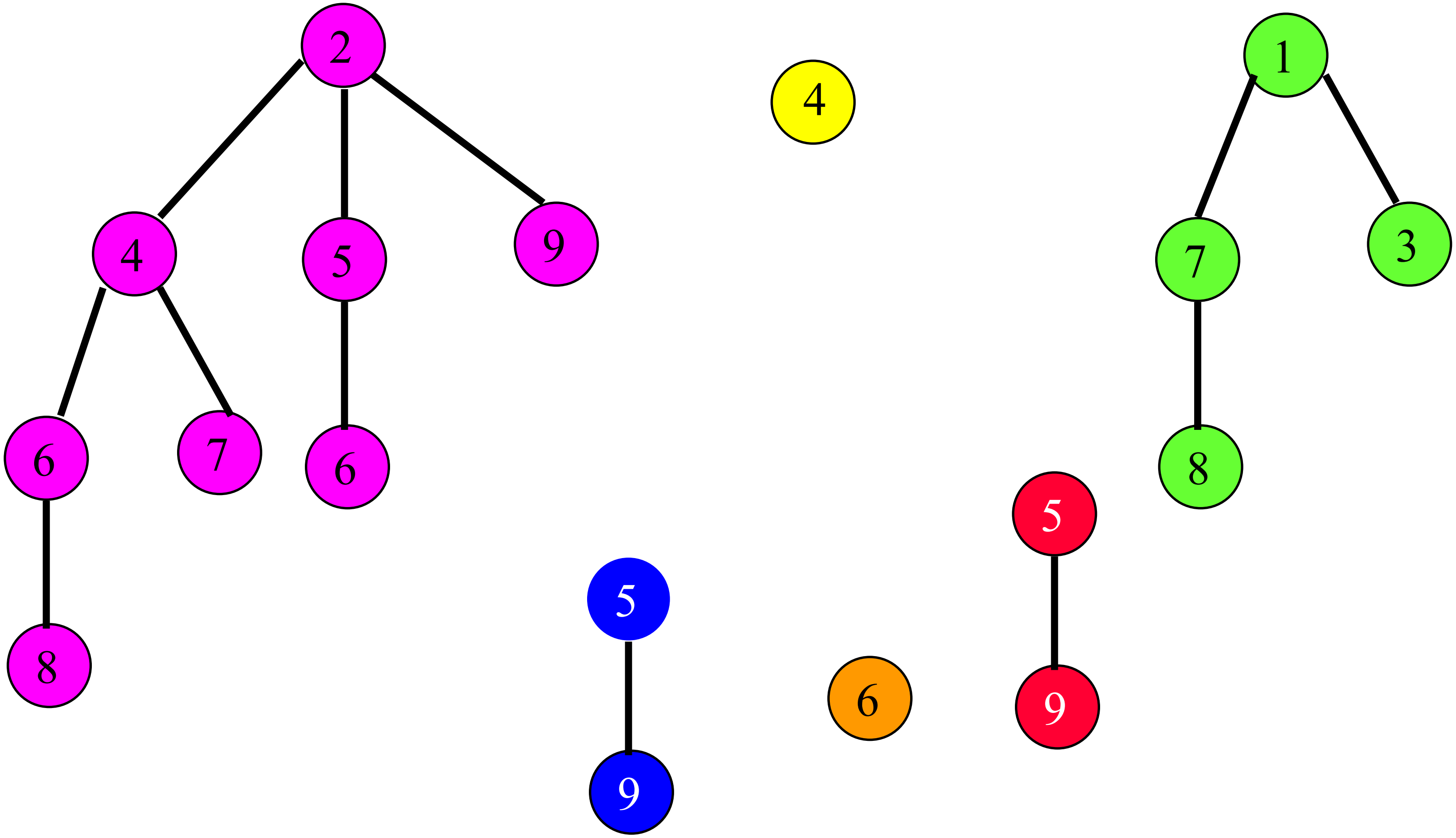
Binomial Heaps

	Leftist trees	Binomial heaps	
		Actual	Amortized
Insert	O(log n)	O(1)	O(1)
Remove min (or max)	O(log n)	O(n)	O(log n)
Meld	O(log n)	O(1)	O(1)

Min Binomial Heap

- Collection of min trees.

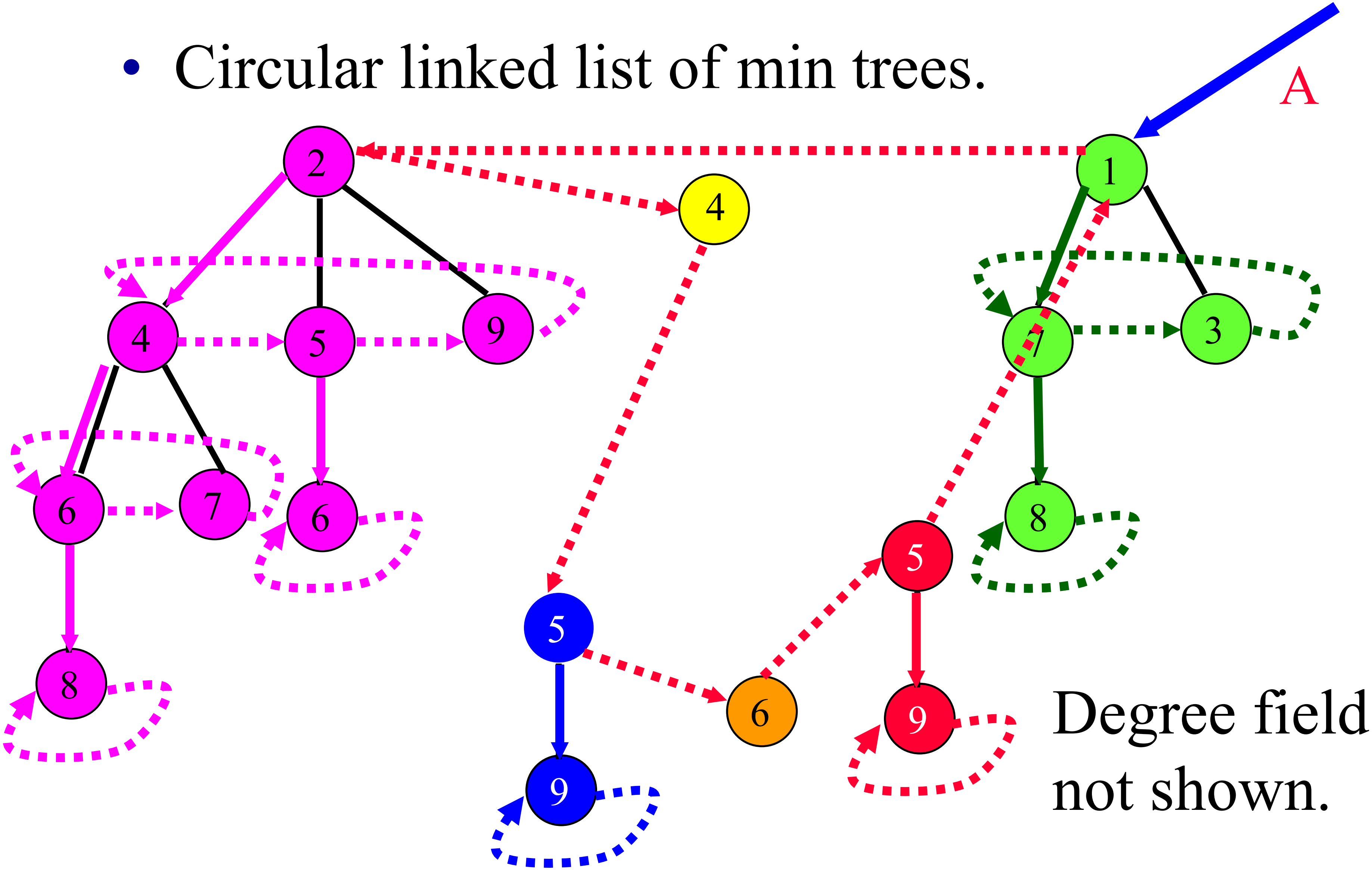


Node Structure

- Degree
  - Number of children.
- Child
  - Pointer to one of the node’s children.
  - Null iff node has no child.
- Sibling
  - Used for circular linked list of siblings.
- Data

Binomial Heap Representation

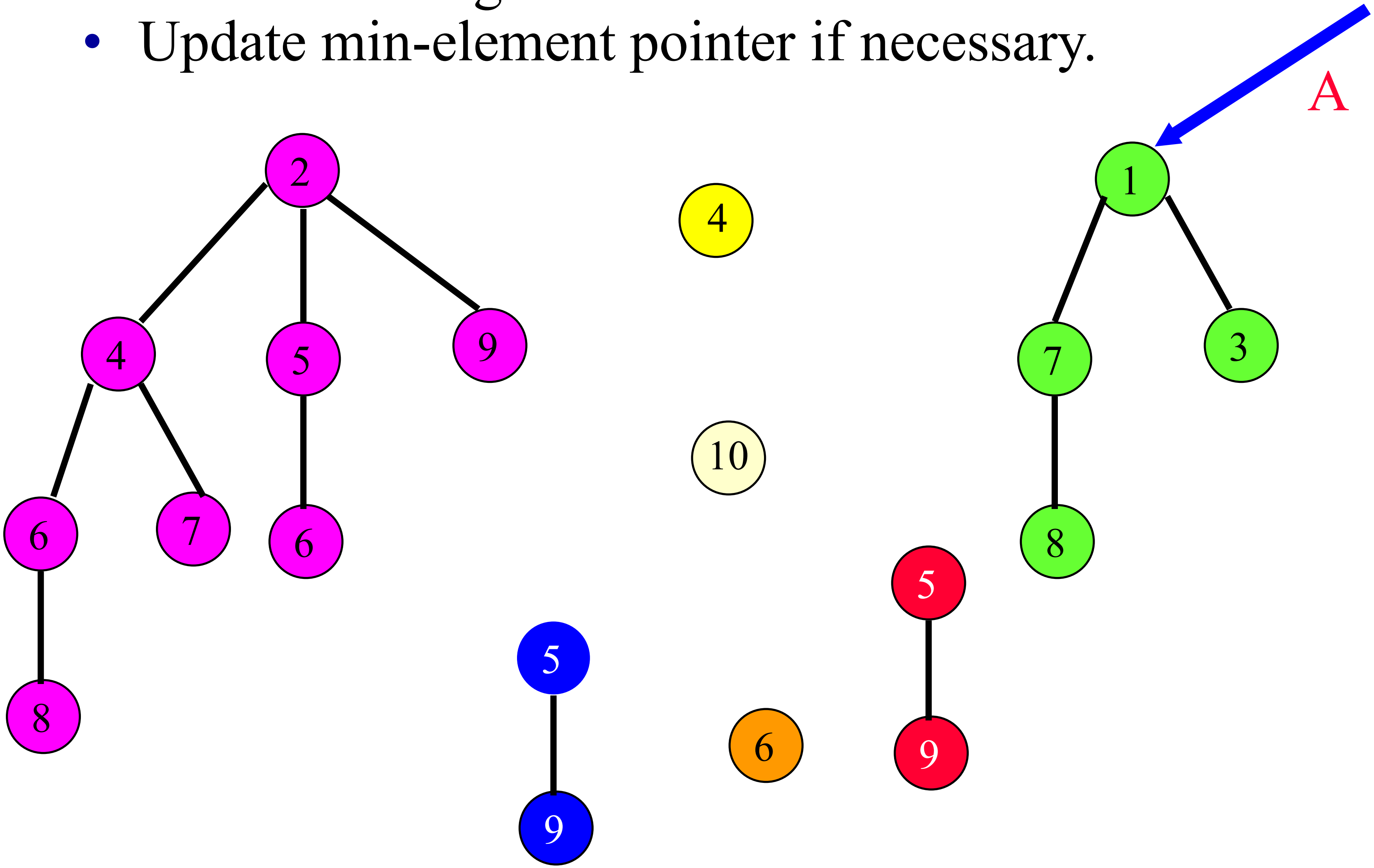
- Circular linked list of min trees.



