Name: ____________________

GSI's name: ____________________

Lab section: ____________________

Statistics 153 (Introduction to Time Series) Homework 2

Due on 27 September, 2018

14 September, 2018

Remember: Your homework must be stapled when you hand it in. Late assignments handed in after 3:50pm on Thursday 27th lecture will not be accepted!

Computer exercises:

1. Go to `https://arxiv.org/stats/monthly_submissions` and download the data for number of new arXiv submissions received during each month since August 1991.

   (a) Argue whether or not you would expect such data to have a constant variance over time. How can you transform the data such that the variance is approximately constant over time (homogeneous)?

   (1 Point)

   (b) Difference both, the original and the transformed data. Which one looks more like white noise?

   (1 Point)

   (c) Based on the differenced data, provide a forecast for the number of new arXiv submissions for September (Remark: Ignore the data for the partial submissions in September).

   (1 Point)

2. The data file *retail* in the R package TSA lists total U.K. (United Kingdom) retail sales (in billions of pounds) from January 1986 through March 2007. For the purpose of this exercise, take the square root of the observations and work with the transformed data. We will call the transformed data (y_t).

   (a) Make a time series plot of (y_t). Is there any trend or seasonality?

   (1 Point)

   (b) Use least squares to fit the following model

$$y_t = \beta_0 + \beta_1 t + \sum_{j=1}^{6} \left[\beta_{2j} \cos\left(\frac{2\pi j t}{12}\right) + \beta_{2j+1} \sin\left(\frac{2\pi j t}{12}\right) \right] + w_t, \tag{1}$$

where (w_t) is zero mean white noise. Note that $\sin(2\pi(6)/12) = 0$, so you have to only estimate $(\beta_0, ..., \beta_{12})$. Plot (y_t) and the fitted values on the same graph.

(1 Point)

(c) Use least squares to fit the following model

$$y_t = \beta_0 + \beta_1 t + \beta_2 I(t \text{ is January}) + ... + \beta_{12} I(t \text{ is November}) + w_t, \tag{2}$$

where (w_t) is zero mean white noise. The indicator variable $I(A)$ takes values 1 if A and 0 otherwise. Plot (y_t) and the fitted values on the same graph. (Hint: You may find the function `seasonaldummy()` in the `forecast` package useful.)

(1 Point)

(d) Compare the fitted values from (b) and those from (c). What do you notice?

(1 Point)

(e) Plot the following three versions of (y_t):

i. (∇y_t): the first difference of the data

ii. $(\nabla_{12} y_t) = (y_t - y_{12})$: the seasonal difference of the data (with 12 months being a season)

iii. $(\nabla\nabla_{12} y_t)$: the first difference of the seasonal difference of the data (with 12 months being a season)

Which one look more like white noise?

(1 Point)

Theoretical exercises:

3. *(Stationarity does not imply strict stationarity)* Consider (w_t) i.i.d. Normal$(0, 1)$. Define

$$x_t = \begin{cases} w_t & \text{if } t = 1, 3, 5, 7, ... \\ \frac{1}{\sqrt{2}}(w_{t-1}^2 - 1) & \text{if } t = 2, 4, 6, 8, ... \end{cases}. \tag{3}$$

(a) Compute the mean function for x_t.

(1 Point)

(b) Compute the autocovariance function for x_t.

(1 Point)

(c) Explain why (x_t) is zero mean white noise with variance 1, and hence weakly stationary.

(1 Point)

(d) Are x_1 and x_2 identically distributed? If not, find their corresponding density functions.

(1 Point)

(e) Is (x_t) strictly stationary? Why or why not?

(1 Point)

4. *(Check invertibility of an MA process)* For each of the following MA process, check if it is invertible.

   (a) $x_t = w_t - w_{t-1} + \frac{3}{16} w_{t-2}$;

   (1 Point)

   (b) $x_t = 2w_{t-2} + 0.4w_{t-1} + w_t$;

   (1 Point)

   (c) $x_t = \left(1 - \frac{1}{2}B\right)\left(1 - \frac{3}{2}B\right)\left(1 - \frac{5}{2}B\right) w_t$.

   (1 Point)

5. *(Linear combinations of observations from a weakly stationary process)* Let (X_t) be a weakly stationary process with mean μ and autocovariance function $\gamma(k) = \mathrm{Cov}(X_t, X_{t+k})$. Consider a derived series (Y_t) defined as

$$Y_t = \sum_{j=a}^{b} c_j X_{t+j}, \tag{4}$$

   where a and b are integers with $a \le b$, and $(c_a, ..., c_b)$ are all fixed real numbers.

   (Remark: This is a special case of a general *linear filter*, where we take $a = -\infty$ and $b = \infty$ with some regularity conditions on $\{c_j\}_{-\infty}^{\infty}$. We will discuss linear filters later in the course.)

   (a) Show that $\{Y_t\}$ is a weakly stationary series.

   (1 Point)

   (b) Show that order k ($k \ge 1$) differencing (that is, $\nabla^k X_t$) can be put in the form of (4). Identify the corresponding a, b, and $(c_a, ..., c_b)$. (Hint: Note that $\nabla = 1 - B$, where B is the backshift operator: $BX_t = X_{t-1}$.)

   (1 Point)

   (c) Recall that smoothing via simple averaging with parameter q corresponds to computing for each t

$$\frac{1}{2q+1} \sum_{j=-q}^{q} X_{t+j}. \tag{5}$$

   Show that smoothing via simple averaging with parameter q can be put in the form of (4). Identify the corresponding a, b, and $(c_a, ..., c_b)$.

   (1 Point)

   (d) Is the kth differenced version of a weakly stationary process always weakly stationary? Is the smoothed (via simple averaging) version of a weakly stationary process always weakly stationary?

   (1 Point)