71117408_梅洛瑜

1.4：

a) $f(k) = \varepsilon(k+2)$

b) $f(k) = \varepsilon(k-3)$

c) $f(k) = \varepsilon(-k+2)$

d) $f(k) = (-1)^k \varepsilon(k)$

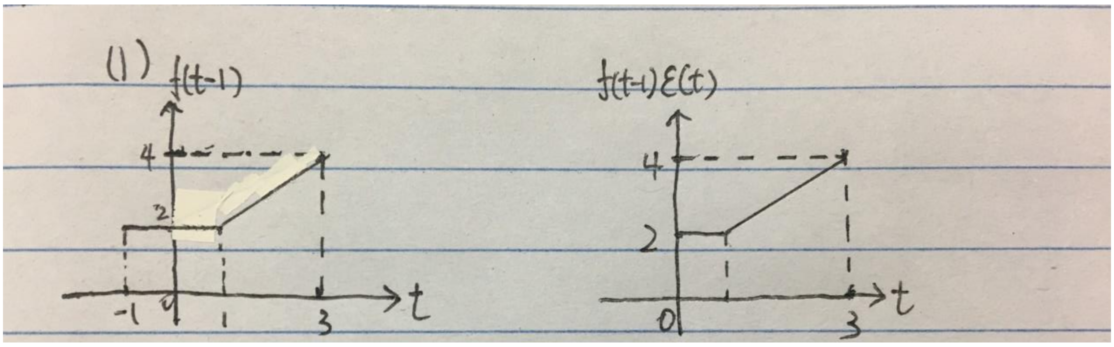
1.5：

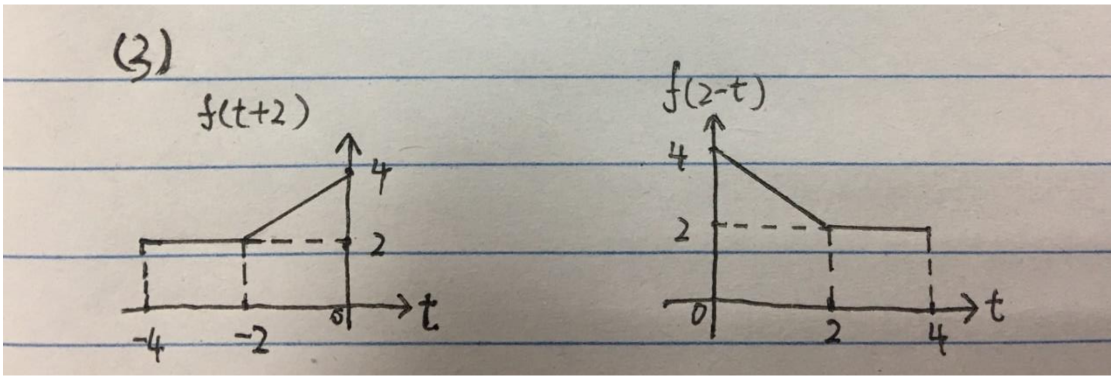
（1）序列为周期序列，T=0.3

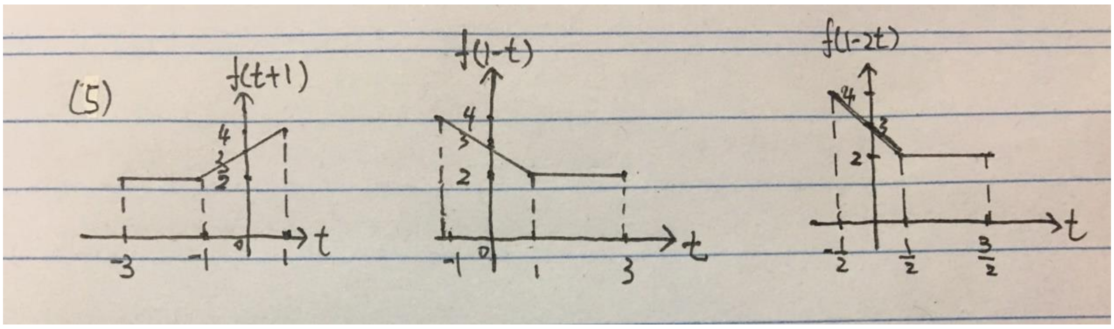
（2）对于 k=1，不存在周期 T 使$f_3(1) = f_3(1+mT)(m = 0, \pm1, \pm2 \dots)$

