第八章　角度调制与解调

电子工程与信息科学系

内容提要

- 基本概念
- 调制信号通过非线性电路
- 调制信号通过网络
- 调频波的产生
- 鉴频原理与 电路

§8.1 基本概念

- 角度调制信号的数学表达式与波形
  - 调相波
  - 调频波
- 调频波频谱
  - 频谱
  - 带宽

§8.1.1 角度调制信号的数学表达式与波形

1. 调相波

$$u(t)=U\cos(\omega t+\varphi)$$

$$u_{PM}(t)=U_{PM}\cos\left[\omega_0 t+m_P s(t)\right] \quad \text{或} u_{PM}(t)=U_{PM}\cos(\omega_0 t+m_P\cos\omega_m t)$$

$$\varphi(t)=k_P U_\Omega s(t)=m_P s(t)$$

与基带信号成线性关系

调相波的瞬时频率

$$\omega(t)=\frac{d[\omega_0 t+m_P s(t)]}{dt}=\omega_0+m_P\frac{ds(t)}{dt}$$

$$\text{或}\,\omega(t)=\omega_0-m_P\omega_m\sin\omega_m t$$

§8.1.1 角度调制信号的数学表达式与波形

1. 调相波

$$u_{PM}(t) = U_{PM}\cos\left[\omega_0 t + m_P s(t)\right]$$

$$\varphi(t) = k_P U_\Omega s(t) = m_P s(t)$$

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① $s(t)$：**基带信号** $s(t)$或$\cos\omega_m t$

② k_P：**调相增益，单位基带信号电压幅度引起的相位变化**

③ m_P：**调相指数，或最大相移** $m_p = k_p U_\Omega$

④ $\Delta\omega$：**最大频移** $\Delta\omega = m_p \omega_m$

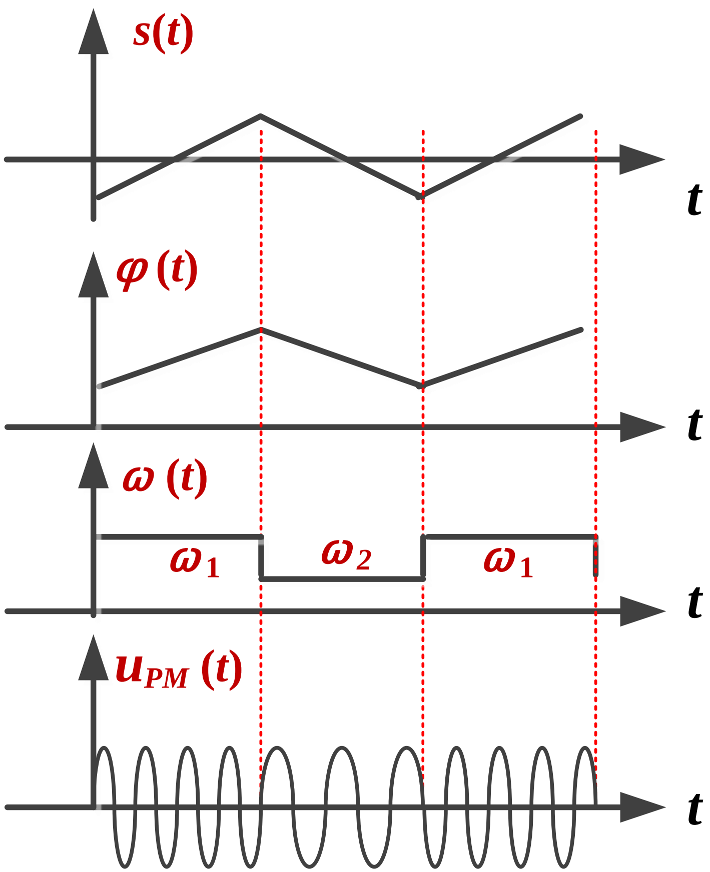
$$\omega(t) = \frac{d\varphi}{dt} = \frac{d}{dt}\left(\omega_0 t + m_P\cos\omega_m t\right) = \omega_0 - m_P\omega_m\sin\omega_m t$$

§8.1.1 角度调制信号的数学表达式与波形

1. 调相波

$$u_{PM}(t) = U_{PM}\cos\left[\omega_0 t + m_P s(t)\right]$$

$$\omega(t) = \frac{d[\omega_0 t + m_P s(t)]}{dt} = \omega_0 + m_P\frac{ds(t)}{dt}$$



§8.1.1 角度调制信号的数学表达式与波形

2. 调频波

$$\omega(t)=\omega_0+k_f U_\Omega s(t)=\omega_0+\triangle\omega\, s(t)$$

$$\varphi(t)=\int_0^t\left[\omega_0+\triangle\omega\, s(\tau)\right]d\tau=\omega_0 t+\triangle\omega\int_0^t s(\tau)\,d\tau$$

$$u_{FM}(t)=U_{FM}\cos\left[\omega_0 t+\triangle\omega\int_0^t s(\tau)\,d\tau\right]$$

或$u_{FM}(t)=U_{FM}\cos\left[\omega_0 t+\dfrac{\triangle\omega}{\omega_m}\sin\omega_m t\right]$

$$=U_{FM}\cos\left[\omega_0 t+m_f\sin\omega_m t\right] \quad ; \quad m_f=\frac{\triangle\omega}{\omega_m}=\frac{k_f V_\Omega}{\omega_m}$$

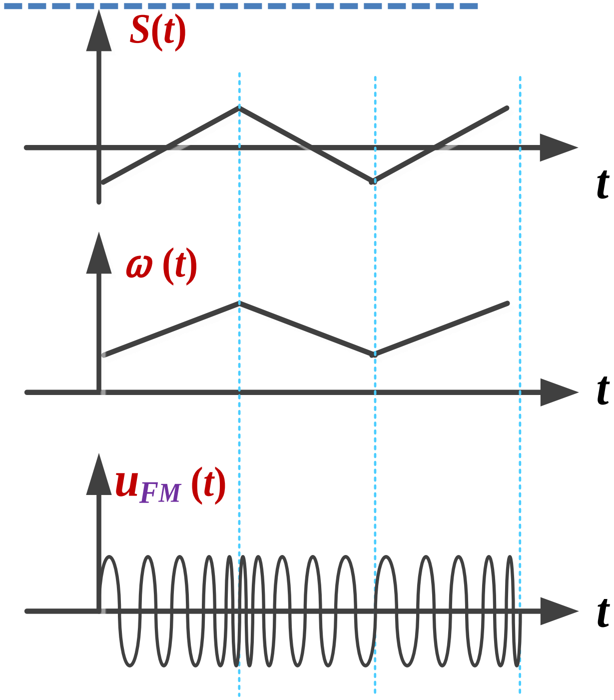
§8.1.1 角度调制信号的数学表达式与波形

2. 调频波

$$u_{FM}(t)=U_{FM}\cos\left[\omega_0 t+\Delta\omega\int_0^t s(\tau)d\tau\right]$$

$$或u_{FM}(t)=U_{FM}\cos\left[\omega_0 t+\frac{\Delta\omega}{\omega_m}\sin\omega_m t\right]$$

$$=U_{FM}\cos\left[\omega_0 t+m_f\sin\omega_m t\right] \quad ; \quad m_f=\frac{\Delta\omega}{\omega_m}$$



① s(t)：基带信号 $s(t)$或$\cos\omega_m t$

② k_f：**调频增益**，单位基带信号电压幅度引起的**频率**变化

③ m_f：调频指数，**最大相移**

④ ⊿ω：**最大频移** $\Delta\omega=k_f U_\Omega=m_f\omega_m$

调相波和调频波的参数比较

调制方式	PM	FM
数学表达式	$u_{PM}(t)=U_{PM}\cos\left[\omega_0 t+m_P s(t)\right]$	$u_{FM}(t)=U_{FM}\cos\left[\omega_0 t+\Delta\omega\int_0^t s(\tau)d\tau\right]$
调制增益	调相增益k_P	调频增益k_f
调制指数	调相指数m_P, $m_p=k_pU_\Omega$	调频指数m_f
最大频移	$\Delta\omega=m_p\omega_m$	$\Delta\omega=k_fU_\Omega=m_f\omega_m$

§8.1.2 调频波频谱

1. 频谱

$$\frac{u_{FM}(t)}{U_{FM}}=\cos\left[\omega_0 t+m_f\sin\omega_m t\right]=\cos\omega_0 t\cos\left(m_f\sin\omega_m t\right)-\sin\omega_0 t\sin\left(m_f\sin\omega_m t\right)$$

$$\cos\left(m_f\sin\omega_m t\right)=J_0\left(m_f\right)+2\sum_{n=1}^{\infty}J_{2n}\left(m_f\right)\cos\left(2n\omega_m t\right)$$ 偶数阶贝塞尔函数

$$\sin\left(m_f\sin\omega_m t\right)=2\sum_{n=0}^{\infty}J_{2n+1}\left(m_f\right)\sin\left[\left(2n+1\right)\omega_m t\right]$$ 奇数阶贝塞尔函数

$$\frac{u_{FM}(t)}{U_{FM}}=J_0\left(m_f\right)\cos\omega_0 t+J_1\left(m_f\right)\left[\cos\left(\omega_0+\omega_m\right)t-\cos\left(\omega_0-\omega_m\right)t\right]$$
$$+J_2\left(m_f\right)\left[\cos\left(\omega_0+2\omega_m\right)t+\cos\left(\omega_0-2\omega_m\right)t\right]+\cdots\cdots$$

$$u_{FM}(t)=U_{FM}\sum_{n=-\infty}^{\infty}J_n\left(m_f\right)\cos\left(\omega_0+n\omega_m\right)t \qquad \left\{J_{-(2n+1)}=-J_{2n+1},\quad J_{2n}=J_{-2n}\right\}$$

§8.1.2 调频波频谱

2. 带宽

$$u_{FM}(t) = U_{FM}\sum_{n=-\infty}^{\infty} J_n(m_f)\cos(\omega_0 + n\omega_m)t$$

根据$J_n(m_f)$ 的性质（收敛）

对任意给定 $\varepsilon > 0$，总存在自然数 N，当 $n > N$时，$J_n(m_f) < \varepsilon$

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① **令 $\varepsilon = 0.1$，$N_{0.1}$，$J_n(m_f) < 0.1$的项都可以忽略**

$$BW_{0.1} = 2N_{0.1}\omega_m$$

② **令 $\varepsilon = 0.01$，$N_{0.01}$，** $BW_{0.01} = 2N_{0.01}\omega_m$

③ **Carson公式定义带宽** $BW_{carson} = 2(m_f + 1)\omega_m = BW_{CR}$

④ $m_f \ll 1$，$BW_{Carson} \simeq 2\omega_m$

$m_f \gg 1$，$BW_{Carson} \simeq 2m_f\omega_m$

⑤ **Carson公式亦适用于调相波的带宽估计**

§8.1.2 调频波频谱

3. 功率关系

平均功率：$$\frac{\overline{u_{FM}^2}}{R_L}=\frac{U_{FM}^2}{R_L}\sum_{n=-\infty}^{\infty}J_n^2\left(m_f\right)\cdot\frac{1}{2}=\frac{1}{2}\cdot\frac{U_{FM}^2}{R_L}$$

平均功率不随时间的变化而变化，恒包络特性

效率：边带信号的功率除以总功率

当m_f=2.405，5.52等值时，$J_0(m_f)$=0，理论上可得100%的效率

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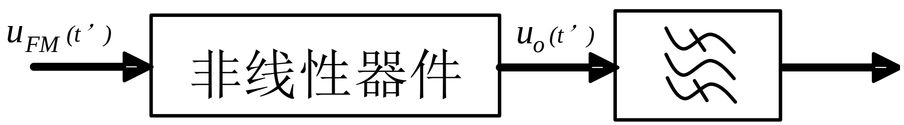
§8. 2　调频信号通过非线性电路

1.　FM通过倍频网络

$$u_{FM}(t)=U_{FM}\cos\left[\omega_0 t+\Delta\omega\int_0^t s(\tau)d\tau\right]$$ **不是周期信号**

令 $\boxed{t'=t+\dfrac{\Delta\omega}{\omega_0}\int_0^t s(\tau)d\tau}$

$$u_{FM}(t')=U_{FM}\cos\omega_0 t'$$ **为周期信号**



$u_{FM}(t')$和$u_o(t')$在时间域t'内应有相同的周期，将$u_o(t')$展开成傅里叶级数

$$u_o(t')=a_0+\sum_{n=1}^{\infty}a_n\cos(n\omega_0 t') \qquad u_o(t)=a_0+\sum_{n=1}^{\infty}a_n\cos\left[n\omega_0 t+n\Delta\omega\int_0^t s(\tau)d\tau\right]$$