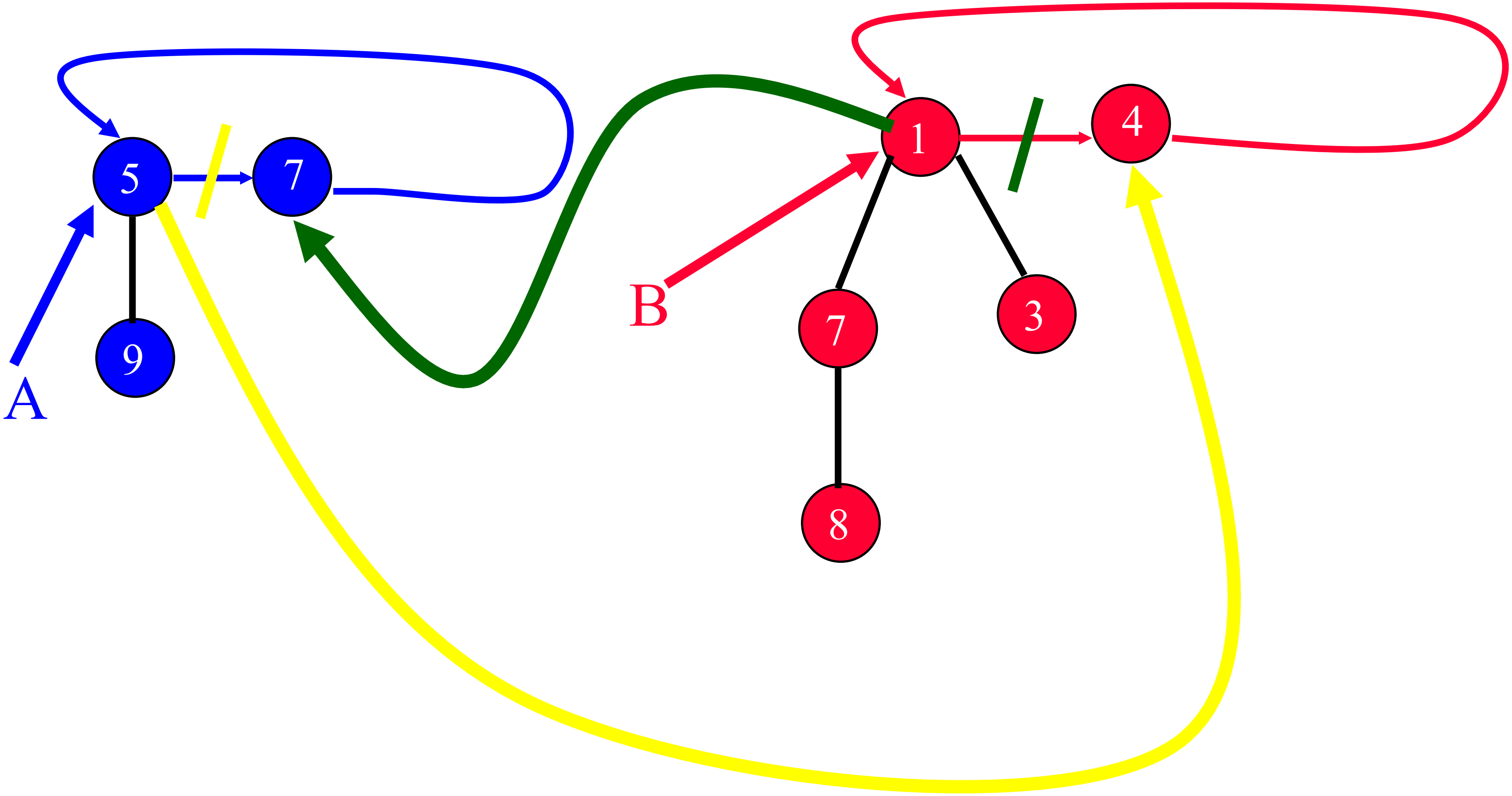
Degree fields not shown.

Insert 10

- Add a new single-node min tree to the collection.
- Update min-element pointer if necessary.

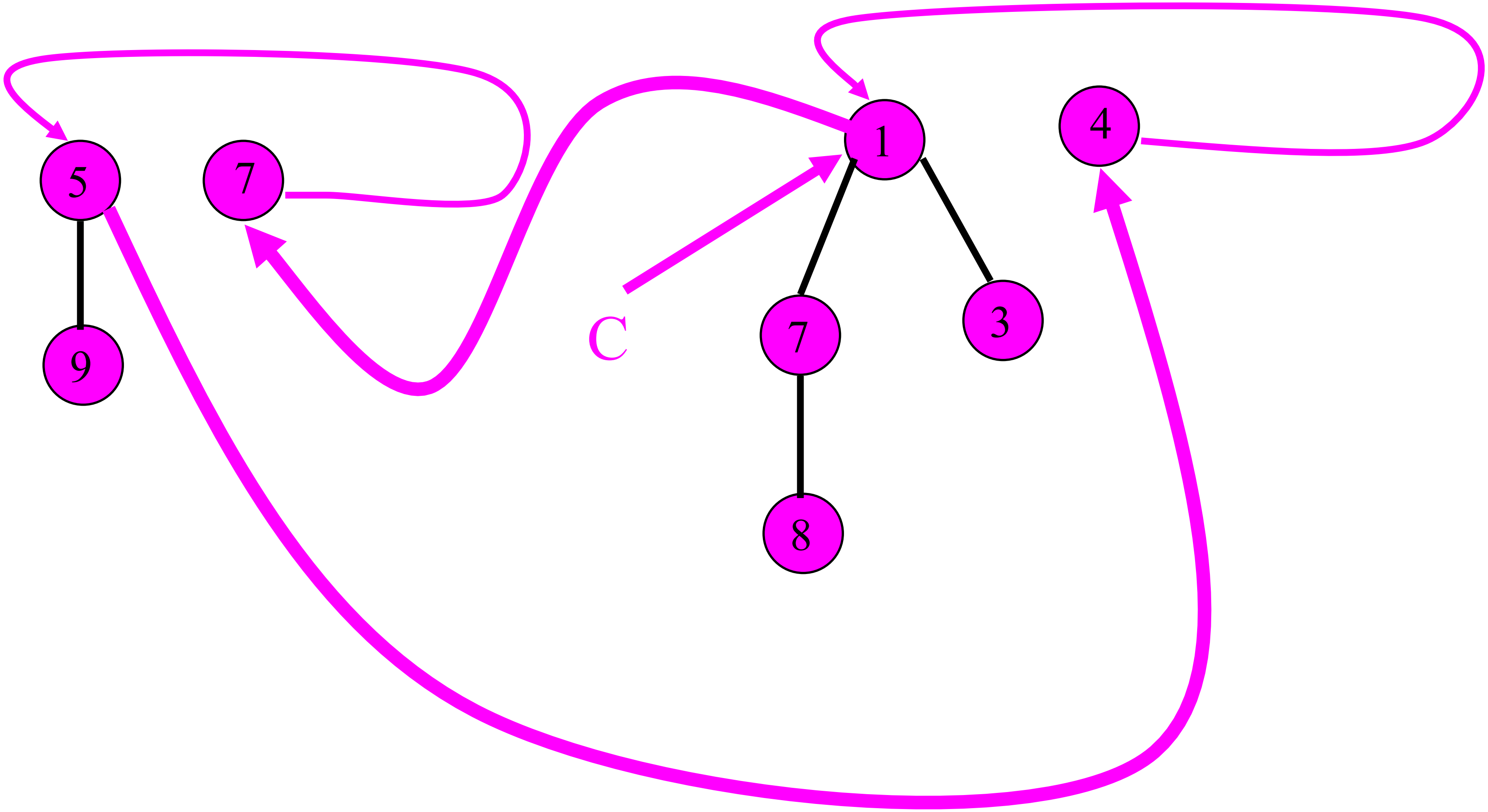


Meld



- Combine the 2 top-level circular lists.
- Set min-element pointer.

Meld

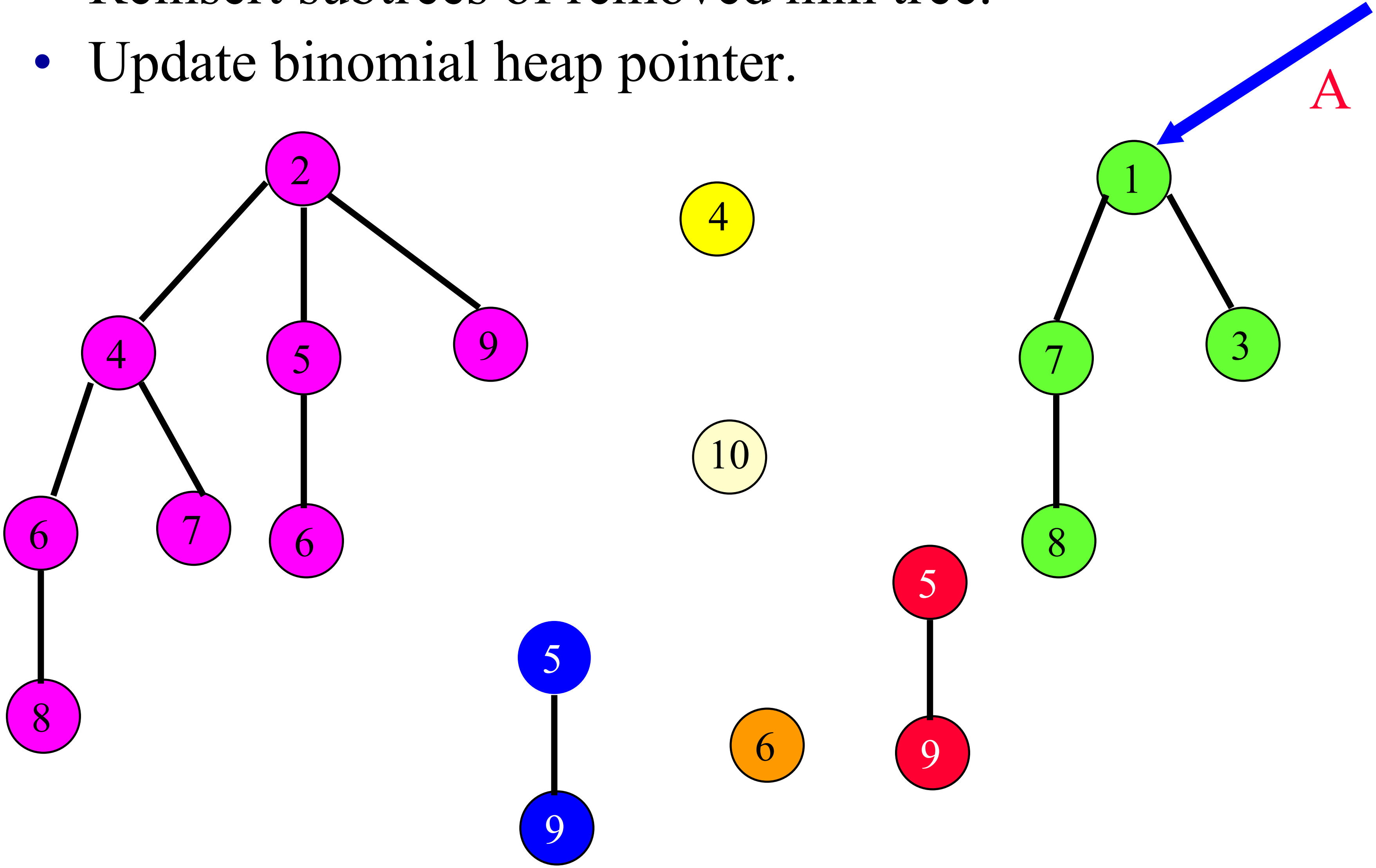


Remove Min

- Empty binomial heap => fail.