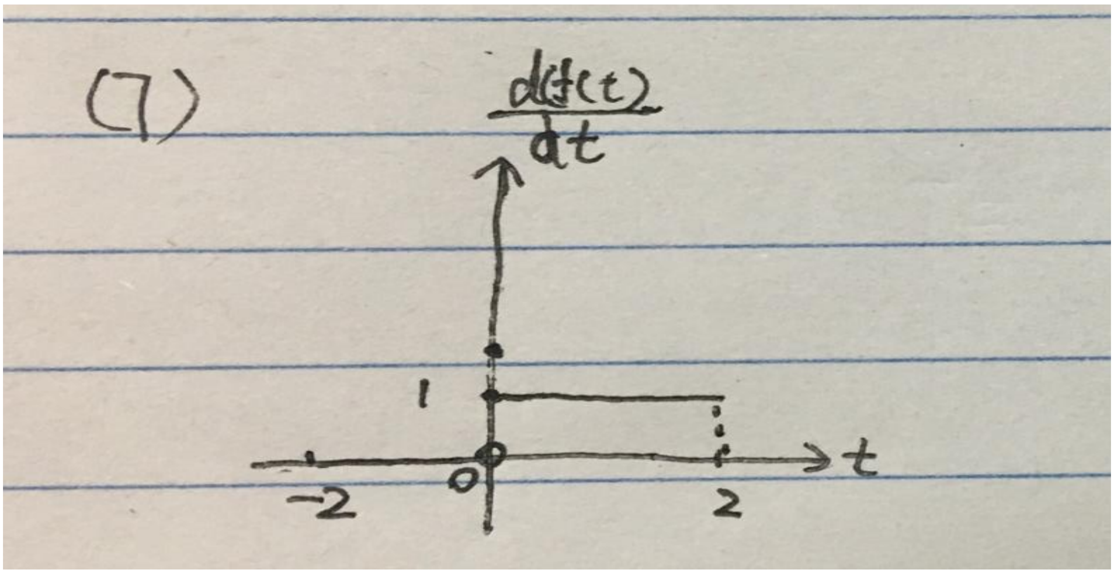
（3）$\cos t$周期$T_1 = 2\pi$，$\sin(\pi t)$周期$T_2 = 2$，$\frac{T_1}{T_2} = \pi$不为有理数，故 f5 不具有周期性

1.6：









1.10:

(1)

$$\frac{d^2}{dt^2}[\cos t+\sin 2t]\varepsilon(t) = \frac{d}{dt}\{[-\text{sint}+2\cos(2\text{t})]\varepsilon(\text{t})+[\cos t+\sin 2t]\sigma(t)\}$$

$$=\frac{d}{dt}\{[-\text{sint}+2\cos(2\text{t})]\varepsilon(\text{t})+\sigma(t)\}$$

$$=[-\cos t-4\sin(2t)]\varepsilon(t)+[-\text{sint}+2\cos(2\text{t})]\sigma(t)+\sigma^2(t)$$

$$=[-\cos t-4\sin(2t)]\varepsilon(t)+2\sigma(t)+\sigma^2(t)$$

(2)

$$(1+t)\frac{d}{dt}[e^{-t}\delta(t)]=(1+\text{t})*[-e^{-t}\delta(t)+e^{-t}\delta'(t)]$$

$$=(1+\text{t})*[-\delta(t)+\delta(t)+\delta'(t))]$$

$$=(1+t)*\delta'(t)=\delta'(t)+\delta(t)$$

Tips: $\begin{cases} e^{-at}\delta'(t)=\delta'(t)+a\delta(t) \\ t\delta'(t)=-\delta(t) \end{cases}$

