Name: ID:

Quiz #1

MATH 697: Mathematical Statistics

3 October 2017, 9:00–10:00

Instructions

- This is a closed book exam.
- Answer all questions in the space provided below the questions. Extra paper will be provided if needed.
- There are a total of six questions. Each question is worth 10 marks. There are no optional questions.
- Show all your work. Be explicit about the distributions and assumptions you're using.
- Calculators and translation dictionaries are permitted.
- A table of discrete distributions is provided.

Good Luck!

Problem 1. [10 marks] Define the following. Points will be given for concise answers.

(a) A sample space Ω

(b) An event E

(c) A random variable X

(d) The distribution of a random variable X

Problem 2. [**10 marks**] Three prisoners A, B, and C with apparently equally good records have applied for parole. The parole board has decided to release two, but not all three. The supervisor knows which two are to be released. Prisoner A asks the supervisor for the name of the one prisoner other than himself who is to be released. While his chances of being released before asking are 2/3, he thinks his chances after asking and being told who is to be released are reduced to 1/2, since now either A and B or B and C are to be released. He is, however, mistaken. Explain why he is mistaken using numbers and probability formulas. *hint: an event is a combination of the two prisoners that will be released and what the supervisor tells prisoner A. Consider all possibilities for this event.*

Problem 3. [**10 marks**] A study of the residents of a region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was ten times the probability of death due to lung cancer, given that the person did not smoke. If the probability of death due to lung cancer in the region is .006, what is the probability of death due to lung cancer given that the person is a smoker?

Problem 4. A study is to be conducted in a hospital to determine the attitudes of nurses toward various administrative procedures. A sample of 10 nurses is to be selected from a total of the 90 nurses employed by the hospital.

(a) [**4 marks**] How many different samples of 10 nurses can be selected?

(b) [**6 marks**] Twenty of the 90 nurses are male. If 10 nurses are randomly selected from those employed by the hospital, what is the probability that the sample of ten will include exactly 4 male (and 6 female) nurses?

Problem 5.

(a) [**4 marks**] Provide the formula for the cumulative distribution function of the random variable Y shown in the graph above. Note that $R_Y = \{0, 1, 2, 3, 4, 5\}$

(b) [**3 marks**] What is the probability mass function of Y?

(c) [**3 marks**] Find $P(1 < Y \leqslant 5)$

Problem 6. Suppose that a basketball player sinks a basket from a certain position on the court with probability 0.35.

(a) [**4 marks**] What is the probability that the player sinks three baskets in 10 independent throws?

(b) [**3 marks**] What is the probability that the player throws 10 times before obtaining the first basket?

(c) [**3 marks**] What is the probability that the player throws 10 times before obtaining two baskets?

DISCRETE DISTRIBUTIONS

	RANGE	PARAMETERS	MASS FUNCTION	CDF	E(X)	var(X)	MGF
Bernoulli(θ)	$\{0,1\}$	$\theta \in (0,1)$	$\theta^x(1-\theta)^{1-x}$		θ	$\theta(1-\theta)$	$1-\theta+\theta e^t$
Binomial(n,θ)	$\{0,\ldots,n\}$	$n \in \mathbb{N}, \theta \in (0,1)$	$\binom{n}{x}\theta^x(1-\theta)^{n-x}$		$n\theta$	$n\theta(1-\theta)$	$(1-\theta+\theta e^t)^n$
Poisson(λ)	$\{0,1,2,\ldots\}$	$\lambda \in \mathbb{R}^+$	$e^{-\lambda}\dfrac{\lambda^x}{x!}$		λ	λ	$\exp\{\lambda(e^t-1)\}$
Geometric(θ)	$\{0,1,2,\ldots\}$	$\theta \in (0,1)$	$(1-\theta)^x\theta$		$\dfrac{1-\theta}{\theta}$	$\dfrac{1-\theta}{\theta^2}$	$\dfrac{\theta e^t}{1-e^t(1-\theta)}$
NegBinom(r,θ)	$\{r,r+1,\ldots\}$	$r \in \mathbb{N}, \theta \in (0,1)$	$\binom{x-1}{r-1}\theta^r(1-\theta)^{x-r}$		$\dfrac{r}{\theta}$	$\dfrac{r(1-\theta)}{\theta^2}$	$\left\{\dfrac{\theta e^t}{1-e^t(1-\theta)}\right\}^r$
or	$\{0,1,2,\ldots\}$	$r \in \mathbb{N}, \theta \in (0,1)$	$\binom{r+x-1}{x}\theta^r(1-\theta)^x$		$\dfrac{r(1-\theta)}{\theta}$	$\dfrac{r(1-\theta)}{\theta^2}$	$\left\{\dfrac{\theta}{1-e^t(1-\theta)}\right\}^r$

For *continuous* distributions (see next page), define *Euler's gamma function*, for all $\alpha > 0$, by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,\mathrm{d}x.$$

Further note that the *location/scale* transformation $Y = \mu + \sigma X$ gives

$$f_Y(y) = \frac{1}{\sigma} f_X\left(\frac{y-\mu}{\sigma}\right), \qquad F_Y(y) = F_X\left(\frac{y-\mu}{\sigma}\right), \qquad M_Y(t) = e^{\mu t}M_X(\sigma t), \qquad \mathrm{E}(Y) = \mu + \sigma\mathrm{E}(X), \qquad \mathrm{var}(Y) = \sigma^2\mathrm{var}(X).$$