

# An Algorithm of Color Image Mosaic Based on Fourier-Mellin Transform

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**Abstract.** The article proposes an algorithm of color image mosaic based on Fourier-Mellin transform. The algorithm need not accurately control the motion of camera, and need not know the internal camera parameters and detect image feature. The algorithm firstly extracts a color parameter from two color images on the existence of rotation, scaling and translation transform, and then emendates them by Fourier-Mellin algorithm and phase correlation algorithm, finally completes image mosaic after compensating on rotation and proportion zoom. The experimental result indicates that registration precision of the algorithm can reach one pixel level.

**Keywords:** Fourier-Mellin transform, image mosaic, rotation, color parameter.

## 1 Introduction

Image registration is discovering the relations of relative position between the images by using their mutual scenery in the images to compare with and match. These images are obtained from different view angle and an identical scenery in the different time by using different detector. Its goal is eliminating inconsistency in the geometry between the preparative matching image and the reference image, including translation, rotation and scaling, and preparing for the next image processing. Image mosaic is that merges two or many images having overlap region with a big image. The precision of image registration is the most immediate influence on the success of image mosaic.

The methods of image registration may be approximately divided into two kinds ones, which are based on image feature and based on gray-level image. The registration method based on image feature need seek for a series of feature points, then establish feature points' corresponding relationships, and calculate their coordinate transform relations between these images preparing for mosaic. Its shortcoming lies in lacking the adaptability, and is only suitable to such images having the corresponding image feature, but some images do not possibly have such feature. Moreover the computation quantity is rather big, the algorithmic reliability and astringency may have certain problem. The method of image registration based on the gray-level image studied quite early. It takes the foundation of image registration by containing target area of the corresponding images and searching gray-level of pixels the area. It uses some

correlated measure to determine the corresponding pixels in two images, such as covariance or correlative coefficient's maximum value. In 1994, Q.S. Chen et al. proposed the method of image registration based on Fourier-Mellin transform [1-2] that is a kind of classical algorithm. It is by calculating the corresponding position of peak value from which uses inverse Fourier transform of power spectrum of two images to obtain, so as to seek their relative displacement; and by carrying on the logarithm-polar coordinate transform to scope spectrum of the images, i.e., obtains the relative rotation angle and the scale factor similar to the algorithm of calculating the relative displacement in the logarithm-polar coordinate space; thus realizes the image matching. The Fourier-Mellin transform algorithm need not detect image feature, and its request is not very high to photographer and photography condition. Moreover its computation speed is faster and it is strong practicability.

This article proposes one kind of color image mosaic algorithm based on Fourier-Mellin transform. The color image has rich color information. If we transform two color images preparing for mosaic to gray-level images, which use gray value to substitute for tricolor, will lose a large number of useful information. In the very big similar region, it will result in mismatch that the corresponding pixels' tricolor value can be possibly completely identical. Therefore we retain the character of tricolor, and separately pick up a corresponding color component to carry on mosaic in two images, the precision may achieve highly. The algorithm in this article firstly extracts a component in color components of R, G, B from color images, and gains the characteristics of rotation and proportion; secondly, uses the correlation function according to these characteristics to compute two images' parameters of rotation and scale transform in identical scene; then carries on the compensation in two images' rotation angle and zoom; finally, still uses the correlation function to determinate translation parameter. So it completes the entire image mosaic process. In following content, we firstly give the theory of image registration based on Fourier-Mellin transform, then give the algorithm process of panoramic image mosaic, finally carry on the mosaic experiment and the discussion.

## 2 The Principle of Image Registration Based on Fourier-Mellin Transform

### 2.1 Phase Correlation Theory [3]

Suppose two separate images  $f_1$  and  $f_2$  are relative in the spatial field only with displacement  $(x_0, y_0)$ , i.e.,

$$f_1(x, y) = f_2(x - x_0, y - y_0) \quad (1)$$

The corresponding Fourier transforms  $F_1$  and  $F_2$  will be relative, i.e.,

$$F_2(\xi, \eta) = e^{-j2\pi(\xi x_0 + \eta y_0)} F_1(\xi, \eta) \quad (2)$$

The cross-power spectrum of two images  $f_1$  and  $f_2$  is

$$\frac{F_1(\xi, \eta) F_2^*(\xi, \eta)}{|F_1(\xi, \eta) F_2^*(\xi, \eta)|} = e^{-j2\pi(\xi x_0 + \eta y_0)} \quad (3)$$

Where  $F^*$  expresses the complex conjugate of  $F$ . Carry on the inverse Fourier transform to the formula (3), and produce function  $\delta$  in the place  $(x_0, y_0)$ . After measuring the location of the peak of the inverse Fourier transform of the cross-power spectrum phase about the formula (3), we will obtain the displacement  $(x_0, y_0)$  between two images.