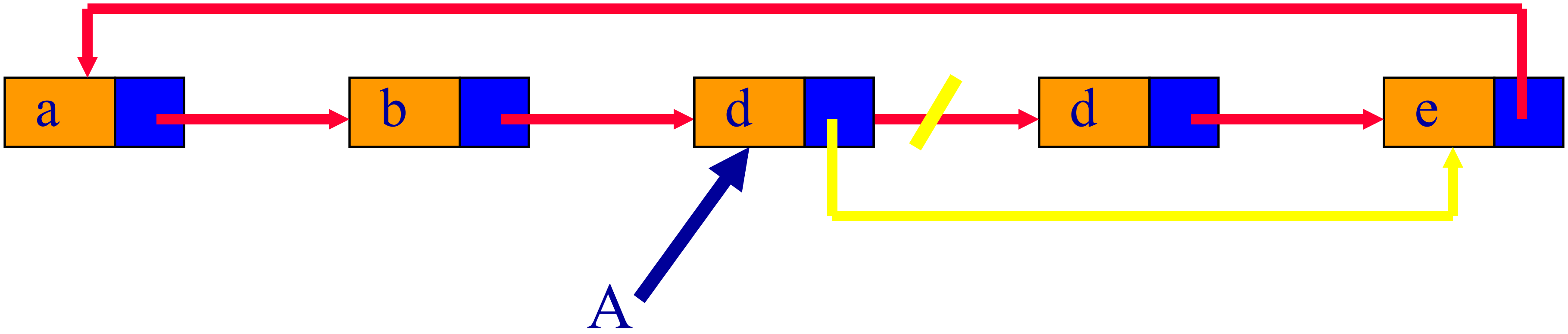
Nonempty Binomial Heap

- Remove a min tree.
- Reinsert subtrees of removed min tree.
- Update binomial heap pointer.



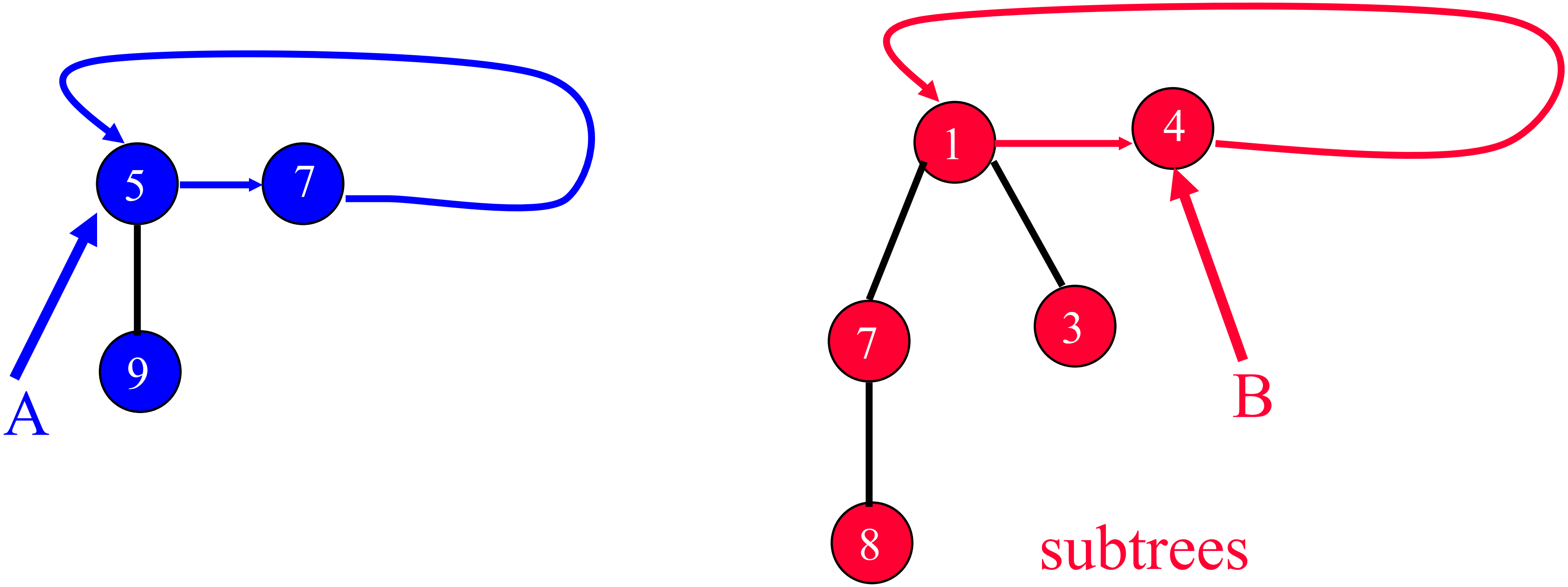
Remove Min Tree

- Same as remove a node from a circular list.



- No next node => empty after remove.
- Otherwise, copy next-node data and remove next node.

Reinsert Subtrees



- Combine the 2 top-level circular lists.
  - Same as in meld operation.

Update Binomial Heap Pointer

- Must examine roots of all min trees to determine the min value.

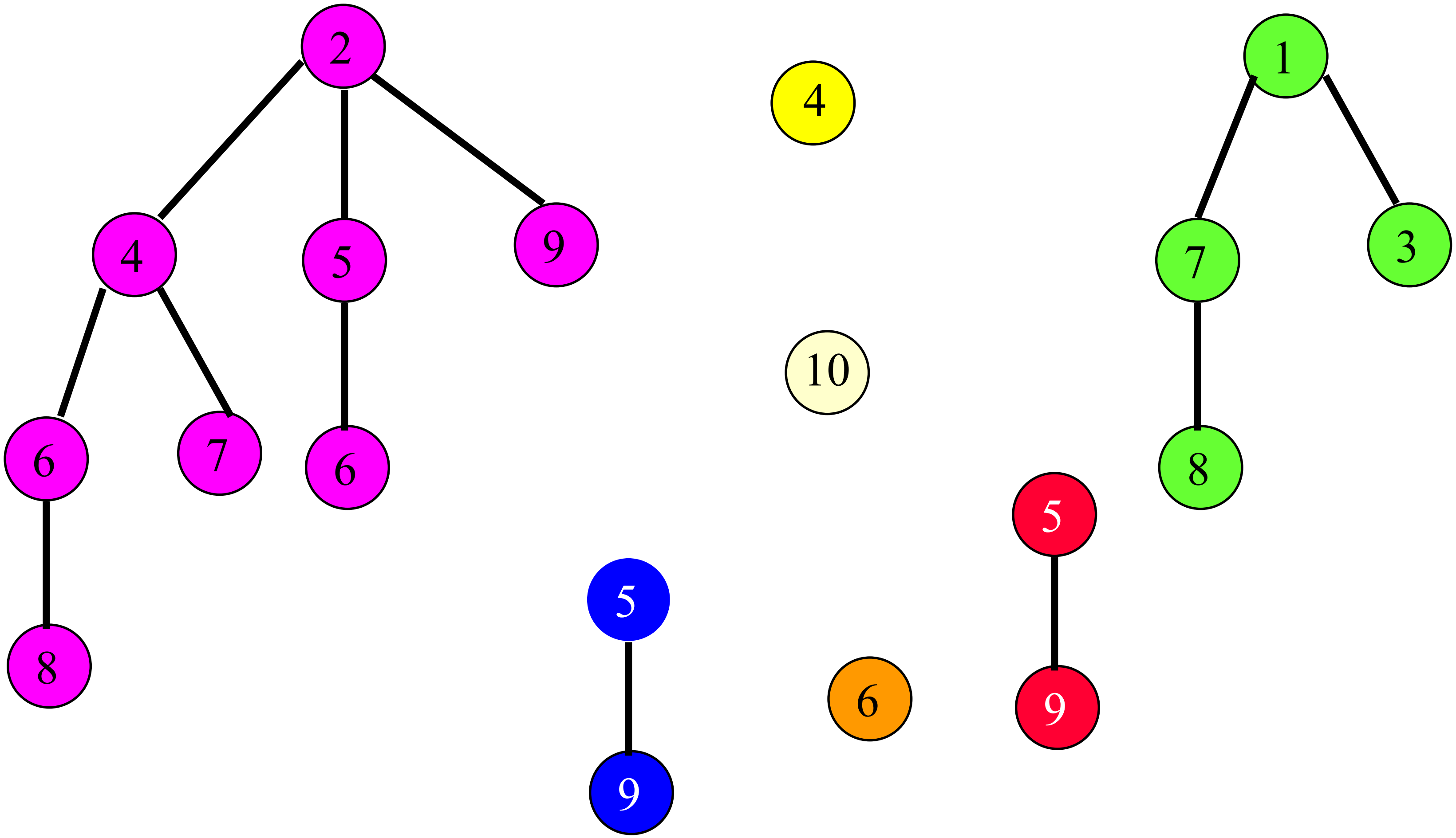
Complexity Of Remove Min

- Remove a min tree.
  - O(1).
- Reinsert subtrees.
  - O(1).
- Update binomial heap pointer.
  - O(s), where s is the number of min trees in final top-level circular list.
  - s = O(n).
- Overall complexity of remove min is O(n).

Enhanced Remove Min

- During reinsert of subtrees, pairwise combine min trees whose roots have equal degree.

