Supplementary Material

DESCRIPTION: This Supplementary Material includes the full matrix details for Simulations in the paper "Bipartite synchronization for multi-level networks with antagonistic interactions".

I. Matricies in Simulation 1

The nonidentical mho-transformed intra-layer coupling matrices are defined as:

$$
\begin{aligned}
(H^{\underline{1}})^1 &= \begin{pmatrix} -7 & 4 & -3 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & -2 \end{pmatrix}, (H^{\underline{1}})^2 = \begin{pmatrix} 1 & -2 & 0 & -1 \\ 2 & -6 & -4 & 0 \\ 0 & 0 & -3 & 3 \\ -3 & 0 & 0 & -3 \end{pmatrix}, \\
(H^{\underline{2}})^1 &= \begin{pmatrix} -3 & -2 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -4 & -4 \\ -1 & 3 & 0 & -4 \end{pmatrix}, (H^{\underline{2}})^2 = \begin{pmatrix} -2 & 0 & 2 & 0 \\ -2 & -6 & -4 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\
(H^{\underline{3}})^1 &= \begin{pmatrix} -5 & 0 & -4 & 1 \\ -4 & -8 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & -2 & -3 \end{pmatrix}, (H^{\underline{3}})^2 = \begin{pmatrix} -2 & -2 & 0 & 0 \\ 2 & 3 & -2 & -1 \\ 0 & 0 & -5 & -5 \\ 1 & 0 & 0 & -1 \end{pmatrix},
\end{aligned}
\tag{1}
$$

the mho-transformed inter-layer coupling matrices G^{w_2} are defined as:

$$
G^1 = \left(\begin{array}{cccc|cccc|cccc}
-2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
\hline
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),
$$

$$
G^2 = \left(\begin{array}{cccc|cccc|cccc}
-1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\
\hline
2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),
$$

$$
G^3 = \left(\begin{array}{cccc|cccc|cccc}
-3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 \\
\hline
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & -4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & -4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),
$$

and the mho-transformed inter-block coupling matrices F^{w_3} are defined as:

$$F^1 = \begin{pmatrix} -I_{NL\times NL} & I_{NL\times NL} \\ I_{NL\times NL} & -I_{NL\times NL} \end{pmatrix}, F^2 = \begin{pmatrix} -2I_{NL\times NL} & 2I_{NL\times NL} \\ 4I_{NL\times NL} & -4I_{NL\times NL} \end{pmatrix}.$$

After using gauge transformation $\mathcal{T} = \text{diag}(1, 1, -1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, -1, 1, -1, 1, -1, -1, 1)$, the gauge-transformed intra-layer coupling matrices $\tilde{\mathbb{H}}^{w_1} = \mathcal{T}(I_2 \otimes \text{diag}((H^{\underline{1}})^{w_1}, (H^{\underline{2}})^{w_1}, (H^{\underline{3}})^{w_1}))\mathcal{T}$ are

$$\tilde{\mathbb{H}}^1 = \left(\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc}
-7 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & -8 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & -8 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & -3
\end{array}\right),$$

$$\tilde{\mathbb{H}}^2 = \left(\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc}
1 & -2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & -6 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & -6 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 3 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 & 4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 3 & -2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1
\end{array}\right),$$

the gauge-transformed inter-layer coupling matrices $\tilde{\mathbb{G}}^{w_2} = \mathcal{T}(I_2 \otimes G^{w_2})\mathcal{T}$ are

$$\tilde{\mathbb{G}}^1 = \left(\begin{array}{cccccccccccc|cccccccccccc}
-2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),$$

$$\tilde{\mathbb{G}}^2 = \left(\begin{array}{cccccccccccc|cccccccccccc}
-1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),$$

$$\tilde{\mathbb{G}}^3 = \left(\begin{array}{cccccccccccc|cccccccccccc}
-3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),$$

the gauge-transformed inter-block coupling matrices $\tilde{\mathbb{F}}^{w_3} = \mathcal{T}F^{w_3}\mathcal{T}$ are

$$\tilde{\mathbb{F}}^1 = \begin{pmatrix} -I_{NL\times NL} & I_{NL\times NL} \\ I_{NL\times NL} & -I_{NL\times NL} \end{pmatrix}, \tilde{\mathbb{F}}^2 = \begin{pmatrix} -2I_{NL\times NL} & 2I_{NL\times NL} \\ 4I_{NL\times NL} & -4I_{NL\times NL} \end{pmatrix}.$$

We can verify that there also exist negative elements in the non-diagonal parts of $\tilde{\mathbb{H}}^{w_1}$ and $\tilde{\mathbb{G}}^{w_2}$, which means they are all structurally unbalanced. Then, by using formula (7), we can have intra-layer union matrices $\tilde{\mathbb{H}}^{[d]}$:

$$\tilde{\mathbb{H}}^{[1]} = \left(\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc}
-6 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & -7 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -5 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 2 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -5 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -7 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -6 & 5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 2 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -5 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -6 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -4
\end{array}\right),$$

$$\tilde{\mathbb{H}}^{[2]} = \left(\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc}
-6 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & -7 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -5 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 2 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -5 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -7 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -6 & 5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 2 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -5 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -6 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -4
\end{array}\right),$$

$$\tilde{\mathbb{H}}^{[3]} = \left(\begin{array}{cccc|cccc|cccc|cccc|cccc|cccc}
-5 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & -13 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 1 & 1 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -7 & 2 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & -12 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 & 4 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -11 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & -13 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 1 & 1 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 2 & 5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -12 & 9 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & -4 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 & 4 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -11 & 10 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & -5
\end{array}\right),$$

inter-layer union matrices $\tilde{\mathbb{G}}^{[d]}$:

$$\tilde{\mathbb{G}}^{[1]}=\left(\begin{array}{cccccccccccc|cccccccccccc}
-11 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -7 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -8 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -9 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 0 & -10 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -11 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -10 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -10 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & -6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),$$

$$\tilde{\mathbb{G}}^{[2]}=\left(\begin{array}{cccccccccccc|cccccccccccc}
-10 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -6 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -6 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 0 & -7 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -10 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -7 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),$$

$$\tilde{\mathbb{G}}^{[3]}=\left(\begin{array}{cccccccccccc|cccccccccccc}
-9 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -6 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -5 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right),$$

inter-block union matrices $\tilde{\mathbb{F}}^{[d]}$:

$$\tilde{\mathbb{F}}^{[1]}=\begin{pmatrix} -5I_{NL\times NL} & 5I_{NL\times NL} \\ 9I_{NL\times NL} & -9I_{NL\times NL} \end{pmatrix}, \tilde{\mathbb{F}}^{[2]}=\begin{pmatrix} -4I_{NL\times NL} & 4I_{NL\times NL} \\ 6I_{NL\times NL} & -6I_{NL\times NL} \end{pmatrix}, \tilde{\mathbb{F}}^{[3]}=\begin{pmatrix} -3I_{NL\times NL} & 3I_{NL\times NL} \\ 5I_{NL\times NL} & -5I_{NL\times NL} \end{pmatrix}.$$

It is obvious that all non-diagonal elements of $\tilde{\mathbb{H}}^{[d]}$, $\tilde{\mathbb{G}}^{[d]}$, $\tilde{\mathbb{F}}^{[d]}$ are nonnegative, and every matrices is zero-row-sum. Thus, set the coupling strengths $\rho_1 = \rho_2 = \rho_3 = 1$, the $\tilde{\mathcal{N}}^{[d]} = \rho_1 \tilde{\mathbb{H}}^{[d]} + \rho_2 \tilde{\mathbb{G}}^{[d]} + \rho_3 \tilde{\mathbb{F}}^{[d]}$ defined in (8) are:

$$\tilde{\mathcal{N}}^{[1]} = \left(\begin{array}{cccccccccccccccccccccccc}
-22 & 2 & 3 & 1 & 6 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & -19 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -15 & 2 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & -19 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & -15 & 2 & 3 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 1 & -21 & 5 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -14 & 5 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 3 & 0 & -13 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -12 & 2 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 0 & 4 & 0 & 0 & 2 & -20 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 \\
0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 1 & -17 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\
9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -26 & 2 & 3 & 1 & 6 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -23 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -19 & 2 & 0 & 0 & 4 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 & -23 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -19 & 2 & 3 & 0 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 1 & -25 & 5 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -18 & 5 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & -17 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 2 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 4 & 0 & 0 & 2 & -24 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 1 & -21 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -13
\end{array}\right)$$

$$
\tilde{\mathcal{N}}^{[2]} = \begin{pmatrix}
-20 & 2 & 3 & 1 & 3 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & -15 & 4 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -12 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & -15 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & -14 & 2 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 1 & -17 & 5 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -12 & 5 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 3 & 0 & -11 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -11 & 2 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -14 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & -14 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -22 & 2 & 3 & 1 & 3 & 0 & 0 & 0 & 7 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -17 & 4 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 & -17 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -16 & 2 & 3 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 1 & -19 & 5 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & 5 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & -13 & 0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -13 & 2 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -16 & 2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & -16 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -10
\end{pmatrix}
$$

$$
\tilde{\mathcal{N}}^{[3]} = \left(\begin{array}{cccccccccccccccccccccccc}
-17 & 0 & 3 & 2 & 1 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & -20 & 8 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -14 & 5 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 1 & 1 & -16 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & -15 & 2 & 5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & -18 & 9 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -12 & 6 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 3 & 0 & -10 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -12 & 4 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -10 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -14 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -19 & 0 & 3 & 2 & 1 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & -22 & 8 & 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 5 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 1 & 1 & -18 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -17 & 2 & 5 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -20 & 9 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & 6 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & -12 & 0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & 4 & 4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & -12 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -16 & 10 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & -10
\end{array}\right)
$$

It can be seen that $\tilde{\mathcal{N}}^{[d]}$ are Metzler and irreducible, $d = 1, 2, 3$, hence Assumption 2 holds. Then, according to Lemma 1, the NLEs for $\tilde{\mathcal{N}}^{[d]}$ are:

$$\begin{aligned}
\xi^{[1]} =&(0.0083, 0.0315, 0.0446, 0.0070, 0.0128, 0.0289, 0.0722, 0.0495, 0.0515, 0.0406, 0.0962, 0.1997, 0.0046,\\
&0.0175, 0.0248, 0.0039, 0.0071, 0.0161, 0.0401, 0.0275, 0.0286, 0.0226, 0.0534, 0.1110)^T,\\
\xi^{[2]} =&(0.0087, 0.0247, 0.0406, 0.0082, 0.0100, 0.0257, 0.0589, 0.0479, 0.0498, 0.0479, 0.0957, 0.1821, 0.0058,\\
&0.0165, 0.0270, 0.0054, 0.0066, 0.0171, 0.0393, 0.0319, 0.0332, 0.0319, 0.0638, 0.1214)^T,\\
\xi^{[3]} =&(0.0074, 0.0058, 0.0066, 0.0037, 0.0133, 0.0366, 0.0462, 0.0422, 0.0594, 0.0947, 0.0780, 0.2311, 0.0045,\\
&0.0035, 0.0039, 0.0022, 0.0080, 0.0220, 0.0277, 0.0253, 0.0357, 0.0568, 0.0468, 0.1387)^T.
\end{aligned}$$

Since $L_\phi = 2.0528$, the matrices $\mathbf{M}_1^{[1]} = L_\phi \mathbf{\Pi}^{[1]} + \tilde{\mathcal{N}}_{\Xi}^{[1]}$ defined in (10) are:

$$\mathbf{M}_1^{[1]} = \left(\begin{matrix}
-0.1655 & 0.0550 & 0.0117 & 0.0145 & 0.0375 & -0.0005 & -0.0012 & -0.0008 & 0.0198 & -0.0007 & -0.0016 & -0.0034\\
0.0550 & -0.5353 & 0.0601 & 0.0030 & -0.0008 & 0.1478 & -0.0047 & -0.0032 & -0.0033 & 0.1664 & -0.0062 & -0.0129\\
0.0117 & 0.0601 & -0.5819 & 0.0475 & -0.0012 & -0.0026 & 0.0826 & -0.0045 & -0.0047 & -0.0037 & 0.2247 & -0.0183\\
0.0145 & 0.0030 & 0.0475 & -0.1182 & -0.0002 & -0.0004 & -0.0010 & 0.0167 & -0.0007 & -0.0006 & -0.0014 & 0.0111\\
0.0375 & -0.0008 & -0.0012 & -0.0002 & -0.1662 & 0.0265 & 0.0173 & 0.0234 & 0.0179 & -0.0011 & -0.0025 & -0.0053\\
-0.0005 & 0.1478 & -0.0026 & -0.0004 & 0.0265 & -0.5493 & 0.0680 & 0.0713 & -0.0031 & 0.1366 & -0.0057 & -0.0118\\
-0.0012 & -0.0047 & 0.0826 & -0.0010 & 0.0173 & 0.0680 & -0.8736 & 0.1732 & -0.0076 & -0.0060 & 0.2745 & -0.0296\\
-0.0008 & -0.0032 & -0.0045 & 0.0167 & 0.0234 & 0.0713 & 0.1732 & -0.5469 & -0.0052 & -0.0041 & -0.0098 & 0.0787\\
0.0198 & -0.0033 & -0.0047 & -0.0007 & 0.0179 & -0.0031 & -0.0076 & -0.0052 & -0.5182 & 0.0879 & 0.0929 & 0.1045\\
-0.0007 & 0.1664 & -0.0037 & -0.0006 & -0.0011 & 0.1366 & -0.0060 & -0.0041 & 0.0879 & -0.7321 & 0.0807 & 0.1035\\
-0.0016 & -0.0062 & 0.2247 & -0.0014 & -0.0025 & -0.0057 & 0.2745 & -0.0098 & 0.0929 & 0.0807 & -1.4567 & 0.4008\\
-0.0034 & -0.0129 & -0.0183 & 0.0111 & -0.0053 & -0.0118 & -0.0296 & 0.0787 & 0.1045 & 0.1035 & 0.4008 & -1.4695\\
0.0414 & -0.0003 & -0.0004 & -0.0001 & -0.0001 & -0.0003 & -0.0007 & -0.0005 & -0.0005 & -0.0004 & -0.0009 & -0.0019\\
-0.0003 & 0.1562 & -0.0016 & -0.0003 & -0.0005 & -0.0010 & -0.0026 & -0.0018 & -0.0018 & -0.0015 & -0.0035 & -0.0072\\
-0.0004 & -0.0016 & 0.2209 & -0.0004 & -0.0007 & -0.0015 & -0.0037 & -0.0025 & -0.0026 & -0.0021 & -0.0049 & -0.0102\\
-0.0001 & -0.0003 & -0.0004 & 0.0348 & -0.0001 & -0.0002 & -0.0006 & -0.0004 & -0.0004 & -0.0003 & -0.0008 & -0.0016\\
-0.0001 & -0.0005 & -0.0007 & -0.0001 & 0.0639 & -0.0004 & -0.0011 & -0.0007 & -0.0008 & -0.0006 & -0.0014 & -0.0029\\
-0.0003 & -0.0010 & -0.0015 & -0.0002 & -0.0004 & 0.1435 & -0.0024 & -0.0016 & -0.0017 & -0.0013 & -0.0032 & -0.0066\\
-0.0007 & -0.0026 & -0.0037 & -0.0006 & -0.0011 & -0.0024 & 0.3552 & -0.0041 & -0.0042 & -0.0033 & -0.0079 & -0.0165\\
-0.0005 & -0.0018 & -0.0025 & -0.0004 & -0.0007 & -0.0016 & -0.0041 & 0.2447 & -0.0029 & -0.0023 & -0.0054 & -0.0113\\
-0.0005 & -0.0018 & -0.0026 & -0.0004 & -0.0008 & -0.0017 & -0.0042 & -0.0029 & 0.2547 & -0.0024 & -0.0057 & -0.0117\\
-0.0004 & -0.0015 & -0.0021 & -0.0003 & -0.0006 & -0.0013 & -0.0033 & -0.0023 & -0.0024 & 0.2011 & -0.0045 & -0.0092\\
-0.0009 & -0.0035 & -0.0049 & -0.0008 & -0.0014 & -0.0032 & -0.0079 & -0.0054 & -0.0057 & -0.0045 & 0.4704 & -0.0219\\
-0.0019 & -0.0072 & -0.0102 & -0.0016 & -0.0029 & -0.0066 & -0.0165 & -0.0113 & -0.0117 & -0.0092 & -0.0219 & 0.9532
\end{matrix}\right.$$

