习题课 2

zqy576086420

January 2024

1 重要知识点梳理

1. 法拉第电磁感应定律:

$$\epsilon = -N\frac{d\Phi}{dt}$$

2. 动生电动势:

$$\epsilon = \int_a^b (\vec{v}\times\vec{B})\cdot d\vec{l}$$

3. 感生电动势和涡旋电场

$$\epsilon = -\frac{d}{dt}\int\int \vec{B}\cdot d\vec{S} = \int_L \vec{E}\cdot d\vec{l}$$

4. 互感和自感

$$\Phi = MI$$

$$\Phi = LI$$

$$M^2 = L_1L_2$$

5. 自感串并联:

$$L = L_1 + L_2 \pm 2M$$

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 \mp 2M}$$

6. 暂态过程

$$\epsilon_L = -L\frac{dI}{dt}$$

$$\epsilon_C = \frac{\int idt}{C}$$

RL 暂态过程

$$I = I_0(1-e^{-t/\tau}), I_0 = \frac{\epsilon}{R}, \tau = \frac{L}{R}$$

$$I = I_0e^{-t/\tau}$$

7. 磁场能

$$W = \sum\frac{1}{2}L_iI_i^2 + \sum\frac{1}{2}M_{ik}I_iI_k$$

$$W = \int_V \frac{1}{2}\vec{B} \cdot \vec{H} dV$$

电磁场能（坡印廷矢量）

$$\vec{S} = \vec{E} \times \vec{H}$$

8. 麦克斯韦方程组

$$\oint_S \vec{D} \cdot d\vec{S} = \sum q_0$$

$$\oint_l \vec{E} \cdot d\vec{l} = -\int\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oint_S \vec{B} \cdot \vec{S} = 0$$

$$\oint_l \vec{H} \cdot d\vec{l} = \sum I_0 + \int\int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

$$\nabla \cdot \vec{D} = \rho_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{j_0} + \frac{\partial \vec{D}}{\partial t}$$

9. 各向同性电磁介质本构方程

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\vec{j_0} = \sigma \vec{E}$$

10. 边值关系

$$(\vec{H}_1 - \vec{H}_2) \times \vec{n} = \vec{i}_0$$

$$(\vec{B}_1 - \vec{B}_2) \cdot \vec{n} = 0$$

$$(\vec{D}_1 - \vec{D}_2) \cdot \vec{n} = \sigma_0$$

$$(\vec{E}_1 - \vec{E}_2) \times \vec{n} = 0$$

2 第 20、21 次作业

5.19

平均半径为 $R_a = 11cm$，根据磁路基尔霍夫定理：

$$\frac{B}{\mu_0\mu_r}(2\pi R_a - \delta) + \frac{B}{\mu_0}\delta = NI$$

直接求得：

$$\delta = 0.01m \;\; B = 0.71T$$

$$\delta = 0.02m \;\; B = 0.37T$$

5.20

考虑左半边磁路：

$$R_1 = \frac{1.4 \times 2 + 1.8}{\mu_0\mu_r \times 0.5^2} + \frac{1.8 - 0.15}{\mu_0\mu_r \times \pi(0.25)^2}$$

$$R_2 = \frac{0.15}{\mu_0 \times \pi(0.25)^2}$$

左边磁路的磁感应强度为中间磁路的一半，列出磁路定理:

$$NI = \frac{1}{2}BS_1 \times R_1 + BS_2 \times R_2 = 1.2 \times 10^5$$

5.21

$$B(\pi(R + \frac{a}{2}) + 2l + x + d + a) = \mu_0\mu_r NI$$

$$B = 0.31T$$

$$F = \frac{B^2 S}{\mu_0} = 197N$$

6.1

$$t > ltan\alpha/2v \;\; \epsilon = Blv$$

$$t > ltan\alpha/2v \;\; \epsilon = Bv^2 ttan\alpha$$

6.3

（1）无线长直导线的磁场为 $B(r) = \frac{\mu_0 I}{2\pi r}$，则线圈内的磁通量为：

$$\phi = \int_a^b B(r)ldr = \frac{mu_0 I_0 l}{2\pi} ln\frac{b}{a} sin\omega t$$

则电动势为：

$$\epsilon = -\frac{d\phi}{dt} = \frac{mu_0 I_0 l\omega}{2\pi} ln\frac{b}{a} cos\omega t$$

(2) 线圈在 t 时刻的磁通量为：

$$\phi = \int_{a+vt}^{b+vt} B(r)ldr = \frac{mu_0 I_0 l}{2\pi} ln\frac{b+vt}{a+vt} sin\omega t$$