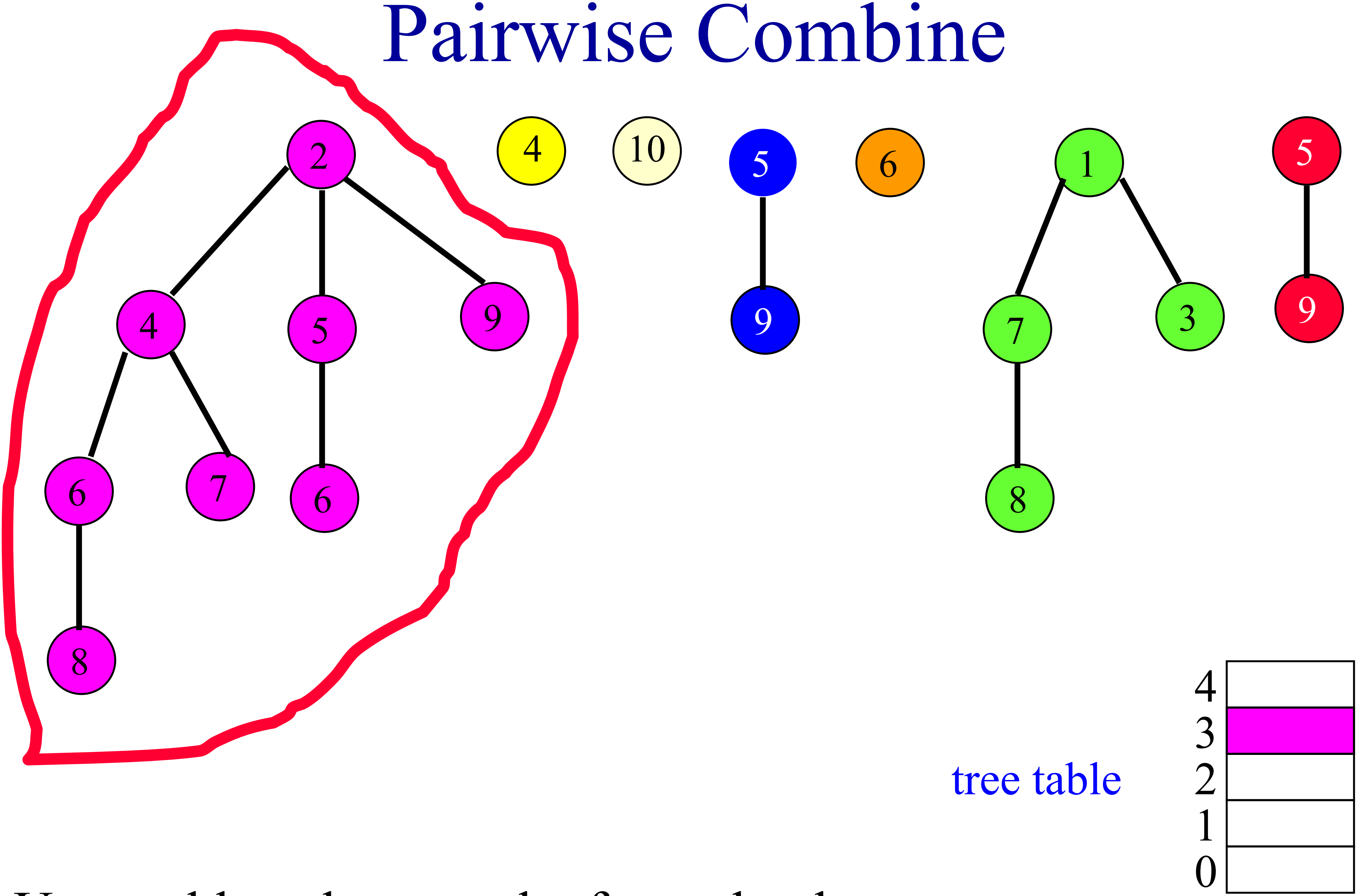
Pairwise Combine



Examine the s = 7 trees in some order.

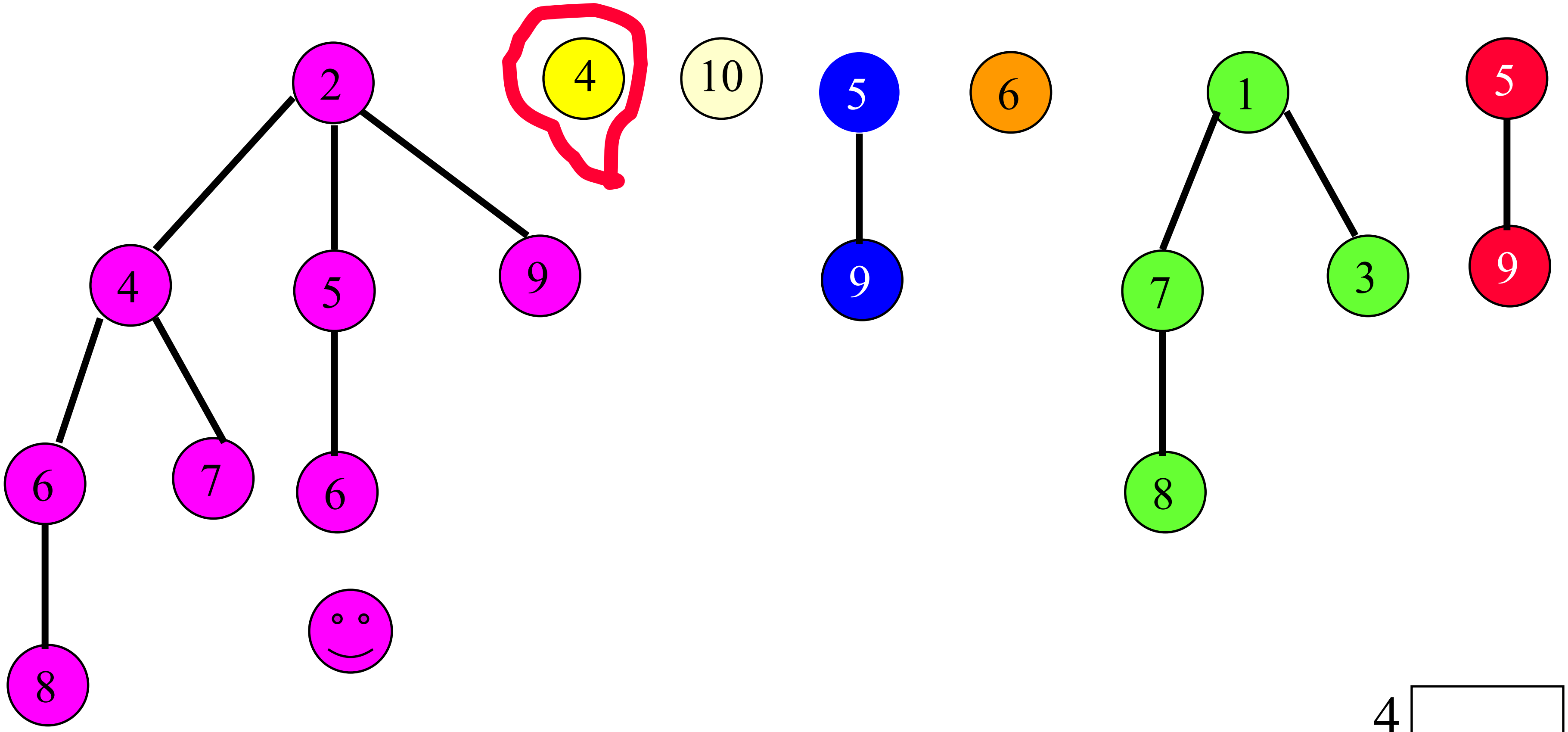
Determined by the 2 top-level circular lists.

Pairwise Combine



Use a table to keep track of trees by degree.