$$\left.\begin{matrix}
0.0414 & -0.0003 & -0.0004 & -0.0001 & -0.0001 & -0.0003 & -0.0007 & -0.0005 & -0.0005 & -0.0004 & -0.0009 & -0.0019\\
-0.0003 & 0.1562 & -0.0016 & -0.0003 & -0.0005 & -0.0010 & -0.0026 & -0.0018 & -0.0018 & -0.0015 & -0.0035 & -0.0072\\
-0.0004 & -0.0016 & 0.2209 & -0.0004 & -0.0007 & -0.0015 & -0.0037 & -0.0025 & -0.0026 & -0.0021 & -0.0049 & -0.0102\\
-0.0001 & -0.0003 & -0.0004 & 0.0348 & -0.0001 & -0.0002 & -0.0006 & -0.0004 & -0.0004 & -0.0003 & -0.0008 & -0.0016\\
-0.0001 & -0.0005 & -0.0007 & -0.0001 & 0.0639 & -0.0004 & -0.0011 & -0.0007 & -0.0008 & -0.0006 & -0.0014 & -0.0029\\
-0.0003 & -0.0010 & -0.0015 & -0.0002 & -0.0004 & 0.1435 & -0.0024 & -0.0016 & -0.0017 & -0.0013 & -0.0032 & -0.0066\\
-0.0007 & -0.0026 & -0.0037 & -0.0006 & -0.0011 & -0.0024 & 0.3552 & -0.0041 & -0.0042 & -0.0033 & -0.0079 & -0.0165\\
-0.0005 & -0.0018 & -0.0025 & -0.0004 & -0.0007 & -0.0016 & -0.0041 & 0.2447 & -0.0029 & -0.0023 & -0.0054 & -0.0113\\
-0.0005 & -0.0018 & -0.0026 & -0.0004 & -0.0008 & -0.0017 & -0.0042 & -0.0029 & 0.2547 & -0.0024 & -0.0057 & -0.0117\\
-0.0004 & -0.0015 & -0.0021 & -0.0003 & -0.0006 & -0.0013 & -0.0033 & -0.0023 & -0.0024 & 0.2011 & -0.0045 & -0.0092\\
-0.0009 & -0.0035 & -0.0049 & -0.0008 & -0.0014 & -0.0032 & -0.0079 & -0.0054 & -0.0057 & -0.0045 & 0.4704 & -0.0219\\
-0.0019 & -0.0072 & -0.0102 & -0.0016 & -0.0029 & -0.0066 & -0.0165 & -0.0113 & -0.0117 & -0.0092 & -0.0219 & 0.9532\\
-0.1103 & 0.0307 & 0.0067 & 0.0081 & 0.0209 & -0.0002 & -0.0004 & -0.0003 & 0.0112 & -0.0002 & -0.0005 & -0.0010\\
0.0307 & -0.3668 & 0.0341 & 0.0018 & -0.0003 & 0.0826 & -0.0014 & -0.0010 & -0.0010 & 0.0931 & -0.0019 & -0.0040\\
0.0067 & 0.0341 & -0.4214 & 0.0265 & -0.0004 & -0.0008 & 0.0475 & -0.0014 & -0.0015 & -0.0011 & 0.1270 & -0.0056\\
0.0081 & 0.0018 & 0.0265 & -0.0811 & -0.0001 & -0.0001 & -0.0003 & 0.0095 & -0.0002 & -0.0002 & -0.0004 & 0.0069\\
0.0209 & -0.0003 & -0.0004 & -0.0001 & -0.1207 & 0.0149 & 0.0101 & 0.0133 & 0.0103 & -0.0003 & -0.0008 & -0.0016\\
-0.0002 & 0.0826 & -0.0008 & -0.0001 & 0.0149 & -0.3689 & 0.0388 & 0.0403 & -0.0009 & 0.0765 & -0.0018 & -0.0037\\
-0.0004 & -0.0014 & 0.0475 & -0.0003 & 0.0101 & 0.0388 & -0.6432 & 0.0980 & -0.0024 & -0.0019 & 0.1560 & -0.0091\\
-0.0003 & -0.0010 & -0.0014 & 0.0095 & 0.0133 & 0.0403 & 0.0980 & -0.4126 & -0.0016 & -0.0013 & -0.0030 & 0.0487\\
0.0112 & -0.0010 & -0.0015 & -0.0002 & 0.0103 & -0.0009 & -0.0024 & -0.0016 & -0.4011 & 0.0499 & 0.0541 & 0.0633\\
-0.0002 & 0.0931 & -0.0011 & -0.0002 & -0.0003 & 0.0765 & -0.0019 & -0.0013 & 0.0499 & -0.4961 & 0.0468 & 0.0616\\
-0.0005 & -0.0019 & 0.1270 & -0.0004 & -0.0008 & -0.0018 & 0.1560 & -0.0030 & 0.0541 & 0.0468 & -1.0184 & 0.2324\\
-0.0010 & -0.0040 & -0.0056 & 0.0069 & -0.0016 & -0.0037 & -0.0091 & 0.0487 & 0.0633 & 0.0616 & 0.2324 & -1.2400
\end{matrix}\right)$$

and the eigenvalues of $\mathbf{M}_1^{[1]}$ are:

$$\begin{aligned}
&-2.5131, -1.7259, -1.1469, -0.9555, -0.9498, -0.8056, -0.6946, -0.6681, -0.6547, -0.4875, -0.4447, -0.4022,\\
&-0.3836, -0.3269, -0.2342, -0.2111, -0.1677, -0.1474, -0.1316, -0.1285, -0.0962, -0.0618, -0.0567, 0;
\end{aligned}$$

the matrices $\mathbf{M}_1^{[2]} = L_\phi \mathbf{\Pi}^{[2]} + \tilde{\mathcal{N}}_{\Xi}^{[2]}$ defined in (10) are:

$$\mathbf{M}_1^{[2]} = \left(\begin{matrix}
-0.1554 & 0.0453 & 0.0123 & 0.0164 & 0.0327 & -0.0005 & -0.0010 & -0.0009 & 0.0294 & -0.0009 & -0.0017 & -0.0032 \\
0.0453 & -0.3212 & 0.0474 & 0.0037 & -0.0005 & 0.0871 & -0.0030 & -0.0024 & -0.0025 & 0.0818 & -0.0049 & -0.0092 \\
0.0123 & 0.0474 & -0.4068 & 0.0440 & -0.0008 & -0.0021 & 0.0559 & -0.0040 & -0.0041 & -0.0040 & 0.1486 & -0.0152 \\
0.0164 & 0.0037 & 0.0440 & -0.1058 & -0.0002 & -0.0004 & -0.0010 & 0.0196 & -0.0008 & -0.0008 & -0.0016 & 0.0010 \\
0.0327 & -0.0005 & -0.0008 & -0.0002 & -0.1191 & 0.0223 & 0.0137 & 0.0230 & 0.0040 & -0.0010 & -0.0020 & -0.0037 \\
-0.0005 & 0.0871 & -0.0021 & -0.0004 & 0.0223 & -0.3849 & 0.0610 & 0.0693 & -0.0026 & 0.0839 & -0.0050 & -0.0096 \\
-0.0010 & -0.0030 & 0.0559 & -0.0010 & 0.0137 & 0.0610 & -0.5931 & 0.1415 & -0.0060 & -0.0058 & 0.1725 & -0.0220 \\
-0.0009 & -0.0024 & -0.0040 & 0.0196 & 0.0230 & 0.0693 & 0.1415 & -0.4334 & -0.0049 & -0.0047 & -0.0094 & 0.0540 \\
0.0294 & -0.0025 & -0.0041 & -0.0008 & 0.0040 & -0.0026 & -0.0060 & -0.0049 & -0.4504 & 0.0928 & 0.0898 & 0.0973 \\
-0.0009 & 0.0818 & -0.0040 & -0.0008 & -0.0010 & 0.0839 & -0.0058 & -0.0047 & 0.0928 & -0.5770 & 0.0864 & 0.0971 \\
-0.0017 & -0.0049 & 0.1486 & -0.0016 & -0.0020 & -0.0050 & 0.1725 & -0.0094 & 0.0898 & 0.0864 & -1.1627 & 0.3856 \\
-0.0032 & -0.0092 & -0.0152 & 0.0010 & -0.0037 & -0.0096 & -0.0220 & 0.0540 & 0.0973 & 0.0971 & 0.3856 & -1.1509 \\
0.0345 & -0.0003 & -0.0005 & -0.0001 & -0.0001 & -0.0003 & -0.0007 & -0.0006 & -0.0006 & -0.0006 & -0.0011 & -0.0022 \\
-0.0003 & 0.0980 & -0.0014 & -0.0003 & -0.0003 & -0.0009 & -0.0020 & -0.0016 & -0.0017 & -0.0016 & -0.0032 & -0.0062 \\
-0.0005 & -0.0014 & 0.1600 & -0.0005 & -0.0006 & -0.0014 & -0.0033 & -0.0027 & -0.0028 & -0.0027 & -0.0053 & -0.0101 \\
-0.0001 & -0.0003 & -0.0005 & 0.0325 & -0.0001 & -0.0003 & -0.0007 & -0.0005 & -0.0006 & -0.0005 & -0.0011 & -0.0020 \\
-0.0001 & -0.0003 & -0.0006 & -0.0001 & 0.0397 & -0.0003 & -0.0008 & -0.0007 & -0.0007 & -0.0007 & -0.0013 & -0.0025 \\
-0.0003 & -0.0009 & -0.0014 & -0.0003 & -0.0003 & 0.1017 & -0.0021 & -0.0017 & -0.0017 & -0.0017 & -0.0034 & -0.0064 \\
-0.0007 & -0.0020 & -0.0033 & -0.0007 & -0.0008 & -0.0021 & 0.2309 & -0.0039 & -0.0040 & -0.0039 & -0.0077 & -0.0147 \\
-0.0006 & -0.0016 & -0.0027 & -0.0005 & -0.0007 & -0.0017 & -0.0039 & 0.1885 & -0.0033 & -0.0031 & -0.0063 & -0.0119 \\
-0.0006 & -0.0017 & -0.0028 & -0.0006 & -0.0007 & -0.0017 & -0.0040 & -0.0033 & 0.1957 & -0.0033 & -0.0065 & -0.0124 \\
-0.0006 & -0.0016 & -0.0027 & -0.0005 & -0.0007 & -0.0017 & -0.0039 & -0.0031 & -0.0033 & 0.1885 & -0.0063 & -0.0119 \\
-0.0011 & -0.0032 & -0.0053 & -0.0011 & -0.0013 & -0.0034 & -0.0077 & -0.0063 & -0.0065 & -0.0063 & 0.3704 & -0.0239 \\
-0.0022 & -0.0062 & -0.0101 & -0.0020 & -0.0025 & -0.0064 & -0.0147 & -0.0119 & -0.0124 & -0.0119 & -0.0239 & 0.6829
\end{matrix}\right.$$

$$\left.\begin{matrix}
0.0345 & -0.0003 & -0.0005 & -0.0001 & -0.0001 & -0.0003 & -0.0007 & -0.0006 & -0.0006 & -0.0006 & -0.0011 & -0.0022 \\
-0.0003 & 0.0980 & -0.0014 & -0.0003 & -0.0003 & -0.0009 & -0.0020 & -0.0016 & -0.0017 & -0.0016 & -0.0032 & -0.0062 \\
-0.0005 & -0.0014 & 0.1600 & -0.0005 & -0.0006 & -0.0014 & -0.0033 & -0.0027 & -0.0028 & -0.0027 & -0.0053 & -0.0101 \\
-0.0001 & -0.0003 & -0.0005 & 0.0325 & -0.0001 & -0.0003 & -0.0007 & -0.0005 & -0.0006 & -0.0005 & -0.0011 & -0.0020 \\
-0.0001 & -0.0003 & -0.0006 & -0.0001 & 0.0397 & -0.0003 & -0.0008 & -0.0007 & -0.0007 & -0.0007 & -0.0013 & -0.0025 \\
-0.0003 & -0.0009 & -0.0014 & -0.0003 & -0.0003 & 0.1017 & -0.0021 & -0.0017 & -0.0017 & -0.0017 & -0.0034 & -0.0064 \\
-0.0007 & -0.0020 & -0.0033 & -0.0007 & -0.0008 & -0.0021 & 0.2309 & -0.0039 & -0.0040 & -0.0039 & -0.0077 & -0.0147 \\
-0.0006 & -0.0016 & -0.0027 & -0.0005 & -0.0007 & -0.0017 & -0.0039 & 0.1885 & -0.0033 & -0.0031 & -0.0063 & -0.0119 \\
-0.0006 & -0.0017 & -0.0028 & -0.0006 & -0.0007 & -0.0017 & -0.0040 & -0.0033 & 0.1957 & -0.0033 & -0.0065 & -0.0124 \\
-0.0006 & -0.0016 & -0.0027 & -0.0005 & -0.0007 & -0.0017 & -0.0039 & -0.0031 & -0.0033 & 0.1885 & -0.0063 & -0.0119 \\
-0.0011 & -0.0032 & -0.0053 & -0.0011 & -0.0013 & -0.0034 & -0.0077 & -0.0063 & -0.0065 & -0.0063 & 0.3704 & -0.0239 \\
-0.0022 & -0.0062 & -0.0101 & -0.0020 & -0.0025 & -0.0064 & -0.0147 & -0.0119 & -0.0124 & -0.0119 & -0.0239 & 0.6829 \\
-0.1151 & 0.0303 & 0.0083 & 0.0110 & 0.0218 & -0.0002 & -0.0005 & -0.0004 & 0.0198 & -0.0004 & -0.0008 & -0.0014 \\
0.0303 & -0.2468 & 0.0320 & 0.0025 & -0.0002 & 0.0583 & -0.0013 & -0.0011 & -0.0011 & 0.0551 & -0.0022 & -0.0041 \\
0.0083 & 0.0320 & -0.3245 & 0.0295 & -0.0004 & -0.0009 & 0.0384 & -0.0018 & -0.0018 & -0.0018 & 0.1008 & -0.0067 \\
0.0110 & 0.0025 & 0.0295 & -0.0814 & -0.0001 & -0.0002 & -0.0004 & 0.0132 & -0.0004 & -0.0004 & -0.0007 & 0.0014 \\
0.0218 & -0.0002 & -0.0004 & -0.0001 & -0.0926 & 0.0150 & 0.0094 & 0.0155 & 0.0029 & -0.0004 & -0.0009 & -0.0017 \\
-0.0002 & 0.0583 & -0.0009 & -0.0002 & 0.0150 & -0.2905 & 0.0414 & 0.0468 & -0.0012 & 0.0565 & -0.0022 & -0.0043 \\
-0.0005 & -0.0013 & 0.0384 & -0.0004 & 0.0094 & 0.0414 & -0.4724 & 0.0956 & -0.0027 & -0.0026 & 0.1176 & -0.0098 \\
-0.0004 & -0.0011 & -0.0018 & 0.0132 & 0.0155 & 0.0468 & 0.0956 & -0.3517 & -0.0022 & -0.0021 & -0.0042 & 0.0400 \\
0.0198 & -0.0011 & -0.0018 & -0.0004 & 0.0029 & -0.0012 & -0.0027 & -0.0022 & -0.3655 & 0.0629 & 0.0620 & 0.0690 \\
-0.0004 & 0.0551 & -0.0018 & -0.0004 & -0.0004 & 0.0565 & -0.0026 & -0.0021 & 0.0629 & -0.4475 & 0.0597 & 0.0687 \\
-0.0008 & -0.0022 & 0.1008 & -0.0007 & -0.0009 & -0.0022 & 0.1176 & -0.0042 & 0.0620 & 0.0597 & -0.8986 & 0.2650 \\
-0.0014 & -0.0041 & -0.0067 & 0.0014 & -0.0017 & -0.0043 & -0.0098 & 0.0400 & 0.0690 & 0.0687 & 0.2650 & -0.9949
\end{matrix}\right)$$