§8. 2 调频信号通过非线性电路

1. FM通过倍频网络

$$u_{FM}(t)=U_{FM}\cos\left[\omega_0 t+\triangle\omega\int_0^t s(\tau)d\tau\right]$$

$$u_o(t)=a_0+\sum_{n=1}^{\infty}a_n\cos\left[n\omega_0 t+n\triangle\omega\int_0^t s(\tau)d\tau\right]$$

ω_m不变

$$\omega_0\to n\omega_0$$

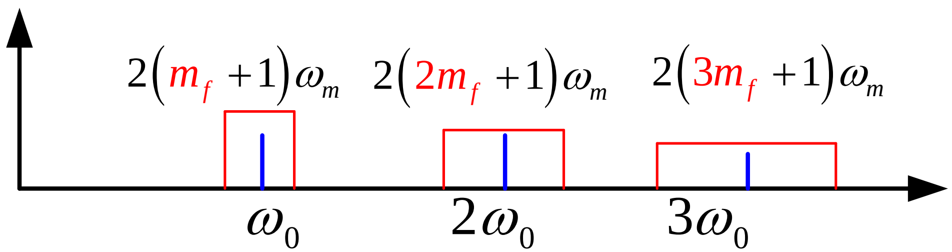
$$\triangle\omega\to n\triangle\omega$$

$$D_n=\frac{n\triangle\omega}{n\omega_0}=\frac{\triangle\omega}{\omega_0}=D_1$$ **相对频偏** $$D=\frac{\triangle\omega}{\omega_0}$$

$$m_f=\frac{\triangle\omega}{\omega_m}\quad\to\quad\frac{n\triangle\omega}{\omega_m}=nm_f$$ **调频指数增加n倍，可用于扩展m_f**

§8. 2 调频信号通过非线性电路

1. FM通过倍频网络



$$(m_f+1)\omega_m+(2m_f+1)\omega_m<\omega_0$$

$$(2m_f+1)\omega_m+(3m_f+1)\omega_m<\omega_0$$

$$\vdots \qquad\qquad \vdots$$

$$(nm_f+1)\omega_m+\left[(n+1)m_f+1\right]\omega_m<\omega_0$$

$$(2n+1)m_f+2<\frac{\omega_0}{\omega_m}=\frac{m_f}{D} \qquad\qquad D<\frac{m_f}{(2n+1)m_f+2}$$

§8. 2　调频信号通过非线性电路

2.　FM通过混频网络

$$\left.\begin{array}{c} \omega_0 \to \omega_I = \omega_L \pm \omega_0 \\ \Delta\omega \to \text{不变} \\ \omega_m \to \text{不变} \end{array}\right\}$$

$m_f = \dfrac{\Delta\omega}{\omega_m}$ 不变，即调角指数不变

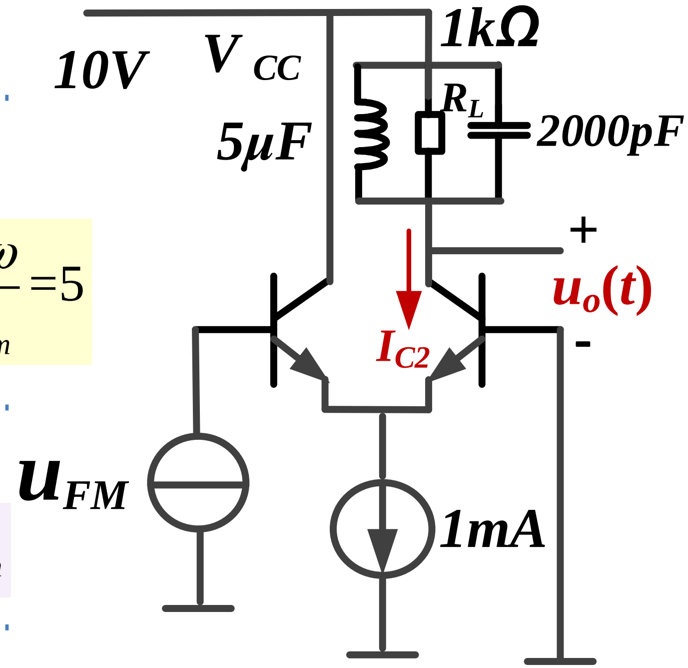
$$D \to D' = \frac{\Delta\omega}{\omega_L \pm \omega_0}$$

高中频： $\omega_I = \omega_L + \omega_0 + \Delta\omega\, s(t)$

低中频： $\omega_I = \omega_L - \omega_0 - \Delta\omega\, s(t)$

§8. 2 调频信号通过非线性电路

例8.2.1 求 $u_o(t)$



调频波参数

$$u_{FM}(t)=100\cos\left[10^7 t+10^5\int_0^t s(\tau)d\tau\right](mV)$$

$$\omega_m=2\times10^4 rad/s$$

$$m_f=\frac{\Delta\omega}{\omega_m}=5$$

输出回路参数

ω_0=10^7rad/s， $\omega_{3\text{dB}}$=5×10^5rad/s

$$2\omega_0>\left(4m_f+2\right)\omega_m$$

i_{C2}中的基波电流幅度为 $I_{C2,1}=I_K a_1\left(\frac{100}{26}\right)=0.55mA$

差分，双曲特性

输出电压： $u_o(t)=V_{CC}+I_{C2,1}R_L\cos\left[10^7 t+10^5\int_0^\tau s(\tau)d\tau\right]$

动态限幅

① 输出信号频率输入信号频率关系变化

② 输出回路带宽是否满足Carson带宽

③ 相位关系（同相or反相）

内容提要

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§8.4.1 概述

1、性能指标

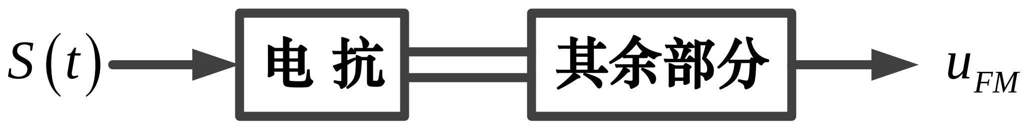
(1)、调频线性 $f - f_0 = \triangle f = k_f U_\Omega$ 调频波的瞬时频率与基带信号幅度成线性

(2)、调频灵敏度 $k_f = \left.\dfrac{\triangle f}{U_\Omega}\right|_{f=f_0}$ 单位基带信号电压引起的频率变化量

(3)、载波频率的稳定度

§8.4.1 概述

2、直接调频电路



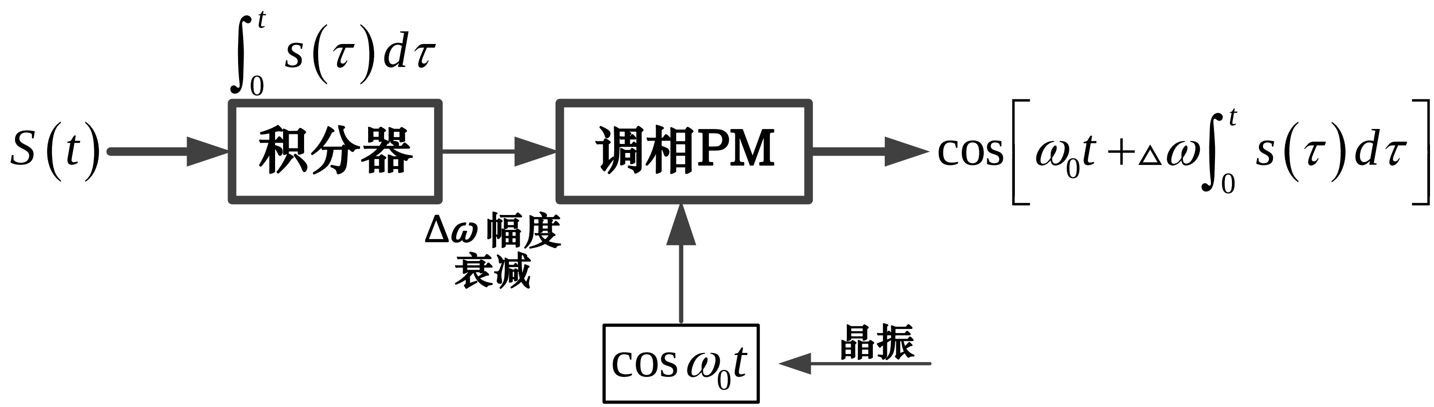
(1)、直接控制振荡器的电抗，如变容二极管等

(2)、电路简单，频率高

(3)、f_0稳定度和准确度差，线性度差

§8.4.1 概述

3、间接调频电路



(1)、工作频率低

(2)、电路复杂

(3)、f_0稳定性和准确性好

(4)、m_f小，$\Delta\omega$小

§8.4.2 变容管调频

1、原理

反偏工作

$$u_C = U_Q + U_\Omega \cdot s(t)$$

$$C_j(u) = C_{j0}\left(1+\frac{u_C}{U_\varphi}\right)^{-\gamma} = C_{jQ}\left[1+M\cdot s(t)\right]^{-\gamma}$$

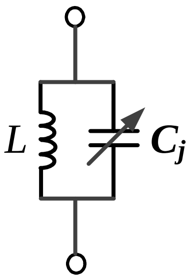
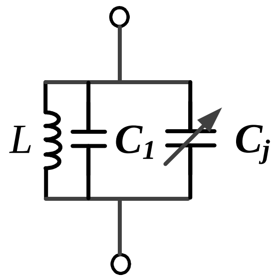
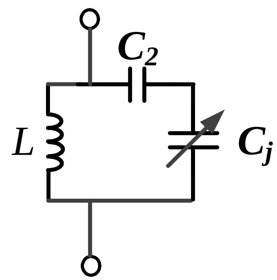
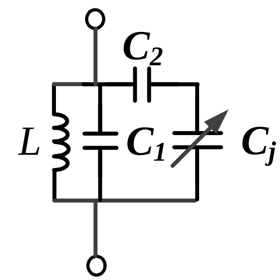
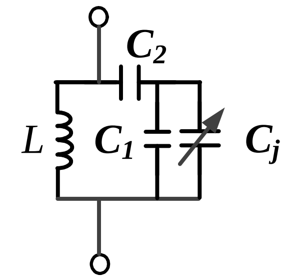
$$C_{jQ}(u) = C_{j0}\left(1+\frac{U_Q}{U_\varphi}\right)^{-\gamma}$$

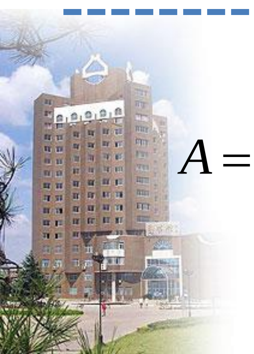
$$M = \frac{U_\Omega}{U_\varphi + U_Q}$$

零偏电容、接触电位差、变容指数、归一化控制电压幅度

§8.4.2 变容管调频

2、C_j在LC谐振回路中的接法

				
$\begin{pmatrix} A=1 \\ B=0 \end{pmatrix}$	$\begin{pmatrix} A=\dfrac{C_{jQ}}{C_{jQ}+C_1} \\ B=0 \end{pmatrix}$	$\begin{pmatrix} A=1 \\ B=\dfrac{C_{jQ}}{C_{jQ}+C_2} \end{pmatrix}$	$\begin{pmatrix} A=\dfrac{C_{jQ}}{C_{jQ}+\dfrac{C_1C_2}{C_1+C_2}} \\ B=\dfrac{C_{jQ}}{C_{jQ}+C_2} \end{pmatrix}$	$\begin{pmatrix} A=\dfrac{C_{jQ}}{C_{jQ}+C_1} \\ B=\dfrac{C_{jQ}}{C_{jQ}+C_1+C_2} \end{pmatrix}$