(3)

令$f(t)=\frac{sin(\pi t)}{t}$, 则$\int_{-\infty}^{\infty}\frac{sin(\pi t)}{t}\delta(t)\,dt=\int_{-\infty}^{\infty}f(t)\delta(t)\,dt=\text{f}(0)=1$

(4)

$$\int_{-\infty}^{\infty}e^{-2t}\left[\delta(t)+\delta'^{(t)}\right]dt=\int_{-\infty}^{\infty}e^{-2t}\delta'^{(t)}\,dt+\int_{-\infty}^{\infty}e^{-2t}\delta(t)\,dt=2+1=3$$

(5)

$$\int_{-\infty}^{\infty}[t^2+sin\left(\frac{\pi}{4}t\right)\delta(t+2)]\,dt=[t^2+\sin\left(\frac{\pi}{4}t\right)]\Big|_{t=-2}=3$$

(7)

$$\int_{-\infty}^{\infty}(t^3+2t^2-2t+1)\delta'^{(t-1)}\,dt=[t^3+2t^2-2t+1]'|_{t=1}=5$$

1.23:

(1)

$$\begin{cases} y_{zs}(t) = \int_0^t sin\, x f(x)\, dx \\ y_{zi}(t) = e^{-t}x(0) \end{cases}$$ 故$y(t) = y_{zi}(t) + y_{zs}(t)$满足可分解性

同时,

$e^{-t}[x_1(0) + x_2(0)] = e^{-t}x_1(0) + e^{-t}x_2(0)$，满足零输入线性

$\int_0^t sin\, x[f_1(x) + f_2(x)]\, dx = \int_0^t sin\, x f_1(x) dx + \int_0^t \sin x\, f_2(x) dx$，满足零状态线性　故系统为线性的

(3)

$$\begin{cases} y_{zs}(t) = \sin[x(0) * t] \\ y_{zi}(t) = \int_0^t f(x)\, dx \end{cases}$$ 故$y(t) = y_{zi}(t) + y_{zs}(t)$满足可分解性

但, $sin\big[\big(x_1(0) + x_2(0)\big) * t\big] \neq \sin[x_1(0) * t] + \sin[x_2(0) * t]$不满足零输入特性，故为非线性系统

(5)

$$\begin{cases} y_{zs}(k) = kx(0) \\ y_{zi}(k) = \sum_{j=0}^{k} f(j) \end{cases}$$ 故$y(t) = y_{zi}(t) + y_{zs}(t)$满足可分解性

且,

$k[x_1(0) + x_2(0)] = kx_1(0) + kx_2(0)$满足零输入线性

$\sum_{j=0}^{k}[f_1(j) + f_2(j)] = \sum_{j=0}^{k} f_1(j) + \sum_{j=0}^{k} f_2(j)$满足零状态线性

故为线性系统

1.25:

(3)

$af_1(t)\, cos(2\pi t) + bf_2(t)\, cos(2\pi t) = af_1(t)\, cos(2\pi t) + bf_2(t)\, cos(2\pi t)$系统满足齐次性与可加性

$y_{zs}(t - t_d) \neq f(t - t_1)\, cos(2\pi t)$ 时变

t<0，$f(t) = 0$，$y_{zs}(t) = f(t)\, cos(2\pi t) = 0$　因果

$|f(t)| < \infty$，$|y_{zs}(t)| = |f(t)\, cos(2\pi t)| < \infty$　稳定

故为线性时变因果稳定系统

1.32

由 LT1 系统为线性时不变因果稳定系统

故$y_{(t)} = \varepsilon(t) - \varepsilon(t-1) - \varepsilon(t-2) + \varepsilon(t-3) = f(t) - f(t-1) - f(t-2) + f(t-3)$

2.14

根据$h(t)$定义有:

$h'(t) + 2h(t) = \sigma'(t) - \sigma(t)$

$h'(0_-) = h(0_-) = 0$

先求出$h'(0_+)$，令:

$h'(t) = a\sigma'(t) + b\sigma(t) + r_1(t)$

$h(t) = a\sigma(t) + r_2(t)$

故$\begin{cases} a=1 \\ b=-2 \end{cases}$，对$h'(t)$从 0-到 0+求积分

$$h(0_+)-h(0_-)=b$$

故$h(0_+)=b=-2$

当 t>0 时有$h'^{(t)}+2h(t)=0$，微分方程的特征根为-2

故系统的冲激响应为：$h(t)=C_1e^{-2t}$

带入初始条件$h(0_+)=-2$，得出$C_1=-2$

故冲激响应为：$\mathrm{h}(t)=\sigma(t)-2e^{-2t}$

阶跃响应为冲击响应的积分：$g(t)=\int_{-\infty}^{t}h(\tau)\,d\tau=\varepsilon(t)-2e^{-2t}$

2.17

(1)

$$y_{zs}(t)=\int_{-\infty}^{\infty}f_1(\tau)f_2(t-\tau)\,d\tau=\int_{-\infty}^{\infty}\tau\varepsilon(t)\varepsilon(t-\tau)\,d\tau=\int_0^t\tau\,d\tau=\frac{1}{2}\tau^2$$

(3)

$$y_{zs}(t)=\int_{-\infty}^{\infty}f_1(\tau)f_2(t-\tau)\,d\tau=\int_{-\infty}^{\infty}e^{-2\tau}\varepsilon(\tau)e^{-2(t-\tau)}\varepsilon(t-\tau)\,d\tau$$

$$=\int_{-\infty}^{\infty}e^{-2t}\varepsilon(\tau)\varepsilon(t-\tau)\,d\tau=\int_0^t e^{-2t}\,d\tau=te^{-2t}$$

(5)

$$y_{zs}(t)=\int_{-\infty}^{\infty}f_1(t-\tau)f_2(\tau)\,d\tau=\int_{-\infty}^{\infty}(t-\tau)\varepsilon(t-\tau)e^{-2\tau}\varepsilon(\tau)\,d\tau$$

$$=\int_0^t te^{-2\tau}\,d\tau-\int_0^t\tau e^{-2\tau}\,d\tau=-\frac{1}{2}te^{-2t}+-\frac{1}{2}te^{-2t}+\frac{1}{4}e^{-2t}=\frac{1}{4}e^{-2t}$$

(7)

$$y_{zs}(t)=\int_{-\infty}^{\infty}f_1(t-\tau)f_2(\tau)\,d\tau=\int_{-\infty}^{\infty}\sin(\pi\tau)\,\varepsilon(\tau)\,[\varepsilon(t-\tau)-\varepsilon(\tau)]d\tau$$

$$=\int_0^t\sin(\pi\tau)\,d\tau-\int_0^{t-4}\sin(\pi\tau)\,d\tau=\frac{1}{\pi}[\cos(\pi t)-\cos[\pi(t-4)]]$$

(9)

$$y_{zs}(t)=\int_{-\infty}^{\infty}f_1(\tau)f_2(t-\tau)\,d\tau=\int_{-\infty}^{\infty}\tau\varepsilon(\tau-1)\varepsilon(t-\tau+3)\,d\tau$$

$$=\int_1^{t+3}\tau\,d\tau=\frac{1}{2}t^2+3t+4$$

2.28

$$y(t)=h_b\left(f(t)+h_a\big(f(t)\big)+h_a\Big(h_a\big(f(t)\big)\Big)\right)$$

$$=\varepsilon\left(f(t)+\delta(f(t)-1)+\delta\big(\delta(f(t)-1)\big)\right)$$

$$-\varepsilon\big(f(t)+\delta(f(t)-1)+\delta\big(\delta(f(t)-1)\big)-3\big)$$

$$=\varepsilon\big(f(t)\big)+f(t)-1+\delta(f(t)-1)-\varepsilon\big(f(t)\big)-f(t)+1-\delta(f(t)-1)+3=3$$