and eigenvalues of $\mathbf{M}_1^{[2]} = L_\phi \mathbf{\Pi}^{[2]} + \tilde{\mathcal{N}}_{\Xi}^{[2]}$ are:

$$-2.0066, -1.2946, -0.8675, -0.8241, -0.7767, -0.5934, -0.5403, -0.4921, -0.4542, -0.4022, -0.3612, -0.3074,$$
$$-0.2931, -0.2917, -0.1911, -0.1722, -0.1439, -0.1313, -0.1120, -0.1017, -0.0841, -0.0530, -0.0478, 0;$$

the matrices $\mathbf{M}_1^{[3]} = L_\phi \mathbf{\Pi}^{[3]} + \tilde{\mathcal{N}}_{\Xi}^{[3]}$ defined in (10) are:

$$
\mathbf{M}_1^{[3]} = \left(\begin{matrix}
-0.1113 & 0.0144 & 0.0111 & 0.0184 & 0.0301 & -0.0006 & -0.0007 & -0.0006 & 0.0288 & -0.0014 & -0.0012 & -0.0035 \\
0.0144 & -0.1039 & 0.0231 & 0.0018 & -0.0002 & 0.0082 & -0.0005 & -0.0005 & -0.0007 & 0.0491 & -0.0009 & -0.0027 \\
0.0111 & 0.0231 & -0.0786 & 0.0182 & -0.0002 & -0.0005 & 0.0092 & -0.0006 & -0.0008 & -0.0013 & 0.0088 & -0.0031 \\
0.0184 & 0.0018 & 0.0182 & -0.0512 & -0.0001 & -0.0003 & -0.0003 & 0.0089 & -0.0004 & -0.0007 & -0.0006 & -0.0017 \\
0.0301 & -0.0002 & -0.0002 & -0.0001 & -0.1724 & 0.0672 & 0.0320 & 0.0200 & 0.0050 & -0.0026 & -0.0021 & -0.0063 \\
-0.0006 & 0.0082 & -0.0005 & -0.0003 & 0.0672 & -0.5866 & 0.1613 & 0.0601 & -0.0045 & 0.2371 & -0.0059 & -0.0174 \\
-0.0007 & -0.0005 & 0.0092 & -0.0003 & 0.0320 & 0.1613 & -0.4638 & 0.1345 & -0.0056 & -0.0090 & 0.0619 & -0.0219 \\
-0.0006 & -0.0005 & -0.0006 & 0.0089 & 0.0200 & 0.0601 & 0.1345 & -0.3391 & -0.0052 & -0.0082 & -0.0068 & 0.0433 \\
0.0288 & -0.0007 & -0.0008 & -0.0004 & 0.0050 & -0.0045 & -0.0056 & -0.0052 & -0.5986 & 0.1073 & 0.1094 & 0.2326 \\
-0.0014 & 0.0491 & -0.0013 & -0.0007 & -0.0026 & 0.2371 & -0.0090 & -0.0082 & 0.1073 & -0.7706 & 0.0239 & 0.1653 \\
-0.0012 & -0.0009 & 0.0088 & -0.0006 & -0.0021 & -0.0059 & 0.0619 & -0.0068 & 0.1094 & 0.0239 & -0.9447 & 0.5842 \\
-0.0035 & -0.0027 & -0.0031 & -0.0017 & -0.0063 & -0.0174 & -0.0219 & 0.0433 & 0.2326 & 0.1653 & 0.5842 & -1.4842 \\
0.0222 & -0.0001 & -0.0001 & -0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0004 & -0.0005 & -0.0009 & -0.0007 & -0.0021 \\
-0.0001 & 0.0173 & -0.0000 & -0.0000 & -0.0001 & -0.0003 & -0.0003 & -0.0003 & -0.0004 & -0.0007 & -0.0006 & -0.0016 \\
-0.0001 & -0.0000 & 0.0197 & -0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0003 & -0.0005 & -0.0008 & -0.0006 & -0.0019 \\
-0.0000 & -0.0000 & -0.0000 & 0.0110 & -0.0001 & -0.0002 & -0.0002 & -0.0002 & -0.0003 & -0.0004 & -0.0004 & -0.0010 \\
-0.0001 & -0.0001 & -0.0001 & -0.0001 & 0.0397 & -0.0006 & -0.0008 & -0.0007 & -0.0010 & -0.0015 & -0.0013 & -0.0038 \\
-0.0003 & -0.0003 & -0.0003 & -0.0002 & -0.0006 & 0.1082 & -0.0021 & -0.0019 & -0.0027 & -0.0043 & -0.0035 & -0.0104 \\
-0.0004 & -0.0003 & -0.0004 & -0.0002 & -0.0008 & -0.0021 & 0.1359 & -0.0024 & -0.0034 & -0.0054 & -0.0044 & -0.0131 \\
-0.0004 & -0.0003 & -0.0003 & -0.0002 & -0.0007 & -0.0019 & -0.0024 & 0.1244 & -0.0031 & -0.0049 & -0.0041 & -0.0120 \\
-0.0005 & -0.0004 & -0.0005 & -0.0003 & -0.0010 & -0.0027 & -0.0034 & -0.0031 & 0.1740 & -0.0069 & -0.0057 & -0.0169 \\
-0.0009 & -0.0007 & -0.0008 & -0.0004 & -0.0015 & -0.0043 & -0.0054 & -0.0049 & -0.0069 & 0.2729 & -0.0091 & -0.0269 \\
-0.0007 & -0.0006 & -0.0006 & -0.0004 & -0.0013 & -0.0035 & -0.0044 & -0.0041 & -0.0057 & -0.0091 & 0.2266 & -0.0222 \\
-0.0021 & -0.0016 & -0.0019 & -0.0010 & -0.0038 & -0.0104 & -0.0131 & -0.0120 & -0.0169 & -0.0269 & -0.0222 & 0.6276
\end{matrix}\right.
$$

$$
\left.\begin{matrix}
0.0222 & -0.0001 & -0.0001 & -0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0004 & -0.0005 & -0.0009 & -0.0007 & -0.0021 \\
-0.0001 & 0.0173 & -0.0000 & -0.0000 & -0.0001 & -0.0003 & -0.0003 & -0.0003 & -0.0004 & -0.0007 & -0.0006 & -0.0016 \\
-0.0001 & -0.0000 & 0.0197 & -0.0000 & -0.0001 & -0.0003 & -0.0004 & -0.0003 & -0.0005 & -0.0008 & -0.0006 & -0.0019 \\
-0.0000 & -0.0000 & -0.0000 & 0.0110 & -0.0001 & -0.0002 & -0.0002 & -0.0002 & -0.0003 & -0.0004 & -0.0004 & -0.0010 \\
-0.0001 & -0.0001 & -0.0001 & -0.0001 & 0.0397 & -0.0006 & -0.0008 & -0.0007 & -0.0010 & -0.0015 & -0.0013 & -0.0038 \\
-0.0003 & -0.0003 & -0.0003 & -0.0002 & -0.0006 & 0.1082 & -0.0021 & -0.0019 & -0.0027 & -0.0043 & -0.0035 & -0.0104 \\
-0.0004 & -0.0003 & -0.0004 & -0.0002 & -0.0008 & -0.0021 & 0.1359 & -0.0024 & -0.0034 & -0.0054 & -0.0044 & -0.0131 \\
-0.0004 & -0.0003 & -0.0003 & -0.0002 & -0.0007 & -0.0019 & -0.0024 & 0.1244 & -0.0031 & -0.0049 & -0.0041 & -0.0120 \\
-0.0005 & -0.0004 & -0.0005 & -0.0003 & -0.0010 & -0.0027 & -0.0034 & -0.0031 & 0.1740 & -0.0069 & -0.0057 & -0.0169 \\
-0.0009 & -0.0007 & -0.0008 & -0.0004 & -0.0015 & -0.0043 & -0.0054 & -0.0049 & -0.0069 & 0.2729 & -0.0091 & -0.0269 \\
-0.0007 & -0.0006 & -0.0006 & -0.0004 & -0.0013 & -0.0035 & -0.0044 & -0.0041 & -0.0057 & -0.0091 & 0.2266 & -0.0222 \\
-0.0021 & -0.0016 & -0.0019 & -0.0010 & -0.0038 & -0.0104 & -0.0131 & -0.0120 & -0.0169 & -0.0269 & -0.0222 & 0.6276 \\
-0.0756 & 0.0086 & 0.0067 & 0.0110 & 0.0181 & -0.0002 & -0.0003 & -0.0002 & 0.0175 & -0.0005 & -0.0004 & -0.0013 \\
0.0086 & -0.0692 & 0.0139 & 0.0011 & -0.0001 & 0.0050 & -0.0002 & -0.0002 & -0.0003 & 0.0297 & -0.0003 & -0.0010 \\
0.0067 & 0.0139 & -0.0550 & 0.0109 & -0.0001 & -0.0002 & 0.0057 & -0.0002 & -0.0003 & -0.0005 & 0.0055 & -0.0011 \\
0.0110 & 0.0011 & 0.0109 & -0.0351 & -0.0000 & -0.0001 & -0.0001 & 0.0054 & -0.0002 & -0.0003 & -0.0002 & -0.0006 \\
0.0181 & -0.0001 & -0.0001 & -0.0000 & -0.1193 & 0.0406 & 0.0195 & 0.0122 & 0.0034 & -0.0009 & -0.0008 & -0.0023 \\
-0.0002 & 0.0050 & -0.0002 & -0.0001 & 0.0406 & -0.3952 & 0.0976 & 0.0368 & -0.0016 & 0.1440 & -0.0021 & -0.0063 \\
-0.0003 & -0.0002 & 0.0057 & -0.0001 & 0.0195 & 0.0976 & -0.3326 & 0.0817 & -0.0020 & -0.0032 & 0.0389 & -0.0079 \\
-0.0002 & -0.0002 & -0.0002 & 0.0054 & 0.0122 & 0.0368 & 0.0817 & -0.2532 & -0.0019 & -0.0030 & -0.0024 & 0.0308 \\
0.0175 & -0.0003 & -0.0003 & -0.0002 & 0.0034 & -0.0016 & -0.0020 & -0.0019 & -0.4288 & 0.0672 & 0.0679 & 0.1464 \\
-0.0005 & 0.0297 & -0.0005 & -0.0003 & -0.0009 & 0.1440 & -0.0032 & -0.0030 & 0.0672 & -0.5715 & 0.0180 & 0.1100 \\
-0.0004 & -0.0003 & 0.0055 & -0.0002 & -0.0008 & -0.0021 & 0.0389 & -0.0024 & 0.0679 & 0.0180 & -0.6575 & 0.3594 \\
-0.0013 & -0.0010 & -0.0011 & -0.0006 & -0.0023 & -0.0063 & -0.0079 & 0.0308 & 0.1464 & 0.1100 & 0.3594 & -1.1416
\end{matrix}\right)
$$

and eigenvalues of $\mathbf{M}_1^{[3]} = L_\phi \mathbf{\Pi}^{[3]} + \tilde{\mathcal{N}}_{\Xi}^{[3]}$ are:

$$
\begin{aligned}
&-2.2486, -1.1133, -1.0719, -0.9171, -0.6959, -0.5790, -0.5517, -0.4887, -0.4188, -0.3148, -0.3109, -0.2284, \\
&-0.1807, -0.1271, -0.1231, -0.1123, -0.0822, -0.0713, -0.0623, -0.0456, -0.0410, -0.0375, -0.0173, 0.
\end{aligned}
$$

Then, according to Theorem 1, $\max_d \lambda_2(\mathbf{M}_1^{[d]}) = \max(-0.0567, -0.0478, -0.0173) = -0.0173 < 0$, thus Bi-Syn under nonidentical node groups for each layer can be achieved exponentially.