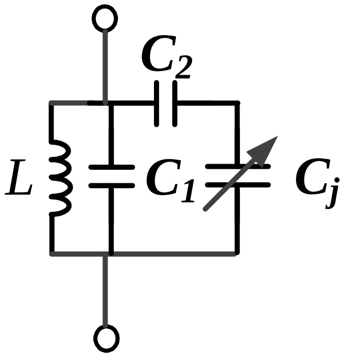


$$A=\frac{C_{jQ}}{C_{jQ}+C_{open}}, \quad B=\frac{C_{jQ}}{C_{jQ}+C_{short}}$$

***Copen*：L开路时，从C_j向外看的电容**

***Cshort*：L短路时，从C_j向外看的电容**

§8.4.2 变容管调频



$$\omega=\left[L\left(C_1+\frac{C_2C_j}{C_2+C_j}\right)\right]^{-1/2}=\omega_0\sqrt{\frac{B+(1-B)\left[1+M\cdot s(t)\right]^{\gamma}}{A+(1-A)\left[1+M\cdot s(t)\right]^{\gamma}}}$$

$$\text{其中}\omega_0=\left[L\left(C_1+\frac{C_2C_{jQ}}{C_2+C_{jQ}}\right)\right]^{-1/2}$$

$$\omega(t)=\omega_0\left[1+D_1s(t)+D_2s^2(t)+\cdots\cdots\right]$$

$$\text{或}\omega(t)=\omega_0\left[\left(1+\frac{1}{2}D_2+\cdots\right)+\left(D_1\cos\omega_mt+\cdots\right)+\left(\frac{1}{2}D_2\cos2\omega_mt+\cdots\right)+\cdots\right]$$

$$D_1=\frac{1}{2}M\gamma(A-B)\qquad D_2=\frac{1}{8}M\gamma^2(A-B)(3\gamma A+\gamma B-2\gamma-2)$$

§8.4.2 变容管调频

① 最大频偏 $\Delta\omega = D_1\omega_0$

$$\omega(t) = \omega_0\left[1 + D_1 s(t) + D_2 s^2(t) + \cdots\cdots\right]$$

$$或\omega(t) = \omega_0\left[\left(1+\frac{1}{2}D_2+\cdots\right)+\left(D_1\cos\omega_m t+\cdots\right)+\left(\frac{1}{2}D_2\cos 2\omega_m t+\cdots\right)+\cdots\right]$$

② 调频增益 $k_f = \dfrac{\Delta\omega}{U_\Omega} = \dfrac{D_1\omega_0}{U_\Omega}$

③ 中心频率的飘移 $\varepsilon = \dfrac{\overline{\omega(t)} - \omega_0}{\omega_0}$

$$\overline{\omega(t)} = \omega_0\left[1 + D_1\overline{s(t)} + D_2\overline{s^2(t)} + \cdots\cdots\right] = \omega_0\left[1 + D_2\overline{s^2(t)} + \cdots\cdots\right]$$

$$\varepsilon = \frac{\overline{\omega(t)} - \omega_0}{\omega_0} = D_2\overline{s^2(t)}或\varepsilon = \frac{D_2}{2}$$

④ 调频非线性系数：

最大的非线性频移/最大的线性频移

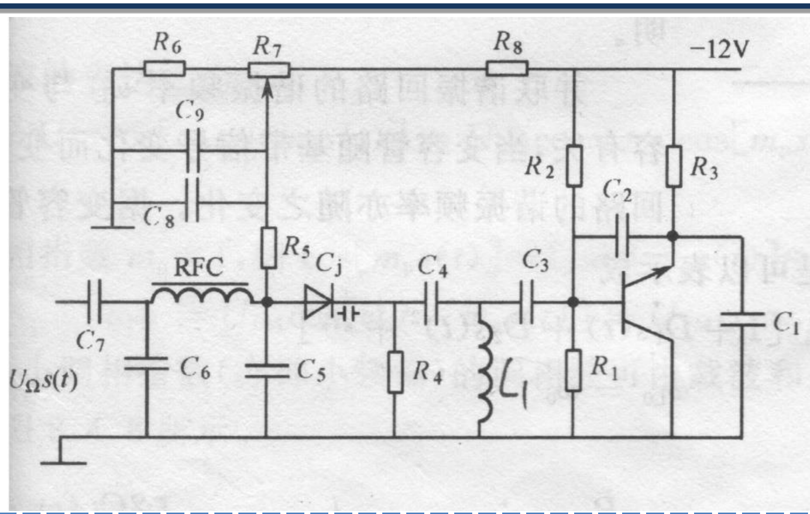
$$\delta = \frac{D_2}{2D_1} \quad \Leftrightarrow \quad D_2小，D_1大$$

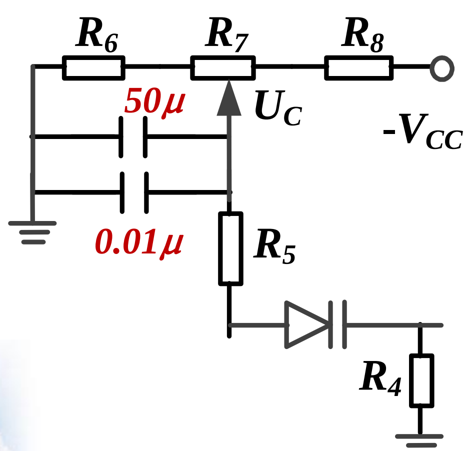
$$D_1 = \frac{1}{2}M\gamma(A-B)$$

$$D_2 = \frac{1}{8}M\gamma^2(A-B)(3\gamma A + \gamma B - 2\gamma - 2)$$

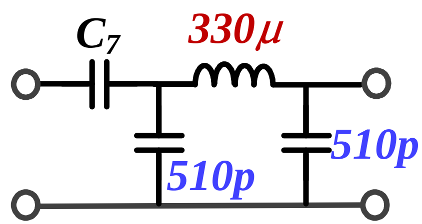
§8.4.2 变容管调频

例8.4.1

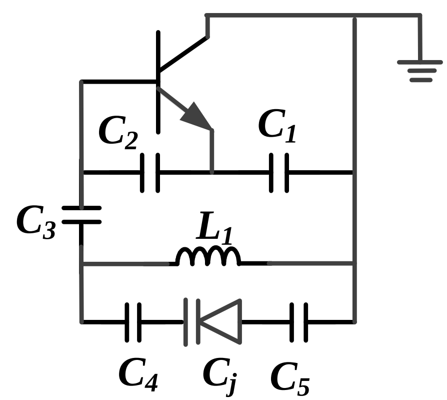




C_j的偏置电路

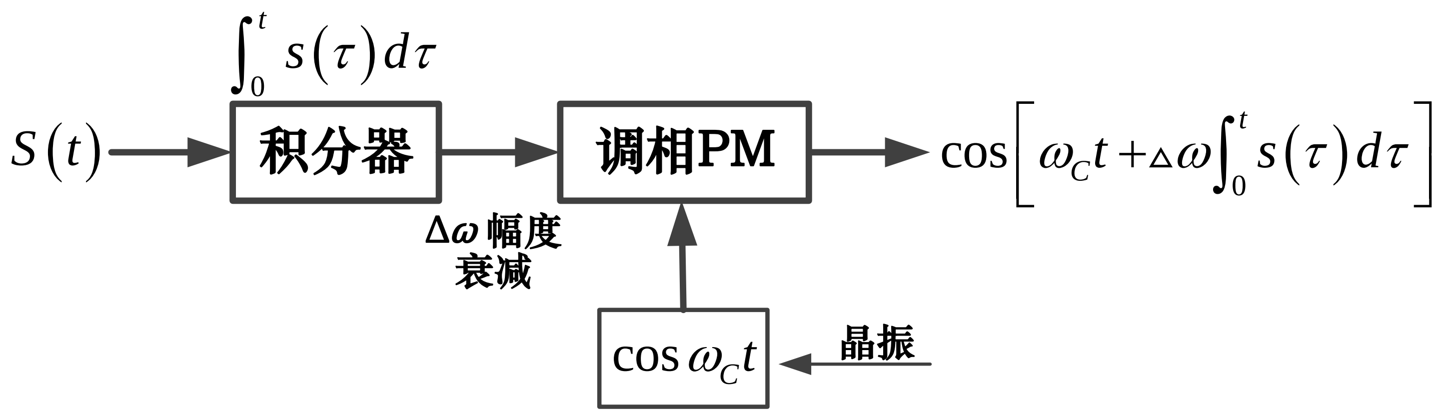


音频控制部分



共集 Siller 电路

§8.4.3 间接调频



$$u_{FM} = U_{FM}\cos\left[\omega_C t + y(t)\right]$$ ⟹ **调相电路即是将两输入信号相位相加**

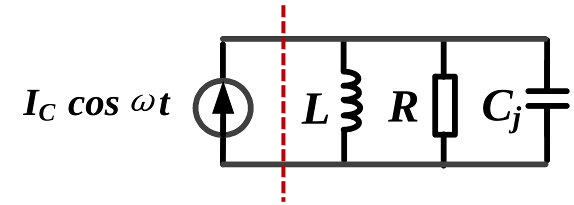
$$= U_{FM}\cos\left[\omega_C t + \Delta\omega\int_0^t s(\tau)d\tau\right]$$ ⟹ **由此表达式看，已实现了调频**

实现：积分器较容易；窄带调相电路关键

§8.4.3 间接调频

1、谐振回路调相

$$C_j(t) = C_{jQ}\left[1 + M\cdot s(t)\right]^{-\gamma} \qquad M = \frac{U_\Omega}{U_\varphi + U_\Omega}$$



回路对激励电流呈现的阻抗：

$$Z = \frac{R}{1 + jQ\left(\dfrac{\omega_C}{\omega_0(t)} - \dfrac{\omega_0(t)}{\omega_C}\right)} \qquad \omega_0(t) = \frac{1}{\sqrt{LC_j(t)}} = \omega_0\left[1 + D_1 s(t)\right]$$

$$\frac{\omega_C}{\omega_0(t)} - \frac{\omega_0(t)}{\omega_C} = \frac{\left[\omega_C + \omega_0(t)\right]\left[\omega_C - \omega_0(t)\right]}{\omega_C\omega_0(t)} \approx \frac{2\omega_C\left[\omega_C - \omega_0(t)\right]}{\omega_C\omega_0(t)} = 2\frac{\omega_C - \omega_0(t)}{\omega_0(t)}$$

$$Z = \frac{R}{1 + 2jQ\dfrac{-D_1 s(t)}{1 + D_1 s(t)}} \qquad \begin{aligned}&\omega_C = \omega_0\\ &\omega_C - \omega_0(t) = -D_1 s(t)\omega_0\end{aligned}$$

§8.4.3 间接调频

$$Z=\frac{R}{1+2jQ\dfrac{-D_1 s(t)}{1+D_1 s(t)}}=|Z|e^{j\varphi}$$

$$|Z|=\frac{R}{\sqrt{1+\left[\dfrac{2QD_1 s(t)}{1+D_1 s(t)}\right]^2}}\approx R \quad , \quad \left[2QD_1 s(t)\ll 1\right]$$

D_1很小，M很小，m_p也很小

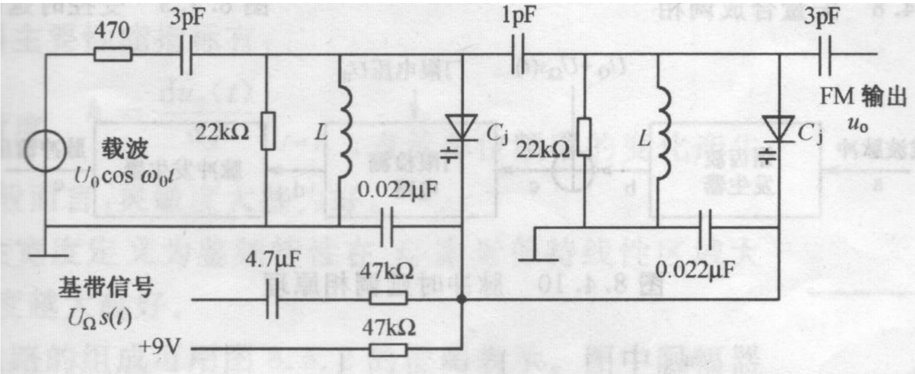
$$\varphi=-tg^{-1}\frac{-2QD_1 s(t)}{1+D_1 s(t)}\approx 2QD_1 s(t)=QM\gamma(A-B)\cdot s(t)=m_p s(t)$$

