DALITE Q5 - Bootstrap, Tests of Significance and Small Sample Inference for One Mean. Solutions.

EPIB607 - Inferential Statistics[a]

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This DALITE quiz will cover the bootstrap, an introduction to significance testing, and inference for a single mean using the t distribution.

Hypothesis testing | Bootstrap | t distribution | One sample mean | Normal calculations | Confidence intervals | Central Limit Theorem (CLT)

1. Hypothesis tests 1

The average human gestation time is 266 days from conception. A researcher suspects that proper nutrition plays an important role and that poor women with inadequate food intake would have shorter gestation times even when given vitamin supplements. A random sample of 20 poor women given vitamin supplements throughout the pregnancy has a mean gestation time from conception of $\bar{y}$=256 days. The null hypothesis for the researcher's test is

a. **H_0: $\mu = 266$ (Correct)**
b. H_0: $\mu = 256$
c. H_0: $\mu < 266$
d. H_0: $\bar{y} = 266$

1.1. Correct rationales.

- The null hypothesis is usually a statement of "no effect" or "no difference."
- The null hypothesis is the statement of no effect, so if there was no effect, the average gestation time would be the population mean.
- The null hypothesis of this study is that the average gestation period of women with proper nutrition and poor women with inadequate food intake is the same that is 266 days. Hence, the population mean (all poor women) have the gestation period of 266 days.

1.2. Incorrect rationales.

- The null hypothesis is "the accepted fact", which in this case is that poorer nutrion will yield results lower than the mean (<266) or H null: u<266
- The researcher is testing that the hypothesis that poor women with inadequate food intake will have lower gestation time

2. Hypothesis tests 2

The average human gestation time is 266 days from conception. A researcher suspects that proper nutrition plays an important role and that poor women with inadequate food intake would have shorter gestation times even when given vitamin supplements. A random sample of 20 poor women given vitamin supplements throughout the pregnancy has a mean gestation time from conception of $\bar{y}$=256 days. The researcher's alternative hypothesis for the test is

a. H_a: $\mu \neq 256$
b. **H_a: $\mu < 266$ (Correct)**
c. H_a: $\mu < 256$

2.1. Correct rationales.

- The alternative hypothesis is the point of view of the researchers. In this scenario, the researchers are suspecting that there will be a shorter gestation time for poor women in comparison to an average woman. The alternative hypothesis should be one-sided because the investigators only want to know if gestation time will be shorter and not if it is overall different (shorter or greater) from the average gestation time.
- Researcher has specified a specific direction - that the sample would generate a mean gestation time less than 266. Therefore the alternative hypothesis is that μ is less than 266.

2.2. Incorrect rationales.

3. Hypothesis test 3

The average human gestation time is 266 days from conception. A researcher suspects that proper nutrition plays an important role and that poor women with inadequate food intake would have shorter gestation times even when given vitamin supplements. A random sample of 20 poor women given vitamin supplements throughout the pregnancy has a mean gestation time from conception of $\bar{y}$=256 days. Human gestation times are approximately Normal with standard deviation $\sigma = 16$ days. The p-value for the researcher's test is (provide your calculation in the rationale)

a. more than 0.1
b. **less than 0.01 (Correct)**
c. less than 0.001
d. less than 0.05
e. less than 0.025

3.1. Correct rationales.

- $Z = (y - \mu)/(\sigma/sqrt(n))$ = -2.795 → `pnorm(-2.795, mean = 0 , sd=1 ) = 0.00026`
- `mosaic::xpnorm(q= (256 - 266)/(16/sqrt(20)))`
- `stats::pnorm(q = 256, mean = 266, sd = 16/(sqrt(20)), lower.tail = T)`

3.2. Incorrect rationales.

- $t* = (256 - 266)/(16/\sqrt(20))$ = -2.795. degrees of freedom = n-1 = 19. p-value is between 0.01 and 0.005. This is a one tailed t-test so divide by 2 and p-value is between 0.005 and 0.0025. Therefore, less than 0.01.
- $(256 - 266)/16 = -0.625$ → `pnorm(-0.625)` corresponds to a p value of 0.27
- $t = (\bar{y} - \mu)/(\sigma/\sqrt{n})$ then use t distribution with df=19