II. Matricies in Simulation 2

The mho-transformed intra-layer coupling matrices are identical for each layer and the same as $(H^{\underline{1}})^1$, $(H^{\underline{1}})^2$ defined in (1), the mho-transformed inter-layer coupling matrices G^{w_2} are set as

$$
G^1 = \begin{pmatrix} -1 & -1 & -2 \\ 3 & -3 & 0 \\ -3 & -2 & -5 \end{pmatrix}, G^2 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -2 & -2 \\ -4 & -2 & -6 \end{pmatrix}, G^3 = \begin{pmatrix} -2 & 2 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix},
$$

the mho-transformed inter-block coupling matrices F^{w_3} are set as

$$
F^1 = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}, F^2 = \begin{pmatrix} -2 & -2 \\ -4 & -4 \end{pmatrix}.
$$

After using gauge transformation matrices $\mathcal{S}=\mathrm{diag}(1,-1)$, $\mathcal{R}=\mathrm{diag}(1,1,-1)$, $\mathcal{P}=\mathrm{diag}(1,1,-1,-1)$, which are defined in (5), we have the gauge-transformed intra-layer coupling matrices $\tilde{H}^{w_1}=\mathcal{P}H^{w_1}\mathcal{P}$:

$$\tilde{H}^1=\begin{pmatrix}-7 & 4 & 3 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 0 & 1 & 1 & -2\end{pmatrix}, \tilde{H}^2=\begin{pmatrix}1 & -2 & 0 & 1\\ 2 & -6 & 4 & 0\\ 0 & 0 & -3 & 3\\ 3 & 0 & 0 & -3\end{pmatrix},$$

gauge-transformed inter-layer coupling matrices $\tilde{G}^{w_2}=\mathcal{R}G^{w_2}\mathcal{R}$:

$$\tilde{G}^1=\begin{pmatrix}-1 & -1 & 2\\ 3 & -3 & 0\\ 3 & 2 & -5\end{pmatrix}, \tilde{G}^2=\begin{pmatrix}-1 & 1 & 0\\ 0 & -2 & 2\\ 4 & 2 & -6\end{pmatrix}, \tilde{G}^3=\begin{pmatrix}-2 & 2 & 0\\ -1 & 0 & 1\\ 0 & 0 & 0\end{pmatrix},$$

and gauge-transformed inter-block coupling matrices $\tilde{F}^{w_3}=\mathcal{S}F^{w_3}\mathcal{S}$:

$$\tilde{F}^1=\begin{pmatrix}-1 & 1\\ -1 & 1\end{pmatrix}, \tilde{F}^2=\begin{pmatrix}-2 & 2\\ 4 & -4\end{pmatrix}.$$

We can verify that there also exist negative elements in the non-diagonal parts of $\tilde{H}^{w_1}$, $\tilde{G}^{w_2}$, and $\tilde{F}^{w_3}$, which means they are all structurally unbalanced. Then, by using formula (7), we can have the intra-layer union matrices

$$\tilde{H}^{[1]}=\tilde{H}^{[2]}=\begin{pmatrix}-6 & 2 & 3 & 1\\ 3 & -7 & 4 & 0\\ 0 & 0 & -2 & 2\\ 3 & 1 & 1 & -5\end{pmatrix}, \tilde{H}^{[3]}=\begin{pmatrix}-5 & 0 & 3 & 2\\ 5 & -13 & 8 & 0\\ 0 & 0 & -5 & 5\\ 6 & 1 & 1 & -8\end{pmatrix},$$

the inter-layer union matrices $\tilde{G}^{[d]}$:

$$\tilde{G}^{[1]}=\begin{pmatrix}-7 & 3 & 4\\ 4 & -8 & 4\\ 10 & 6 & -16\end{pmatrix}, \tilde{G}^{[2]}=\begin{pmatrix}-7 & 5 & 2\\ 1 & -7 & 6\\ 11 & 6 & -17\end{pmatrix}, \tilde{G}^{[3]}=\begin{pmatrix}-6 & 2 & 4\\ 5 & -10 & 5\\ 14 & 8 & -22\end{pmatrix},$$

and the inter-block union matrices $\tilde{F}^{[d]}$:

$$\tilde{F}^{[1]}=\begin{pmatrix}-5 & 5\\ 7 & -7\end{pmatrix}, \tilde{F}^{[2]}=\begin{pmatrix}-4 & 4\\ 2 & -2\end{pmatrix}, \tilde{F}^{[3]}=\begin{pmatrix}-3 & 3\\ 3 & -3\end{pmatrix}.$$

It is obvious that all non-diagonal elements of $\tilde{H}^{[d]}$, $\tilde{G}^{[d]}$, $\tilde{F}^{[d]}$ are nonnegative, and every matrices is zero-row-sum. Thus, set the coupling strengths $\rho_1=\rho_2=\rho_3=1$, the $\tilde{\mathcal{N}}^{[d]}=\rho_1(I_2\otimes I_3\otimes\tilde{H}^{[d]})+\rho_2(I_2\otimes\tilde{G}^{[d]}\otimes I_4)+\rho_3(\tilde{F}^{[d]}\otimes I_3\otimes I_4)$ defined in (8) are:

$$
\tilde{\mathcal{N}}^{[1]} = \left(\begin{array}{cccccccccccc}
-18 & 2 & 3 & 1 & 3 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
3 & -19 & 4 & 0 & 0 & 3 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & -14 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 4 & 0 \\
3 & 1 & 1 & -17 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 4 \\
4 & 0 & 0 & 0 & -19 & 2 & 3 & 1 & 4 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 3 & -20 & 4 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 0 & 0 & -15 & 2 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 3 & 1 & 1 & -18 & 0 & 0 & 0 & 4 \\
10 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -27 & 2 & 3 & 1 \\
0 & 10 & 0 & 0 & 0 & 6 & 0 & 0 & 3 & -28 & 4 & 0 \\
0 & 0 & 10 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -23 & 2 \\
0 & 0 & 0 & 10 & 0 & 0 & 0 & 6 & 3 & 1 & 1 & -26 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7
\end{array} \right.
$$

$$
\left. \begin{array}{cccccccccccc}
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\
-20 & 2 & 3 & 1 & 3 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
3 & -21 & 4 & 0 & 0 & 3 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & -16 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 4 & 0 \\
3 & 1 & 1 & -19 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 4 \\
4 & 0 & 0 & 0 & -21 & 2 & 3 & 1 & 4 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 3 & -22 & 4 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 & 0 & 0 & -17 & 2 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 & 3 & 1 & 1 & -20 & 0 & 0 & 0 & 4 \\
10 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -29 & 2 & 3 & 1 \\
0 & 10 & 0 & 0 & 0 & 6 & 0 & 0 & 3 & -30 & 4 & 0 \\
0 & 0 & 10 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -25 & 2 \\
0 & 0 & 0 & 10 & 0 & 0 & 0 & 6 & 3 & 1 & 1 & -28
\end{array} \right)
$$

$$\tilde{\mathcal{N}}^{[2]} = \begin{pmatrix}
-17 & 2 & 3 & 1 & 5 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & -18 & 4 & 0 & 0 & 5 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -13 & 2 & 0 & 0 & 5 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & -16 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -17 & 2 & 3 & 1 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 3 & -18 & 4 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -13 & 2 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 3 & 1 & 1 & -16 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\
11 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -27 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
0 & 11 & 0 & 0 & 0 & 6 & 0 & 0 & 3 & -28 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 11 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -23 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 11 & 0 & 0 & 0 & 6 & 3 & 1 & 1 & -26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\
2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -15 & 2 & 3 & 1 & 5 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -16 & 4 & 0 & 0 & 5 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -11 & 2 & 0 & 0 & 5 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 & -14 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -15 & 2 & 3 & 1 & 6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 3 & -16 & 4 & 0 & 0 & 6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -11 & 2 & 0 & 0 & 6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 1 & 1 & -14 & 0 & 0 & 0 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 11 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -25 & 2 & 3 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 11 & 0 & 0 & 0 & 6 & 0 & 0 & 3 & -26 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 11 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & -21 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 11 & 0 & 0 & 0 & 6 & 3 & 1 & 1 & -24
\end{pmatrix}$$

$$\tilde{\mathcal{N}}^{[3]} = \left(\begin{array}{cccccccccccccccccccccccc}
-14 & 0 & 3 & 2 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & -22 & 8 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -14 & 5 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 1 & 1 & -17 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & -18 & 0 & 3 & 2 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 & 5 & -26 & 8 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 0 & 0 & 0 & -18 & 5 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 5 & 6 & 1 & 1 & -21 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
14 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & -30 & 0 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & 14 & 0 & 0 & 0 & 8 & 0 & 0 & 5 & -38 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 14 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & -30 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 14 & 0 & 0 & 0 & 8 & 6 & 1 & 1 & -33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & 0 & 3 & 2 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & -22 & 8 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & 5 & 0 & 0 & 2 & 0 & 0 & 0 & 4 & 0 \\
0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 1 & 1 & -17 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 4 \\
0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & -18 & 0 & 3 & 2 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 5 & -26 & 8 & 0 & 0 & 5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & -18 & 5 & 0 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 1 & 1 & -21 & 0 & 0 & 0 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 14 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & -30 & 0 & 3 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 14 & 0 & 0 & 0 & 8 & 0 & 0 & 5 & -38 & 8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 14 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & -30 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 14 & 0 & 0 & 0 & 8 & 6 & 1 & 1 & -33
\end{array}\right)$$

It can be seen that $\tilde{\mathcal{N}}^{[d]}$ are Metzler and irreducible, $d = 1, 2, 3$, hence Assumption 2 holds. Then, according to Lemma 1, the NLEs for $\tilde{\mathcal{N}}^{[d]}$ are:

$$
\begin{aligned}
\xi^{[1]} =&(0.0441, 0.0221, 0.1434, 0.0662, 0.0305, 0.0153, 0.0993, 0.0458, 0.0187, 0.0093, 0.0607, 0.0280, 0.0315,\\
&0.0158, 0.1024, 0.0473, 0.0218, 0.0109, 0.0709, 0.0327, 0.0133, 0.0067, 0.0433, 0.0200)^T,\\
\xi^{[2]} =&(0.0198, 0.0099, 0.0642, 0.0296, 0.0231, 0.0115, 0.0751, 0.0346, 0.0105, 0.0052, 0.0340, 0.0157, 0.0395,\\
&0.0198, 0.1285, 0.0593, 0.0462, 0.0231, 0.1501, 0.0693, 0.0210, 0.0105, 0.0681, 0.0314)^T,\\
\xi^{[3]} =&(0.1091, 0.0066, 0.0930, 0.0854, 0.0461, 0.0028, 0.0393, 0.0361, 0.0303, 0.0018, 0.0258, 0.0237, 0.1091,\\
&0.0066, 0.0930, 0.0854, 0.0461, 0.0028, 0.0393, 0.0361, 0.0303, 0.0018, 0.0258, 0.0237)^T.
\end{aligned}
$$

Since $L_\phi = 2.0528$, the matrices $\mathbf{M}_1^{[1]} = L_\phi \mathbf{\Pi}^{[1]} + \tilde{\mathcal{N}}_\Xi^{[1]}$ defined in (10) are:

$$
\mathbf{M}_1^{[1]} = \left(\begin{array}{cccccccccccc}
-0.7076 & 0.0752 & 0.0532 & 0.1153 & 0.1245 & -0.0014 & -0.0090 & -0.0041 & 0.1799 & -0.0008 & -0.0055 & -0.0025\\
0.0752 & -0.3749 & 0.0376 & 0.0301 & -0.0014 & 0.0629 & -0.0045 & -0.0021 & -0.0008 & 0.0904 & -0.0027 & -0.0013\\
0.0532 & 0.0376 & -1.7554 & 0.1570 & -0.0090 & -0.0045 & 0.3844 & -0.0135 & -0.0055 & -0.0027 & 0.5723 & -0.0082\\
0.1153 & 0.0301 & 0.1570 & -0.9982 & -0.0041 & -0.0021 & -0.0135 & 0.1847 & -0.0025 & -0.0013 & -0.0082 & 0.2686\\
0.1245 & -0.0014 & -0.0090 & -0.0041 & -0.5196 & 0.0525 & 0.0396 & 0.0811 & 0.1159 & -0.0006 & -0.0038 & -0.0018\\
-0.0014 & 0.0629 & -0.0045 & -0.0021 & 0.0525 & -0.2746 & 0.0274 & 0.0215 & -0.0006 & 0.0583 & -0.0019 & -0.0009\\
-0.0090 & -0.0045 & 0.3844 & -0.0135 & 0.0396 & 0.0274 & -1.3055 & 0.1128 & -0.0038 & -0.0019 & 0.3682 & -0.0057\\
-0.0041 & -0.0021 & -0.0135 & 0.1847 & 0.0811 & 0.0215 & 0.1128 & -0.7350 & -0.0018 & -0.0009 & -0.0057 & 0.1730\\
0.1799 & -0.0008 & -0.0055 & -0.0025 & 0.1159 & -0.0006 & -0.0038 & -0.0018 & -0.4664 & 0.0323 & 0.0257 & 0.0503\\
-0.0008 & 0.0904 & -0.0027 & -0.0013 & -0.0006 & 0.0583 & -0.0019 & -0.0009 & 0.0323 & -0.2424 & 0.0175 & 0.0135\\
-0.0055 & -0.0027 & 0.5723 & -0.0082 & -0.0038 & -0.0019 & 0.3682 & -0.0057 & 0.0257 & 0.0175 & -1.2784 & 0.0712\\
-0.0025 & -0.0013 & -0.0082 & 0.2686 & -0.0018 & -0.0009 & -0.0057 & 0.1730 & 0.0503 & 0.0135 & 0.0712 & -0.6721\\
0.2178 & -0.0014 & -0.0093 & -0.0043 & -0.0020 & -0.0010 & -0.0064 & -0.0030 & -0.0012 & -0.0006 & -0.0039 & -0.0018\\
-0.0014 & 0.1096 & -0.0046 & -0.0021 & -0.0010 & -0.0005 & -0.0032 & -0.0015 & -0.0006 & -0.0003 & -0.0020 & -0.0009\\
-0.0093 & -0.0046 & 0.6868 & -0.0139 & -0.0064 & -0.0032 & -0.0209 & -0.0096 & -0.0039 & -0.0020 & -0.0128 & -0.0059\\
-0.0043 & -0.0021 & -0.0139 & 0.3245 & -0.0030 & -0.0015 & -0.0096 & -0.0044 & -0.0018 & -0.0009 & -0.0059 & -0.0027\\
-0.0020 & -0.0010 & -0.0064 & -0.0030 & 0.1514 & -0.0007 & -0.0044 & -0.0021 & -0.0008 & -0.0004 & -0.0027 & -0.0013\\
-0.0010 & -0.0005 & -0.0032 & -0.0015 & -0.0007 & 0.0760 & -0.0022 & -0.0010 & -0.0004 & -0.0002 & -0.0014 & -0.0006\\
-0.0064 & -0.0032 & -0.0209 & -0.0096 & -0.0044 & -0.0022 & 0.4819 & -0.0067 & -0.0027 & -0.0014 & -0.0088 & -0.0041\\
-0.0030 & -0.0015 & -0.0096 & -0.0044 & -0.0021 & -0.0010 & -0.0067 & 0.2260 & -0.0013 & -0.0006 & -0.0041 & -0.0019\\
-0.0012 & -0.0006 & -0.0039 & -0.0018 & -0.0008 & -0.0004 & -0.0027 & -0.0013 & 0.0928 & -0.0003 & -0.0017 & -0.0008\\
-0.0006 & -0.0003 & -0.0020 & -0.0009 & -0.0004 & -0.0002 & -0.0014 & -0.0006 & -0.0003 & 0.0465 & -0.0008 & -0.0004\\
-0.0039 & -0.0020 & -0.0128 & -0.0059 & -0.0027 & -0.0014 & -0.0088 & -0.0041 & -0.0017 & -0.0008 & 0.2979 & -0.0025\\
-0.0018 & -0.0009 & -0.0059 & -0.0027 & -0.0013 & -0.0006 & -0.0041 & -0.0019 & -0.0008 & -0.0004 & -0.0025 & 0.1389
\end{array}\right.
$$