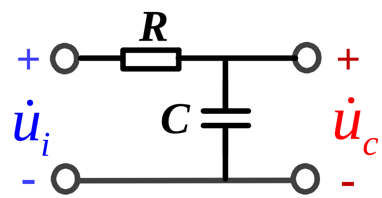
谐振频率ω_0略有偏移时，幅度变化产生寄生调幅，很小，可以忽略

相位为线性变化，形成调相

§8.4.3 间接调频

例：课本P151，图8.4.6为间接调频实例





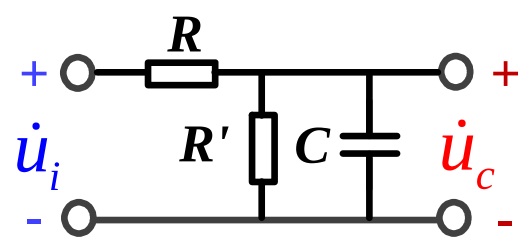
积分器

积分器： $\dfrac{u_c}{u_i}=\dfrac{1}{1+j\omega_m RC}\approx\dfrac{1}{j\omega_m RC}$　　条件：$\omega_m RC\gg 1$

若 $\omega_m RC\gg 1$ 不满足，起不到积分器的作用

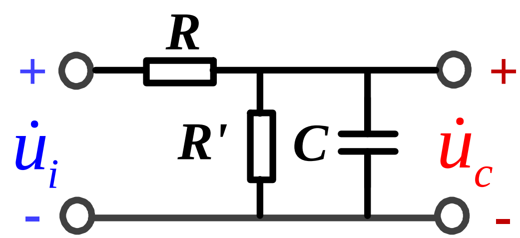
若考虑变容管的偏置电阻 R' 的影响

$$\frac{u_c}{u_i}=\frac{\dfrac{R'}{1+j\omega R'C}}{R+\dfrac{R'}{1+j\omega R'C}}=\frac{R'}{R+R'}\cdot\frac{1}{1+j\omega_m\dfrac{RR'}{R+R'}C}$$



§8.4.3 间接调频

$$\frac{u_c}{u_i}=\frac{R'}{R+R'}\cdot\frac{1}{1+j\omega_m\dfrac{RR'}{R+R'}C}$$



若虚部 $\omega_m C\left(R\,\|\,R'\right)\gg 1$ **，则** $$\frac{u_C}{u_i}\approx\frac{R'}{\left(R+R'\right)}\cdot\frac{1}{j\omega_m\dfrac{RR'}{R+R'}C}=\frac{1}{j\omega_m RC}$$

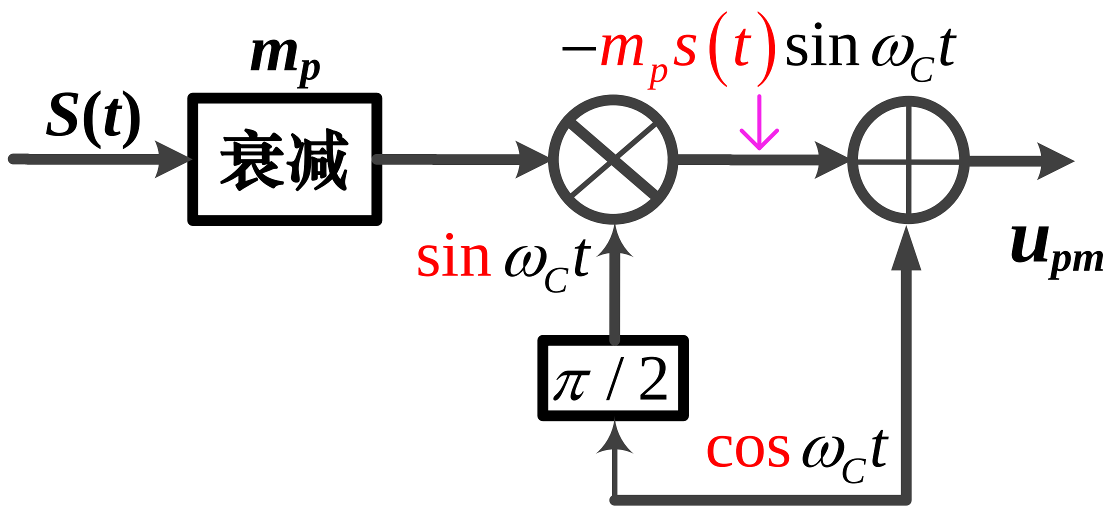
§8.4.3 间接调频

2、矢量合成调相

$$u_{PM} = U_{PM}\cos\left[\omega_C t + m_p s(t)\right]$$

$$\frac{u_{PM}}{U_{PM}} = \cos\omega_C t\cos\left[m_p s(t)\right] - \sin\omega_C t\sin\left[m_p s(t)\right]$$

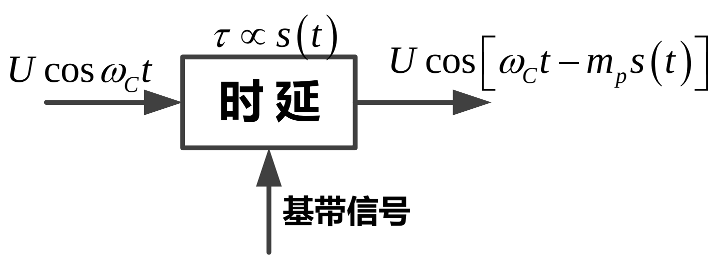
$$\xlongequal{m_p<<1} \cos\omega_C t - m_p s(t)\sin\omega_C t$$



§8.4.3 间接调频

3、时延法调相

$$u_{PM} = U_{PM} \cos\left[\omega_C t + m_p s(t)\right] \qquad m_p \ll 1$$



$$\cos\omega_C(t-\tau)$$
$$= \cos(\omega_C t - \omega_C \tau) = \cos\left[\omega_C t - m_p s(t)\right] \qquad \tau \propto s(t)$$

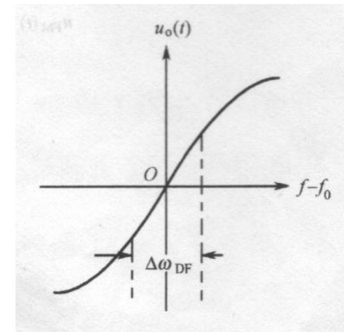
内容提要

- 基本概念
- 调制信号通过非线性电路
- 调制信号通过网络
- 调频波的产生
- **鉴频原理与电路**

§8.5.1 鉴频

鉴频：从调频波中恢复出基带信号

1、消除寄生调幅



① 二极管对

② 动态限幅（如，差分放大等）

2、鉴频电路的要求

① 鉴频线性度：保证信号不失真 $u_o = a\omega + b$

② 鉴频宽度（鉴宽）：$BW_F > BW_{CR}$ 大于*Carson*带宽

保证基带信号信息落入接收带宽内，鉴频后信号不失真

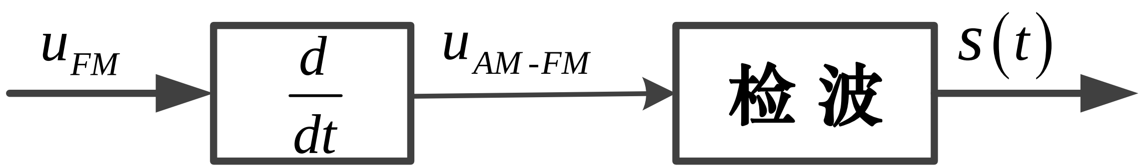
③ 鉴频灵敏度：$k_{Df} = \dfrac{du_o}{d\omega}$ 或 $k_{Df} = \dfrac{du_o}{df}$

输入的频率变化在输出端引起相应的信号电压幅度变化

§8.5.2　鉴频方法概述

1、利用PLL的鉴频鉴相：用VCO跟踪 v_{FM} 的频率

2、先微分再检波



设：

$$\frac{u_{FM}}{U_{FM}} = \cos\left[\omega_C t + \Delta\omega\int_0^t s(\tau)\,d\tau\right]$$

$$\frac{du_{FM}}{dt} = -\sin\underbrace{\left[\omega_C t + \Delta\omega\int_0^t s(\tau)\,d\tau\right]}_{\text{调频}}\cdot\underbrace{\boxed{\left[\omega_C + \Delta\omega\cdot s(t)\right]}}_{\text{调幅}}$$

变换为调幅调频波，检波可得包络信息

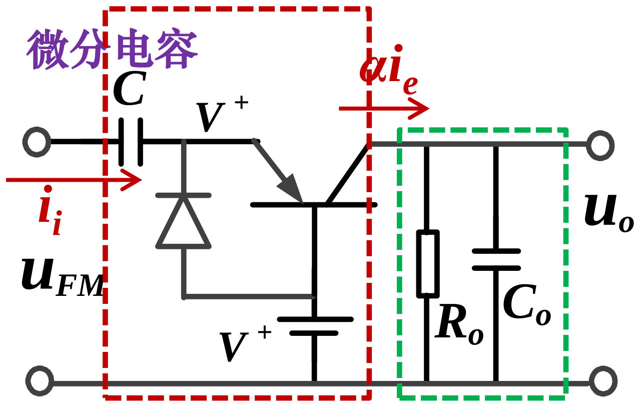
§8.5.3 直接微分鉴频

1、基本电路

1）忽略三极管和二极管导通电压的影响

2）u_{FM}正半周时，三极管发射结导通

3）u_{FM}负半周时，二极管导通，
电容C上不产生自生负偏压



设：$u_{FM}(t) = U\cos\left[\omega_C t + \Delta\omega\int_0^t s(\tau)d\tau\right]$

电容C上的电压：$u_C = u_{FM} - V^+$

C上流过的电流：$i_i = C\dfrac{du_C}{dt} = C\dfrac{d\left(u_{FM} - V^+\right)}{dt} = C\dfrac{du_{FM}}{dt}$

$$= -CU\sin\left[\omega_C t + \Delta\omega\int_0^t s(\tau)d\tau\right]\cdot\left[\omega_C + \Delta\omega\cdot s(t)\right]$$

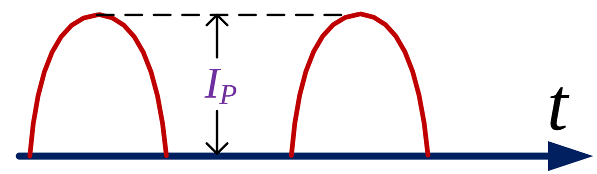
$$= -CU\sin\left[\omega_C t + \Delta\omega\int_0^t s(\tau)d\tau\right]\cdot\omega_i(t)$$

将FM信号 → 幅度随ω变化 的AM- FM信号 → 半波整流

§8.5.3 直接微分鉴频

令三极管： $U_T = 0$

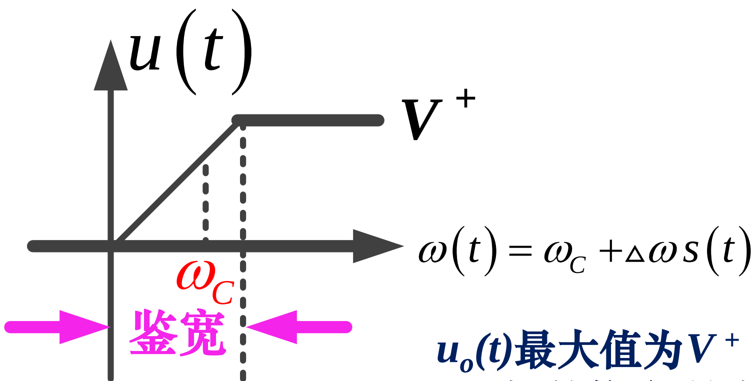
则EB导通角： $\varphi = \dfrac{\pi}{2}$



$$\Rightarrow \quad I_P = \alpha I_i = \alpha C U \omega_i(t)$$

$$\Rightarrow \quad I_{o0} = \frac{I_P}{\pi} = \frac{\alpha C U \omega_i(t)}{\pi}$$

$$\Rightarrow \quad u_o(t) = I_{o0} R_o = \frac{\alpha C U R_o \left[\omega_C + \Delta\omega s(t)\right]}{\pi}$$



$u_o(t)$最大值为V^+
因三极管饱和所致
($U_B \geq U_C$)