则电动势为:

$$\epsilon = -\frac{d\phi}{dt} = \frac{\mu_0 I_0 l}{2\pi}(\omega cos\omega t ln\frac{b+vt}{a+vt} + sin\omega t\frac{(a-b)v}{(a+vt)(b+vt)}$$

(3) 电路中电流为 $I = \epsilon/R$

$$F = (B(a{+}vt){-}B(b{+}vt))Il = \frac{mu_0^2 I_0^2 l^2 sin\omega t}{4\pi^2 R}\frac{b-a}{(a+vt)(b+vt)}(\omega cos\omega t ln\frac{b+vt}{a+vt}{+}sin\omega t\frac{(a-b)v}{(a+vt)(b+vt)})$$

6.4

磁矩在 θ 处产生的磁场为:

$$B_r = \frac{\mu_0 \mu cos\theta}{2\pi R^3}$$

$$B_\theta = \frac{\mu_0 \mu sin\theta}{4\pi R^3}$$

环仅有切割 r 方向磁场才会产生沿环方向的电动势（电场）, 分析 θ 处的电流元:

$$d\epsilon = B_\theta R d\theta R\omega cos\theta$$

则 1/4 圆环的总电动势为:

$$\int_0^{\pi/2} d\epsilon = \frac{\mu_0\mu\omega}{2\pi R}\int_0^{\pi/2} sin\theta cos\theta d\theta = \frac{\mu_0\mu\omega}{4\pi R}$$

3 第 22 次作业

6.7

长直导线的磁感应强度为 $B = \frac{\mu_0 I}{2\pi r}$, 线圈在 B 点离导线距离为 x 处的磁通量为:

$$\phi = \int_x^{a+x} B\cdot dr\cdot\frac{b}{a}(r-x) = \frac{mu_0 Ib}{2\pi a}(a - xln\frac{a+x}{x})$$

设 $x = vt$, 则线圈的磁感应强度为:

$$\epsilon = -\frac{d\phi}{dx}\frac{dx}{dt} = \frac{mu_0 Ib}{2\pi a}(vln\frac{a+x}{x} + xv(\frac{1}{a+x} - \frac{1}{x}))$$

代入 x=vt:

$$\epsilon = \frac{mu_0 Ib}{2\pi a}(vln\frac{a+d}{d} + \frac{av}{a+d})$$

6.14

将 O 与 a、b、c 连线形成闭合三角形，感应电动势只在 ab 杆上产生，分别求出各三角形的磁通量变化即得相应边上得磁感应强度:

$$\phi_{ac} = (B_0 + kt)\cdot\frac{\sqrt{3}}{4}R^2$$

$$\phi_{ab} = (B_0 + kt)\cdot(\frac{\sqrt{3}}{4}R^2 + \frac{\pi}{12}R^2)$$

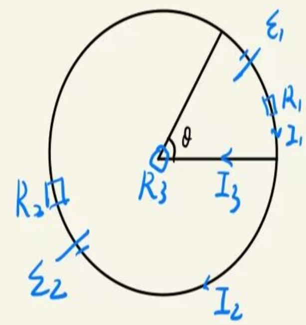
则有:

$$U_{ac} = \frac{\sqrt{3}}{4}kR^2$$

$$U_{ab} = \frac{\sqrt{3}}{4}kR^2 + \frac{\pi}{12}kR^2$$

6.15

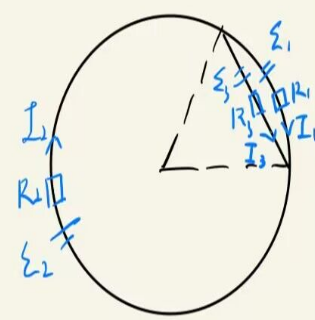
(a) 列出基尔霍夫方程:



$$\begin{cases} I_1R_1 + I_2R_2 - \varepsilon_1 - \varepsilon_2 = 0 \\ I_1R_1 + I_3R_3 - \varepsilon_1 = 0 \\ I_1 = I_2 + I_3 \\ R_1 = \frac{3}{2}R, \quad R_2 = \frac{15}{2}R, \quad R_3 = R \\ \varepsilon_1 = \frac{\pi}{6}ka^2, \quad \varepsilon_2 = \frac{5}{6}\pi ka^2 \end{cases} \Rightarrow \begin{cases} I_1 = I_2 = \frac{\pi ka^2}{9R} \\ I_3 = 0 \end{cases}$$

$$\Rightarrow V = I_3R_3 = 0$$

(b)



$$\begin{cases} I_1R_1 + I_2R_2 - \varepsilon_1 - \varepsilon_2 = 0 \\ I_1R_1 - I_3R_3 - \varepsilon_1 + \varepsilon_3 = 0 \\ I_1 + I_3 = I_2 \\ \varepsilon_3 = \frac{\sqrt{3}}{4}ka^2 \end{cases}$$

$$\Rightarrow \begin{cases} I_1 = \left(\frac{\pi}{9} - \frac{5\sqrt{3}}{54}\right)\frac{ka^2}{R} \\ I_2 = \left(\frac{\pi}{9} + \frac{\sqrt{3}}{54}\right)\frac{ka^2}{R} \\ I_3 = \frac{\sqrt{3}}{9}\frac{ka^2}{R} \end{cases} \Rightarrow V = \frac{\sqrt{3}}{9}ka^2$$