$$
\left.\begin{array}{cccccccccccc}
0.2178 & -0.0014 & -0.0093 & -0.0043 & -0.0020 & -0.0010 & -0.0064 & -0.0030 & -0.0012 & -0.0006 & -0.0039 & -0.0018\\
-0.0014 & 0.1096 & -0.0046 & -0.0021 & -0.0010 & -0.0005 & -0.0032 & -0.0015 & -0.0006 & -0.0003 & -0.0020 & -0.0009\\
-0.0093 & -0.0046 & 0.6868 & -0.0139 & -0.0064 & -0.0032 & -0.0209 & -0.0096 & -0.0039 & -0.0020 & -0.0128 & -0.0059\\
-0.0043 & -0.0021 & -0.0139 & 0.3245 & -0.0030 & -0.0015 & -0.0096 & -0.0044 & -0.0018 & -0.0009 & -0.0059 & -0.0027\\
-0.0020 & -0.0010 & -0.0064 & -0.0030 & 0.1514 & -0.0007 & -0.0044 & -0.0021 & -0.0008 & -0.0004 & -0.0027 & -0.0013\\
-0.0010 & -0.0005 & -0.0032 & -0.0015 & -0.0007 & 0.0760 & -0.0022 & -0.0010 & -0.0004 & -0.0002 & -0.0014 & -0.0006\\
-0.0064 & -0.0032 & -0.0209 & -0.0096 & -0.0044 & -0.0022 & 0.4819 & -0.0067 & -0.0027 & -0.0014 & -0.0088 & -0.0041\\
-0.0030 & -0.0015 & -0.0096 & -0.0044 & -0.0021 & -0.0010 & -0.0067 & 0.2260 & -0.0013 & -0.0006 & -0.0041 & -0.0019\\
-0.0012 & -0.0006 & -0.0039 & -0.0018 & -0.0008 & -0.0004 & -0.0027 & -0.0013 & 0.0928 & -0.0003 & -0.0017 & -0.0008\\
-0.0006 & -0.0003 & -0.0020 & -0.0009 & -0.0004 & -0.0002 & -0.0014 & -0.0006 & -0.0003 & 0.0465 & -0.0008 & -0.0004\\
-0.0039 & -0.0020 & -0.0128 & -0.0059 & -0.0027 & -0.0014 & -0.0088 & -0.0041 & -0.0017 & -0.0008 & 0.2979 & -0.0025\\
-0.0018 & -0.0009 & -0.0059 & -0.0027 & -0.0013 & -0.0006 & -0.0041 & -0.0019 & -0.0008 & -0.0004 & -0.0025 & 0.1389\\
-0.5676 & 0.0541 & 0.0406 & 0.0836 & 0.0895 & -0.0007 & -0.0046 & -0.0021 & 0.1288 & -0.0004 & -0.0028 & -0.0013\\
0.0541 & -0.2991 & 0.0282 & 0.0221 & -0.0007 & 0.0451 & -0.0023 & -0.0011 & -0.0004 & 0.0646 & -0.0014 & -0.0006\\
0.0406 & 0.0282 & -1.4501 & 0.1161 & -0.0046 & -0.0023 & 0.2805 & -0.0069 & -0.0028 & -0.0014 & 0.4124 & -0.0042\\
0.0836 & 0.0221 & 0.1161 & -0.8057 & -0.0021 & -0.0011 & -0.0069 & 0.1332 & -0.0013 & -0.0006 & -0.0042 & 0.1926\\
0.0895 & -0.0007 & -0.0046 & -0.0021 & -0.4144 & 0.0377 & 0.0296 & 0.0585 & 0.0830 & -0.0003 & -0.0019 & -0.0009\\
-0.0007 & 0.0451 & -0.0023 & -0.0011 & 0.0377 & -0.2179 & 0.0202 & 0.0156 & -0.0003 & 0.0417 & -0.0010 & -0.0004\\
-0.0046 & -0.0023 & 0.2805 & -0.0069 & 0.0296 & 0.0202 & -1.0702 & 0.0825 & -0.0019 & -0.0010 & 0.2655 & -0.0029\\
-0.0021 & -0.0011 & -0.0069 & 0.1332 & 0.0585 & 0.0156 & 0.0825 & -0.5896 & -0.0009 & -0.0004 & -0.0029 & 0.1241\\
0.1288 & -0.0004 & -0.0028 & -0.0013 & 0.0830 & -0.0003 & -0.0019 & -0.0009 & -0.3597 & 0.0232 & 0.0188 & 0.0361\\
-0.0004 & 0.0646 & -0.0014 & -0.0006 & -0.0003 & 0.0417 & -0.0010 & -0.0004 & 0.0232 & -0.1864 & 0.0127 & 0.0097\\
-0.0028 & -0.0014 & 0.4124 & -0.0042 & -0.0019 & -0.0010 & 0.2655 & -0.0029 & 0.0188 & 0.0127 & -0.9982 & 0.0516\\
-0.0013 & -0.0006 & -0.0042 & 0.1926 & -0.0009 & -0.0004 & -0.0029 & 0.1241 & 0.0361 & 0.0097 & 0.0516 & -0.5198
\end{array}\right),
$$

and the eigenvalues of $\mathbf{M}_1^{[1]}$ are:

$$
\begin{aligned}
&-2.6245, -1.9474, -1.3839, -1.3378, -1.0221, -1.0049, -0.9440, -0.9232, -0.7437, -0.6851, -0.5710, -0.5297,\\
&-0.5000, -0.4791, -0.3813, -0.3634, -0.3411, -0.2580, -0.1967, -0.1955, -0.1836, -0.1206, -0.0718, 0;
\end{aligned}
$$

the matrices $\mathbf{M}_1^{[2]} = L_\phi \mathbf{\Pi}^{[2]} + \tilde{\mathcal{N}}_\Xi^{[2]}$ defined in (10) are:

$$
\mathbf{M}_1^{[2]} = \left(\begin{array}{cccccccccccc}
-0.2962 & 0.0342 & 0.0270 & 0.0531 & 0.0600 & -0.0005 & -0.0030 & -0.0014 & 0.0770 & -0.0002 & -0.0014 & -0.0006 \\
0.0342 & -0.1578 & 0.0185 & 0.0142 & -0.0005 & 0.0302 & -0.0015 & -0.0007 & -0.0002 & 0.0386 & -0.0007 & -0.0003 \\
0.0270 & 0.0185 & -0.7116 & 0.0751 & -0.0030 & -0.0015 & 0.1882 & -0.0046 & -0.0014 & -0.0007 & 0.2470 & -0.0021 \\
0.0531 & 0.0142 & 0.0751 & -0.4152 & -0.0014 & -0.0007 & -0.0046 & 0.0893 & -0.0006 & -0.0003 & -0.0021 & 0.1151 \\
0.0600 & -0.0005 & -0.0030 & -0.0014 & -0.3463 & 0.0399 & 0.0311 & 0.0619 & 0.1002 & -0.0002 & -0.0016 & -0.0007 \\
-0.0005 & 0.0302 & -0.0015 & -0.0007 & 0.0399 & -0.1844 & 0.0213 & 0.0165 & -0.0002 & 0.0502 & -0.0008 & -0.0004 \\
-0.0030 & -0.0015 & 0.1882 & -0.0046 & 0.0311 & 0.0213 & -0.8333 & 0.0870 & -0.0016 & -0.0008 & 0.3221 & -0.0024 \\
-0.0014 & -0.0007 & -0.0046 & 0.0893 & 0.0619 & 0.0165 & 0.0870 & -0.4856 & -0.0007 & -0.0004 & -0.0024 & 0.1500 \\
0.0770 & -0.0002 & -0.0014 & -0.0006 & 0.1002 & -0.0002 & -0.0016 & -0.0007 & -0.2616 & 0.0182 & 0.0150 & 0.0285 \\
-0.0002 & 0.0386 & -0.0007 & -0.0003 & -0.0002 & 0.0502 & -0.0008 & -0.0004 & 0.0182 & -0.1360 & 0.0101 & 0.0077 \\
-0.0014 & -0.0007 & 0.2470 & -0.0021 & -0.0016 & -0.0008 & 0.3221 & -0.0024 & 0.0150 & 0.0101 & -0.7156 & 0.0408 \\
-0.0006 & -0.0003 & -0.0021 & 0.1151 & -0.0007 & -0.0004 & -0.0024 & 0.1500 & 0.0285 & 0.0077 & 0.0408 & -0.3768 \\
0.0774 & -0.0008 & -0.0052 & -0.0024 & -0.0019 & -0.0009 & -0.0061 & -0.0028 & -0.0008 & -0.0004 & -0.0028 & -0.0013 \\
-0.0008 & 0.0391 & -0.0026 & -0.0012 & -0.0009 & -0.0005 & -0.0030 & -0.0014 & -0.0004 & -0.0002 & -0.0014 & -0.0006 \\
-0.0052 & -0.0026 & 0.2400 & -0.0078 & -0.0061 & -0.0030 & -0.0198 & -0.0091 & -0.0028 & -0.0014 & -0.0090 & -0.0041 \\
-0.0024 & -0.0012 & -0.0078 & 0.1150 & -0.0028 & -0.0014 & -0.0091 & -0.0042 & -0.0013 & -0.0006 & -0.0041 & -0.0019 \\
-0.0019 & -0.0009 & -0.0061 & -0.0028 & 0.0902 & -0.0011 & -0.0071 & -0.0033 & -0.0010 & -0.0005 & -0.0032 & -0.0015 \\
-0.0009 & -0.0005 & -0.0030 & -0.0014 & -0.0011 & 0.0456 & -0.0036 & -0.0016 & -0.0005 & -0.0002 & -0.0016 & -0.0007 \\
-0.0061 & -0.0030 & -0.0198 & -0.0091 & -0.0071 & -0.0036 & 0.2771 & -0.0107 & -0.0032 & -0.0016 & -0.0105 & -0.0048 \\
-0.0028 & -0.0014 & -0.0091 & -0.0042 & -0.0033 & -0.0016 & -0.0107 & 0.1336 & -0.0015 & -0.0007 & -0.0048 & -0.0022 \\
-0.0008 & -0.0004 & -0.0028 & -0.0013 & -0.0010 & -0.0005 & -0.0032 & -0.0015 & 0.0415 & -0.0002 & -0.0015 & -0.0007 \\
-0.0004 & -0.0002 & -0.0014 & -0.0006 & -0.0005 & -0.0002 & -0.0016 & -0.0007 & -0.0002 & 0.0208 & -0.0007 & -0.0003 \\
-0.0028 & -0.0014 & -0.0090 & -0.0041 & -0.0032 & -0.0016 & -0.0105 & -0.0048 & -0.0015 & -0.0007 & 0.1314 & -0.0022 \\
-0.0013 & -0.0006 & -0.0041 & -0.0019 & -0.0015 & -0.0007 & -0.0048 & -0.0022 & -0.0007 & -0.0003 & -0.0022 & 0.0618
\end{array} \right.
$$

$$
\left. \begin{array}{cccccccccccc}
0.0774 & -0.0008 & -0.0052 & -0.0024 & -0.0019 & -0.0009 & -0.0061 & -0.0028 & -0.0008 & -0.0004 & -0.0028 & -0.0013 \\
-0.0008 & 0.0391 & -0.0026 & -0.0012 & -0.0009 & -0.0005 & -0.0030 & -0.0014 & -0.0004 & -0.0002 & -0.0014 & -0.0006 \\
-0.0052 & -0.0026 & 0.2400 & -0.0078 & -0.0061 & -0.0030 & -0.0198 & -0.0091 & -0.0028 & -0.0014 & -0.0090 & -0.0041 \\
-0.0024 & -0.0012 & -0.0078 & 0.1150 & -0.0028 & -0.0014 & -0.0091 & -0.0042 & -0.0013 & -0.0006 & -0.0041 & -0.0019 \\
-0.0019 & -0.0009 & -0.0061 & -0.0028 & 0.0902 & -0.0011 & -0.0071 & -0.0033 & -0.0010 & -0.0005 & -0.0032 & -0.0015 \\
-0.0009 & -0.0005 & -0.0030 & -0.0014 & -0.0011 & 0.0456 & -0.0036 & -0.0016 & -0.0005 & -0.0002 & -0.0016 & -0.0007 \\
-0.0061 & -0.0030 & -0.0198 & -0.0091 & -0.0071 & -0.0036 & 0.2771 & -0.0107 & -0.0032 & -0.0016 & -0.0105 & -0.0048 \\
-0.0028 & -0.0014 & -0.0091 & -0.0042 & -0.0033 & -0.0016 & -0.0107 & 0.1336 & -0.0015 & -0.0007 & -0.0048 & -0.0022 \\
-0.0008 & -0.0004 & -0.0028 & -0.0013 & -0.0010 & -0.0005 & -0.0032 & -0.0015 & 0.0415 & -0.0002 & -0.0015 & -0.0007 \\
-0.0004 & -0.0002 & -0.0014 & -0.0006 & -0.0005 & -0.0002 & -0.0016 & -0.0007 & -0.0002 & 0.0208 & -0.0007 & -0.0003 \\
-0.0028 & -0.0014 & -0.0090 & -0.0041 & -0.0032 & -0.0016 & -0.0105 & -0.0048 & -0.0015 & -0.0007 & 0.1314 & -0.0022 \\
-0.0013 & -0.0006 & -0.0041 & -0.0019 & -0.0015 & -0.0007 & -0.0048 & -0.0022 & -0.0007 & -0.0003 & -0.0022 & 0.0618 \\
-0.5149 & 0.0676 & 0.0489 & 0.1039 & 0.1182 & -0.0019 & -0.0122 & -0.0056 & 0.1531 & -0.0008 & -0.0055 & -0.0025 \\
0.0676 & -0.2764 & 0.0343 & 0.0272 & -0.0019 & 0.0600 & -0.0061 & -0.0028 & -0.0008 & 0.0770 & -0.0028 & -0.0013 \\
0.0489 & 0.0343 & -1.1832 & 0.1425 & -0.0122 & -0.0061 & 0.3566 & -0.0183 & -0.0055 & -0.0028 & 0.4850 & -0.0083 \\
0.1039 & 0.0272 & 0.1425 & -0.7155 & -0.0056 & -0.0028 & -0.0183 & 0.1744 & -0.0025 & -0.0013 & -0.0083 & 0.2283 \\
0.1182 & -0.0019 & -0.0122 & -0.0056 & -0.6024 & 0.0786 & 0.0551 & 0.1205 & 0.1994 & -0.0010 & -0.0065 & -0.0030 \\
-0.0019 & 0.0600 & -0.0061 & -0.0028 & 0.0786 & -0.3232 & 0.0391 & 0.0314 & -0.0010 & 0.1002 & -0.0032 & -0.0015 \\
-0.0122 & -0.0061 & 0.3566 & -0.0183 & 0.0551 & 0.0391 & -1.3894 & 0.1634 & -0.0065 & -0.0032 & 0.6337 & -0.0097 \\
-0.0056 & -0.0028 & -0.0183 & 0.1744 & 0.1205 & 0.0314 & 0.1634 & -0.8376 & -0.0030 & -0.0015 & -0.0097 & 0.2977 \\
0.1531 & -0.0008 & -0.0055 & -0.0025 & 0.1994 & -0.0010 & -0.0065 & -0.0030 & -0.4817 & 0.0362 & 0.0285 & 0.0563 \\
-0.0008 & 0.0770 & -0.0028 & -0.0013 & -0.0010 & 0.1002 & -0.0032 & -0.0015 & 0.0362 & -0.2511 & 0.0195 & 0.0150 \\
-0.0055 & -0.0028 & 0.4850 & -0.0083 & -0.0065 & -0.0032 & 0.6337 & -0.0097 & 0.0285 & 0.0195 & -1.2997 & 0.0794 \\
-0.0025 & -0.0013 & -0.0083 & 0.2283 & -0.0030 & -0.0015 & -0.0097 & 0.2977 & 0.0563 & 0.0150 & 0.0794 & -0.6918
\end{array} \right),
$$