R_oC_o组成LPF，截止频率ω_h： $\omega_h >> \omega_m$

$\Rightarrow$ 让 $s(t)$ 信号完整通过，滤除载波信号

结论：当$\omega(t)=\omega_C$时，（即$s(t)=0$），$u_o \neq 0$，有直流分量。

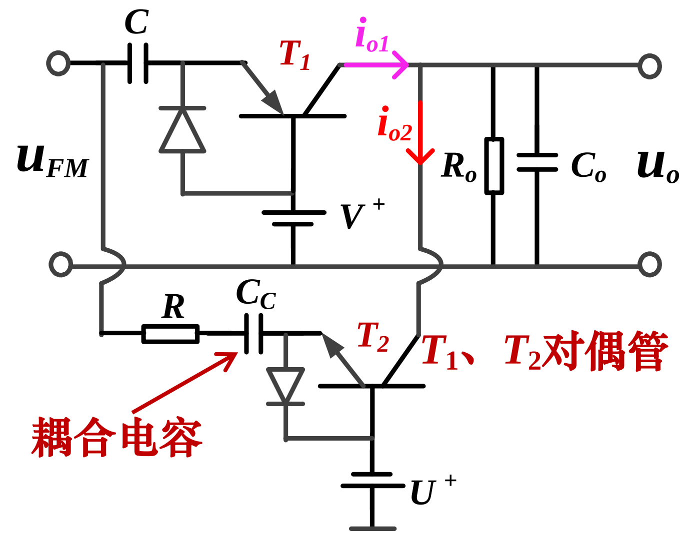
鉴频增益 $\quad k = \dfrac{\alpha C R_o}{\pi}$

§8.5.3 直接微分鉴频

2、改进电路

增加对偶电路：正半周T_1导通，负半周T_2导通

R上的峰值电流 $I_P=\dfrac{U}{R}$**，固定值，和$s(t)$无关**



$$u_o=\left(\bar{i}_{o1}-\bar{i}_{o2}\right)R_o=\frac{\alpha CR_oU}{\pi}\left[\omega_C+\Delta\omega\cdot s(t)\right]-\frac{\alpha R_oU}{\pi R}$$

$$=\frac{\alpha CR_oU}{\pi}\left[\omega_C+\Delta\omega\cdot s(t)-\frac{1}{RC}\right]$$

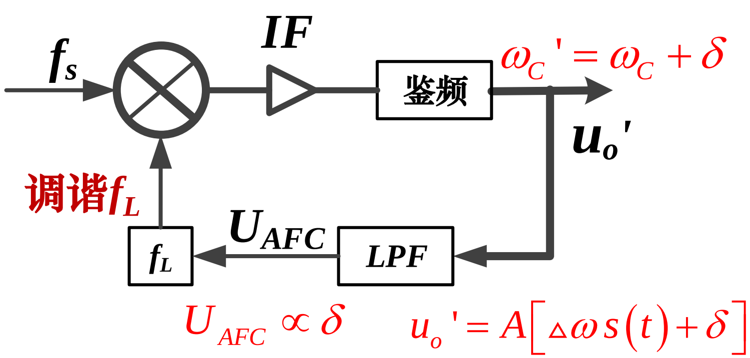
令 $RC=\dfrac{1}{\omega_C}$ **（R在T_2支路，C在T_1支路）**

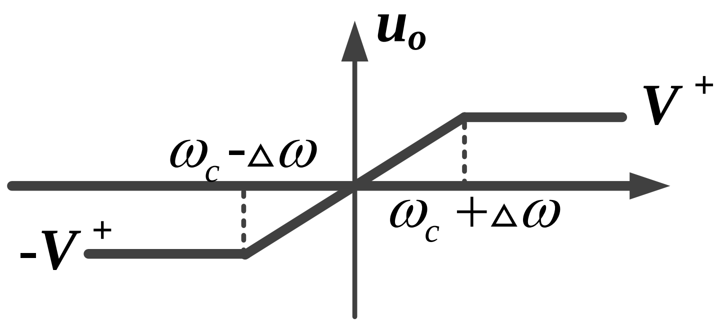
$$u_o(t)=\frac{\alpha CUR_o}{\pi}\cdot\Delta\omega\cdot s(t)$$

§8.5.3 直接微分鉴频

特点： ① $\omega(t)=\omega_c$ 时 $u_o(t)=0$

因 $\frac{1}{RC}$ 稳定性较差(10^{-2})，若与ω_C不匹配，会带来误差





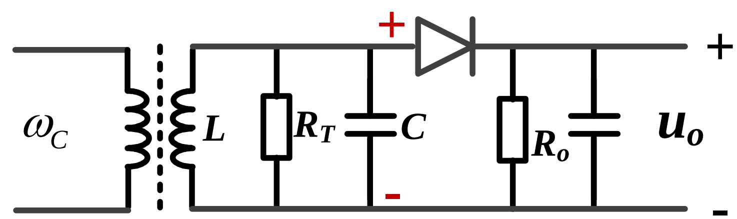
引入AFC，改变混频器的本振，使ω_C随 $\frac{1}{RC}$ 变化，调整电路输出到理想状态

② **鉴频灵敏度**

$$k_{df单}\le\frac{U^+}{\omega_c+\Delta\omega} \qquad k_{df双}\le\frac{2U^+}{2\Delta\omega}=\frac{U^+}{\Delta\omega}$$

§8.5.4 斜率鉴频器

斜率鉴频器：利用线性网络幅频特性



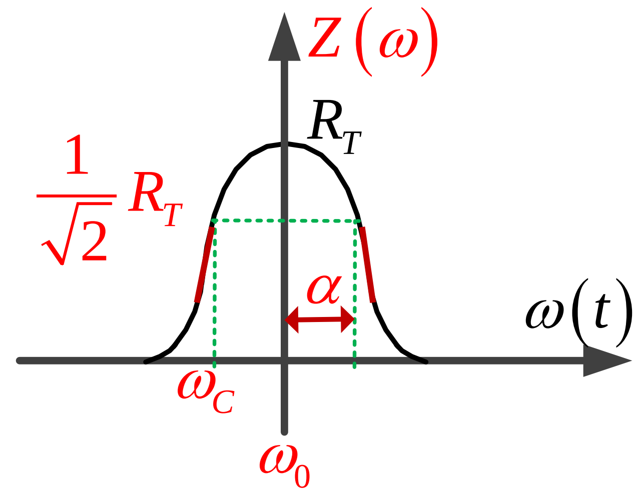
1、基本电路

传输函数$H(\omega)$或$Z(\omega)$的幅度随ω作线性变化，

即将将FM信号 → 幅度随ω变化 的AM- FM信号

$$Z(\omega)=\frac{R_T}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

$$\boxed{\omega=\omega_c+\Delta\omega s(t)}$$



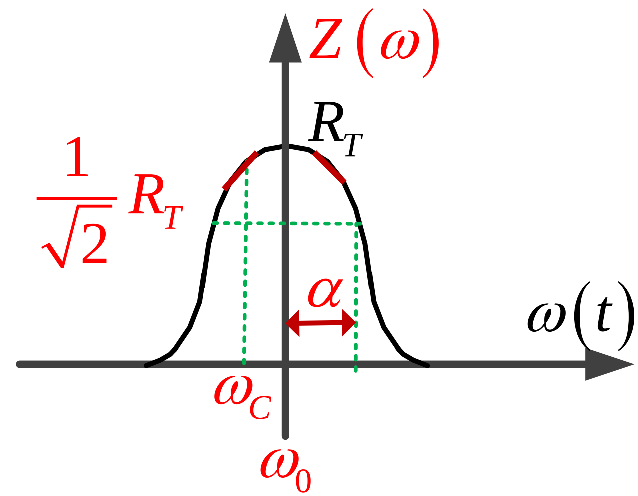
$$\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}=\frac{(\omega+\omega_0)(\omega-\omega_0)}{\omega\omega_0}\approx\frac{2\omega(\omega-\omega_0)}{\omega\omega_0}=2\frac{\omega-\omega_0}{\omega_0}$$

又$Q=\frac{\omega_0}{\Delta\omega_{3dB}}=\frac{\omega_0}{2\alpha}$　　得到$|Z(\omega)|=\frac{R_T}{\sqrt{1+\left(2Q\frac{\omega-\omega_0}{\omega_0}\right)^2}}=\frac{R_T}{\sqrt{1+\left(\frac{\omega-\omega_0}{\alpha}\right)^2}}$

寻找阻抗幅度随ω作线性变化大的区域

§8.5.4 斜率鉴频器

$$|Z(\omega)| = \frac{R_T}{\sqrt{1+\left(2Q\dfrac{\omega-\omega_0}{\omega_0}\right)^2}} = \frac{R_T}{\sqrt{1+\left(\dfrac{\omega-\omega_0}{\alpha}\right)^2}}$$



$$\boxed{\omega = \omega_c + \Delta\omega s(t)}$$

令： $\xi = \dfrac{\omega-\omega_0}{\alpha} = \dfrac{\omega_C-\omega_0}{\alpha} + \dfrac{\Delta\omega\cdot s(t)}{\alpha} = \xi_0 + \xi(t)$

$\xi_0 = \dfrac{\omega_C-\omega_0}{\alpha}$ $\quad \boxed{\xi(t) = \dfrac{\Delta\omega\cdot s(t)}{\alpha}}$ $\quad \boxed{当\dfrac{\Delta\omega}{\alpha} \ll 1时}$

$$|Z(\xi)| = |Z(\xi_0)| + \frac{d|Z|}{d\xi}\underline{(\xi-\xi_0)} + \boxed{\frac{d^2|Z|}{d\xi^2}\cdot\frac{1}{2}(\xi-\xi_0)^2} + \cdots$$

$\dfrac{d|Z|}{d\xi}$ **最大** ← **最大的非线性项，令其为0**

得到 $\xi_0 = \pm\dfrac{1}{\sqrt{2}}$，即 $\omega_C = \omega_0 \pm \dfrac{\alpha}{\sqrt{2}}$ **线性最好**

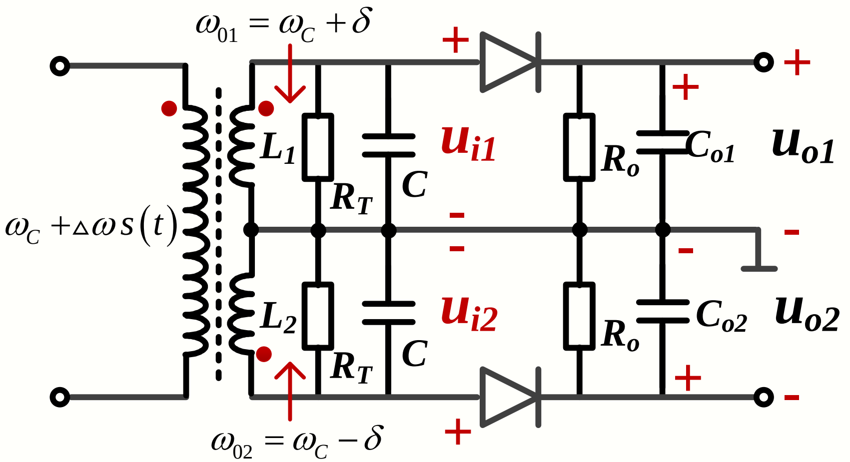
§8.5.4 斜率鉴频器

2、改进电路平衡鉴频器：多加一个支路，消除直流的影响

上下两个支路输入端谐振频率：

$$\begin{cases}\omega_{01}=\omega_C+\delta \\ \omega_{02}=\omega_C-\delta\end{cases}$$

（调节L_1、L_2）



$$\boxed{\omega=\omega_c+\Delta\omega s(t)}$$

$$u_o(t)=u_{o1}(t)-u_{o2}(t)=k(U_{i1}-U_{i2})=kI(|Z_1|-|Z_2|)$$

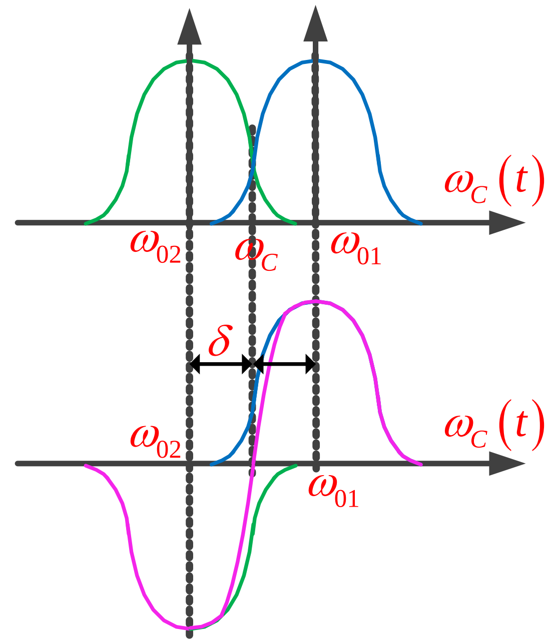
令：$$\frac{\omega-\omega_{01}}{\alpha}=\frac{\omega-\omega_C-\delta}{\alpha}=\frac{\Delta\omega}{\alpha}s(t)-\frac{\delta}{\alpha}=x-a$$

$$\boxed{x=\frac{\Delta\omega}{\alpha}s(t)} \qquad \boxed{a=\frac{\delta}{\alpha}}$$

$$|Z(\omega)|=\frac{R_T}{\sqrt{1+\left(\frac{\omega-\omega_0}{\alpha}\right)^2}} \Rightarrow \begin{cases}\boxed{|Z_1|=\dfrac{R_T}{\sqrt{(1+x^2+a^2)-2ax}}} \\ \boxed{|Z_2|=\dfrac{R_T}{\sqrt{(1+x^2+a^2)+2ax}}}\end{cases}$$