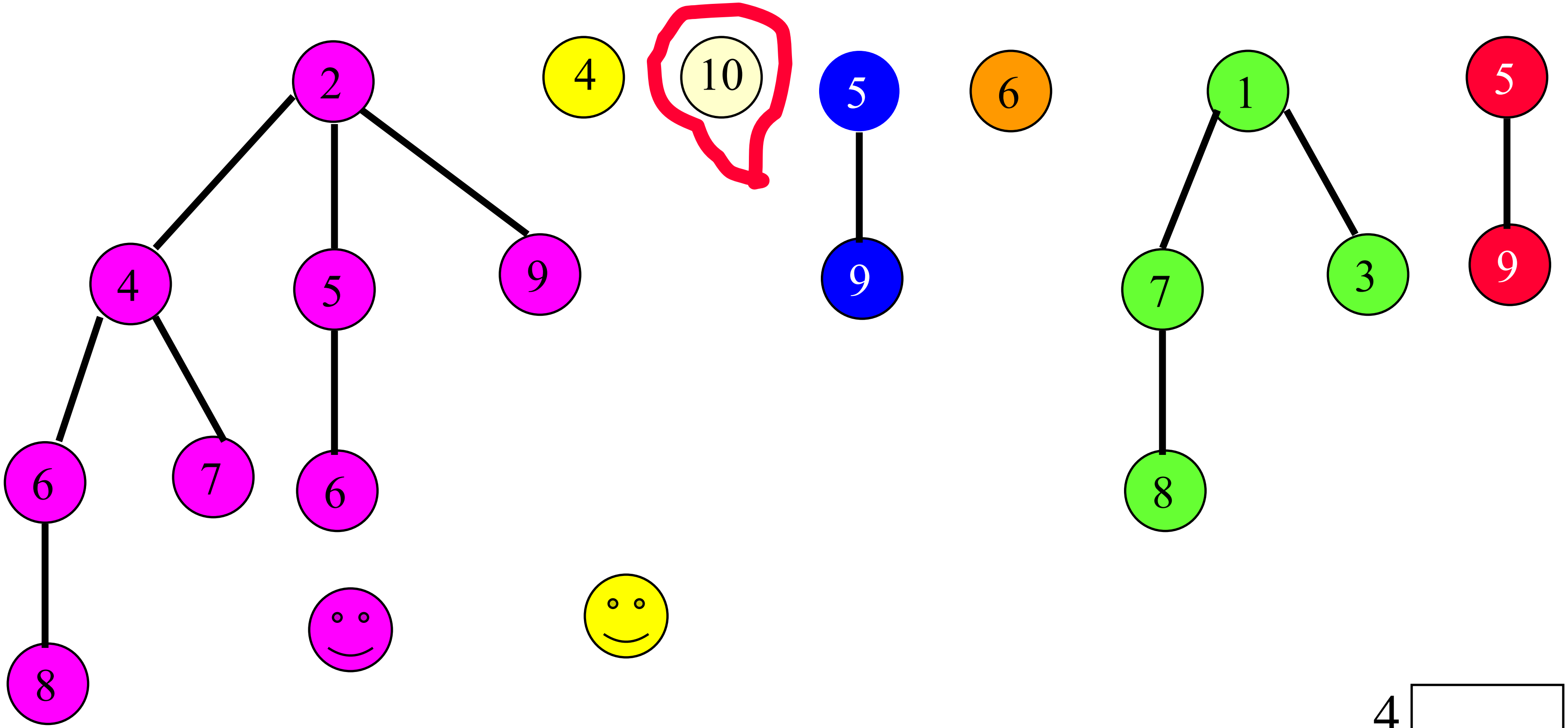
Pairwise Combine

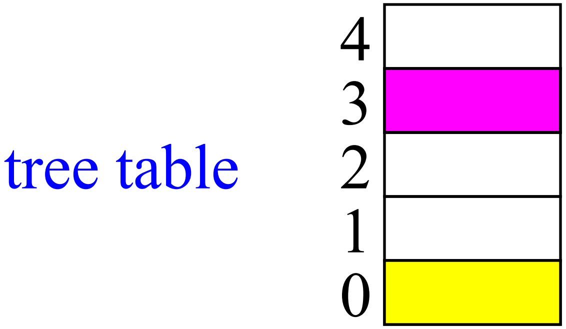


tree table

4	
3	🟪
2	
1	
0	🟨

Pairwise Combine

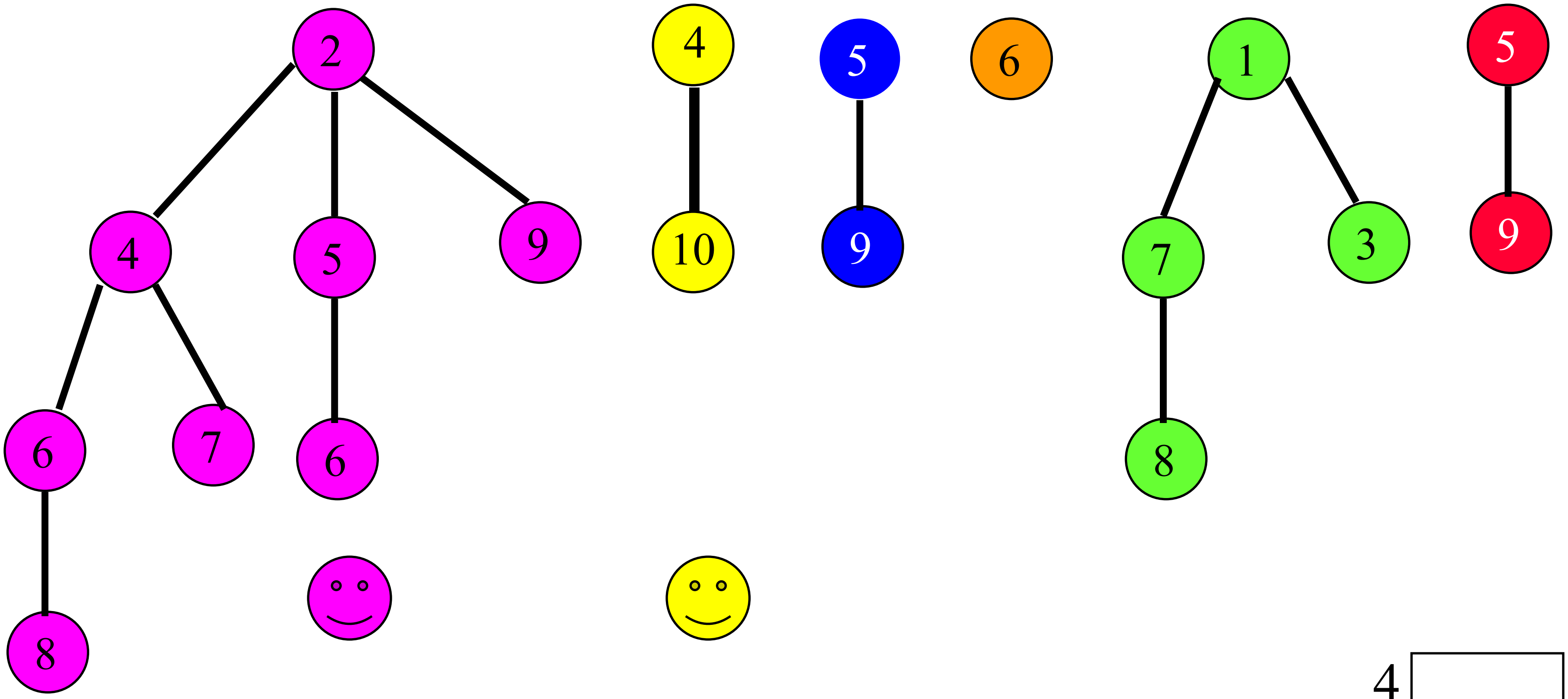




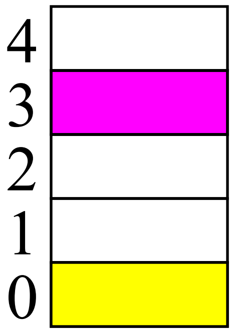
Combine 2 min trees of degree 0.

Make the one with larger root a subtree of other.

Pairwise Combine

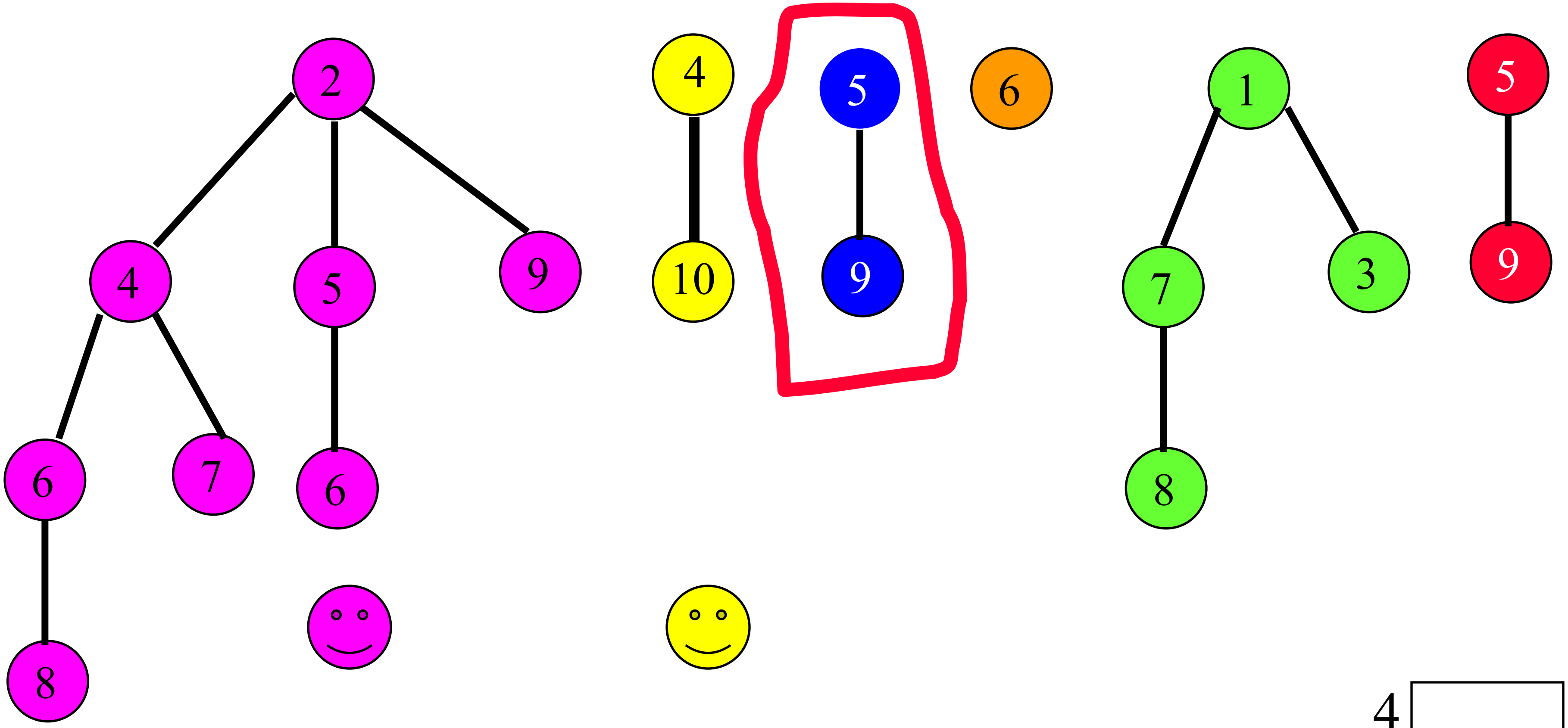


tree table



Update tree table.

Pairwise Combine



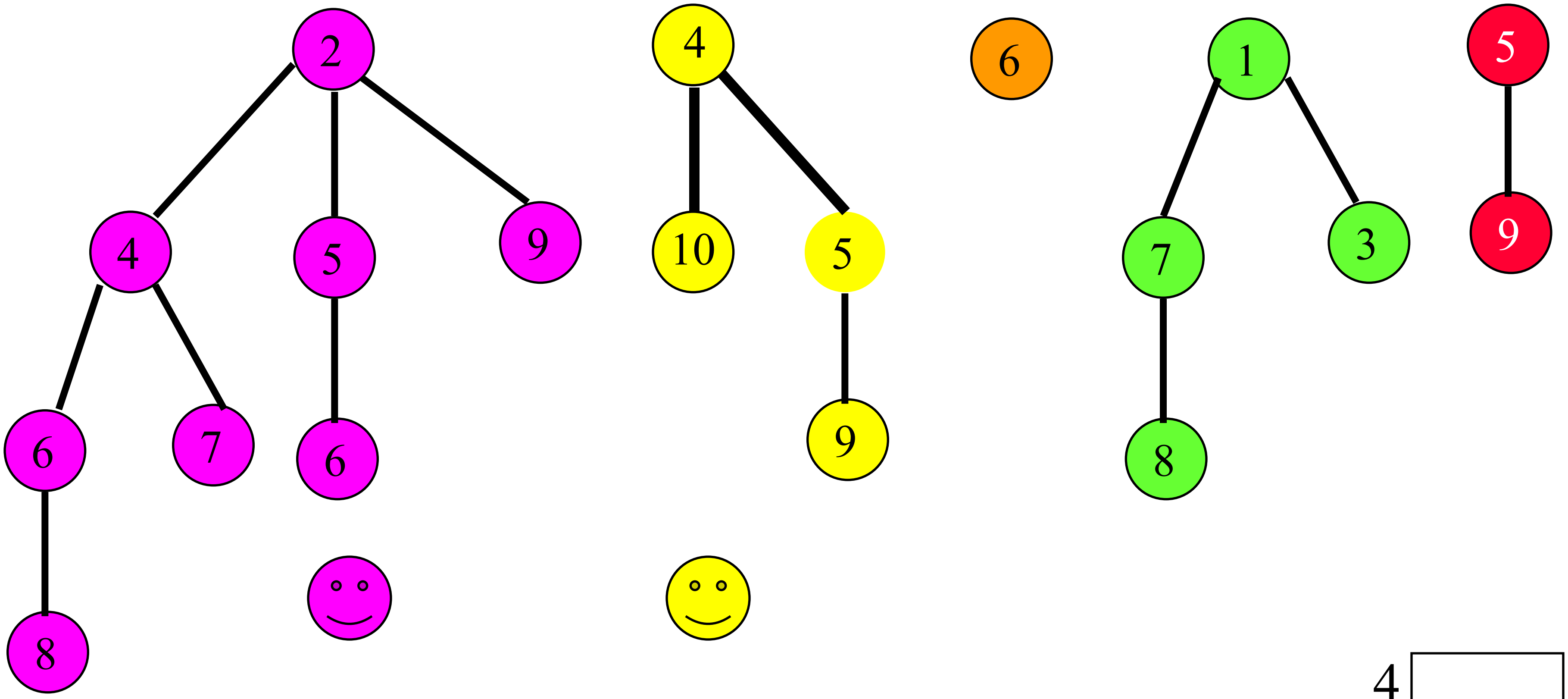
tree table

4	
3	🟪
2	
1	🟨
0	

Combine 2 min trees of degree 1.

Make the one with larger root a subtree of other.