and the eigenvalues of $\mathbf{M}_1^{[2]}$ are:

$$
\begin{aligned}
&-2.0614, -1.7009, -1.1133, -1.0642, -0.9498, -0.8492, -0.7489, -0.6360, -0.5771, -0.5043, -0.4816, -0.3951, \\
&-0.3827, -0.3406, -0.3335, -0.3139, -0.2162, -0.1970, -0.1771, -0.1666, -0.1119, -0.1007, -0.0655, 0;
\end{aligned}
$$

the matrices $\mathbf{M}_1^{[3]} = L_\phi \mathbf{\Pi}^{[3]} + \tilde{\mathcal{N}}_{\Xi}^{[3]}$ defined in (10) are:

$$\mathbf{M}_1^{[3]} = \left(\begin{array}{cccccccccccc}
-1.3276 & 0.0150 & 0.1428 & 0.3462 & 0.2139 & -0.0006 & -0.0088 & -0.0081 & 0.4235 & -0.0004 & -0.0058 & -0.0053 \\
0.0150 & -0.1312 & 0.0250 & 0.0416 & -0.0006 & 0.0135 & -0.0005 & -0.0005 & -0.0004 & 0.0259 & -0.0003 & -0.0003 \\
0.1428 & 0.0250 & -1.1294 & 0.2590 & -0.0088 & -0.0005 & 0.1838 & -0.0069 & -0.0058 & -0.0003 & 0.3621 & -0.0045 \\
0.3462 & 0.0416 & 0.2590 & -1.2918 & -0.0081 & -0.0005 & -0.0069 & 0.1693 & -0.0053 & -0.0003 & -0.0045 & 0.3328 \\
0.2139 & -0.0006 & -0.0088 & -0.0081 & -0.7388 & 0.0067 & 0.0654 & 0.1508 & 0.2335 & -0.0002 & -0.0024 & -0.0022 \\
-0.0006 & 0.0135 & -0.0005 & -0.0005 & 0.0067 & -0.0665 & 0.0109 & 0.0178 & -0.0002 & 0.0142 & -0.0001 & -0.0001 \\
-0.0088 & -0.0005 & 0.1838 & -0.0069 & 0.0654 & 0.0109 & -0.6297 & 0.1133 & -0.0024 & -0.0001 & 0.1995 & -0.0019 \\
-0.0081 & -0.0005 & -0.0069 & 0.1693 & 0.1508 & 0.0178 & 0.1133 & -0.6860 & -0.0022 & -0.0001 & -0.0019 & 0.1833 \\
0.4235 & -0.0004 & -0.0058 & -0.0053 & 0.2335 & -0.0002 & -0.0024 & -0.0022 & -0.8487 & 0.0044 & 0.0438 & 0.1000 \\
-0.0004 & 0.0259 & -0.0003 & -0.0003 & -0.0002 & 0.0142 & -0.0001 & -0.0001 & 0.0044 & -0.0656 & 0.0072 & 0.0118 \\
-0.0058 & -0.0003 & 0.3621 & -0.0045 & -0.0024 & -0.0001 & 0.1995 & -0.0019 & 0.0438 & 0.0072 & -0.7237 & 0.0752 \\
-0.0053 & -0.0003 & -0.0045 & 0.3328 & -0.0022 & -0.0001 & -0.0019 & 0.1833 & 0.1000 & 0.0118 & 0.0752 & -0.7355 \\
0.3028 & -0.0015 & -0.0208 & -0.0191 & -0.0103 & -0.0006 & -0.0088 & -0.0081 & -0.0068 & -0.0004 & -0.0058 & -0.0053 \\
-0.0015 & 0.0196 & -0.0013 & -0.0012 & -0.0006 & -0.0000 & -0.0005 & -0.0005 & -0.0004 & -0.0000 & -0.0003 & -0.0003 \\
-0.0208 & -0.0013 & 0.2614 & -0.0163 & -0.0088 & -0.0005 & -0.0075 & -0.0069 & -0.0058 & -0.0003 & -0.0049 & -0.0045 \\
-0.0191 & -0.0012 & -0.0163 & 0.2413 & -0.0081 & -0.0005 & -0.0069 & -0.0063 & -0.0053 & -0.0003 & -0.0045 & -0.0042 \\
-0.0103 & -0.0006 & -0.0088 & -0.0081 & 0.1338 & -0.0003 & -0.0037 & -0.0034 & -0.0029 & -0.0002 & -0.0024 & -0.0022 \\
-0.0006 & -0.0000 & -0.0005 & -0.0005 & -0.0003 & 0.0083 & -0.0002 & -0.0002 & -0.0002 & -0.0000 & -0.0001 & -0.0001 \\
-0.0088 & -0.0005 & -0.0075 & -0.0069 & -0.0037 & -0.0002 & 0.1147 & -0.0029 & -0.0024 & -0.0001 & -0.0021 & -0.0019 \\
-0.0081 & -0.0005 & -0.0069 & -0.0063 & -0.0034 & -0.0002 & -0.0029 & 0.1055 & -0.0022 & -0.0001 & -0.0019 & -0.0018 \\
-0.0068 & -0.0004 & -0.0058 & -0.0053 & -0.0029 & -0.0002 & -0.0024 & -0.0022 & 0.0890 & -0.0001 & -0.0016 & -0.0015 \\
-0.0004 & -0.0000 & -0.0003 & -0.0003 & -0.0002 & -0.0000 & -0.0001 & -0.0001 & -0.0001 & 0.0055 & -0.0001 & -0.0001 \\
-0.0058 & -0.0003 & -0.0049 & -0.0045 & -0.0024 & -0.0001 & -0.0021 & -0.0019 & -0.0016 & -0.0001 & 0.0762 & -0.0013 \\
-0.0053 & -0.0003 & -0.0045 & -0.0042 & -0.0022 & -0.0001 & -0.0019 & -0.0018 & -0.0015 & -0.0001 & -0.0013 & 0.0700
\end{array} \right.$$

$$\left. \begin{array}{cccccccccccc}
0.3028 & -0.0015 & -0.0208 & -0.0191 & -0.0103 & -0.0006 & -0.0088 & -0.0081 & -0.0068 & -0.0004 & -0.0058 & -0.0053 \\
-0.0015 & 0.0196 & -0.0013 & -0.0012 & -0.0006 & -0.0000 & -0.0005 & -0.0005 & -0.0004 & -0.0000 & -0.0003 & -0.0003 \\
-0.0208 & -0.0013 & 0.2614 & -0.0163 & -0.0088 & -0.0005 & -0.0075 & -0.0069 & -0.0058 & -0.0003 & -0.0049 & -0.0045 \\
-0.0191 & -0.0012 & -0.0163 & 0.2413 & -0.0081 & -0.0005 & -0.0069 & -0.0063 & -0.0053 & -0.0003 & -0.0045 & -0.0042 \\
-0.0103 & -0.0006 & -0.0088 & -0.0081 & 0.1338 & -0.0003 & -0.0037 & -0.0034 & -0.0029 & -0.0002 & -0.0024 & -0.0022 \\
-0.0006 & -0.0000 & -0.0005 & -0.0005 & -0.0003 & 0.0083 & -0.0002 & -0.0002 & -0.0002 & -0.0000 & -0.0001 & -0.0001 \\
-0.0088 & -0.0005 & -0.0075 & -0.0069 & -0.0037 & -0.0002 & 0.1147 & -0.0029 & -0.0024 & -0.0001 & -0.0021 & -0.0019 \\
-0.0081 & -0.0005 & -0.0069 & -0.0063 & -0.0034 & -0.0002 & -0.0029 & 0.1055 & -0.0022 & -0.0001 & -0.0019 & -0.0018 \\
-0.0068 & -0.0004 & -0.0058 & -0.0053 & -0.0029 & -0.0002 & -0.0024 & -0.0022 & 0.0890 & -0.0001 & -0.0016 & -0.0015 \\
-0.0004 & -0.0000 & -0.0003 & -0.0003 & -0.0002 & -0.0000 & -0.0001 & -0.0001 & -0.0001 & 0.0055 & -0.0001 & -0.0001 \\
-0.0058 & -0.0003 & -0.0049 & -0.0045 & -0.0024 & -0.0001 & -0.0021 & -0.0019 & -0.0016 & -0.0001 & 0.0762 & -0.0013 \\
-0.0053 & -0.0003 & -0.0045 & -0.0042 & -0.0022 & -0.0001 & -0.0019 & -0.0018 & -0.0015 & -0.0001 & -0.0013 & 0.0700 \\
-1.3276 & 0.0150 & 0.1428 & 0.3462 & 0.2139 & -0.0006 & -0.0088 & -0.0081 & 0.4235 & -0.0004 & -0.0058 & -0.0053 \\
0.0150 & -0.1312 & 0.0250 & 0.0416 & -0.0006 & 0.0135 & -0.0005 & -0.0005 & -0.0004 & 0.0259 & -0.0003 & -0.0003 \\
0.1428 & 0.0250 & -1.1294 & 0.2590 & -0.0088 & -0.0005 & 0.1838 & -0.0069 & -0.0058 & -0.0003 & 0.3621 & -0.0045 \\
0.3462 & 0.0416 & 0.2590 & -1.2918 & -0.0081 & -0.0005 & -0.0069 & 0.1693 & -0.0053 & -0.0003 & -0.0045 & 0.3328 \\
0.2139 & -0.0006 & -0.0088 & -0.0081 & -0.7388 & 0.0067 & 0.0654 & 0.1508 & 0.2335 & -0.0002 & -0.0024 & -0.0022 \\
-0.0006 & 0.0135 & -0.0005 & -0.0005 & 0.0067 & -0.0665 & 0.0109 & 0.0178 & -0.0002 & 0.0142 & -0.0001 & -0.0001 \\
-0.0088 & -0.0005 & 0.1838 & -0.0069 & 0.0654 & 0.0109 & -0.6297 & 0.1133 & -0.0024 & -0.0001 & 0.1995 & -0.0019 \\
-0.0081 & -0.0005 & -0.0069 & 0.1693 & 0.1508 & 0.0178 & 0.1133 & -0.6860 & -0.0022 & -0.0001 & -0.0019 & 0.1833 \\
0.4235 & -0.0004 & -0.0058 & -0.0053 & 0.2335 & -0.0002 & -0.0024 & -0.0022 & -0.8487 & 0.0044 & 0.0438 & 0.1000 \\
-0.0004 & 0.0259 & -0.0003 & -0.0003 & -0.0002 & 0.0142 & -0.0001 & -0.0001 & 0.0044 & -0.0656 & 0.0072 & 0.0118 \\
-0.0058 & -0.0003 & 0.3621 & -0.0045 & -0.0024 & -0.0001 & 0.1995 & -0.0019 & 0.0438 & 0.0072 & -0.7237 & 0.0752 \\
-0.0053 & -0.0003 & -0.0045 & 0.3328 & -0.0022 & -0.0001 & -0.0019 & 0.1833 & 0.1000 & 0.0118 & 0.0752 & -0.7355
\end{array} \right),$$

and the eigenvalues of $\mathbf{M}_1^{[3]}$ are:

$$\begin{aligned}
&-2.1091, -1.7927, -1.5835, -1.3025, -1.2451, -1.1834, -1.0418, -0.9698, -0.8666, -0.8213, -0.7818, -0.6388, \\
&-0.5214, -0.5023, -0.4060, -0.2360, -0.2217, -0.1521, -0.1234, -0.0861, -0.0710, -0.0477, -0.0449, 0.
\end{aligned}$$

Then, according to Theorem 1, $\max_d \lambda_2(\mathbf{M}_1^{[d]}) = \max(-0.0718, -0.0655, -0.0449) = -0.0449 < 0$, thus complete Bi-Syn would be realized exponentially.

III. Matricies in Simulation 3

The matrices in this simulation are the same as Simulation 1. Suppose only the first node on first layer in the first block is pinned, and the coupling strengths are set as $\rho_1 = \rho_2 = \rho_3 = 50$, the control strength is set as $\nu = 500$, then $\mathcal{U}_c$ defined in (16) is $\mathcal{U}_c = \mathrm{diag}(500, 0, 0, \ldots, 0) \in \mathbb{R}^{12}$, and $\hat{\mathcal{N}}^{[d]} = \rho_1 \tilde{\mathbb{H}}^{[d]} + \rho_2 \tilde{\mathbb{G}}^{[d]} + \rho_3 \tilde{\mathbb{F}}^{[d]} - \mathcal{U}_c$.