3.6

(5)

零输入响应：相当于$f(k)=0$时的解，$\lambda^2+2\lambda+1=0$故$y_{z_i}(k)=[C_1+C_2](-1)^k$

n=1 时$0+0+y_{z_i}(-1)=3$，故$y_{z_i}(-1)=C_1+C_2=3$，故$y_{z_i}(k)=3*(-1)^k$

零状态响应：$y_{zs}(k)+2y_{zs}(k-1)+y_{zs}(k-2)=\mathrm{f(k)}$，$y_{zs}(-1)=3, y_{zs}(-2)=-5$

代入求初值：$y_{zs}(k)=-2y_{zs}(k-1)-y_{zs}(k-2)+3*\left(\frac{1}{2}\right)^k$

$$y_{zs}(0)=-2y_{zs}(-1)-y_{zs}(-2)+3=2$$
$$y_{zs}(1)=-2y_{zs}(0)-y_{zs}(-1)+3=-4$$

求齐次解和特解：$y_{zs}(k)=-2\mathrm{C}_{\mathrm{zs1}}(-1)^k-C_{zs2}(-2)^k+3*\left(\frac{1}{2}\right)^k$

代入初始值，$\begin{cases} y_{zs}(0)=-2C_{zs1}-C_{zs2}+3=2 \\ y_{zs}(1)=2C_{zs1}+2C_{zs2}+\frac{3}{2}=-4 \end{cases}$得出$\begin{cases} C_{zs_1}=-\frac{11}{4} \\ C_{zs_2}=-\frac{13}{2} \end{cases}$

故$\mathrm{y_{zs}}(k)=\frac{11}{2}(-1)^k+\frac{13}{2}(-2)^k+3*\left(\frac{1}{2}\right)^k$

全响应：$\mathrm{y(k)}=\mathrm{y_{zi}}(k)+y_{zs}(k)=\frac{17}{2}(-1)^k+\frac{13}{2}(-2)^k+3*\left(\frac{1}{2}\right)^k$

3.8

(5)

由h(k)定义可知

$\mathrm{h(k)-4h(k-1)+8h(k-2)}=\delta(k)$

$\mathrm{h(-1)=h(-2)=0}$

递推求出初始h(0)与h(1)

$\mathrm{h(0)=4h(-1)-8h(-2)}+\delta(k)=1$

$\mathrm{h(1)=4(0)-8h(-1)}+\delta(k)=1$

对于 k>0，h(k)满足齐次线性方程

$\mathrm{h(k)}-4h(k-1)+8h(k-2)=0$

相应特征方程：$(\lambda+1)(\lambda-2)=0$

$h(k)=C_1(-1)^k+C_2(2)^k$ $\quad\begin{cases} h(0)=C_1+C_2=1 \\ h(1)=-C_1+2C_2=1 \end{cases}$故$C_1=-\frac{1}{3}$ $C_2=\frac{2}{3}$

$h(k)=-\frac{1}{3}(-1)^k+\frac{2}{3}C_2(2)^k$

3.26

$y_{zs}=\left[\frac{6}{5}*2^{\mathrm{k}}-\frac{1}{5}\left(\frac{1}{3}\right)^k\right]\varepsilon(k)=[C_1(\lambda_1)^k-C_2(\lambda_2)^k]\varepsilon(k)=[C_1f(k)-C_2h(k)]\varepsilon(k)$

故$f(k)=2^k\varepsilon(k)$

4.7

(a)

