

71117408_梅洛瑜

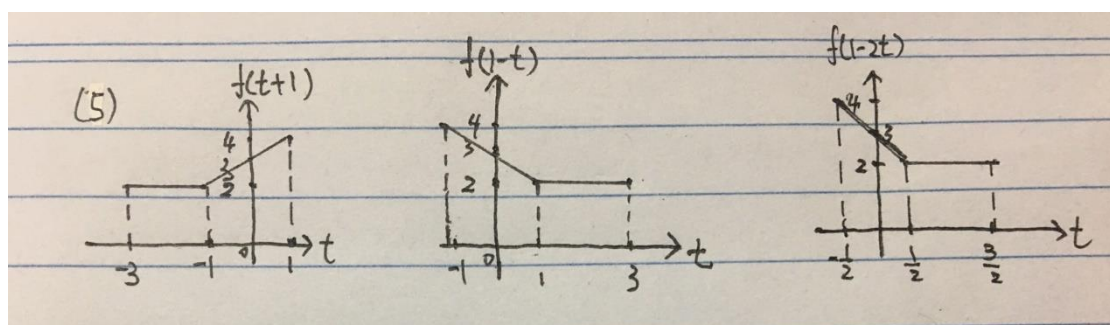
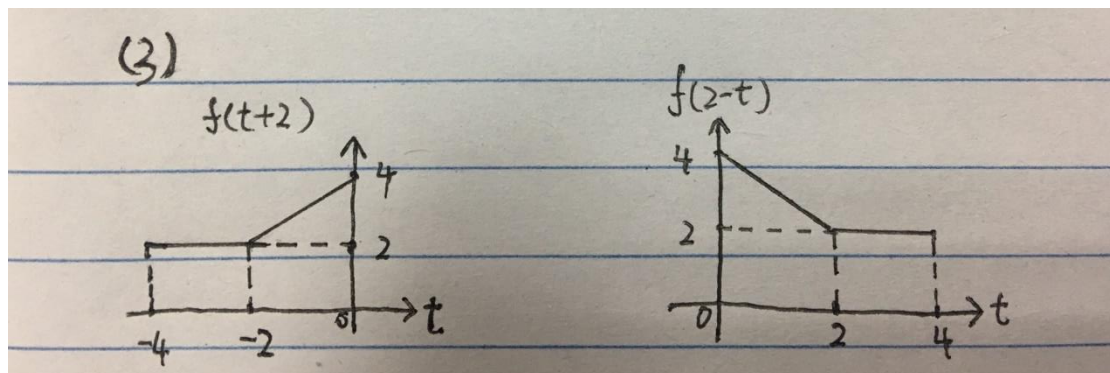
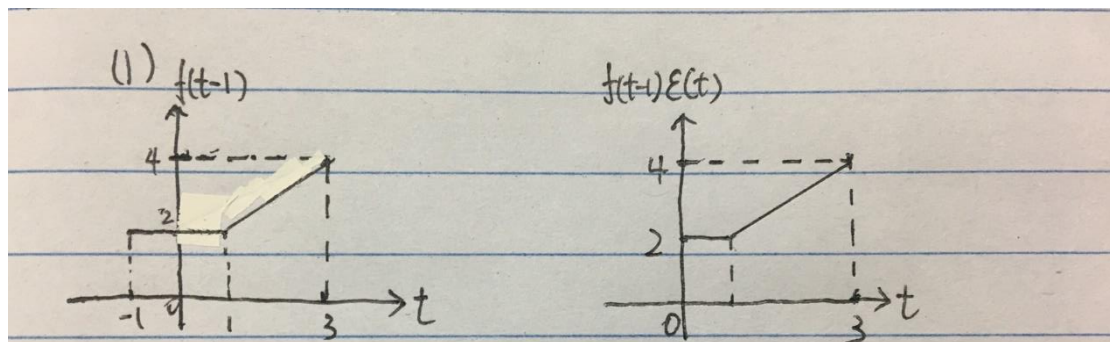
1.4:

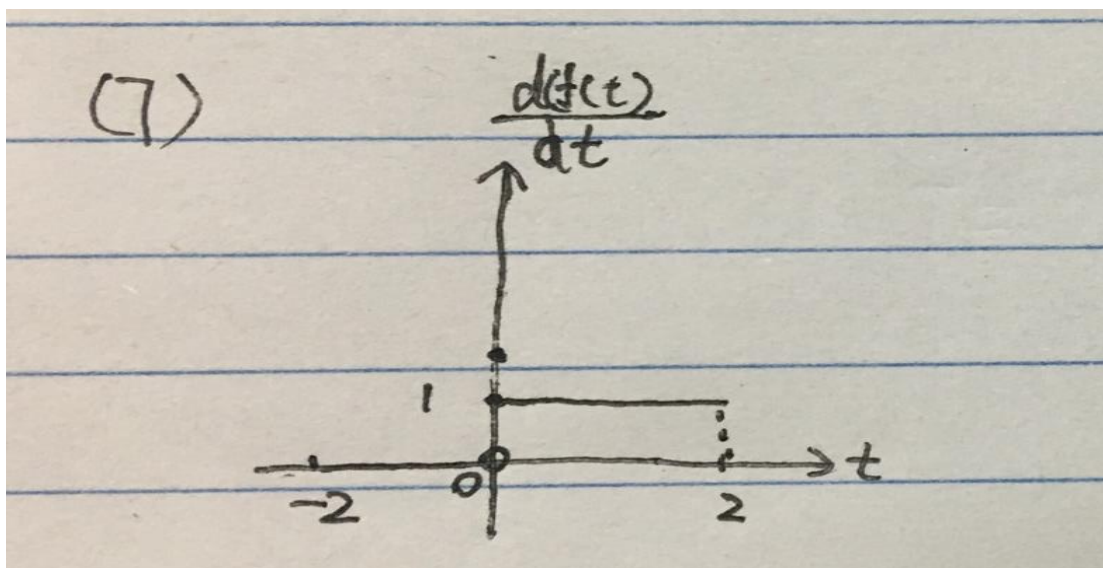
- a) $f(k) = \varepsilon(k+2)$
- b) $f(k) = \varepsilon(k-3)$
- c) $f(k) = \varepsilon(-k+2)$
- d) $f(k) = (-1)^k \varepsilon(k)$

1.5:

- (1) 序列为周期序列, $T=0.3$
- (2) 对于 $k=1$, 不存在周期 T 使 $f_3(1) = f_3(1+mT)$ ($m=0, \pm 1, \pm 2 \dots$)
- (3) $\cos t$ 周期 $T_1 = 2\pi$, $\sin(\pi t)$ 周期 $T_2 = 2$, $\frac{T_1}{T_2} = \pi$ 不为有理数, 故 f_5 不具有周期性

1.6:





1.10:

(1)

$$\begin{aligned}\frac{d^2}{dt^2}[\cos t + \sin 2t]\varepsilon(t) &= \frac{d}{dt}\{[-\sin t + 2\cos(2t)]\varepsilon(t) + [\cos t + \sin 2t]\sigma(t)\} \\ &= \frac{d}{dt}\{[-\sin t + 2\cos(2t)]\varepsilon(t) + \sigma(t)\} \\ &= [-\cos t - 4\sin(2t)]\varepsilon(t) + [-\sin t + 2\cos(2t)]\sigma(t) + \sigma^2(t) \\ &= [-\cos t - 4\sin(2t)]\varepsilon(t) + 2\sigma(t) + \sigma^2(t)\end{aligned}$$

(2)

$$\begin{aligned}(1+t)\frac{d}{dt}[e^{-t}\delta(t)] &= (1+t) * [-e^{-t}\delta(t) + e^{-t}\delta'(t)] \\ &= (1+t) * [-\delta(t) + \delta(t) + \delta'(t)] \\ &= (1+t) * \delta'(t) = \delta'(t) + \delta(t)\end{aligned}$$

$$\text{Tips: } \begin{cases} e^{-at}\delta'(t) = \delta'(t) + a\delta(t) \\ t\delta'(t) = -\delta(t) \end{cases}$$

(3)

$$\text{令 } f(t) = \frac{\sin(\pi t)}{t}, \text{ 则 } \int_{-\infty}^{\infty} \frac{\sin(\pi t)}{t} \delta(t) dt = \int_{-\infty}^{\infty} f(t) \delta(t) dt = f(0) = 1$$

(4)

$$\int_{-\infty}^{\infty} e^{-2t}[\delta(t) + \delta'(t)] dt = \int_{-\infty}^{\infty} e^{-2t}\delta'(t) dt + \int_{-\infty}^{\infty} e^{-2t}\delta(t) dt = 2 + 1 = 3$$

(5)

$$\int_{-\infty}^{\infty} [t^2 + \sin\left(\frac{\pi}{4}t\right)]\delta(t+2) dt = [t^2 + \sin\left(\frac{\pi}{4}t\right)]\Big|_{t=-2} = 3$$