4. Hypothesis tests 4

Average human gestation time is 266 days, when counted from conception. A hospital gives a 90% confidence interval for the mean gestation time from conception among its patients. That interval is 264 ± 5 days. Is the mean gestation time in that hospital significantly different from 266 days?

a. **It is not significantly different at the 10% level and therefore is also not significantly different at the 5% level. (Correct)**
b. It is not significantly different at the 10% level but might be significantly different at the 5% level. c. It is significantly different at the 10% level

4.1. Correct rationales.

- The average human gestation time falls within the interval of the hospital which is 259-269 days, therefore the mean gestation time is not significantly different at the 10% level and since the 5% level is even more specific, then it is not significantly different at the 5% level either.
- 266 is included in the 259-269 range, therefore the mean gestation time in the hospital is not significantly different from 266 days at the 10% level. Looking at the significance at the 5% level would make the confidence interval bigger, meaning there's not significant different there either

4.2. Incorrect rationales.

- Because 266 falls at the upper tail of the 90% confidence interval, therefore it is significant different at 10% significance level
- It is significantly different at the 10% level, since the CI does not contain the value zero.
- The 90% CI includes 266, but the 95% CI may not include 266, in which case a value like 266 would only be observed less than or equal to 5% of the time.
- To be significantly different, p-value should be less than 5%.
- As 266 falls within 264+/- 5 days, it is not significant at the 10% level, but it may not fall under this range because when the level of confidence is increased, the confidence interval becomes smaller.

5. One sample mean

We prefer the t procedures to the z procedures for inference about a population mean because

a. z can be used only for large samples
b. **z requires that you know the population standard deviation σ (Correct)**
c. z requires that you can regard your data as an SRS from the population

5.1. Correct rationales.

- The reason we can use z on large samples is because when we have large samples, we can use the sample sd as an estimate for the population sd. The core reason for using t is because we don't know the population sd and we can't estimate the sd with the small sample size.
- because you don't know population sd, only sample sd for t procedures
- Z test needs to have a known population standard deviation sigma and either a normal distribution or the sample size n is large. If the population standard deviation is unknown and the sample size is small then t test statistic can be used.

5.2. Incorrect rationales.

- df are smaller for t-test, and you do not need a SD value

6. One sample mean 2

Because t procedures are robust, the most important condition for their safe use is that

a. the population standard deviation σ is known
b. the population distribution is exactly Normal
c. **the data can be regarded as an SRS from the population (Correct)**
d. the CLT hasn't kicked in yet
e. the sample size is small

6.1. Correct rationales.

- The point of the t test is that sigma is not known (rules out A) The CLT says that the population distribution doesn't matter (rules out B) We want the CLT to kick in (rules out D) The larger the sample size the better(rules out E).

6.2. Incorrect rationales.

- We should only use t procedures if the CLT is not applicable.
- In order to use the T procedure comfortably, the sample size must be large and the population distribution must be normal.

7. Bootstrap

Which of the following statements *best* describes the utility of the bootstrap

a. The bootstrap frees us from the requirement of using simple formulas to derive confidence intervals
b. The bootstrap allows us to simulate a sampling distribution
c. **The bootstrap frees us from the assumption of a Gaussian sampling distribution for the mean (as per the CLT) (Correct)**
d. The bootstrap tells us if the sampling distribution is asymmetric

7.1. Correct rationales.

- Computer intensive methods can solve most problems without assuming that the data have a Gaussian distribution.
- The bootstrap frees us from the assumption that the data conform to a bell-shaped normal distribution.

7.2. Incorrect rationales.

- due to the nature of bootstrap, you can resample things MANY times, which means that CLT will always kick in giving a gaussian distribution instead of assuming one exists
- Bootstrap is useful in because it does not require the CLT and **the population** does not have to be normally distributed
- The bootstrap doesn't rely on CLT, and gives us freedom from having to assume the normal distribution of **the population**.
- The bootstrap allows us to derive estimates when often-used theories do not apply. This is especially useful when n is small (e.g. CLT requires $n > 30$ to assume normality).