Pairwise Combine

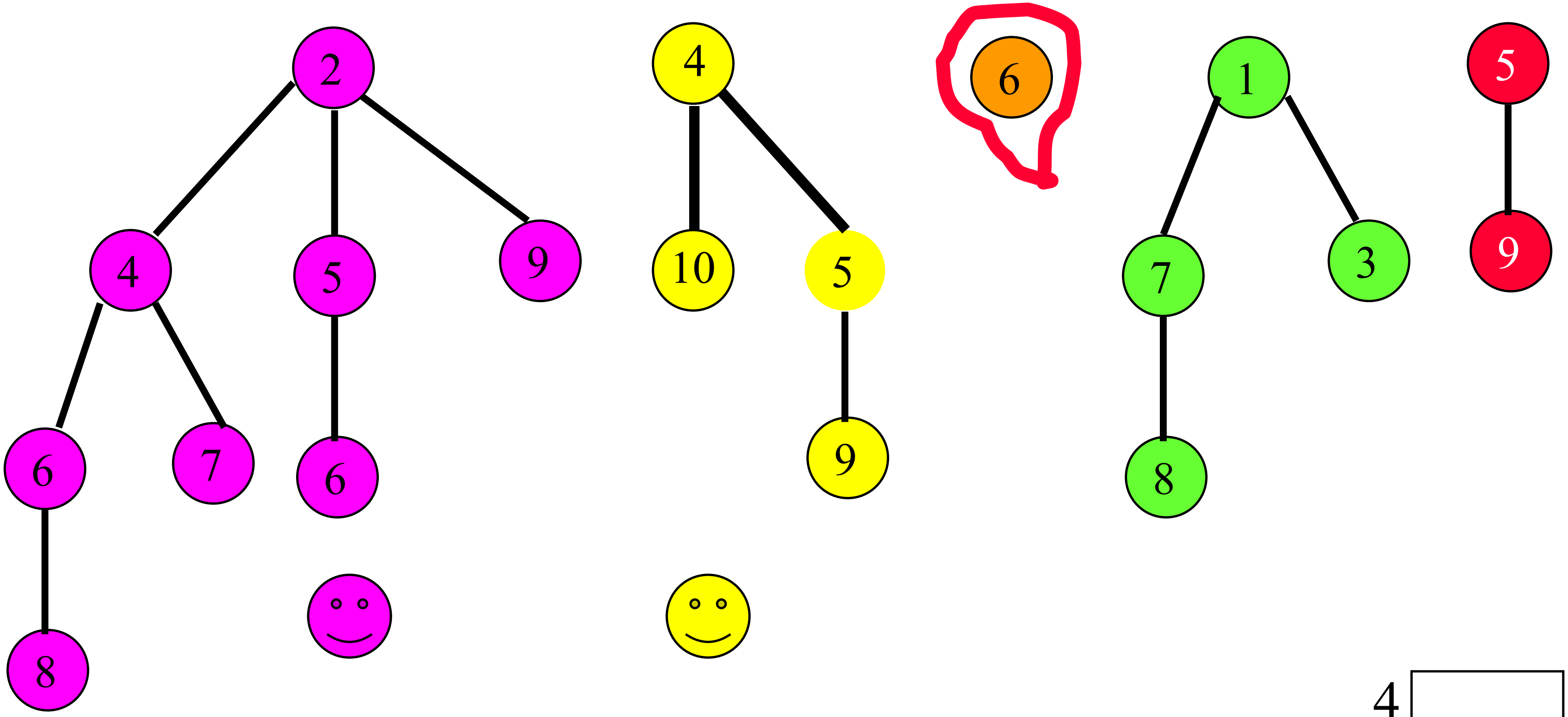


tree table

4	
3	🟪
2	
1	🟨
0	

Update tree table.

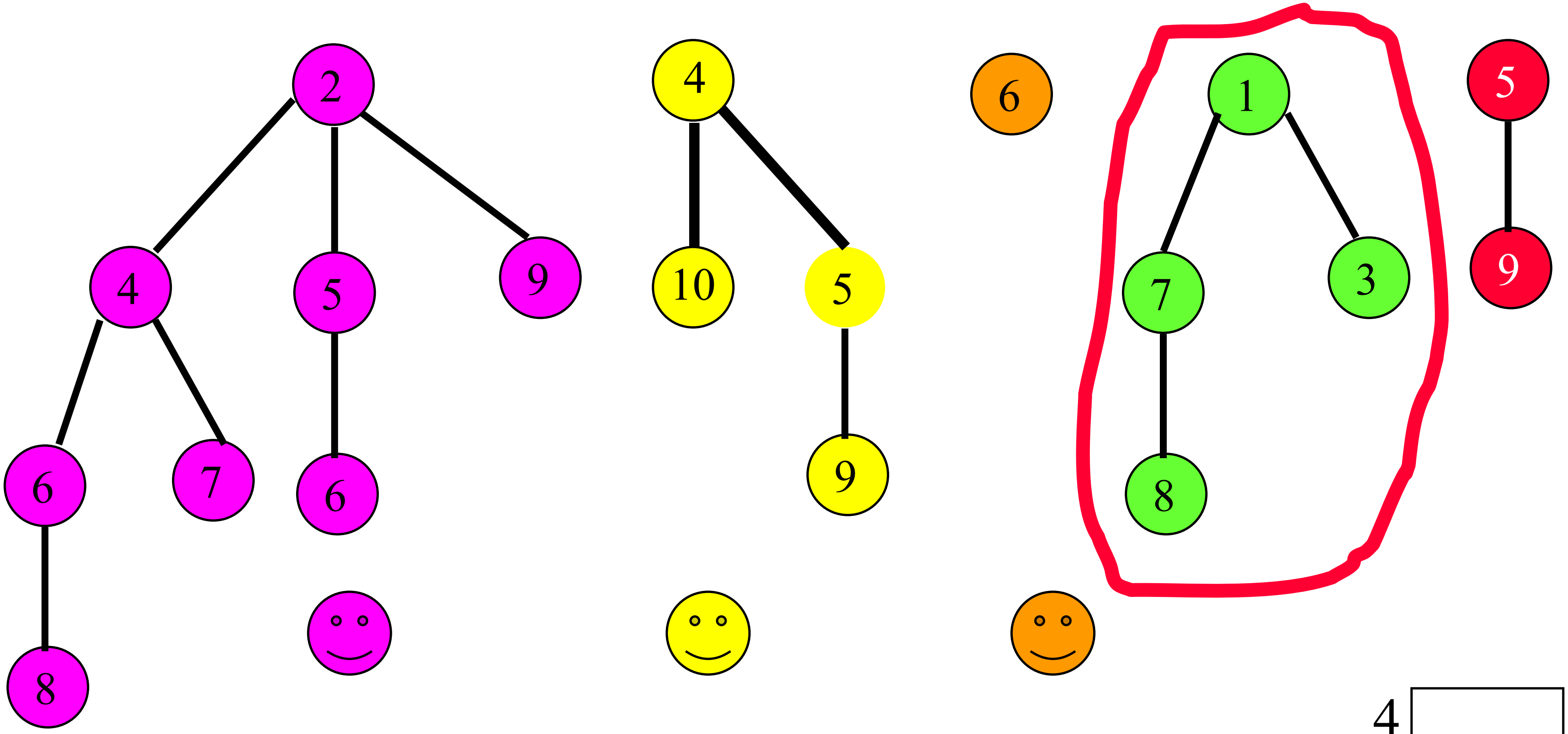
Pairwise Combine



tree table

4	
3	🟪
2	🟨
1	
0	🟧

Pairwise Combine



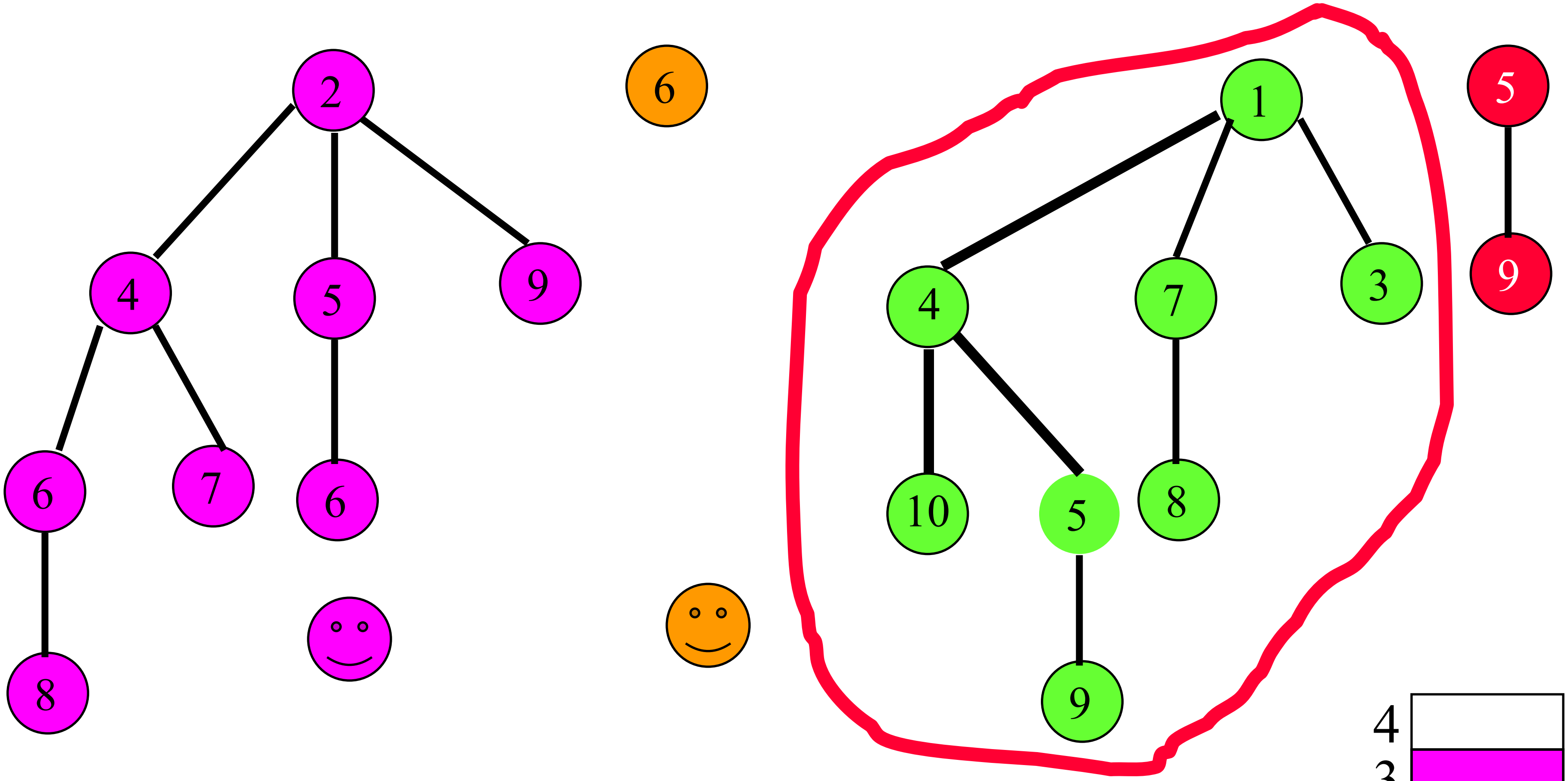
tree table

4	
3	🟪
2	🟨
1	
0	🟧

Combine 2 min trees of degree 2.

Make the one with larger root a subtree of other.