## 2.2 Computing Relative Rotation Angle and Scale Factor [4] Using Fourier-Mellin Transform between Two Images

If two matching images are respectively  $f_1$  and  $f_2$ , which  $f_2$  is a replica of  $f_1$  with displacement  $(x_0, y_0)$  and rotation angle  $\theta_0$ , we have the following equation

$$f_2(x, y) = f_1[a(x \cos \theta_0 + y \sin \theta_0) - x_0, a(-x \sin \theta_0 + y \cos \theta_0) - y_0] \quad (4)$$

In the equation (4),  $a$  is scale factor of two images. According to the Fourier translation property and the Fourier rotation property, we get the following equation

$$F_2(\xi, \eta) = a^{-2} [F_1[a^{-1}(\xi \cos \theta_0 + \eta \sin \theta_0), a^{-1}(-\xi \sin \theta_0 + \eta \cos \theta_0)] \exp\{-j\varphi_{f_2}(\xi, \eta)\}] \quad (5)$$

In the equation (5),  $\varphi_{f_2}(\xi, \eta)$  is the spectrum phase of  $f_2$ , whose value mainly depends on translation, rotation and scale, etc. Therefore, by taking module to the equation (5), we get their relation of the Fourier power spectrum, i.e.,

$$|F_2(\xi, \eta)| = a^{-2} |F_1[a^{-1}(\xi \cos \theta_0 + \eta \sin \theta_0), a^{-1}(-\xi \sin \theta_0 + \eta \cos \theta_0)]| \quad (6)$$

The equation (6) has invariable translation property, i.e., revolving an angle to a figure causes also the power spectrum to revolve a same angle, simultaneously the scale factor  $a$  changes power spectrum  $a^{-1}$  times. Another interesting property is that spectrum center  $\xi = \eta = 0$  is invariable on rotation and scaling. So as to shift rotation and scale transform to translation form, we first carry on the polar coordinate transform to the frequency spectrum. The polar coordinate transform can turn the original coordinate  $(\xi, \eta)$  to  $(\rho, \theta)$ . Therefore, we get the next relations:

$$\begin{cases} a^{-1}(\xi \cos \theta_0 + \eta \sin \theta_0) = \rho/a \cos(\theta - \theta_0) \\ a^{-1}(-\xi \sin \theta_0 + \eta \cos \theta_0) = \rho/a \sin(\theta - \theta_0) \end{cases} \quad (7)$$

If make  $S(\rho, \theta) = |F_2(\rho \cos \theta, \rho \sin \theta)|$  and  $R(\rho, \theta) = |F_1(\rho \cos \theta, \rho \sin \theta)|$ , the formula (8) may be gotten with the above deduction.

$$S(\rho, \theta) = a^{-2} R(\rho/a, \theta - \theta_0) \quad (8)$$

Therefore, the revolving difference is converted to the translation difference by the above polar coordinate transform. But scale factor's transform need also take the

logarithmic transform to achieve. If make  $\lambda = \log \rho$  and  $k = \log a$ , the formula (8) becomes

$$S_l(\log \rho, \theta) = a^{-2} R_l(\log \rho - \log a, \theta - \theta_0) \longrightarrow S_l(\lambda, \theta) = a^{-2} R_l(\lambda - k, \theta - \theta_0) \quad (9)$$

In the formula (9), the expression with the subscript  $l$  is the one by logarithmic transform. So the formula (9) is called Fourier-Mellin transform, which  $R_l$  is called the Fourier-Mellin invariable descriptor of  $f_1$ . By the above transform, the factors of rotation and scale are similarly converted into translation form. Reusing the formula (2), (3) we may calculate the scale factor  $a$  and the rotation angle  $\theta_0$ . According to  $a$  and  $\theta_0$ , we carry on the inverse transform to  $f_2$  and obtain  $f_3$ . Consequently we can calculate the displacement  $(x_0, y_0)$  between  $f_1$  and  $f_3$  with the formula (2), (3).

### 3 The Algorithm of Color Image Mosaic

The algorithm process of color image mosaic is as follows [5]:

a) Extract a color component  $f_1'$  and  $f_2'$  which respectively come from two image  $f_1$  and  $f_2$ , carry on the Fourier transform to  $f_1'$  and  $f_2'$ , and compute  $F_1(\xi, \eta)$  and  $F_2(\xi, \eta)$ 's absolute value  $|F_1(\xi, \eta)|$  and  $|F_2(\xi, \eta)|$ .

b) Make  $|F_1(\xi, \eta)|$  and  $|F_2(\xi, \eta)|$  pass the high-pass filter, for eliminating the low-frequency noise. And the filter can be used in the following function:

$$H(\xi, \eta) = (1.0 - X(\xi, \eta)) * (2.0 - X(\xi, \eta)) \quad (10)$$

Where  $X(\xi, \eta) = \cos(\pi\xi)\cos(\pi\eta)$ , and  $-0.5 \leq \xi, \eta \leq 0.5$ .