(7)

$$\int_{-\infty}^{\infty} (t^3 + 2t^2 - 2t + 1)\delta'(t-1) dt = [t^3 + 2t^2 - 2t + 1]'\Big|_{t=1} = 5$$

1.23:

(1)

$$\begin{cases} y_{zs}(t) = \int_0^t \sin x f(x) dx \\ y_{zi}(t) = e^{-t} x(0) \end{cases} \quad \text{故 } y(t) = y_{zi}(t) + y_{zs}(t) \text{ 满足可分解性}$$

同时,

$$e^{-t}[x_1(0) + x_2(0)] = e^{-t}x_1(0) + e^{-t}x_2(0), \text{ 满足零输入线性}$$

$$\int_0^t \sin x [f_1(x) + f_2(x)] dx = \int_0^t \sin x f_1(x) dx + \int_0^t \sin x f_2(x) dx, \text{ 满足零状态线性}$$

故系统为线性的

(3)

$$\begin{cases} y_{zs}(t) = \sin[x(0) * t] \\ y_{zi}(t) = \int_0^t f(x) dx \end{cases} \quad \text{故 } y(t) = y_{zi}(t) + y_{zs}(t) \text{ 满足可分解性}$$

但, $\sin[(x_1(0) + x_2(0)) * t] \neq \sin[x_1(0) * t] + \sin[x_2(0) * t]$ 不满足零输入特性, 故为非线性系统

(5)

$$\begin{cases} y_{zs}(k) = kx(0) \\ y_{zi}(k) = \sum_{j=0}^k f(j) \end{cases} \quad \text{故 } y(t) = y_{zi}(t) + y_{zs}(t) \text{ 满足可分解性}$$

且,

$$k[x_1(0) + x_2(0)] = kx_1(0) + kx_2(0) \text{ 满足零输入线性}$$

$$\sum_{j=0}^k [f_1(j) + f_2(j)] = \sum_{j=0}^k f_1(j) + \sum_{j=0}^k f_2(j) \text{ 满足零状态线性}$$

故为线性系统

1.25:

(3)

$$af_1(t) \cos(2\pi t) + bf_2(t) \cos(2\pi t) = af_1(t) \cos(2\pi t) + bf_2(t) \cos(2\pi t) \text{ 系统满足齐次性与可加性}$$

$$y_{zs}(t - t_d) \neq f(t - t_1) \cos(2\pi t) \text{ 时变}$$

$$t < 0, f(t) = 0, y_{zs}(t) = f(t) \cos(2\pi t) = 0 \quad \text{因果}$$

$$|f(t)| < \infty, |y_{zs}(t)| = |f(t) \cos(2\pi t)| < \infty \quad \text{稳定}$$

故为线性时变因果稳定系统

1.32

由 LT1 系统为线性时不变因果稳定系统

$$\text{故 } y(t) = \varepsilon(t) - \varepsilon(t-1) - \varepsilon(t-2) + \varepsilon(t-3) = f(t) - f(t-1) - f(t-2) + f(t-3)$$

2.14

根据 $h(t)$ 定义有:

$$h'(t) + 2h(t) = \sigma'(t) - \sigma(t)$$

$$h'(0_-) = h(0_-) = 0$$

先求出 $h'(0_+)$, 令:

$$h'(t) = a\sigma'(t) + b\sigma(t) + r_1(t)$$

$$h(t) = a\sigma(t) + r_2(t)$$

故 $\begin{cases} a = 1 \\ b = -2 \end{cases}$, 对 $h'(t)$ 从 0^- 到 0^+ 求积分

$$h(0_+) - h(0_-) = b$$

$$\text{故 } h(0_+) = b = -2$$

当 $t > 0$ 时有 $h'(t) + 2h(t) = 0$, 微分方程的特征根为 -2

故系统的冲激响应为: $h(t) = C_1 e^{-2t}$

带入初始条件 $h(0_+) = -2$, 得出 $C_1 = -2$

$$\text{故冲激响应为: } h(t) = \sigma(t) - 2e^{-2t}$$

$$\text{阶跃响应为冲击响应的积分: } g(t) = \int_{-\infty}^t h(\tau) d\tau = \varepsilon(t) - 2e^{-2t}$$

2.17

(1)

$$y_{zs}(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{\infty} \tau \varepsilon(t) \varepsilon(t - \tau) d\tau = \int_0^t \tau d\tau = \frac{1}{2} \tau^2$$