T=4，$\Omega=\frac{2\pi}{T}=\frac{\pi}{2}$，在一个周期内$f(t)=\begin{cases} 0\ (\frac{T}{4}<t<\frac{3}{4}T) \\ 1\ (|t|<\frac{T}{4}) \end{cases}$

$$a_n = \frac{1}{2}\int_{-1}^{1} cos\left(\frac{\pi}{2}nT\right) = \frac{2}{n\pi} sin\left(\frac{\pi}{2}n\right)\ (n = 0,1,2\ldots)$$

$$b_n = \frac{1}{2}\int_{-1}^{1} sin\left(\frac{\pi}{2}nT\right) dt = 0\ (n = 1,2\ldots)$$

(b)

T=2，$\Omega = \frac{2\pi}{T} = \pi$，在一个周期内$f(t) = \begin{cases} 0(-\frac{T}{2} < t < 0) \\ \sin(\pi t)(0 \le t < \frac{T}{2}) \end{cases}$

$$a_n = \int_{-1}^{1} f(t)\, cos(n\pi t)\, dt = \int_{0}^{1} \cos(n\pi t)\, dt = \frac{1}{n\pi}\sin(n\pi)\ \ (n = 0,1,2\ldots)$$

$$b_n = \int_{-1}^{1} f(t) \sin(n\pi t)\, dt = \int_{0}^{1} \sin(n\pi t)\, dt = -\frac{1}{n\pi}\cos(n\pi)\ \ (n = 1,2\ldots)$$

4.17

由对称性：$f(t) \Leftrightarrow F(j\omega) \to F(jt) \Leftrightarrow 2\pi f(-\omega)$

$e^{-\alpha|t|} \Leftrightarrow \frac{2\alpha}{\alpha^2+\omega^2} \to \frac{2\alpha}{\alpha^2+t^2} \Leftrightarrow 2\pi e^{-\alpha|\omega|}$

故$\mathrm{f(t)} = \frac{2\alpha}{\alpha^2+t^2} \Leftrightarrow 2\pi e^{-\alpha|\omega|}\ (-\infty < \omega < +\infty)$

4.20

$f(-\mathrm{t}) \Leftrightarrow \mathrm{F}(-\mathrm{j}\omega)$

$f(-t+3) \Leftrightarrow e^{-3j\omega} F(-j\omega)$

$f(-2t+3) \Leftrightarrow \frac{e^{-3j\omega}}{2} F\left(-\frac{\mathrm{j}\omega}{2}\right)$

$\mathrm{e}^{jt} f(-2t+3) \Leftrightarrow \frac{e^{-3j\omega}}{2} F\left(-\frac{\mathrm{j}(\omega-1)}{2}\right)$

4.18

$\delta(t) \Leftrightarrow 1$

$\delta(t-1) \Leftrightarrow e^{-j\omega}$

$\mathrm{f(t)} = \mathrm{e}^{-jt}\delta(t-1) \Leftrightarrow e^{-j(\omega-1)}$

4.30

将等式两边取傅里叶变换

$(\mathrm{j}\omega)^2 Y(\mathrm{j}\omega) + 5\mathrm{j}\omega\mathrm{Y}(\mathrm{j}\omega) + 6\mathrm{Y}(\mathrm{j}\omega) = \mathrm{j}\omega\mathrm{F}(\mathrm{j}\omega) + 4\mathrm{F}(\mathrm{j}\omega)$

$\mathrm{H}(\mathrm{j}\omega) = \frac{\mathrm{Y}(\mathrm{j}\omega)}{H(\mathrm{j}\omega)} = \frac{(\mathrm{j}\omega)^2+5\mathrm{j}\omega+6}{\mathrm{j}\omega+4}$

4.54

$$\mathrm{DTFT}[\mathrm{f}_2(k)] = \sum_{k=-\infty}^{\infty} f_2(k) e^{-jk\theta} = \sum_{k=0}^{3} k[\varepsilon(k) - \varepsilon(k-4)] e^{-jk\theta}$$

$$= 6\cos\left(\frac{\theta}{2}\right) e^{-j\frac{5}{2}\theta} + 2j sin\left(\frac{\theta}{2}\right) e^{-j\frac{3}{2}\theta}$$