§8.5.4 斜率鉴频器

$$u_o(t)=u_{o1}(t)-u_{o2}(t)=k\left(U_{i1}-U_{i1}\right)=kI\left(|Z_1|-|Z_2|\right)$$



$$|Z_1|=\frac{R_T}{\sqrt{\left(1+x^2+a^2\right)-2ax}}$$

$$|Z_2|=\frac{R_T}{\sqrt{\left(1+x^2+a^2\right)+2ax}}$$

$$\frac{|Z_T|}{R_T}=\frac{|Z_1|-|Z_2|}{R_T}=\left|\frac{Z_T}{R_T}\right|'\cdot x+\frac{1}{3!}\left|\frac{Z_T}{R_T}\right|'''x^3+\cdots$$

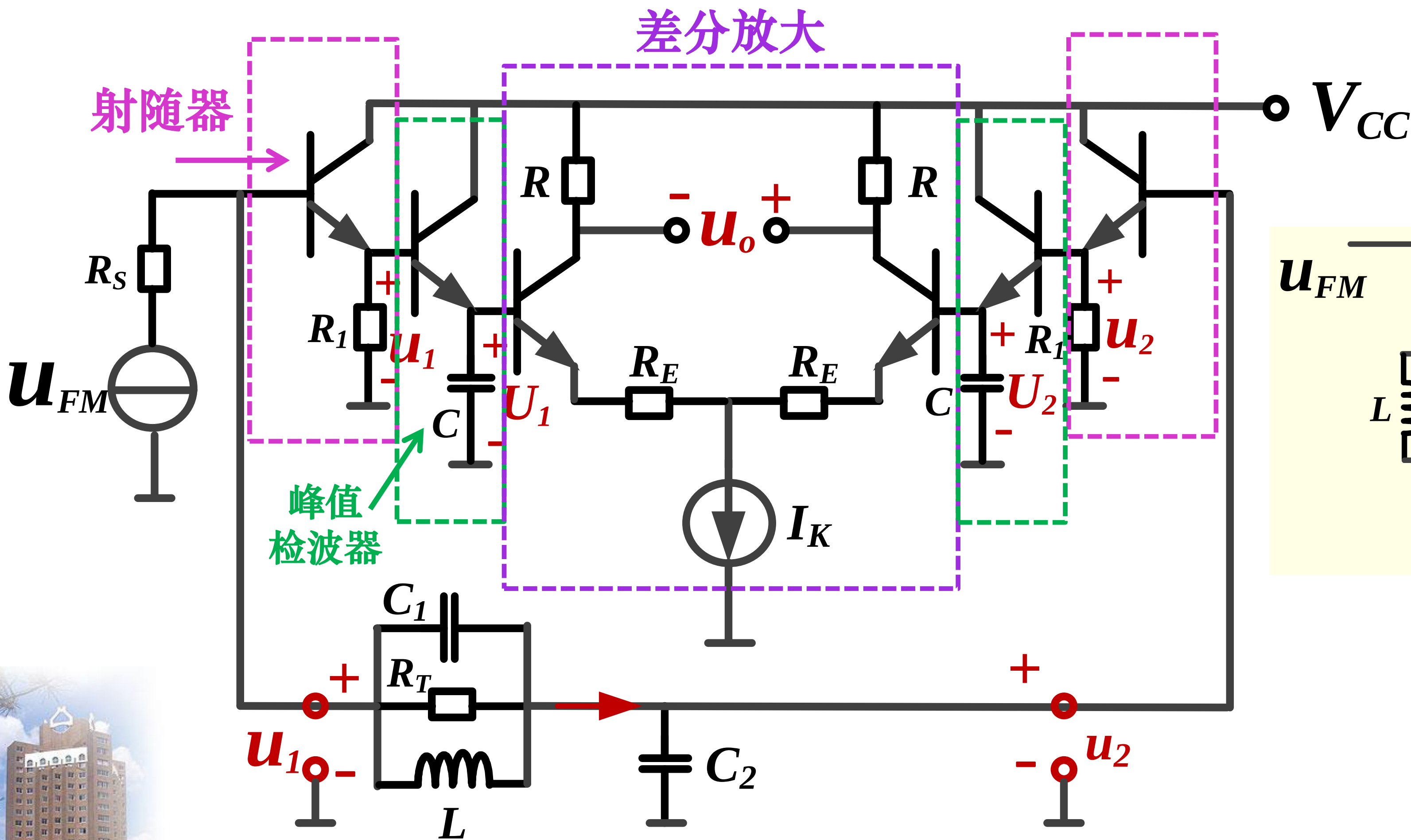
令 $\left|\frac{Z_T}{R_T}\right|'''=0$, $\Rightarrow$ $a=\frac{\delta}{\alpha}=\sqrt{\frac{3}{2}}$ $\Rightarrow$ $\frac{|Z_T|}{R_T}\doteq\frac{4}{5}\sqrt{\frac{3}{5}}\cdot x$

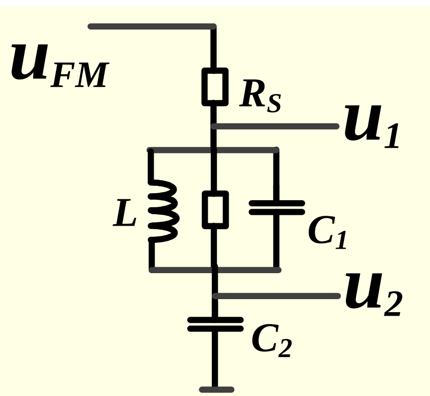
$\Rightarrow$ $$\boxed{u_o(t)=IR_T\cdot\frac{4}{5}\sqrt{\frac{3}{5}}\cdot\frac{\Delta\omega}{\alpha}s(t)}$$

若$R_{T1}\neq R_{T2}$或$\delta_1\neq\delta_2$，将导致u_o在ω_c处不为零，线性变差

§8.5.4 斜率鉴频器

3、用于IC的斜率鉴频器模块

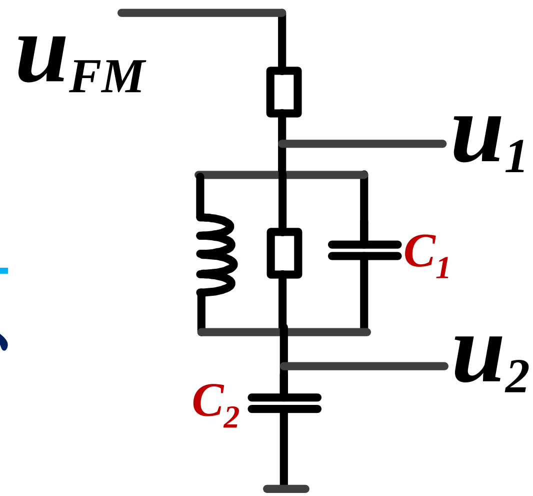




§8.5.4 斜率鉴频器

$$\omega_1^2=\frac{1}{LC_1},\quad \omega_2^2=\frac{1}{L\left(C_1+C_2\right)},$$

$$\omega_1>\omega_2$$



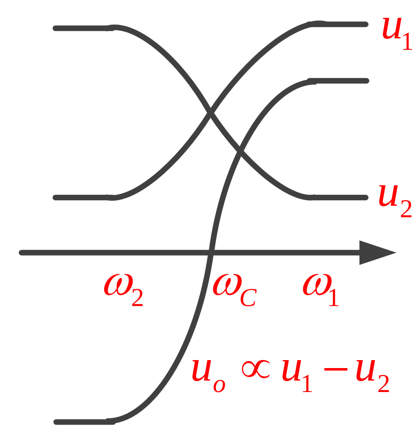
① $\omega=\omega_1$，并联谐振RLC的Z最大，u_1最大，u_2最小

② $\omega<\omega_1$，并联RLC感性，与C_2串联谐振

③ $\omega\doteq\omega_2$，串联谐振，Z最小，u_1最小，u_2最大

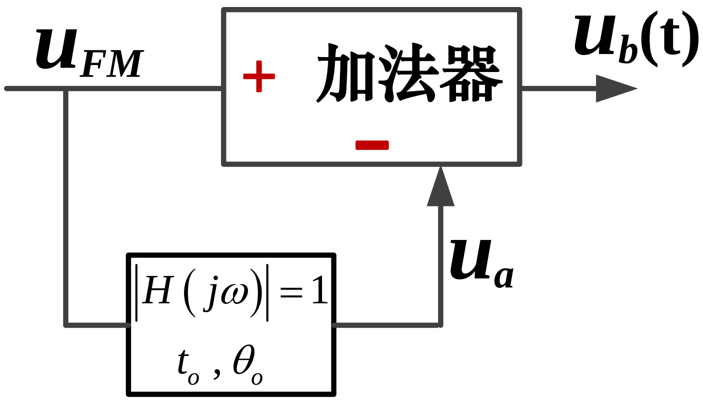
$$u_o(t)\propto u_1(t)-u_2(t)$$

调节L的大小，使当$s(t)$=0时，$u_o(t)=0$



§8.5.5 相位鉴频器

1、原理



$$u_{FM}=U_{FM}\cos\left[\omega_C t+\triangle\omega\int_0^t s(\tau)d\tau\right]$$

$$u_b=U_{FM}\cos\left[\omega_C t+\triangle\omega\int_0^t s(\tau)d\tau\right]$$

$$-U_{FM}\cos\left[\omega_C t+\triangle\omega\int_0^{t-t_0} s(\tau)d\tau+\theta_0\right]$$

t_o：延时时间

θ_o：载波相移

$$u_b=-2U_{FM}\sin\left\{\frac{1}{2}\left[\triangle\omega\int_{t-t_0}^{t} s(\tau)d\tau-\theta_0\right]\right\}\cdot\sin\left\{\omega_C t+\triangle\omega\int_0^t s(\tau)d\tau-\frac{1}{2}\left[\triangle\omega\int_{t-t_0}^{t} s(\tau)d\tau-\theta_0\right]\right\}$$

高频波的包络　　　　**高频波**

§8.5.5 相位鉴频器

积分中值定理： $\int_a^b f(x)dx=(b-a)f(\xi),\quad \xi\in[a,b]$

$$\Rightarrow\int_{t-t_0}^{t}s(\tau)d\tau=t_0s\left(t-\frac{t_0}{2}\right)\qquad 取\xi=t-\frac{t_0}{2}$$

令 $\boxed{\Delta\omega t_0s\left(t-\frac{t_0}{2}\right)=\theta}$

$$\Rightarrow b(t)=2U\sin\left\{\frac{1}{2}\left[\Delta\omega t_0\,s\left(t-\frac{t_0}{2}\right)-\theta_0\right]\right\}$$

$$=2U\sin\left[\frac{1}{2}(\theta-\theta_0)\right]=2U\left[\sin\frac{\theta}{2}\cos\frac{\theta_0}{2}-\cos\frac{\theta}{2}\sin\frac{\theta_0}{2}\right]$$