(3)

$$\begin{aligned} y_{zs}(t) &= \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{\infty} e^{-2\tau} \varepsilon(\tau) e^{-2(t-\tau)} \varepsilon(t - \tau) d\tau \\ &= \int_{-\infty}^{\infty} e^{-2t} \varepsilon(\tau) \varepsilon(t - \tau) d\tau = \int_0^t e^{-2t} d\tau = t e^{-2t} \end{aligned}$$

(5)

$$\begin{aligned} y_{zs}(t) &= \int_{-\infty}^{\infty} f_1(t - \tau) f_2(\tau) d\tau = \int_{-\infty}^{\infty} (t - \tau) \varepsilon(t - \tau) e^{-2\tau} \varepsilon(\tau) d\tau \\ &= \int_0^t t e^{-2\tau} d\tau - \int_0^t \tau e^{-2\tau} d\tau = -\frac{1}{2} t e^{-2t} + \left[-\frac{1}{2} \tau e^{-2\tau} + \frac{1}{4} e^{-2\tau} \right]_0^t = \frac{1}{4} e^{-2t} \end{aligned}$$

(7)

$$\begin{aligned} y_{zs}(t) &= \int_{-\infty}^{\infty} f_1(t - \tau) f_2(\tau) d\tau = \int_{-\infty}^{\infty} \sin(\pi\tau) \varepsilon(\tau) [\varepsilon(t - \tau) - \varepsilon(\tau)] d\tau \\ &= \int_0^t \sin(\pi\tau) d\tau - \int_0^{t-4} \sin(\pi\tau) d\tau = \frac{1}{\pi} [\cos(\pi t) - \cos[\pi(t - 4)]] \end{aligned}$$

(9)

$$\begin{aligned} y_{zs}(t) &= \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{\infty} \tau \varepsilon(\tau - 1) \varepsilon(t - \tau + 3) d\tau \\ &= \int_1^{t+3} \tau d\tau = \frac{1}{2} t^2 + 3t + 4 \end{aligned}$$

2.28

$$\begin{aligned} y(t) &= h_b \left(f(t) + h_a(f(t)) + h_a \left(h_a(f(t)) \right) \right) \\ &= \varepsilon \left(f(t) + \delta(f(t) - 1) + \delta(\delta(f(t) - 1)) \right) \\ &\quad - \varepsilon(f(t) + \delta(f(t) - 1) + \delta(\delta(f(t) - 1)) - 3) \\ &= \varepsilon(f(t)) + f(t) - 1 + \delta(f(t) - 1) - \varepsilon(f(t)) - f(t) + 1 - \delta(f(t) - 1) + 3 = 3 \end{aligned}$$

3.6

(5)

零输入响应：相当于 $f(k) = 0$ 时的解， $\lambda^2 + 2\lambda + 1 = 0$ 故 $y_{zi}(k) = [C_1 + C_2](-1)^k$

$n=1$ 时 $0 + 0 + y_{zi}(-1) = 3$ ，故 $y_{zi}(-1) = C_1 + C_2 = 3$ ，故 $y_{zi}(k) = 3 * (-1)^k$

零状态响应： $y_{zs}(k) + 2y_{zs}(k-1) + y_{zs}(k-2) = f(k)$ ， $y_{zs}(-1) = 3, y_{zs}(-2) = -5$

代入求初值： $y_{zs}(k) = -2y_{zs}(k-1) - y_{zs}(k-2) + 3 * \left(\frac{1}{2}\right)^k$

$$y_{zs}(0) = -2y_{zs}(-1) - y_{zs}(-2) + 3 = 2$$

$$y_{zs}(1) = -2y_{zs}(0) - y_{zs}(-1) + 3 = -4$$

求齐次解和特解： $y_{zs}(k) = -2C_{zs1}(-1)^k - C_{zs2}(-2)^k + 3 * \left(\frac{1}{2}\right)^k$

代入初始值， $\begin{cases} y_{zs}(0) = -2C_{zs1} - C_{zs2} + 3 = 2 \\ y_{zs}(1) = 2C_{zs1} + 2C_{zs2} + \frac{3}{2} = -4 \end{cases}$ 得出 $\begin{cases} C_{zs1} = -\frac{11}{4} \\ C_{zs2} = -\frac{13}{2} \end{cases}$

$$\text{故 } y_{zs}(k) = \frac{11}{2}(-1)^k + \frac{13}{2}(-2)^k + 3 * \left(\frac{1}{2}\right)^k$$