Pairwise Combine



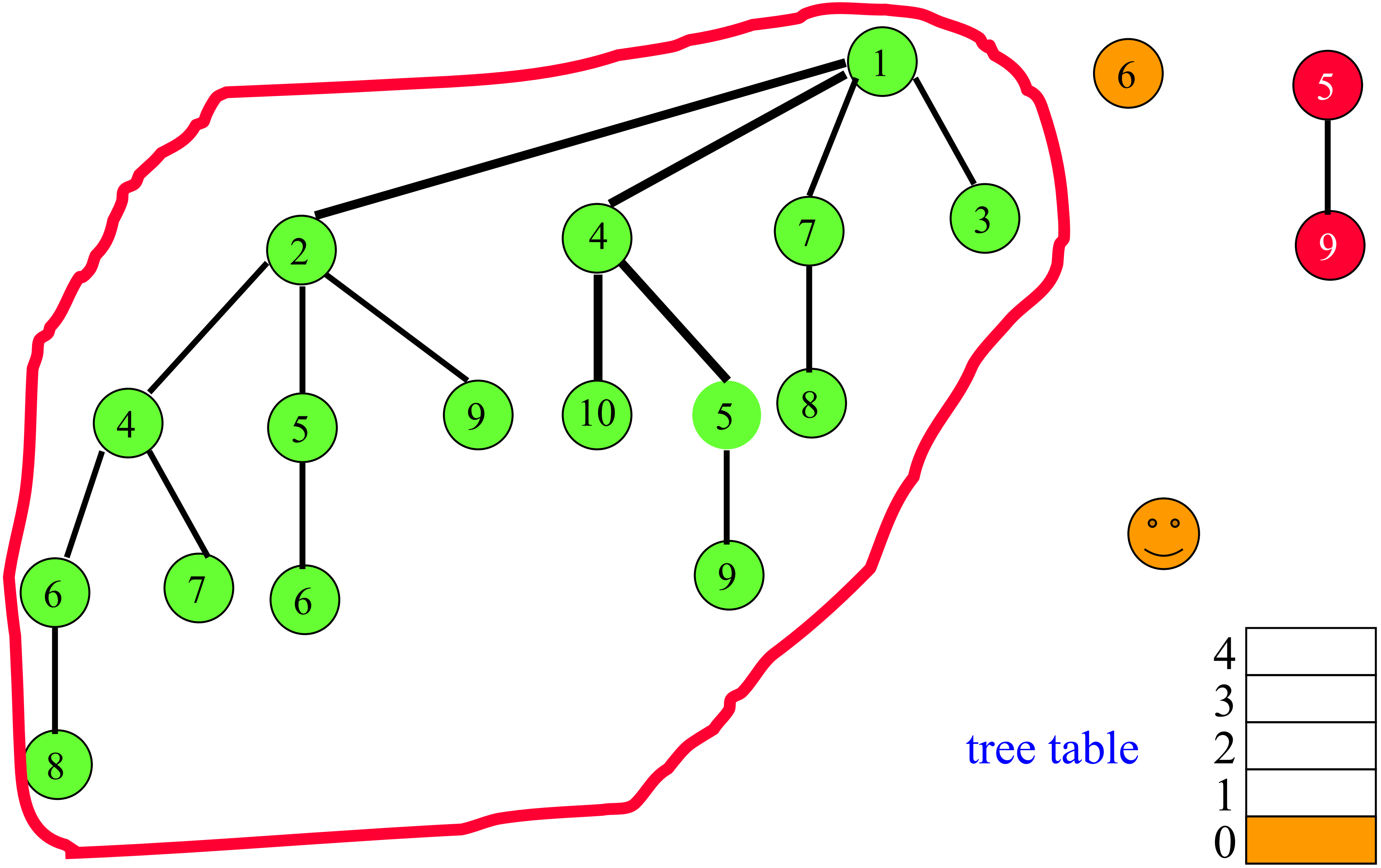
tree table

4	
3	🟪
2	
1	
0	🟧

Combine 2 min trees of degree 3.

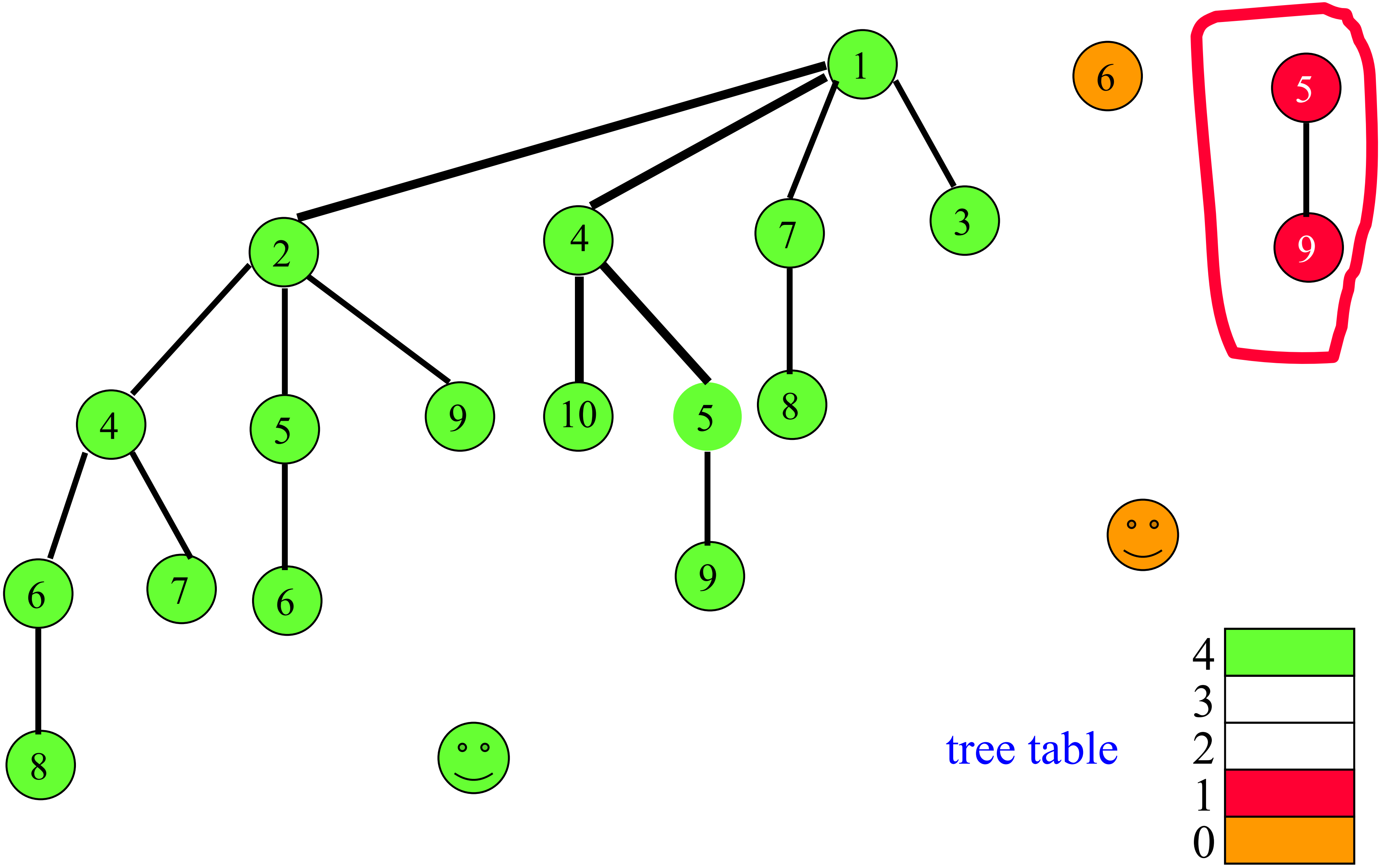
Make the one with larger root a subtree of other.

Pairwise Combine



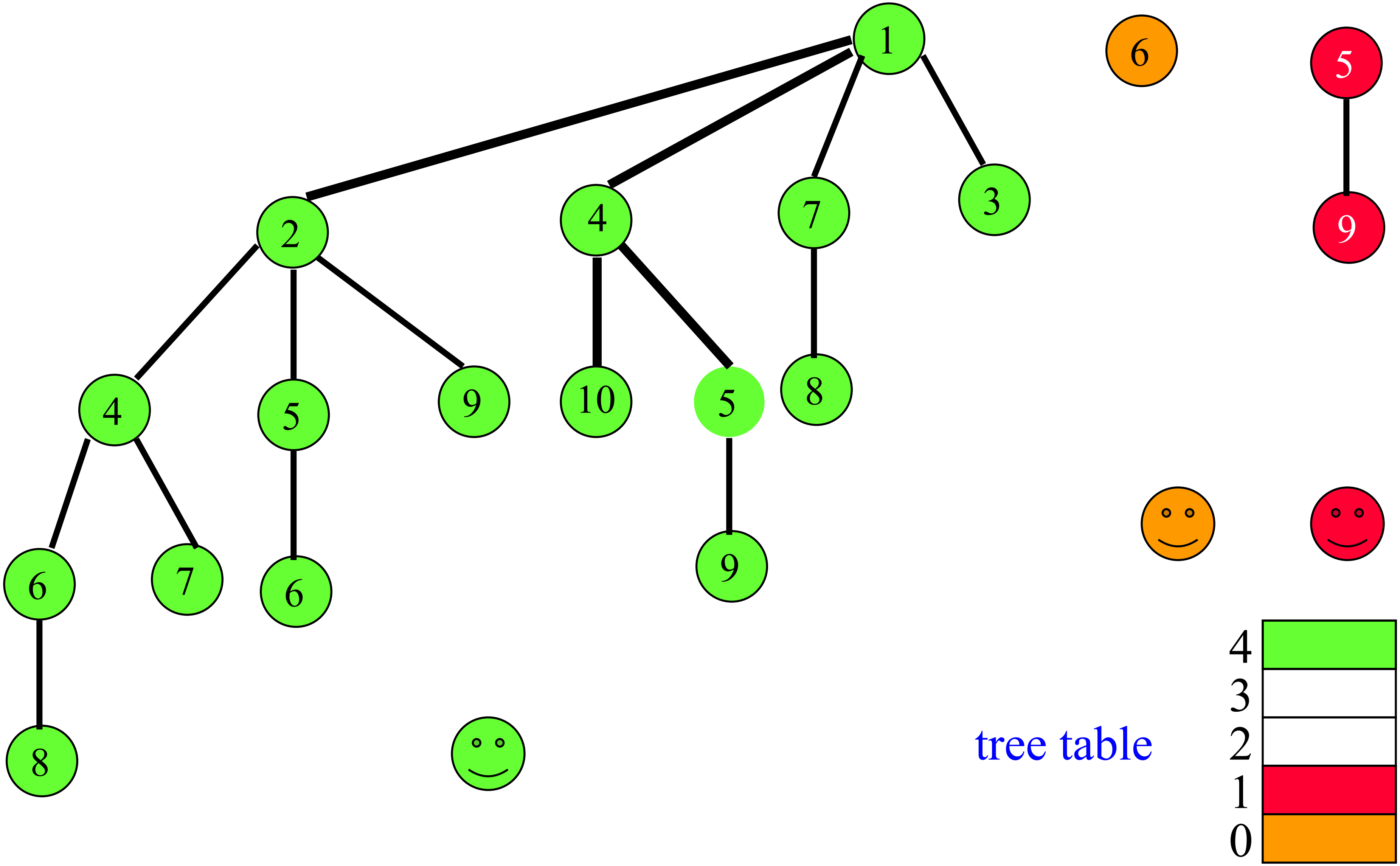
Update tree table.

Pairwise Combine



tree table

Pairwise Combine



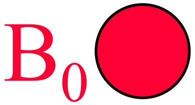
Create circular list of remaining trees.

Complexity Of Remove Min

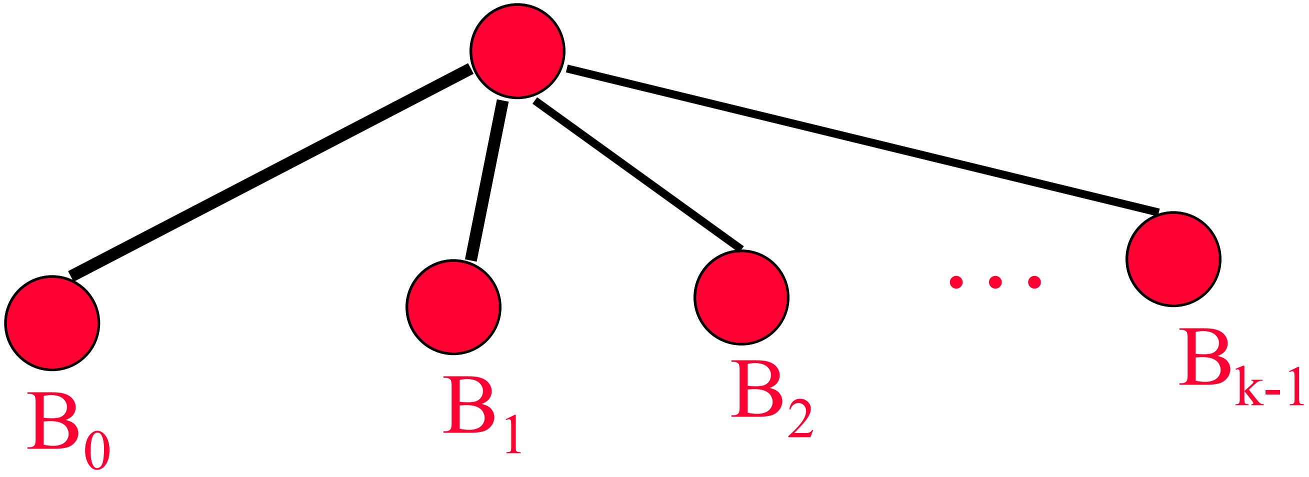
- Create and initialize tree table.
  - O(MaxDegree).
  - Done once only.
- Examine s min trees and pairwise combine.
  - O(s).
- Collect remaining trees from tree table, reset table entries to null, and set binomial heap pointer.
  - O(MaxDegree).
- Overall complexity of remove min.
  - O(MaxDegree + s).