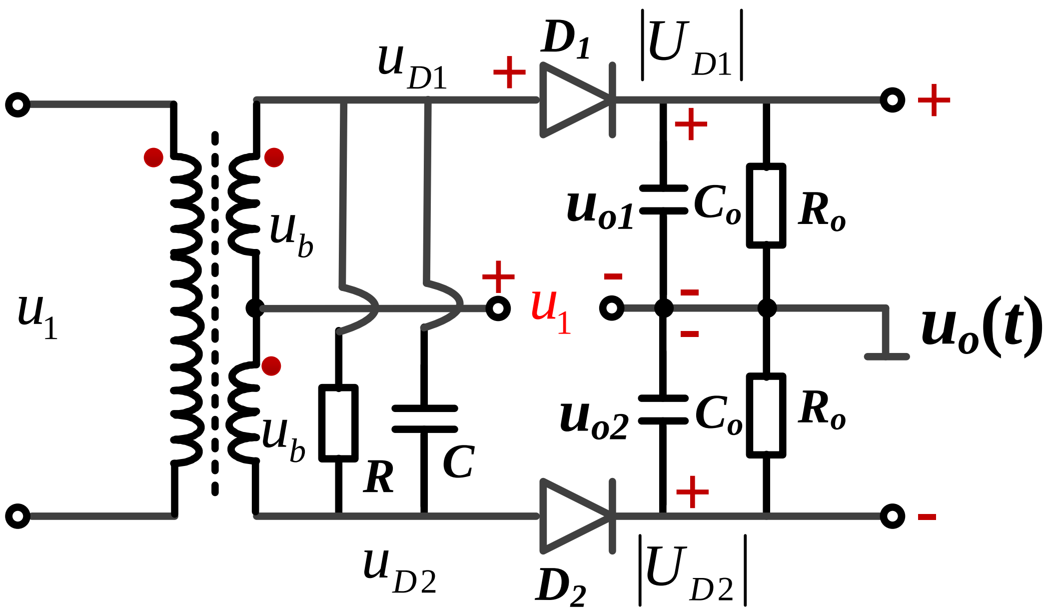
$$\overset{\theta\ll 2}{=\!=\!=}2U\left[\frac{\theta}{2}\cdot\cos\frac{\theta_0}{2}-\sin\frac{\theta_0}{2}\right]=\underbrace{U\cdot\Delta\omega t_0s\left(t-\frac{t_0}{2}\right)}_{包含基带信号}\cos\frac{\theta_0}{2}-\underbrace{2U\sin\frac{\theta_0}{2}}_{直流}$$

§8.5.5 相位鉴频器

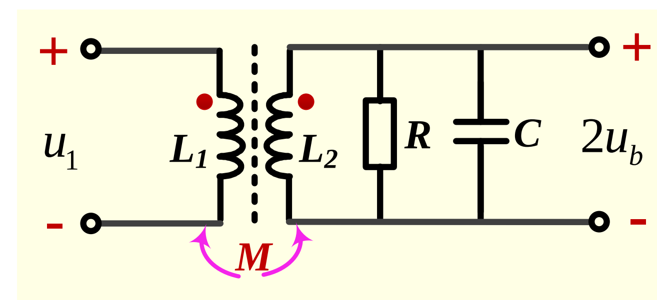
2、相位鉴频器电路

$$u_{D1}=u_1+u_b,\quad u_{D2}=u_1-u_b$$

$$u_o=u_{o1}-u_{o2}=|U_{D1}|-|U_{D2}|$$



移相网络



D_1、D_2峰值包络检波

*R*可以包含多个部分：

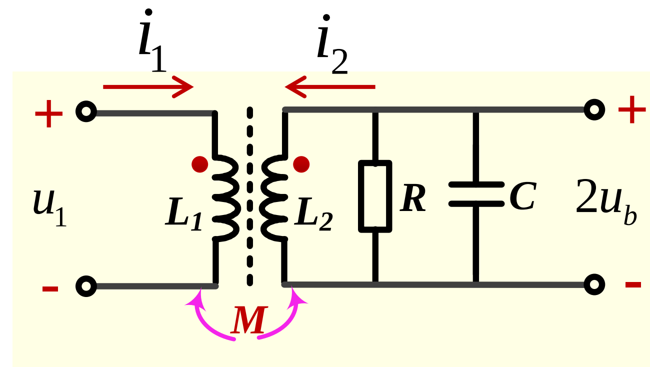
1）L_1通过互感过来；

2）检波负载；

3）L_2中的直流电阻

§8.5.5 相位鉴频器

$$
\begin{cases}
u_1 = i_1 \cdot j\omega L_1 + j\omega M \cdot i_2 \\
2u_b = i_1 \cdot j\omega M + j\omega L_2 \cdot i_2 \\
2u_b = -i_2 \cdot \dfrac{R}{1 + j\omega RC}
\end{cases}
$$



$$
L_1 = L_2 = L\,, \quad \alpha = \frac{1}{2RC}\,, \quad M = k\sqrt{L_1 L_2} = kL\,, \quad \omega_0^2 = \frac{1}{\left(1-k^2\right)LC}
$$

$$
H(j\omega) = \frac{u_b}{u_1} = \frac{\dfrac{k}{2}\omega_0^2}{\omega_0^2 - \omega^2 + 2\alpha j\omega} \approx \frac{\dfrac{k}{2}\omega_0^2 RC}{2\omega RC\left(\omega_0 - \omega\right) + j\omega}
$$

$$
\omega_0^2 - \omega^2 \approx 2\omega\left(\omega_0 - \omega\right)
$$

$$
H(j\omega) = -j\frac{kQ_T}{2} \cdot \frac{1}{1 + j\dfrac{\omega - \omega_0}{\alpha}}
$$

$$
Q_T = \frac{\omega_0}{2\alpha} = \omega_0 RC
$$

§8.5.5 相位鉴频器

$$H(j\omega) = -j\frac{kQ_T}{2}\cdot\frac{1}{1+j\dfrac{\omega-\omega_0}{\alpha}}$$

若满足

$\triangle\omega \ll \alpha$

$$\Rightarrow \quad |H(j\omega)| = \frac{kQ_T}{2}\cdot\frac{1}{\sqrt{1+\left[\dfrac{\triangle\omega}{\alpha}\cdot s(t)\right]^2}} \approx \frac{kQ_T}{2} = 1$$

令 $\dfrac{kQ_T}{2} = 1$

$$\boxed{\varphi(j\omega) = -\frac{\pi}{2} - \tan^{-1}\left[\frac{\triangle\omega}{\alpha}s(t)\right] \approx -\frac{\pi}{2} - \frac{\triangle\omega}{\alpha}s(t)}$$

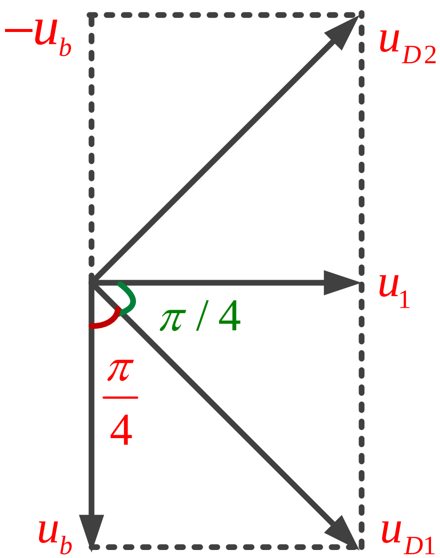
令 $u_1 = U\cos\left[\omega_C t + \triangle\omega\int_0^t s(\tau)d\tau\right]$

则 $u_b = U\cos\left[\omega_C t + \triangle\omega\int_0^t s(\tau)d\tau - \dfrac{\pi}{2} - \dfrac{\triangle\omega}{\alpha}s(t)\right]$

§8.5.5 相位鉴频器

$$u_b = U\cos\left[\omega_C t + \triangle\omega\int_0^t s(\tau)d\tau - \frac{\pi}{2} - \frac{\triangle\omega}{\alpha}s(t)\right]$$

矢量合成



① 当 $s(t)=0$ 时:

$$\omega_C = \omega_0\ , \quad \varphi = -\frac{\pi}{2}$$

$$|u_1| = |u_b| \qquad u_b = u_1 e^{j\varphi} = u_1 e^{-j\frac{\pi}{2}}$$

$$u_{D1} = u_1 + u_b$$

$$u_{D2} = u_1 - u_b$$

$$u_o = u_{o1} - u_{o2} = |U_{D1}| - |U_{D2}| = 0$$

§8.5.5 相位鉴频器

$$u_b = U\cos\left[\omega_C t + \Delta\omega\int_0^t s(\tau)d\tau - \frac{\pi}{2} - \frac{\Delta\omega}{\alpha}s(t)\right]$$

② $\boxed{s(t)>0}$ $\quad \omega(t)>\omega_0\,,\quad \varphi = -\frac{\pi}{2} - \frac{\Delta\omega}{\alpha}\cdot s(t) < -\frac{\pi}{2}$

令 $\frac{\Delta\omega}{\alpha}\cdot s(t) = \theta$

$$|U_{D1}| = \sqrt{2U^2 - 2U^2\sin\theta} \qquad |U_{D2}| = \sqrt{2U^2 + 2U^2\sin\theta}$$

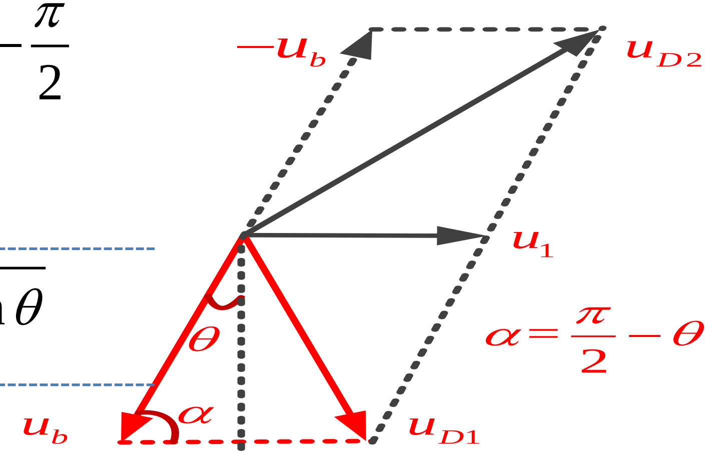
$$1 = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} \qquad \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$U_{D1} = \sqrt{2}U\left|\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right| = \sqrt{2}U\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)$$

$$U_{D2} = \sqrt{2}U\left|\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right| = \sqrt{2}U\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

$$U_o = |U_{D1}| - |U_{D2}| = -2\sqrt{2}U\sin\frac{\theta}{2} \approx -\sqrt{2}U\theta = -\sqrt{2}U\cdot\frac{\Delta\omega}{\alpha}\cdot s(t)$$

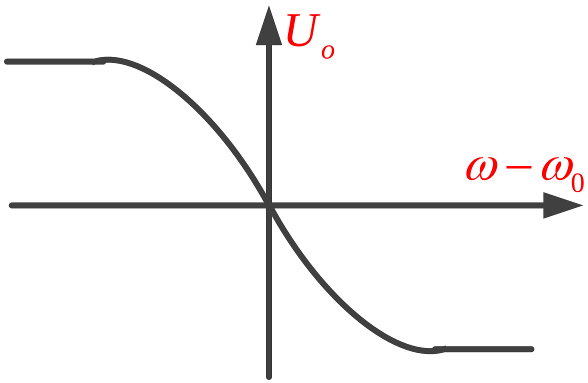
矢量合成



§8.5.5 相位鉴频器

③ $\omega(t) < \omega_0$ 　　同理

$$U_o = -2\sqrt{2}U\sin\left[\frac{\triangle\omega}{2\alpha}\cdot s(t)\right] \approx -\sqrt{2}U\cdot\frac{\triangle\omega}{\alpha}s(t)$$

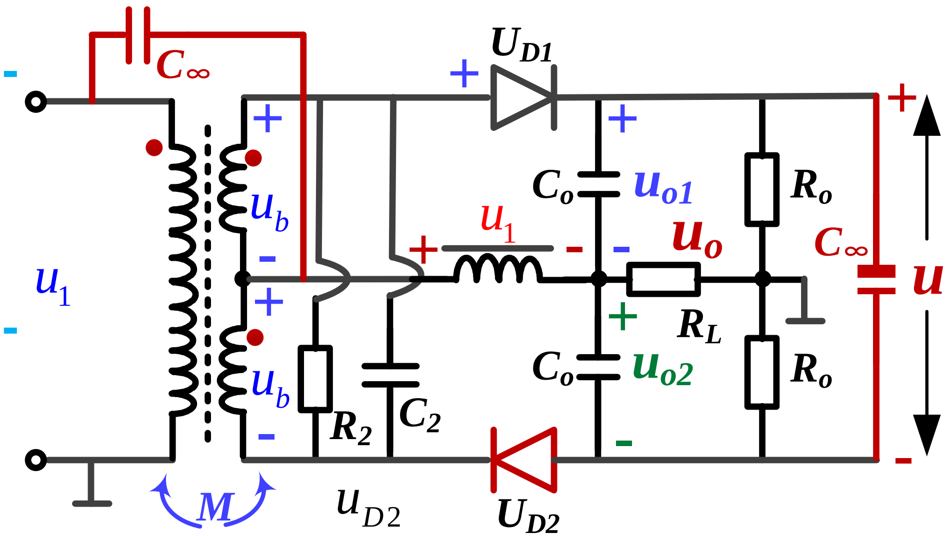


$$k_f = -\frac{\sqrt{2}U}{\alpha}$$

§8.5.5 相位鉴频器

3、比例鉴频器

1）D2反接
2）接入C_∞，滤除u上的小的寄生调幅
3）输出u_o在R_L，对地输出



$$u_{o1} \propto U_{D_1}, \quad u_{o2} \propto U_{D_2}$$

$$u_{D1} = u_1 + u_b,$$

$$u_{o1} = U_{D1} = \sqrt{2}U\sqrt{1-\sin\theta} = \sqrt{2}U\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)$$