4 第 23 次作业

6.25

(1)

$$L = \frac{N \cdot \mu_0 n I \cdot \frac{\pi d^2}{4}}{I} = \frac{\mu_0 N^2 \pi d^2}{4l} = 9.87 \times 10^{-4} H$$

$$R = N \cdot \rho \cdot \pi d = 7.76\Omega$$

(2) 列出回路方程:

$$IR - L\frac{dI}{dt} - E = 0$$

$$I(0) = 0$$

解得:

$$I = \frac{E}{R}(1 - e^{-\frac{R}{L}t})$$

则:

$$\frac{dI}{dt}|_{t=0} = \frac{E}{L} = 2.03 \times 10^3 A/s$$

$$I|_{t->\infty} = \frac{E}{R} = 0.258A$$

$$\tau = \frac{L}{R} = 1.27 \times 10^{-4} s$$

$$t = ln2 \times \tau = 8.80 \times 10^{-4} s$$

$$W = \frac{1}{2} L I^2 = 3.28 \times 10^{-5} J$$

$$w = \frac{W}{\frac{1}{4}\pi d^2 l} = 4.18 J/m^3$$

6.26

(1) 磁感应强度为 $B = \frac{mu_0 I}{2\pi r}$, 磁场能密度为 $w = \frac{B^2}{2\mu_0}$, 则磁场能量为:

$$W = \int_a^b w \cdot 2\pi r dr = \frac{\mu_0 I^2}{4\pi} ln\frac{b}{a}$$

则自感系数为:

$$L = \frac{2W}{I^2} = \frac{\mu_0}{2\pi} ln\frac{b}{a}$$

(2)

$$W' = \frac{\mu_0 I^2}{4\pi} ln\frac{2b}{a}$$

$$\Delta W = W' - W = \frac{\mu_0 I^2}{4\pi} ln2$$

(3) 外圆柱面处的磁感应强度为 $B = \frac{\mu_0 I}{4\pi r}$, 取长条形微元 $rd\theta$ 分析受力

$$dF = B \cdot ird\theta = \frac{\mu_0 I}{4\pi r} \cdot \frac{I}{2\pi} d\theta$$

在外圆柱面半径扩大过程中磁场力对该微元做的总功为:

$$dW = \int_b^{2b} F dr$$

沿整个圆柱面积分即为总功:

$$W = int_0^{2\pi} dW = \int_b^{2b} \frac{\mu_0 I}{4\pi r} \cdot \frac{I}{2\pi} dr \int_0^{2\pi} d\theta = \frac{mu_0 I^2}{4\pi} ln2$$

电源做功为:

$$W_E = \int I\epsilon dt = \int I d\phi = \int I^2 dL = \frac{\mu_0 I^2}{2\pi}(ln\frac{2b}{a} - ln\frac{b}{a}) = \frac{\mu_0 I^2}{2\pi} ln2$$

$$W_E = W + \Delta W$$

6.33

(1) 列出基尔霍夫方程:

$$iR_1 - L\frac{di_1}{dt} - U = 0$$

$$i_2 R_2 + L\frac{di_1}{dt} = 0$$

$$i = i_1 + i_2$$

解得:

$$i_1 = \frac{U}{R_1}(1 - e^{-t/\tau})$$

$$i_2 = \frac{U}{R_1 + R_2} e^{-t/\tau}$$

$$\tau = \frac{L(R_1 + R_2)}{R_1 R_2}$$

电阻 2 消耗的焦耳热为:

$$Q = \int_0^\infty i_2^2 R_2 dt = 220J$$

(2) 电阻 2 消耗的焦耳热等于自感线圈储能:

$$Q = W = \frac{1}{2} L(\frac{U}{R_1})^2 = 2420J$$

5 补充习题

5. 一个薄的圆柱型带电导体壳长为 l，半径为 a，l»a，壳表面的电荷密度为 σ，此圆柱壳以 $\omega = kt$ 的角速度绕其中心转动，k>0，为常数，忽略边缘效应。求:(1) 圆柱体内外磁感应强度；(2) 圆柱体内外涡旋电场强度；（3）圆柱体内部空间贮存的总能量；（4）圆柱壳表面流入圆柱体内的能量