Since $L_\phi = 2.0528$, the $\mathbf{M}_2^{[1]} = L_\phi \Xi^{[1]} + \hat{\mathcal{N}}_\Xi^{[1]}$ are

$$\mathbf{M}_2^{[1]} = \begin{pmatrix}
-13.2473 & 2.7746 & 0.6218 & 0.7298 & 1.8842 & 0 & 0 & 0 & 1.0363 & 0 & 0 & 0 \\
2.7746 & -29.8299 & 3.1468 & 0.1742 & 0 & 7.4817 & 0 & 0 & 0 & 8.4503 & 0 & 0 \\
0.6218 & 3.1468 & -33.3791 & 2.4056 & 0 & 0 & 4.4628 & 0 & 0 & 0 & 11.6769 & 0 \\
0.7298 & 0.1742 & 2.4056 & -6.6049 & 0 & 0 & 0 & 0.8709 & 0 & 0 & 0 & 0.6968 \\
1.8842 & 0 & 0 & 0 & -9.5838 & 1.3632 & 0.9610 & 1.2374 & 0.9610 & 0 & 0 & 0 \\
0 & 7.4817 & 0 & 0 & 1.3632 & -30.2849 & 3.6124 & 3.7121 & 0 & 6.9500 & 0 & 0 \\
0 & 0 & 4.4628 & 0 & 0.9610 & 3.6124 & -50.4084 & 9.0280 & 0 & 0 & 14.4366 & 0 \\
0 & 0 & 0 & 0.8709 & 1.2374 & 3.7121 & 9.0280 & -32.0699 & 0 & 0 & 0 & 4.9495 \\
1.0363 & 0 & 0 & 0 & 0.9610 & 0 & 0 & 0 & -30.8224 & 4.6074 & 5.1547 & 6.2821 \\
0 & 8.4503 & 0 & 0 & 0 & 6.9500 & 0 & 0 & 4.6074 & -40.5179 & 4.4348 & 6.0084 \\
0 & 0 & 11.6769 & 0 & 0 & 0 & 14.4366 & 0 & 5.1547 & 4.4348 & -81.5632 & 22.0104 \\
0 & 0 & 0 & 0.6968 & 0 & 0 & 0 & 4.9495 & 6.2821 & 6.0084 & 22.0104 & -89.4709 \\
2.0725 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 7.8670 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 11.1569 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.7419 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.2034 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 7.2248 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 18.0559 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.3736 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.8868 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.1503 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 24.0472 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 49.9339 \\
2.0725 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 7.8670 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 11.1569 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.7419 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.2034 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 7.2248 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 18.0559 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.3736 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.8868 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.1503 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 24.0472 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 49.9339 \\
-5.9779 & 1.5414 & 0.3454 & 0.4055 & 1.0468 & 0 & 0 & 0 & 0.5757 & 0 & 0 & 0 \\
1.5414 & -20.0686 & 1.7482 & 0.0968 & 0 & 4.1565 & 0 & 0 & 0 & 4.6946 & 0 & 0 \\
0.3454 & 1.7482 & -23.5026 & 1.3364 & 0 & 0 & 2.4793 & 0 & 0 & 0 & 6.4872 & 0 \\
0.4055 & 0.0968 & 1.3364 & -4.4435 & 0 & 0 & 0 & 0.4839 & 0 & 0 & 0 & 0.3871 \\
1.0468 & 0 & 0 & 0 & -6.7481 & 0.7573 & 0.5339 & 0.6874 & 0.5339 & 0 & 0 & 0 \\
0 & 4.1565 & 0 & 0 & 0.7573 & -20.0359 & 2.0069 & 2.0623 & 0 & 3.8611 & 0 & 0 \\
0 & 0 & 2.4793 & 0 & 0.5339 & 2.0069 & -36.0295 & 5.0155 & 0 & 0 & 8.0203 & 0 \\
0 & 0 & 0 & 0.4839 & 0.6874 & 2.0623 & 5.0155 & -23.3160 & 0 & 0 & 0 & 2.7497 \\
0.5757 & 0 & 0 & 0 & 0.5339 & 0 & 0 & 0 & -22.8510 & 2.5597 & 2.8637 & 3.4900 \\
0 & 4.6946 & 0 & 0 & 0 & 3.8611 & 0 & 0 & 2.5597 & -27.0212 & 2.4638 & 3.3380 \\
0 & 0 & 6.4872 & 0 & 0 & 0 & 8.0203 & 0 & 2.8637 & 2.4638 & -56.0005 & 12.2280 \\
0 & 0 & 0 & 0.3871 & 0 & 0 & 0 & 2.7497 & 3.4900 & 3.3380 & 12.2280 & -71.8989
\end{pmatrix},$$

and the eigenvalues of $\mathbf{M}_2^{[1]}$ are:

$$-140.3306, -95.5296, -63.2703, -55.0019, -51.5197, -43.6114, -38.8913, -37.4673, -36.7901, -28.8680,$$
$$-26.3217, -23.6743, -21.9577, -21.3501, -14.7542, -13.8427, -11.1661, -10.6161, -9.6770, -7.3445,$$
$$-5.8989, -4.2851, -3.4811, -0.0267;$$

the $\mathbf{M}_2^{[2]} = L_\phi \Xi^{[2]} + \hat{\mathcal{N}}_\Xi^{[2]}$ are

$$\mathbf{M}_2^{[2]} = \begin{pmatrix}
-12.9592 & 2.2858 & 0.6488 & 0.8283 & 1.6441 & 0 & 0 & 0 & 1.5140 & 0 & 0 & 0 \\
2.2858 & -18.4820 & 2.4710 & 0.2040 & 0 & 4.4192 & 0 & 0 & 0 & 4.2106 & 0 & 0 \\
0.6488 & 2.4710 & -24.2499 & 2.2318 & 0 & 0 & 3.0416 & 0 & 0 & 0 & 7.8288 & 0 \\
0.8283 & 0.2040 & 2.2318 & -6.1034 & 0 & 0 & 0 & 1.0200 & 0 & 0 & 0 & 0.2040 \\
1.6441 & 0 & 0 & 0 & -6.9462 & 1.1391 & 0.7464 & 1.1977 & 0.2488 & 0 & 0 & 0 \\
0 & 4.4192 & 0 & 0 & 1.1391 & -21.7579 & 3.2074 & 3.5932 & 0 & 4.3197 & 0 & 0 \\
0 & 0 & 3.0416 & 0 & 0.7464 & 3.2074 & -35.2271 & 7.3642 & 0 & 0 & 9.2057 & 0 \\
0 & 0 & 0 & 1.0200 & 1.1977 & 3.5932 & 7.3642 & -26.2520 & 0 & 0 & 0 & 3.5932 \\
1.5140 & 0 & 0 & 0 & 0.2488 & 0 & 0 & 0 & -27.2713 & 4.8837 & 4.9770 & 5.7960 \\
0 & 4.2106 & 0 & 0 & 0 & 4.3197 & 0 & 0 & 4.8837 & -33.4346 & 4.7888 & 5.7494 \\
0 & 0 & 7.8288 & 0 & 0 & 0 & 9.2057 & 0 & 4.9770 & 4.7888 & -66.8239 & 21.0715 \\
0 & 0 & 0 & 0.2040 & 0 & 0 & 0 & 3.5932 & 5.7960 & 5.7494 & 21.0715 & -72.4544 \\
1.7303 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4.9421 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 8.1111 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.6320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.9905 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5.1319 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 11.7827 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.5819 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.9540 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.5809 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.1487 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 36.4141 \\
1.7303 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4.9421 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 8.1111 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.6320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.9905 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5.1319 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 11.7827 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.5819 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.9540 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.5809 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.1487 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 36.4141 \\
-6.3324 & 1.5239 & 0.4326 & 0.5522 & 1.0961 & 0 & 0 & 0 & 1.0093 & 0 & 0 & 0 \\
1.5239 & -13.9687 & 1.6474 & 0.1360 & 0 & 2.9461 & 0 & 0 & 0 & 2.8071 & 0 & 0 \\
0.4326 & 1.6474 & -18.8703 & 1.4878 & 0 & 0 & 2.0278 & 0 & 0 & 0 & 5.2192 & 0 \\
0.5522 & 0.1360 & 1.4878 & -4.6129 & 0 & 0 & 0 & 0.6800 & 0 & 0 & 0 & 0.1360 \\
1.0961 & 0 & 0 & 0 & -5.2943 & 0.7594 & 0.4976 & 0.7985 & 0.1659 & 0 & 0 & 0 \\
0 & 2.9461 & 0 & 0 & 0.7594 & -16.2159 & 2.1383 & 2.3955 & 0 & 2.8798 & 0 & 0 \\
0 & 0 & 2.0278 & 0 & 0.4976 & 2.1383 & -27.4123 & 4.9094 & 0 & 0 & 6.1371 & 0 \\
0 & 0 & 0 & 0.6800 & 0.7985 & 2.3955 & 4.9094 & -20.6953 & 0 & 0 & 0 & 2.3955 \\
1.0093 & 0 & 0 & 0 & 0.1659 & 0 & 0 & 0 & -21.4989 & 3.2558 & 3.3180 & 3.8640 \\
0 & 2.8071 & 0 & 0 & 0 & 2.8798 & 0 & 0 & 3.2558 & -25.4834 & 3.1925 & 3.8329 \\
0 & 0 & 5.2192 & 0 & 0 & 0 & 6.1371 & 0 & 3.3180 & 3.1925 & -50.9322 & 14.0476 \\
0 & 0 & 0 & 0.1360 & 0 & 0 & 0 & 2.3955 & 3.8640 & 3.8329 & 14.0476 & -60.4409
\end{pmatrix},$$

and the eigenvalues of $\mathbf{M}_2^{[2]}$ are:

$$
\begin{aligned}
&-114.1354, -74.4553, -52.1771, -46.3436, -43.1885, -33.3038, -31.5761, -28.6410, -25.7231, -25.4635,\\
&-22.2940, -20.7878, -17.3148, -16.9543, -13.3903, -11.8337, -9.8665, -8.6356, -7.7788, -6.8403,\\
&-5.7663, -3.7879, -3.4337, -0.0282;
\end{aligned}
$$

the $\mathbf{M}_2^{[3]} = L_\phi \Xi^{[3]} + \hat{\mathcal{N}}_{\Xi}^{[3]}$ are

$$
\mathbf{M}_2^{[3]} = \begin{pmatrix}
-10.0225 & 0.7230 & 0.5577 & 0.9223 & 1.5148 & 0 & 0 & 0 & 1.4871 & 0 & 0 & 0 & 1.1153 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.7230 & -5.7717 & 1.1567 & 0.0917 & 0 & 0.4338 & 0 & 0 & 0 & 2.5109 & 0 & 0 & 0 & 0.8675 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.5577 & 1.1567 & -4.5839 & 0.9127 & 0 & 0 & 0.4926 & 0 & 0 & 0 & 0.4926 & 0 & 0 & 0 & 0.9852 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.9223 & 0.0917 & 0.9127 & -2.9284 & 0 & 0 & 0 & 0.4587 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5505 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1.5148 & 0 & 0 & 0 & -9.9398 & 3.4102 & 1.6612 & 1.0552 & 0.3322 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.9934 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0.4338 & 0 & 0 & 3.4102 & -32.8742 & 8.2373 & 3.1655 & 0 & 12.2109 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.4916 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0.4926 & 0 & 1.6612 & 8.2373 & -27.6148 & 6.9274 & 0 & 0 & 3.4637 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.9274 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0.4587 & 1.0552 & 3.1655 & 6.9274 & -21.0166 & 0 & 0 & 0 & 3.1655 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.3310 & 0 & 0 & 0 & 0\\
1.4871 & 0 & 0 & 0 & 0.3322 & 0 & 0 & 0 & -35.5466 & 5.9448 & 5.9448 & 13.0426 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.9171 & 0 & 0 & 0\\
0 & 2.5109 & 0 & 0 & 0 & 12.2109 & 0 & 0 & 5.9448 & -47.1315 & 1.9507 & 10.5108 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14.1977 & 0 & 0\\
0 & 0 & 0.4926 & 0 & 0 & 0 & 3.4637 & 0 & 5.9448 & 1.9507 & -54.4588 & 31.0632 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11.7041 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.1655 & 13.0426 & 10.5108 & 31.0632 & -91.9768 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34.6692\\
1.1153 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.2290 & 0.4338 & 0.3346 & 0.5534 & 0.9089 & 0 & 0 & 0 & 0.8922 & 0 & 0 & 0\\
0 & 0.8675 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4338 & -3.8101 & 0.6940 & 0.0550 & 0 & 0.2603 & 0 & 0 & 0 & 1.5065 & 0 & 0\\
0 & 0 & 0.9852 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3346 & 0.6940 & -3.1444 & 0.5476 & 0 & 0 & 0.2955 & 0 & 0 & 0 & 0.2955 & 0\\
0 & 0 & 0 & 0.5505 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5534 & 0.0550 & 0.5476 & -1.9773 & 0 & 0 & 0 & 0.2752 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1.9934 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9089 & 0 & 0 & 0 & -6.7612 & 2.0461 & 0.9967 & 0.6331 & 0.1993 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 5.4916 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2603 & 0 & 0 & 2.0461 & -21.9211 & 4.9424 & 1.8993 & 0 & 7.3266 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 6.9274 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2955 & 0 & 0.9967 & 4.9424 & -19.3398 & 4.1564 & 0 & 0 & 2.0782 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.3310 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2752 & 0.6331 & 1.8993 & 4.1564 & -15.1423 & 0 & 0 & 0 & 1.8993\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.9171 & 0 & 0 & 0 & 0.8922 & 0 & 0 & 0 & 0.1993 & 0 & 0 & 0 & -24.8948 & 3.5669 & 3.5669 & 7.8256\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14.1977 & 0 & 0 & 0 & 1.5065 & 0 & 0 & 0 & 7.3266 & 0 & 0 & 3.5669 & -33.9580 & 1.1704 & 6.3065\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11.7041 & 0 & 0 & 0 & 0.2955 & 0 & 0 & 0 & 2.0782 & 0 & 3.5669 & 1.1704 & -37.3569 & 18.6379\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34.6692 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.8993 & 7.8256 & 6.3065 & 18.6379 & -69.0538
\end{pmatrix},
$$

and the eigenvalues of $\mathbf{M}_2^{[3]}$ are:

$$
\begin{aligned}
&-130.0679, -64.6855, -62.6795, -53.1965, -39.2578, -33.6919, -33.0387, -29.0413, -25.8719, -19.7082,\\
&-18.9186, -15.2009, -11.5528, -8.9303, -8.3310, -6.7062, -5.4528, -4.7164, -4.0231, -3.8018,\\
&-2.6753, -2.5351, -1.3674, -0.0032.
\end{aligned}
$$

Then, according to Theorem 2, $\max_d \lambda_{\max}(\mathbf{M}_2^{[d]}) = \max(-0.0267, -0.0282, -0.0032) = -0.0032 < 0$, thus Bi-Syn with nonidentical node groups for each layer under pinning control would be realized.

IV. Matricies in Simulation 4

The matrices in this simulation are the same as Simulation 2. Suppose only the first node on first layer in the first block is pinned, and the coupling strengths are set as $\rho_1 = \rho_2 = \rho_3 = 20$, the control strength is set as $\nu = 200$, then $\mathcal{U}_c$ defined in (16) is $\mathcal{U}_c = \mathrm{diag}(200, 0, 0, \ldots, 0) \in \mathbb{R}^{12}$, and $\hat{\mathcal{N}}^{[d]} = \rho_1 \tilde{\mathbb{H}}^{[d]} + \rho_2 \tilde{\mathbb{G}}^{[d]} + \rho_3 \tilde{\mathbb{F}}^{[d]} - \mathcal{U}_c$.