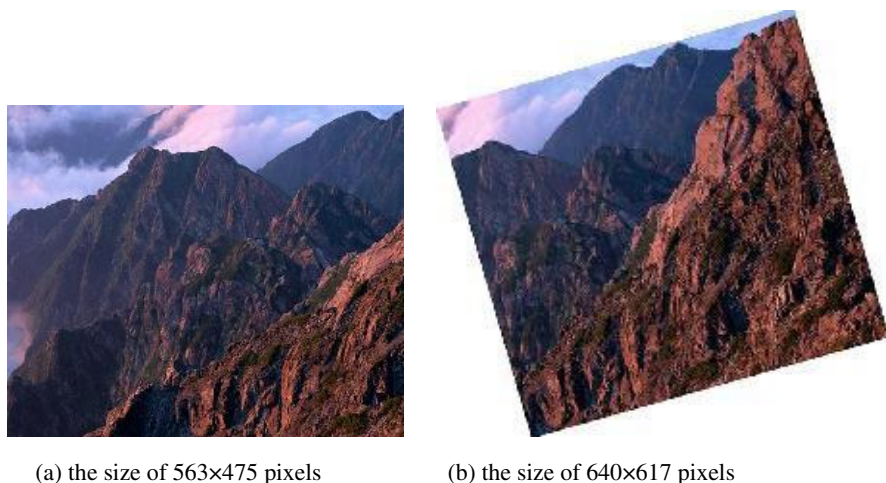
c) Transform the filtered images to the logarithm-polar coordinate form, and calculate their cross-power spectrum. Carry on the inverse Fourier transform to the cross-power spectrum, and find out the location  $(\log a, \theta_0)$  of the peak of the inverse Fourier transform of the cross-power spectrum phase, thus may get the rotation angle  $a$  and the scale factor  $\theta_0$ .

d) Carry on the matching image  $f_2$  to compensate on rotation and proportion zoom, consequently obtain the new image  $f_3$ . The new image  $f_3$  and the original image  $f_1$  are picked up a color component respectively. After calculating the cross-power spectrum, we can get the displacement  $(x_0, y_0)$ .

e) Carry on splicing to two image  $f_1$  and  $f_3$ . If we only adopt simply medial splicing in the overlap image region, the mosaic image could have possibly the quite obvious boundary or the color warp. The vision of the person is suitable with difficulty. Therefore, in the algorithm about the overlap image region, we use the method of image fusion to carry on fusion splicing. Nowadays the methods of color image fusion based on wavelet transform [6] and pyramid fusion technology [7] are applied widely, and have better stability. Due to the length limits, this article will not give unnecessary description in details.

## 4 Experimental Result

The following content is applying this article's algorithm to carry on the experiment of image mosaic. Fig. 1 (a), (b) are original images, which (b) has a certain rotation and scaling to (a). Using the above improved algorithm of image mosaic to measure the rotation angle of (b) is 16.001 degrees, and the scale factor is 1.043. After compensating (b) on revolving and zooming, we obtain Fig. 2. We reuse the principle of phase correlation to get the displacement (322.792, 0.001), which is between Fig. 2 and Fig. 1 (a). Fig. 1 (a) and Fig. 2 are carried on splicing and fusion, then Fig. 3 is obtained.



**Fig. 1.** Original images



**Fig. 2.** The compensated image



Fig. 3. Mosaic result

Table 1. The comparison result of transform parameters

	Rotation angle (degree)	Scale factor	Translation (pixel)	Calculation (time/s)
Original value	16	1.000	(323,0)	26.238
Estimated value	16.001	1.043	(322.792,0.001)	

In order to confirm this algorithm’s accuracy, transform parameters of Fig. 1(b) that are obtained from this algorithm compare with the ones designated in advance. The comparison result is shown in Table 1. We may see from Table 1 that the quite precise transform parameters can be obtained from this algorithm process.

5 Conclusion

This article proposes one kind of color image mosaic algorithm based on Fourier-Mellin transform. When the projective distortion is not very remarkable which is obtained from the camera’s general movement, we carry on geometry matching with image characteristic of the overall frequency domain, and may obtain the quite satisfying panoramic image. This article has made a mass of mosaic experiments. In the experiments, this algorithm can not be applied there is a very serious situation in perspective distortion, and requests all the regions of the images to be spliced have the same rotation, scaling and displacement. Because the algorithm of Fourier-Mellin

transform depends on its own invariable attribute and has its own limitation, it is not too sensitive to the degrees of rotation. But it has certain restriction to the scaling. If the scaling surpasses 1.8, it is very difficult to get the correct matching result under normal circumstances.

**Acknowledgment.** This work was supported in part by the National Natural Science Foundation of China (NSFC) under Grant No. 61071193.

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