全响应： $y(k) = y_{zi}(k) + y_{zs}(k) = \frac{17}{2}(-1)^k + \frac{13}{2}(-2)^k + 3 * \left(\frac{1}{2}\right)^k$

3.8

(5)

由 $h(k)$ 定义可知

$$h(k) - 4h(k-1) + 8h(k-2) = \delta(k)$$

$$h(-1) = h(-2) = 0$$

递推求出初始 $h(0)$ 与 $h(1)$

$$h(0) = 4h(-1) - 8h(-2) + \delta(k) = 1$$

$$h(1) = 4h(0) - 8h(-1) + \delta(k) = 1$$

对于 $k > 0$ ， $h(k)$ 满足齐次线性方程

$$h(k) - 4h(k-1) + 8h(k-2) = 0$$

相应特征方程： $(\lambda + 1)(\lambda - 2) = 0$

$$h(k) = C_1(-1)^k + C_2(2)^k \quad \begin{cases} h(0) = C_1 + C_2 = 1 \\ h(1) = -C_1 + 2C_2 = 1 \end{cases} \text{ 故 } C_1 = -\frac{1}{3} \quad C_2 = \frac{2}{3}$$

$$h(k) = -\frac{1}{3}(-1)^k + \frac{2}{3}C_2(2)^k$$

3.26

$$y_{zs} = \left[\frac{6}{5} * 2^k - \frac{1}{5} \left(\frac{1}{3}\right)^k \right] \varepsilon(k) = [C_1(\lambda_1)^k - C_2(\lambda_2)^k] \varepsilon(k) = [C_1 f(k) - C_2 h(k)] \varepsilon(k)$$

$$\text{故 } f(k) = 2^k \varepsilon(k)$$

4.7

(a)

$$T=4, \quad \Omega = \frac{2\pi}{T} = \frac{\pi}{2}, \quad \text{在一个周期内 } f(t) = \begin{cases} 0 & (\frac{T}{4} < t < \frac{3}{4}T) \\ 1 & (|t| < \frac{T}{4}) \end{cases}$$

$$a_n = \frac{1}{2} \int_{-1}^1 \cos\left(\frac{\pi}{2}nT\right) dt = \frac{2}{n\pi} \sin\left(\frac{\pi}{2}n\right) \quad (n = 0, 1, 2 \dots)$$

$$b_n = \frac{1}{2} \int_{-1}^1 \sin\left(\frac{\pi}{2}nT\right) dt = 0 \quad (n = 1, 2 \dots)$$

(b)

$$T=2, \quad \Omega = \frac{2\pi}{T} = \pi, \quad \text{在一个周期内 } f(t) = \begin{cases} 0 & (-\frac{T}{2} < t < 0) \\ \sin(\pi t) & (0 \leq t < \frac{T}{2}) \end{cases}$$

$$a_n = \int_{-1}^1 f(t) \cos(n\pi t) dt = \int_0^1 \cos(n\pi t) dt = \frac{1}{n\pi} \sin(n\pi) \quad (n = 0, 1, 2 \dots)$$

$$b_n = \int_{-1}^1 f(t) \sin(n\pi t) dt = \int_0^1 \sin(n\pi t) dt = -\frac{1}{n\pi} \cos(n\pi) \quad (n = 1, 2 \dots)$$

4.17

由对称性: $f(t) \Leftrightarrow F(j\omega) \rightarrow F(jt) \Leftrightarrow 2\pi f(-\omega)$

$$e^{-\alpha|t|} \Leftrightarrow \frac{2\alpha}{\alpha^2 + \omega^2} \rightarrow \frac{2\alpha}{\alpha^2 + t^2} \Leftrightarrow 2\pi e^{-\alpha|\omega|}$$

$$\text{故 } f(t) = \frac{2\alpha}{\alpha^2 + t^2} \Leftrightarrow 2\pi e^{-\alpha|\omega|} \quad (-\infty < \omega < +\infty)$$

4.20

$$f(-t) \Leftrightarrow F(-j\omega)$$

$$f(-t+3) \Leftrightarrow e^{-3j\omega} F(-j\omega)$$

$$f(-2t+3) \Leftrightarrow \frac{e^{-3j\omega}}{2} F\left(-\frac{j\omega}{2}\right)$$

$$e^{jt} f(-2t+3) \Leftrightarrow \frac{e^{-3j\omega}}{2} F\left(-\frac{j(\omega-1)}{2}\right)$$