & 0 & 0 & \dots & & -e\\
K_{21}(x) & * & \dots & * & K_{22}(x) & * & & *
\end{array}\right).$$

nen we operate on rows 1, 2, ..., $m_1 - 1$ and $m_1 + 1$, ..., m − 1 in such a way that the reılting matrix has zeros in all places labeled *. Finally we permute the rows and columns nd obtain the matrix

$$\mathrm{Diag}\left(e, \dots, e, \begin{pmatrix} K_{11}(x) & K_{12}(x) \\ K_{21}(x) & K_{22}(x)\end{pmatrix}\right)_{m\times m}.$$

is concludes the proof of Theorem 4.

We shall call a matrix (4) satisfying properties (5)-(8) a quasicanonical form of the trix xE − A. It has already been pointed out that for matrices in R_m a quasicanonical for not unique. For example, for the matrix $A=\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ over $R=\mathbf{Z}/4$ the following are quasinonical forms of the matrix xE − A:

$$\begin{pmatrix}x-1 & 0\\ 0 & x-3\end{pmatrix},\begin{pmatrix}x-1 & 2\\ 0 & x-3\end{pmatrix},\begin{pmatrix}x-1 & 0\\ 2 & x-3\end{pmatrix},\begin{pmatrix}x-3 & 2\\ 2 & x-1\end{pmatrix},\begin{pmatrix}x-3 & 0\\ 0 & x-1\end{pmatrix},\dots$$

the following section we derive conditions which are equivalent to the uniqueness of the asicanonical form for xE − A.

We note that the condition that A is normal is obviously equivalent to the condition at xE − A has a diagonal quasicanonical form. However, as one can see from the above ample, even in this case the matrix (4) is nonunique. Nonetheless, as we shall see below, eorem 4 is sometimes useful in the solution of diverse questions connected with the similty problem of matrices.

Fitting Invariants and Canonically Determined Matrices

For a matrix $\mathfrak{A}(x)\in R[x]_m$ we define the s-th Fitting invariant to be the ideal $_s(xE-A)$ generated in R[x] by all minors of order s of the matrix $\mathfrak{A}(x)$, $s=\overline{1,\ m}$. These variants were introduced in [13] for finitely generated modules over a ring (in our case e invariant $\mathfrak{D}_s(\mathfrak{A}(x))$ corresponds to the (m − s)-th invariant from [13] for the R[x]-dule $\mathfrak{M}(\mathfrak{A}(x))$).

It is well known that for matrices over a field the system of Fitting invariants of the sociated characteristic matrix determines the similarity class of the original matrix. For trices in R_m this is not the case. We can only say that if the matrices A, B ∈ R_m are nilar then

$$\mathfrak{D}_s(xE-A)=\mathfrak{D}_s(xE-B),\quad s=\overline{1,m}. \tag{13}$$

s follows, for example, from Theorems 2 and 3 and the results of [13, Sec. 1]. At the e time, the Fitting invariants of the matrix A ∈ R_m contain a fair amount of information out the matrix A ∈ R_m: sometimes it is possible to use them to establish that the matrix s normal (cf. Sec. 4, Remark 1 to Theorem 11), and sometimes they completely determine the nilarity class of A.

THEOREM 5. For a matrix A ∈ R_m the following statements are equivalent:

(a) (xE − A) ~ Diag($K_1(x)$, ..., $K_m(x)$), where $K_1(x)$, ..., $K_m(x)$ are monic polynomials $K_i(x)|K_{i+1}(x)$ for $i=\overline{1,\ m-1}$.

(b) There exists a unique quasicanonical matrix which is equivalent to xE − A.

(c) All the Fitting invariants of the matrix xE − A are principal ideals.