Binomial Trees

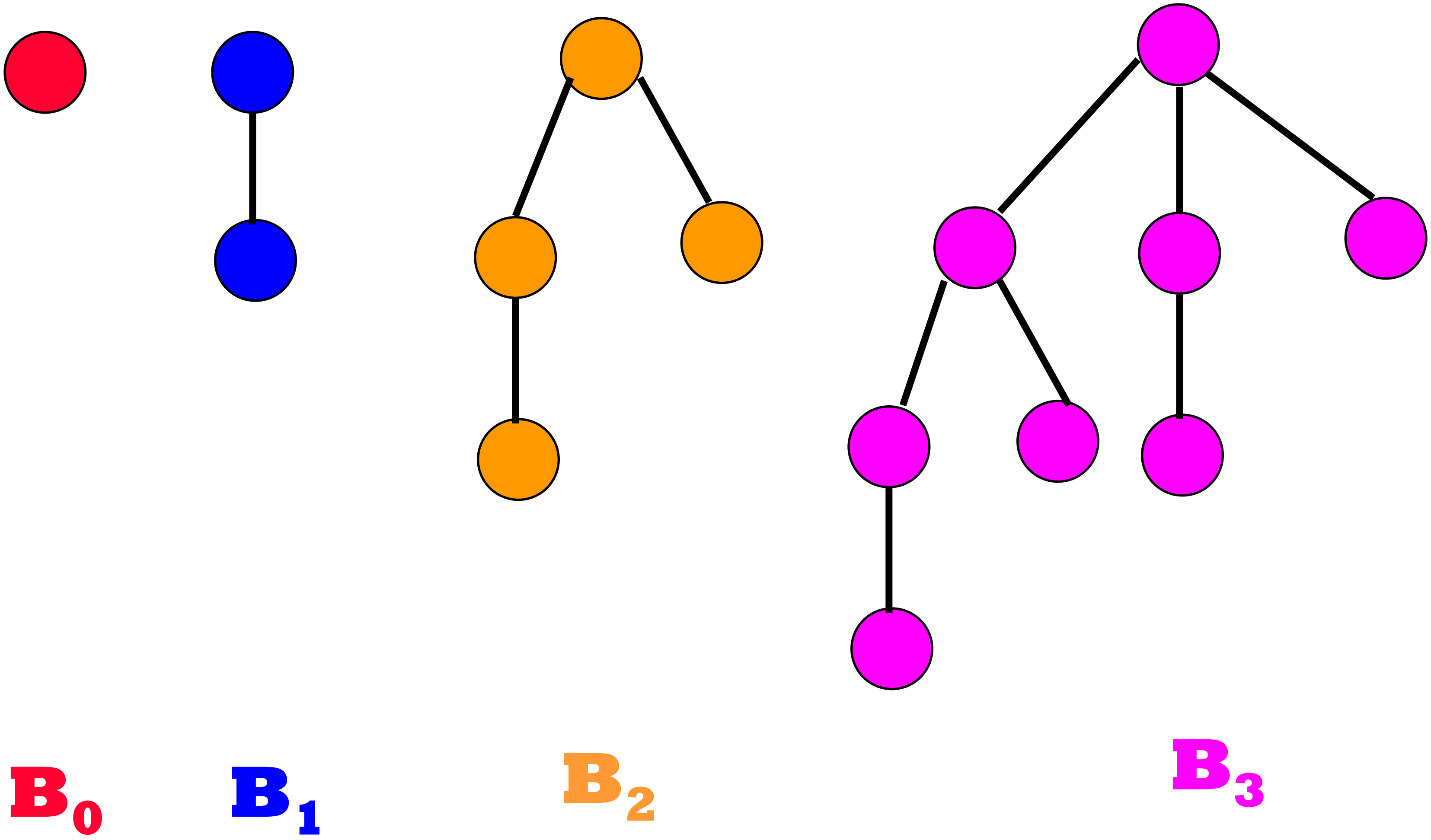
- B_k is degree k binomial tree.



- B_k , $k > 0$, is:

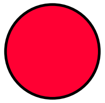


Examples

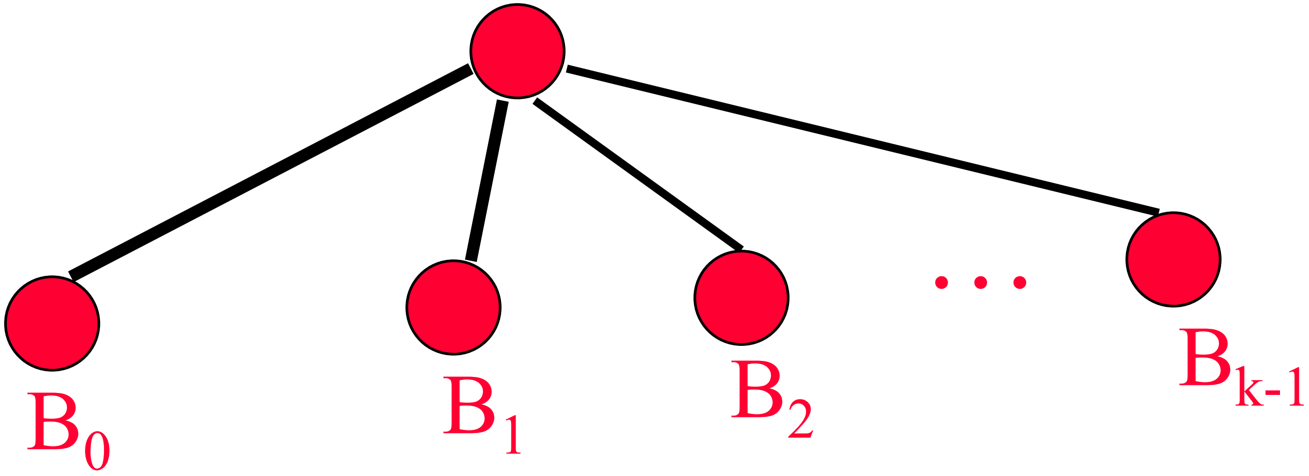


Number Of Nodes In B_k

- N_k = number of nodes in B_k.

B_0  $N_0 = 1$

- B_k , $k > 0$, is:

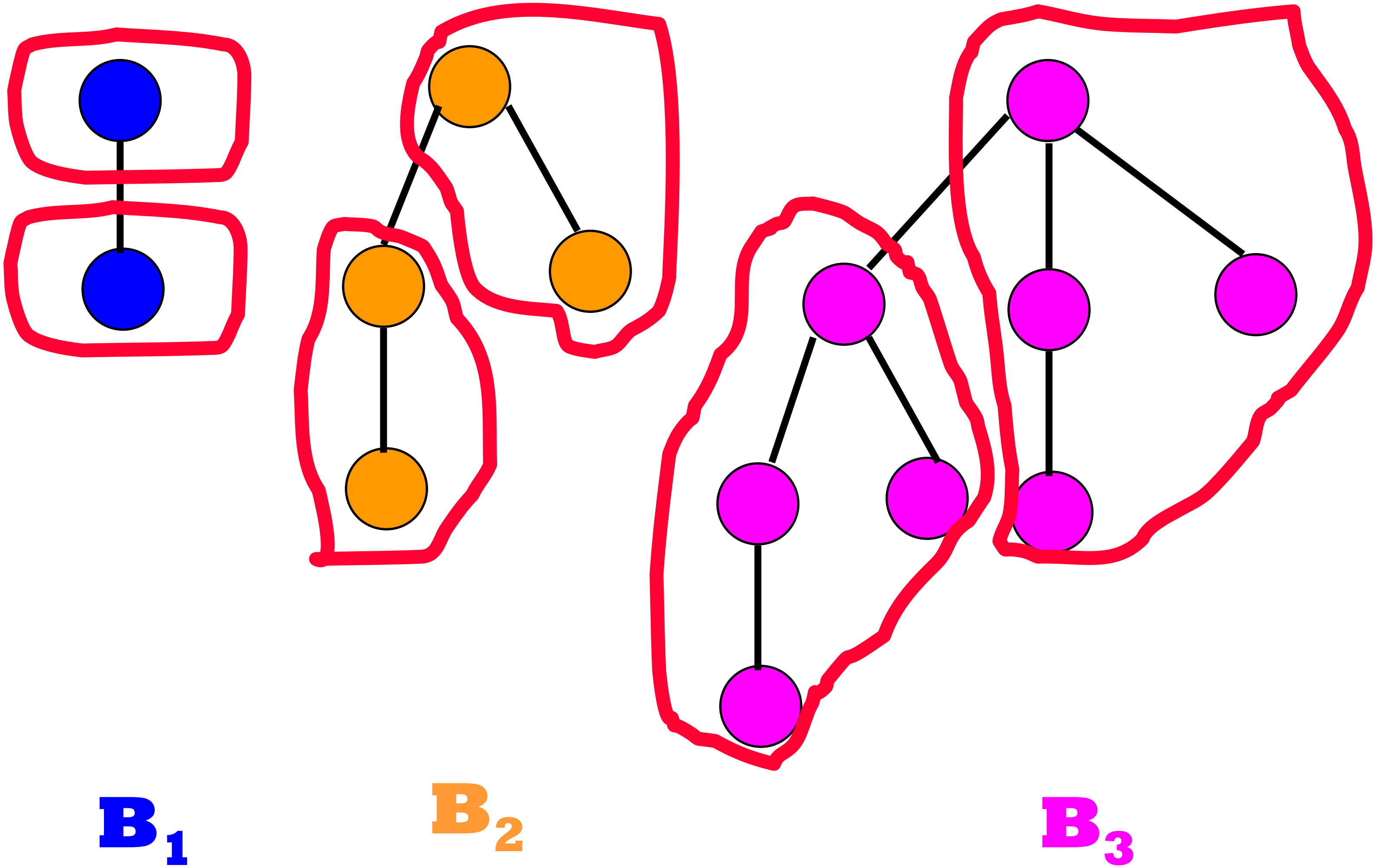


- $N_k = N_0 + N_1 + N_2 + \ldots + N_{k-1} + 1$

  $= 2^k$.

Equivalent Definition

- B_k , $k > 0$, is two B_{k-1}s.
- One of these is a subtree of the other.



N_k And MaxDegree

- $N_0 = 1$
- $N_k = 2N_{k-1}$

  $= 2^k$.
- If we start with zero elements and perform operations as described, then all trees in all binomial heaps are binomial trees.
- So, MaxDegree = O(log n).