Since $L_\phi = 2.0528$, the $\mathbf{M}_2^{[1]} = L_\phi \Xi^{[1]} + \hat{\mathcal{N}}_\Xi^{[1]}$ are

$$\mathbf{M}_2^{[1]} = \left(\begin{array}{cccccccccccc}
-24.6173 & 1.5442 & 1.3236 & 2.4267 & 2.5455 & 0 & 0 & 0 & 3.6315 & 0 & 0 & 0 \\
1.5442 & -8.3377 & 0.8824 & 0.6618 & 0 & 1.2727 & 0 & 0 & 0 & 1.8158 & 0 & 0 \\
1.3236 & 0.8824 & -39.8559 & 3.5297 & 0 & 0 & 8.2727 & 0 & 0 & 0 & 11.8024 & 0 \\
2.4267 & 0.6618 & 3.5297 & -22.3660 & 0 & 0 & 0 & 3.8182 & 0 & 0 & 0 & 5.4473 \\
2.5455 & 0 & 0 & 0 & -11.5446 & 1.0691 & 0.9164 & 1.6800 & 2.3418 & 0 & 0 & 0 \\
0 & 1.2727 & 0 & 0 & 1.0691 & -6.0777 & 0.6109 & 0.4582 & 0 & 1.1709 & 0 & 0 \\
0 & 0 & 8.2727 & 0 & 0.9164 & 0.6109 & -29.5780 & 2.4436 & 0 & 0 & 7.6109 & 0 \\
0 & 0 & 0 & 3.8182 & 1.6800 & 0.4582 & 2.4436 & -16.4005 & 0 & 0 & 0 & 3.5127 \\
3.6315 & 0 & 0 & 0 & 2.3418 & 0 & 0 & 0 & -10.0417 & 0.6533 & 0.5600 & 1.0267 \\
0 & 1.8158 & 0 & 0 & 0 & 1.1709 & 0 & 0 & 0.6533 & -5.2075 & 0.3733 & 0.2800 \\
0 & 0 & 11.8024 & 0 & 0 & 0 & 7.6109 & 0 & 0.5600 & 0.3733 & -27.7821 & 1.4933 \\
0 & 0 & 0 & 5.4473 & 0 & 0 & 0 & 3.5127 & 1.0267 & 0.2800 & 1.4933 & -14.5025 \\
4.4121 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2.2061 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 14.3394 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 6.6182 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.0545 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.5273 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 9.9273 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.5818 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.8667 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9333 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.0667 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.8000
\end{array}\right.$$

$$\left.\begin{array}{cccccccccccc}
4.4121 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2.2061 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 14.3394 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 6.6182 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.0545 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.5273 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 9.9273 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.5818 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.8667 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9333 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.0667 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.8000 \\
-12.5414 & 1.1030 & 0.9455 & 1.7333 & 1.8182 & 0 & 0 & 0 & 2.5939 & 0 & 0 & 0 \\
1.1030 & -6.5858 & 0.6303 & 0.4727 & 0 & 0.9091 & 0 & 0 & 0 & 1.2970 & 0 & 0 \\
0.9455 & 0.6303 & -32.5655 & 2.5212 & 0 & 0 & 5.9091 & 0 & 0 & 0 & 8.4303 & 0 \\
1.7333 & 0.4727 & 2.5212 & -17.8666 & 0 & 0 & 0 & 2.7273 & 0 & 0 & 0 & 3.8909 \\
1.8182 & 0 & 0 & 0 & -9.1188 & 0.7636 & 0.6545 & 1.2000 & 1.6727 & 0 & 0 & 0 \\
0 & 0.9091 & 0 & 0 & 0.7636 & -4.7776 & 0.4364 & 0.3273 & 0 & 0.8364 & 0 & 0 \\
0 & 0 & 5.9091 & 0 & 0.6545 & 0.4364 & -23.9635 & 1.7455 & 0 & 0 & 5.4364 & 0 \\
0 & 0 & 0 & 2.7273 & 1.2000 & 0.3273 & 1.7455 & -13.0237 & 0 & 0 & 0 & 2.5091 \\
2.5939 & 0 & 0 & 0 & 1.6727 & 0 & 0 & 0 & -7.7060 & 0.4667 & 0.4000 & 0.7333 \\
0 & 1.2970 & 0 & 0 & 0 & 0.8364 & 0 & 0 & 0.4667 & -3.9863 & 0.2667 & 0.2000 \\
0 & 0 & 8.4303 & 0 & 0 & 0 & 5.4364 & 0 & 0.4000 & 0.2667 & -21.5777 & 1.0667 \\
0 & 0 & 0 & 3.8909 & 0 & 0 & 0 & 2.5091 & 0.7333 & 0.2000 & 1.0667 & -11.1589
\end{array}\right),$$

and the eigenvalues of $\mathbf{M}_2^{[1]}$ are:

$$\begin{aligned}
&-57.0744, -41.9999, -30.7017, -30.1415, -25.9798, -23.1054, -21.9436, -21.0660, -16.6216, -14.7055,\\
&-12.8300, -11.9800, -11.7926, -10.4264, -8.7811, -8.2656, -7.4546, -5.8235, -5.5672, -4.4517, -4.1420,\\
&-4.0346, -2.1642, -0.1309;
\end{aligned}$$

the $\mathbf{M}_2^{[2]} = L_\phi \Xi^{[2]} + \hat{\mathcal{N}}_\Xi^{[2]}$ are

$$\mathbf{M}_2^{[2]} = \left(\begin{array}{cccccccccccc}
-10.6309 & 0.6917 & 0.5929 & 1.0869 & 1.2190 & 0 & 0 & 0 & 1.5476 & 0 & 0 & 0 \\
0.6917 & -3.5369 & 0.3952 & 0.2964 & 0 & 0.6095 & 0 & 0 & 0 & 0.7738 & 0 & 0 \\
0.5929 & 0.3952 & -16.5670 & 1.5810 & 0 & 0 & 3.9619 & 0 & 0 & 0 & 5.0298 & 0 \\
1.0869 & 0.2964 & 1.5810 & -9.4249 & 0 & 0 & 0 & 1.8286 & 0 & 0 & 0 & 2.3214 \\
1.2190 & 0 & 0 & 0 & -7.8050 & 0.8083 & 0.6929 & 1.2702 & 2.0143 & 0 & 0 & 0 \\
0 & 0.6095 & 0 & 0 & 0.8083 & -4.1334 & 0.4619 & 0.3464 & 0 & 1.0071 & 0 & 0 \\
0 & 0 & 3.9619 & 0 & 0.6929 & 0.4619 & -19.3614 & 1.8476 & 0 & 0 & 6.5464 & 0 \\
0 & 0 & 0 & 1.8286 & 1.2702 & 0.3464 & 1.8476 & -11.0146 & 0 & 0 & 0 & 3.0214 \\
1.5476 & 0 & 0 & 0 & 2.0143 & 0 & 0 & 0 & -5.6356 & 0.3667 & 0.3143 & 0.5762 \\
0 & 0.7738 & 0 & 0 & 0 & 1.0071 & 0 & 0 & 0.3667 & -2.9226 & 0.2095 & 0.1571 \\
0 & 0 & 5.0298 & 0 & 0 & 0 & 6.5464 & 0 & 0.3143 & 0.2095 & -15.5920 & 0.8381 \\
0 & 0 & 0 & 2.3214 & 0 & 0 & 0 & 3.0214 & 0.5762 & 0.1571 & 0.8381 & -8.1392 \\
1.5810 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.7905 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5.1381 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2.3714 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.8476 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.9238 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6.0048 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7714 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8381 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4190 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7238 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.2571
\end{array}\right.$$

$$\left.\begin{array}{cccccccccccc}
1.5810 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.7905 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5.1381 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2.3714 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.8476 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.9238 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 6.0048 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7714 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8381 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4190 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7238 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.2571 \\
-11.7760 & 1.3833 & 1.1857 & 2.1738 & 2.4381 & 0 & 0 & 0 & 3.0952 & 0 & 0 & 0 \\
1.3833 & -6.2832 & 0.7905 & 0.5929 & 0 & 1.2190 & 0 & 0 & 0 & 1.5476 & 0 & 0 \\
1.1857 & 0.7905 & -27.9958 & 3.1619 & 0 & 0 & 7.9238 & 0 & 0 & 0 & 10.0595 & 0 \\
2.1738 & 0.5929 & 3.1619 & -16.4783 & 0 & 0 & 0 & 3.6571 & 0 & 0 & 0 & 4.6429 \\
2.4381 & 0 & 0 & 0 & -13.7623 & 1.6167 & 1.3857 & 2.5405 & 4.0286 & 0 & 0 & 0 \\
0 & 1.2190 & 0 & 0 & 1.6167 & -7.3431 & 0.9238 & 0.6929 & 0 & 2.0143 & 0 & 0 \\
0 & 0 & 7.9238 & 0 & 1.3857 & 0.9238 & -32.7180 & 3.6952 & 0 & 0 & 13.0929 & 0 \\
0 & 0 & 0 & 3.6571 & 2.5405 & 0.6929 & 3.6952 & -19.2578 & 0 & 0 & 0 & 6.0429 \\
3.0952 & 0 & 0 & 0 & 4.0286 & 0 & 0 & 0 & -10.4332 & 0.7333 & 0.6286 & 1.1524 \\
0 & 1.5476 & 0 & 0 & 0 & 2.0143 & 0 & 0 & 0.7333 & -5.4261 & 0.4190 & 0.3143 \\
0 & 0 & 10.0595 & 0 & 0 & 0 & 13.0929 & 0 & 0.6286 & 0.4190 & -28.4602 & 1.6762 \\
0 & 0 & 0 & 4.6429 & 0 & 0 & 0 & 6.0429 & 1.1524 & 0.3143 & 1.6762 & -15.0212
\end{array}\right),$$

and the eigenvalues of $\mathbf{M}_2^{[2]}$ are:

$$
\begin{aligned}
&-45.4428, -38.4642, -24.4552, -23.5140, -21.0096, -19.5663, -16.4723, -14.3814, -13.0428, -12.5415,\\
&-11.6819, -9.5423, -8.7230, -8.3957, -8.2194, -7.2673, -5.4320, -4.9334, -4.4193, -3.8712, -3.6396,\\
&-2.8687, -1.8243, -0.0103;
\end{aligned}
$$

the $\mathbf{M}_2^{[3]} = L_\phi \Xi^{[3]} + \hat{\mathcal{N}}_\Xi^{[3]}$ are

$$
\mathbf{M}_2^{[3]} = \left(\begin{array}{cccccccccccccccccccccccc}
-52.1338 & 0.3285 & 3.2724 & 7.3069 & 4.4843 & 0 & 0 & 0 & 8.6051 & 0 & 0 & 0 & 6.5447 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.3285 & -2.8777 & 0.5257 & 0.8542 & 0 & 0.2701 & 0 & 0 & 0 & 0.5184 & 0 & 0 & 0 & 0.3943 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3.2724 & 0.5257 & -25.8617 & 5.5065 & 0 & 0 & 3.8252 & 0 & 0 & 0 & 7.3402 & 0 & 0 & 0 & 5.5827 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
7.3069 & 0.8542 & 5.5065 & -28.8684 & 0 & 0 & 0 & 3.5118 & 0 & 0 & 0 & 6.7389 & 0 & 0 & 0 & 5.1254 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4.4843 & 0 & 0 & 0 & -16.4854 & 0.1387 & 1.3817 & 3.0852 & 4.7267 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.7633 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0.2701 & 0 & 0 & 0.1387 & -1.4370 & 0.2220 & 0.3607 & 0 & 0.2847 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1665 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 3.8252 & 0 & 1.3817 & 0.2220 & -14.0622 & 2.3250 & 0 & 0 & 4.0320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.3571 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 3.5118 & 3.0852 & 0.3607 & 2.3250 & -15.0743 & 0 & 0 & 0 & 3.7017 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.1640 & 0 & 0 & 0 & 0\\
8.6051 & 0 & 0 & 0 & 4.7267 & 0 & 0 & 0 & -18.1175 & 0.0913 & 0.9090 & 2.0297 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.8180 & 0 & 0 & 0\\
0 & 0.5184 & 0 & 0 & 0 & 0.2847 & 0 & 0 & 0.0913 & -1.3835 & 0.1460 & 0.2373 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1095 & 0 & 0\\
0 & 0 & 7.3402 & 0 & 0 & 0 & 4.0320 & 0 & 0.9090 & 0.1460 & -15.4545 & 1.5296 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.5508 & 0\\
0 & 0 & 0 & 6.7389 & 0 & 0 & 0 & 3.7017 & 2.0297 & 0.2373 & 1.5296 & -15.6122 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.4237\\
6.5447 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -30.3181 & 0.3285 & 3.2724 & 7.3069 & 4.4843 & 0 & 0 & 0 & 8.6051 & 0 & 0 & 0\\
0 & 0.3943 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3285 & -2.8777 & 0.5257 & 0.8542 & 0 & 0.2701 & 0 & 0 & 0 & 0.5184 & 0 & 0\\
0 & 0 & 5.5827 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.2724 & 0.5257 & -25.8617 & 5.5065 & 0 & 0 & 3.8252 & 0 & 0 & 0 & 7.3402 & 0\\
0 & 0 & 0 & 5.1254 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.3069 & 0.8542 & 5.5065 & -28.8684 & 0 & 0 & 0 & 3.5118 & 0 & 0 & 0 & 6.7389\\
0 & 0 & 0 & 0 & 2.7633 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.4843 & 0 & 0 & 0 & -16.4854 & 0.1387 & 1.3817 & 3.0852 & 4.7267 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.1665 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2701 & 0 & 0 & 0.1387 & -1.4370 & 0.2220 & 0.3607 & 0 & 0.2847 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 2.3571 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.8252 & 0 & 1.3817 & 0.2220 & -14.0622 & 2.3250 & 0 & 0 & 4.0320 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.1640 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.5118 & 3.0852 & 0.3607 & 2.3250 & -15.0743 & 0 & 0 & 0 & 3.7017\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.8180 & 0 & 0 & 0 & 8.6051 & 0 & 0 & 0 & 4.7267 & 0 & 0 & 0 & -18.1175 & 0.0913 & 0.9090 & 2.0297\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1095 & 0 & 0 & 0 & 0.5184 & 0 & 0 & 0 & 0.2847 & 0 & 0 & 0.0913 & -1.3835 & 0.1460 & 0.2373\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.5508 & 0 & 0 & 0 & 7.3402 & 0 & 0 & 0 & 4.0320 & 0 & 0.9090 & 0.1460 & -15.4545 & 1.5296\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.4237 & 0 & 0 & 0 & 6.7389 & 0 & 0 & 0 & 3.7017 & 2.0297 & 0.2373 & 1.5296 & -15.6122
\end{array}\right),
$$

and the eigenvalues of $\mathbf{M}_2^{[3]}$ are:

$$
\begin{aligned}
&-58.4289, -40.9372, -37.3061, -30.3065, -28.0805, -25.4497, -22.2070, -20.9684, -19.8709, -17.9844,\\
&-17.1808, -14.5766, -12.5239, -11.7637, -9.9565, -7.0814, -6.2478, -3.3615, -2.7368, -1.8194, -1.5229,\\
&-1.2070, -1.0695, -0.3333.
\end{aligned}
$$

Then, according to Theorem 2, $\max_d \lambda_{\max}(\mathbf{M}_2^{[d]}) = \max(-0.1309, -0.0103, -0.3333) = -0.0103 < 0$, thus complete Bi-Syn under pinning control would be realized.