Name: ____________________

GSI's name: ____________________

Lab section: ____________________

Statistics 153 (Introduction to Time Series) Homework 6

Due on November 29, 2018

Instructions: Homework is due by 3:50pm in lecture on the due date. Please staple your homework when you turn it in. For the computing exercises, be sure to attach all relevant code and plots.

****This homework's score is out of 20 points, and you don't need to solve every problem for a full score****. The points associated with each problem add up to more than 20, so you only need to answer a subset to get a full score (assuming correct solutions). You are welcome to attempt as many problems as you want, but your score will be capped at 20.

1. **Interpretation of Spectral Density**

   Recall that one interpretation of the spectral density is that it gives the relative contributions of sinusoids of various frequencies to the process X_t. That is, $f(\lambda)$ gives the relative strength of sinusoids of frequency λ.

   Let Z_t be white noise with variance 1.

   a. Let $X_t = X_{t-1} - .9X_{t-2} + Z_t$. *(2 points)*
      i. Plot its spectral density (you can use the "arma.spec" function). Which frequencies f are dominant?
      ii. Simulate the process for 50 time steps. If there is periodic behavior, compare the observed period with $\frac{1}{f}$, the inverse of the dominant frequencies from (i).

   b. Let $X'_t = -.3X'_{t-2} - .9X'_{t-4} + Z_t$.Repeat the same procedure as above for this process. *(2 points)*

   c. Let $X''_t = .9X''_{t-5} + Z_t$. Repeat the same procedure as above for this process. *(2 points)*

2. **Spectral Density of AR Processes**

   Let Z_t be a white noise process with variance 1. Consider the AR(3) process:

   $$(1 - .9B^3)X_t = Z_t$$

   a. Compute the transfer and power transfer functions associated with the AR polynomial $(1 - .9B^3)$. Also, compute the spectral density $f_X(\lambda)$. *(3 points)*

   b. Plot the spectral density $f_X(\lambda)$. Do you think X_t will oscillate? If so, what period? *(1 point)*

   c. Simulate X_t for 50 time steps. Is the simulation consistent with your answer to (b)? *(1 point)*

   d. Consider the linear filter with weights $a_{-1} = a_0 = a_1 = \frac{1}{3}$; $a_j = 0$ otherwise. Let Y_t be the time series obtained by applying this filter to X_t. Compute the transfer function, power transfer function, and spectral density $f_Y(\lambda)$. *(3 points)*

   e. Plot the spectral density $f_Y(\lambda)$. Do you think Y_t will oscillate? If so, what period? *(1 point)*

   f. Simulate Y_t by applying the filter from (d) to your simulated X_t from (c). Is the simulation consistent with your answer to (d)? *(1 point)*

3. **DFT and Convolution**

For data $x_0, ..., x_{n-1}$ and $y_0, ..., y_{n-1}$, we define their convolution z_t as:

$$z_t = \sum_{k=0}^{n-1} x_{t-k} y_k$$

where $x_{-m} = x_{n-m}$ for negative values in the sum above. Show that the j^{th} coefficient of DFT of z_t is $b_j^Z = b_j^X b_J^Y$. That is, DFT coefficients of the convolution of x_t and y_t is equal to the product of their DFT coefficients. *(This is analogous to the spectral density result from Shumway Problem 4.8)* *(5 points)*

4. **DFT and Periodic Data**

Suppose the data x_t is $h-$cyclic for some positive integer h: $x_{t+h} = x_t$ for all integers t. Now, suppose the DFT of the first cycle $x_0, .., x_{h-1}$ is $\beta_0, ..., \beta_{h-1}$.

For $n = kh$, show that the DFT of $x_0, ..., x_{n-1}$ is $b_j = b_{mk} = k\beta_m$ for multiples of k and $b_j = 0$ otherwise. *(5 points)*