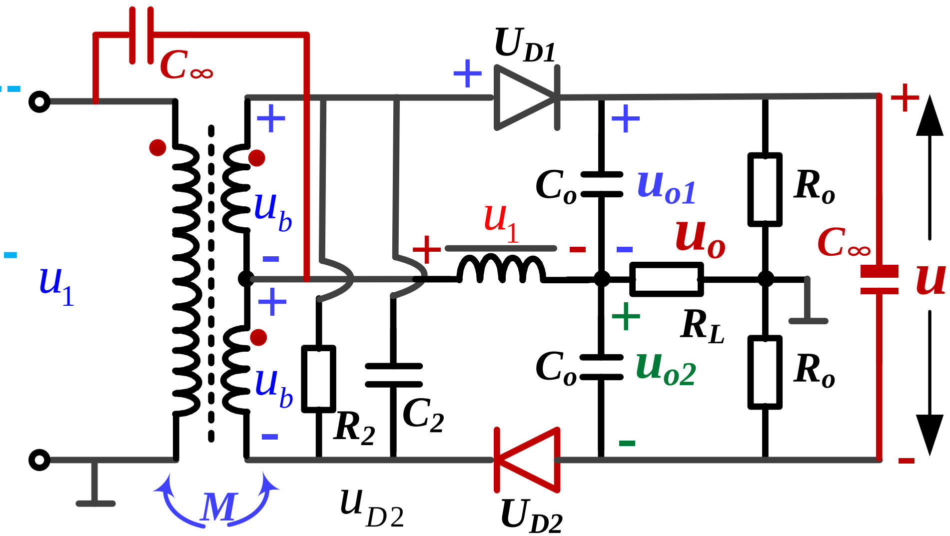
$$u_{D2} = u_b - u_1, \quad u_{o2} = U_{D2} = \sqrt{2}U\sqrt{1+\sin\theta} = \sqrt{2}U\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$$

输出端电容C_∞上的电压：$u = u_{o1} + u_{o2} = U_{D_1} + U_{D_2} = 2\sqrt{2}U\cos\frac{\theta}{2} \xrightarrow{\Delta\omega/\alpha \ll 1} \boxed{2\sqrt{2}U}$

仅和输入电压幅值有关，和电路参数无关

§8.5.5 相位鉴频器

以输出端C_∞负端为参考电平：



$$u_{C_0} = u_{o2} \qquad u_{R_0} = \frac{1}{2}u = \frac{u_{o1}+u_{o2}}{2}$$

$$u_o = u_{C_0} - u_{R_0} = u_{o2} - \frac{u_{o1}+u_{o2}}{2}$$

$$= \frac{1}{2}(u_{o2}-u_{o1}) = \frac{(u_{o2}-u_{o1})(u_{o1}+u_{o2})}{2(u_{o1}+u_{o2})}$$

$$= \frac{1-k}{2(1+k)}(u_{o1}+u_{o2}) = \frac{1-k}{2(1+k)}\cdot u = \sqrt{2}U\cdot\frac{1-k}{1+k} \quad ; \quad k = \frac{u_{o1}}{u_{o2}}$$

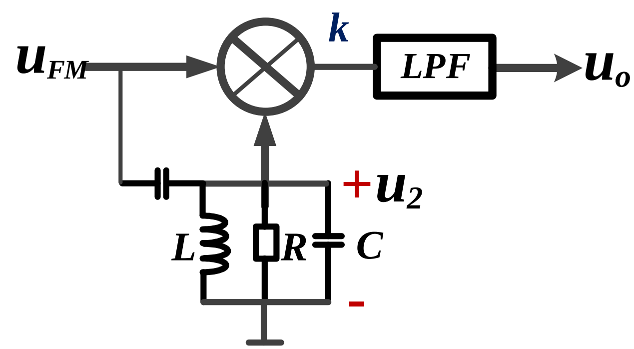
输出电压的大小仅与比例系数相关，寄生调幅被抑制

$$u_o = \frac{1}{2}(u_{o2}-u_{o1}) = \frac{1}{2}\left(|U_{o2}|-|U_{o1}|\right) = \sqrt{2}U\sin\frac{\Delta\omega}{2\alpha}\cdot s(t)$$

输出电压幅度减小一半

§8.5.6 利用乘法器的相位鉴频器

1、基本电路



$$H(j\omega)=\frac{u_2}{u_{FM}}=\frac{j\omega C_1 R}{1+jR\left(\omega C_1+\omega C-\dfrac{1}{\omega L}\right)}$$

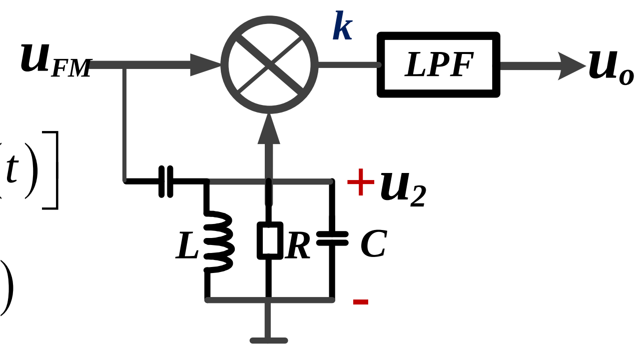
令： $$\omega_0^2=\frac{1}{L(C_1+C)}=\omega_C^2$$

$$H(j\omega)=\frac{j\omega C_1 R}{1+jQ_T\cdot\dfrac{\omega^2-\omega_0^2}{\omega\omega_0}}\approx\frac{j\omega C_1 R}{1+j\dfrac{\Delta\omega}{\alpha}s(t)}$$

条件 $\dfrac{\Delta\omega}{\alpha}\cdot s(t)\ll 1$

$$\left|H(j\omega)\right|\approx\omega C_1 R \qquad \varphi=\frac{\pi}{2}-tg^{-1}\frac{\Delta\omega}{\alpha}\cdot s(t)$$

§8.5.6 利用乘法器的相位鉴频器



$$u_o' = kU_{FM}U_2\cos\left[\omega_0 t + \Delta\omega\int s(\tau)d\tau\right]\cdot\cos\left[\omega_0 t + \Delta\omega\int s(\tau)d\tau - \varphi(t)\right]$$

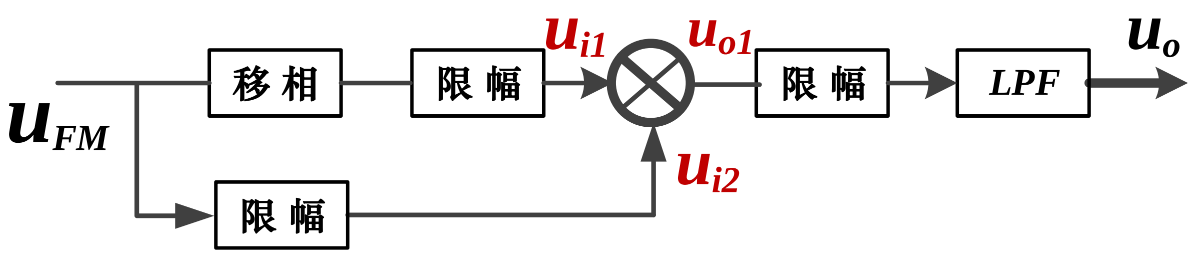
$$= \underbrace{\frac{kU_{FM}U_2}{2}\cos\left[2\omega_0 t + 2\Delta\omega\int s(\tau)d\tau - \varphi(t)\right]}_{\text{高频信号，用LPF滤除}} + \underbrace{\frac{kU_{FM}U_2}{2}\cos\varphi(t)}_{\text{输出}}$$

$$\varphi(t) = \frac{\pi}{2} - tg^{-1}\frac{\Delta\omega}{\alpha}\cdot s(t)$$

$$u_o \propto \cos\left[\frac{\pi}{2} - tg^{-1}\frac{\Delta\omega}{\alpha}\cdot s(t)\right] = \sin\left[tg^{-1}\frac{\Delta\omega}{\alpha}\cdot s(t)\right] \approx \frac{\Delta\omega}{\alpha}\cdot s(t)$$

§8.5.6 利用乘法器的相位鉴频器

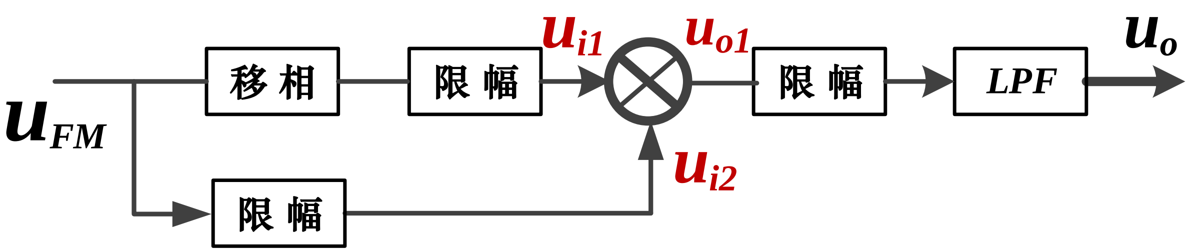
2、符合门鉴频器



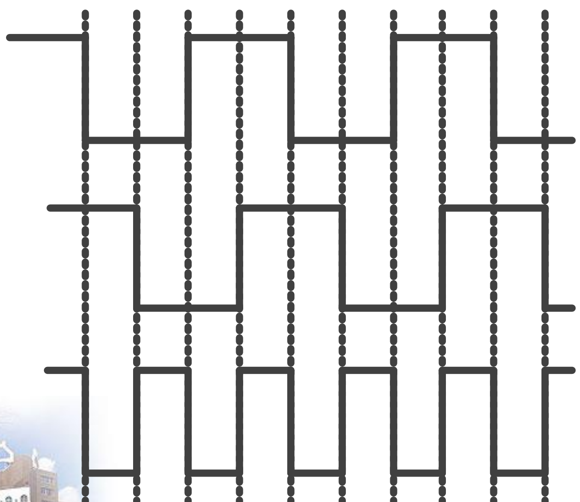
乘法器工作于开关状态，u_{i1}、u_{i2}电平相同时u_o高电平（同或关系）

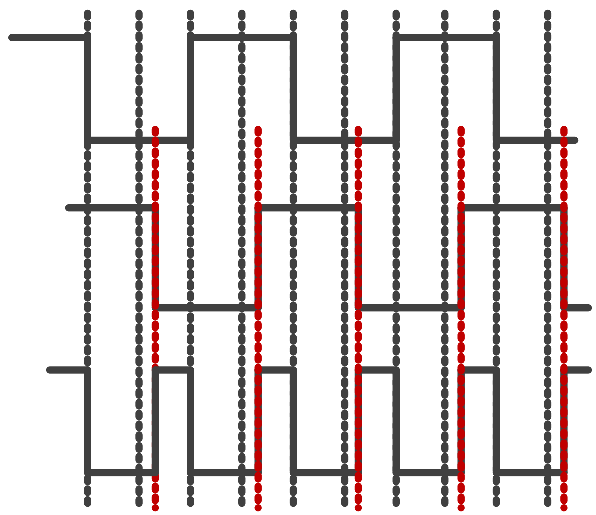
移相器移相 $\varphi=\frac{\pi}{2}-\frac{\Delta\omega}{\alpha}s(t)$

§8.5.6 利用乘法器的相位鉴频器



移相器移相$\frac{\pi}{2}-\frac{\triangle\omega}{\alpha}s(t)$





$s(t)<0$

$s(t)=0, \quad \varphi=\frac{\pi}{2}, \quad u_o=0$

(平均值=0)

$s(t)<0$	**U_{o1}上窄下宽，均值分量<0，幅宽正比于*s(t)***
$s(t)>0$	**U_{o1}上宽下窄，均值分量>0，幅宽正比于*s(t)***