

Q1 There are three Canadian firms which build large bridges, firm 1, firm 2, and firm 3. 20% of Canadian large bridges have been built by firm 1, 30% by firm 2, and the rest by firm 3. 5% of the bridges built by firm 1 have collapsed, while 10% of those by firm 2 have collapsed, and 30% by firm 3 have collapsed.

- (a) What is the probability that a bridge collapses?
- (b) Suppose it is reported in tomorrow's newspaper that a large bridge has collapsed. What is the probability it was built by firm 1?

Q2 Suppose a class contains 60% girls and 40% boys. Suppose that 30% of the girls have long hair, and 20% of the boys have long hair. A student is chosen uniformly at random from the class. What is the probability that the chosen student will have long hair?

Q3 Suppose there are three events A , B , and C . Prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Q4 Suppose $S = \{1, 2, 3\}$, with $P(1) = 1/2$, $P(2) = 1/3$, and $P(3) = 1/6$.

- (a) What is $P(1, 2)$? (in words, what is the probability of a 1 or a 2)
- (b) What is $P(1, 2, 3)$?
- (c) List all events A such that $P(A) = 1/2$

Q5 Suppose $S = \{1, 2, 3\}$, with $P(1) = 1/2$ and $P(1, 2) = 2/3$. What must $P(2)$ be?

Q6 Suppose a card is chosen uniformly at random from a standard 52-card deck.

- (a) What is the probability that the card is a jack?
- (b) What is the probability that the card is a club?
- (c) What is the probability that the card is both a jack and a club?
- (d) What is the probability that the card is either a jack or a club?

Q7 Suppose we roll eight fair six-sided dice.

- (a) What is the probability that all eight dice show a 6?
- (b) What is the probability that all eight dice show the same number?
- (c) What is the probability that the sum of the eight dice is equal to 9?

Q8 Suppose we are dealt five cards from an ordinary 52-card deck. What is the probability that

- (a) we get all four aces, plus the king of spades?
- (b) all five cards are spades?
- (c) we get no pairs (i.e., all five cards are different values)?
- (d) we get a full house (i.e., three cards of a kind, plus a different pair)?

Q9 Suppose that we roll four fair six-sided dice.

- (a) What is the conditional probability that the first die shows 2, conditional on the event that exactly three dice show 2?
- (b) What is the conditional probability that the first die shows 2, conditional on the event that *at least* three dice show 2?

Q10 Consider two urns, labelled urn #1 and urn #2. Suppose, that urn #1 has 5 red and 7 blue balls, that urn #2 has 6 red and 12 blue balls, and that we pick three balls uniformly at random from each of the two urns. Conditional on the fact that all six chosen balls are the same color, what is the conditional probability that this color is red?

Q11 Consider three cards, as follows: One is red on both sides, one is black on both sides, and one is red on one side and black on the other. Suppose the cards are placed in a hat, and one is chosen at random. Suppose further that this card is placed flat on the table, so we can see one side only.

- (a) What is the probability that this one side is red?
- (b) Conditional on this one side being red, what is the probability that the card showing is the one that is red on both sides?

Q12 An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = 0.4$ and $P(B) = 0.7$

- (a) If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.
- (b) What is the probability that at least one of the two projects will be successful?
- (c) Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful?