第2章 信号与系统的时域分析

内容回顾

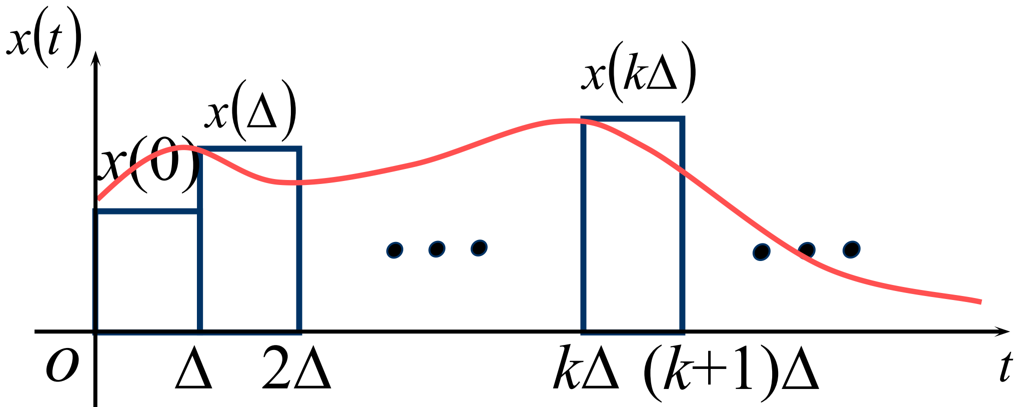
主要内容

CONTENTS

01 时域信号的分解

对子信号的要求

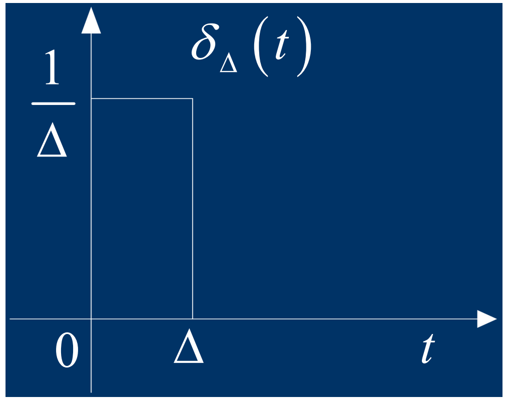
- 完备性：任意信号都可以分解为该子信号的和;
- 简单性：容易求得系统对该子信号的**响应**;
- 相似性：不同子信号的响应具有内在联系,可以类推。



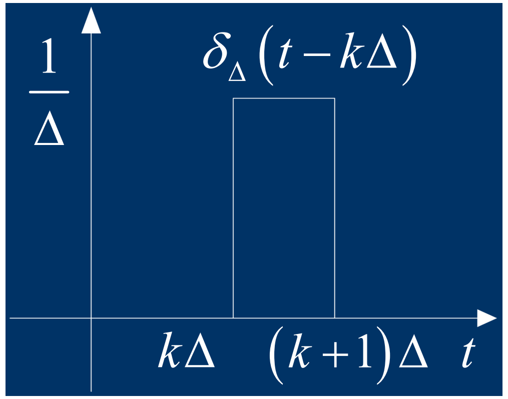
$$x(t) \approx \hat{x}(t) = \sum_{k=-\infty}^{\infty} x_k(t) \xrightarrow{\text{LTI}} y(t) = \sum_{k=-\infty}^{\infty} y_k(t)$$

矩形脉冲信号 $\delta_\Delta(t)$

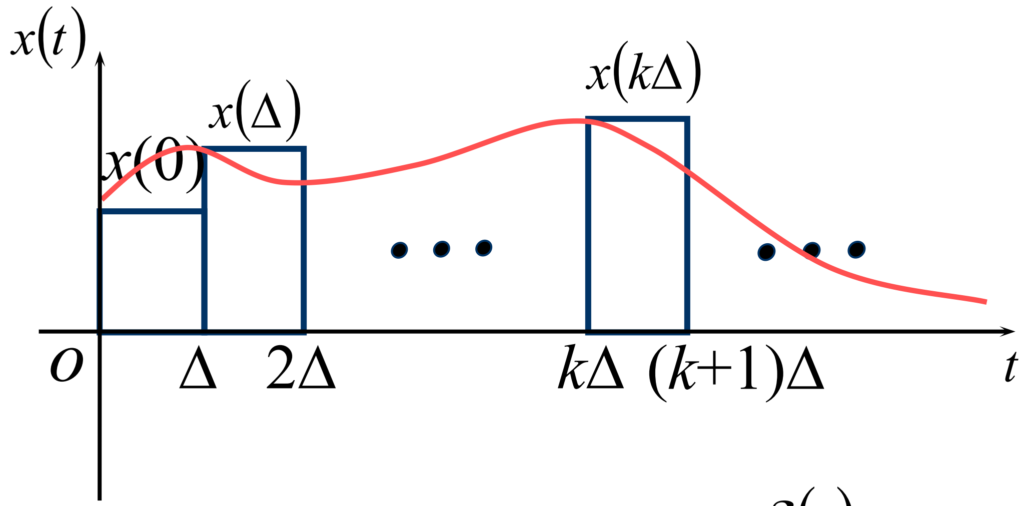
$$\delta_\Delta(t)=\begin{cases}\dfrac{1}{\Delta} & 0<t<\Delta \\ 0 & else\end{cases}$$



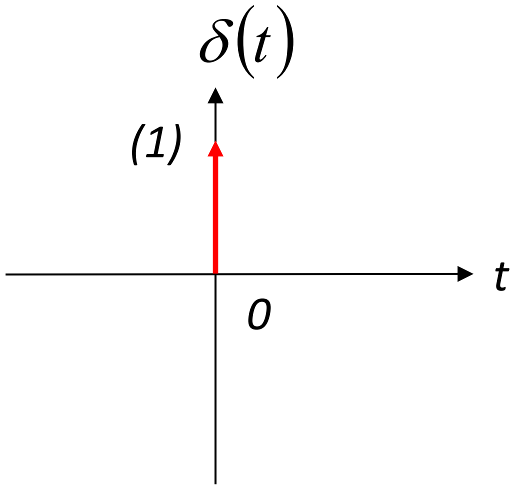
$$\Delta\times\delta_\Delta(t-k\Delta)=\begin{cases}1 & k\Delta<t<(k+1)\Delta \\ 0 & else\end{cases}$$



近似函数：$\hat{x}(t)=\sum_{k=-\infty}^{\infty} x(k\Delta)\,\delta_{\Delta}(t-k\Delta)\times\Delta$



当$\Delta\to 0$时，$\hat{x}(t)\to x(t)$，$\delta_{\Delta}(t)\to\delta(t)$，$\Delta\to d\tau$，$k\Delta\to\tau$



$$x(t)=\lim_{\Delta\to 0}\hat{x}(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$$

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0) \qquad x(t)\text{在}t_0\text{连续}$$

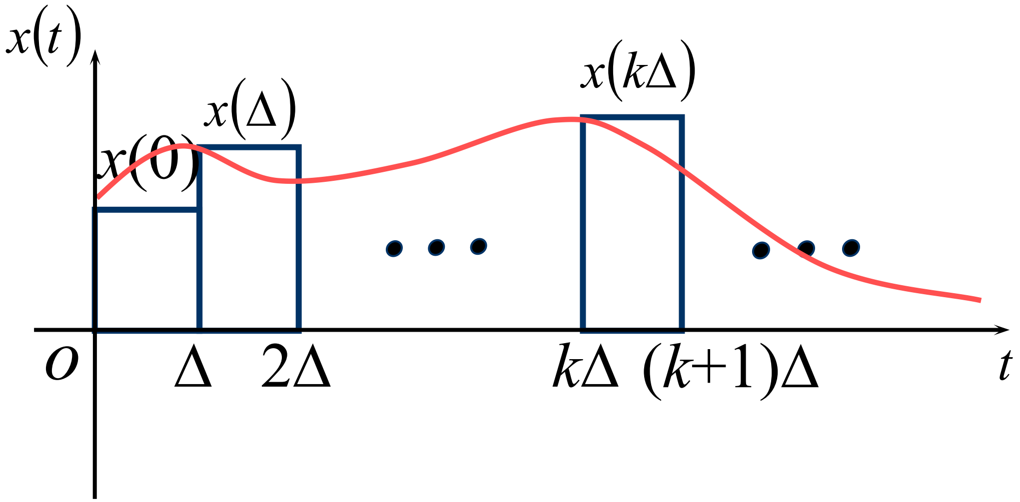
变量代换：$t \to \tau, t_0 \to t$

$$\int_{-\infty}^{\infty} x(\tau)\delta(\tau-t)d\tau = x(t)$$

$\delta(t-\tau) = \delta(\tau-t)$

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)$$

任意连续时间信号，均可以分解成一系列不同加权的单位冲激信号之和。

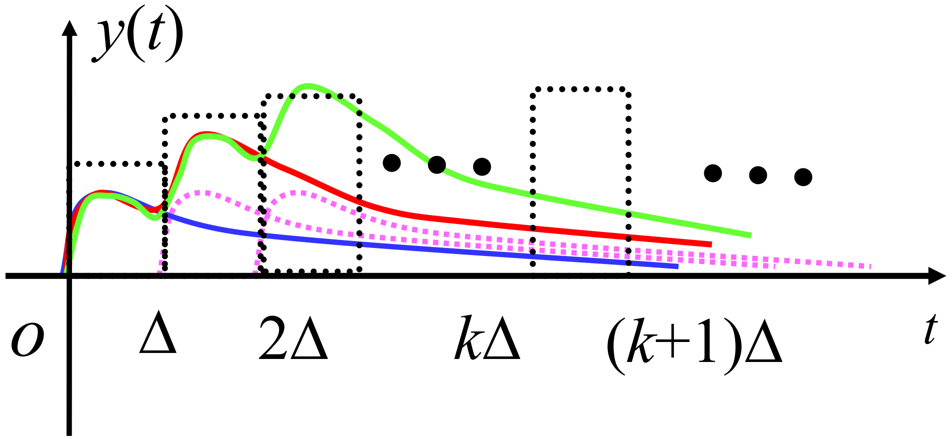


$$\hat{x}(t)=\sum_{k=-\infty}^{\infty}x(k\Delta)\,\delta_{\Delta}(t-k\Delta)\times\Delta$$

假设：对于LTI系统，单位矩形脉冲信号$\delta_{\Delta}(t)$的输出为$h_{\Delta}(t)$

$$\hat{x}(t)\rightarrow\hat{y}(t)$$

则$\hat{y}(t)=\sum_{k=-\infty}^{\infty}x(k\Delta)\,h_{\Delta}(t-k\Delta)\times\Delta$



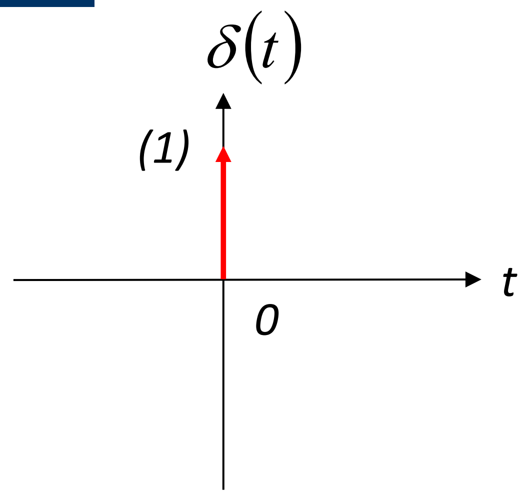
LTI连续时间系统时域分析法

$$\hat{x}(t)=\sum_{k=-\infty}^{\infty}x(k\Delta)\,\delta_{\Delta}(t-k\Delta)\times\Delta \longrightarrow \hat{y}(t)=\sum_{k=-\infty}^{\infty}x(k\Delta)\,h_{\Delta}(t-k\Delta)\times\Delta$$

当$\Delta\to 0$时，$\hat{x}(t)\to x(t)$，$\delta_{\Delta}(t)\to\delta(t)$，$\Delta\to d\tau$，$k\Delta\to\tau$

输入信号：$x(t)=\lim\limits_{\Delta\to 0}\hat{x}(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$

输出信号：$y(t)=\lim\limits_{\Delta\to 0}\hat{y}(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$



卷积积分：$y(t)=x(t)*h(t)$

系统抽象表示

系统解析表示

$$y(t) = x(t) * h(t)$$

卷积积分？ $h(t)=?$

02 卷积积分

定义

$$f_1(t)*f_2(t)=\int_{-\infty}^{\infty}f_1(\tau)f_2(t-\tau)d\tau$$

因果系统中的定义

$$y(t)=\int_0^t x(\tau)h(t-\tau)d\tau=x(t)*h(t)$$

τ为积分变量（激励作用时刻）

t为参变量（观察响应时刻）

解析法：直接根据卷积定义基于积分运算规则计算。

已知：某因果线性时不变系统

$x(t)=\mathrm{e}^{-t}u(t)$ $h(t)=\mathrm{e}^{-2t}u(t)$ 求: $y(t)$

$$
\begin{aligned}
y(t) &= x(t)*h(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty}e^{-\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)d\tau \\
&= \int_{0}^{t}e^{-\tau}e^{-2(t-\tau)}d\tau = e^{-2t}\int_{0}^{t}e^{\tau}d\tau \\
&= e^{-2t}e^{\tau}\Big|_{0}^{t} = u(t)\left(e^{-t}-e^{-2t}\right)
\end{aligned}
$$

1、变换：改变图形中的横坐标，自变量由t变为τ;

2、反转：将其中一个信号反转;

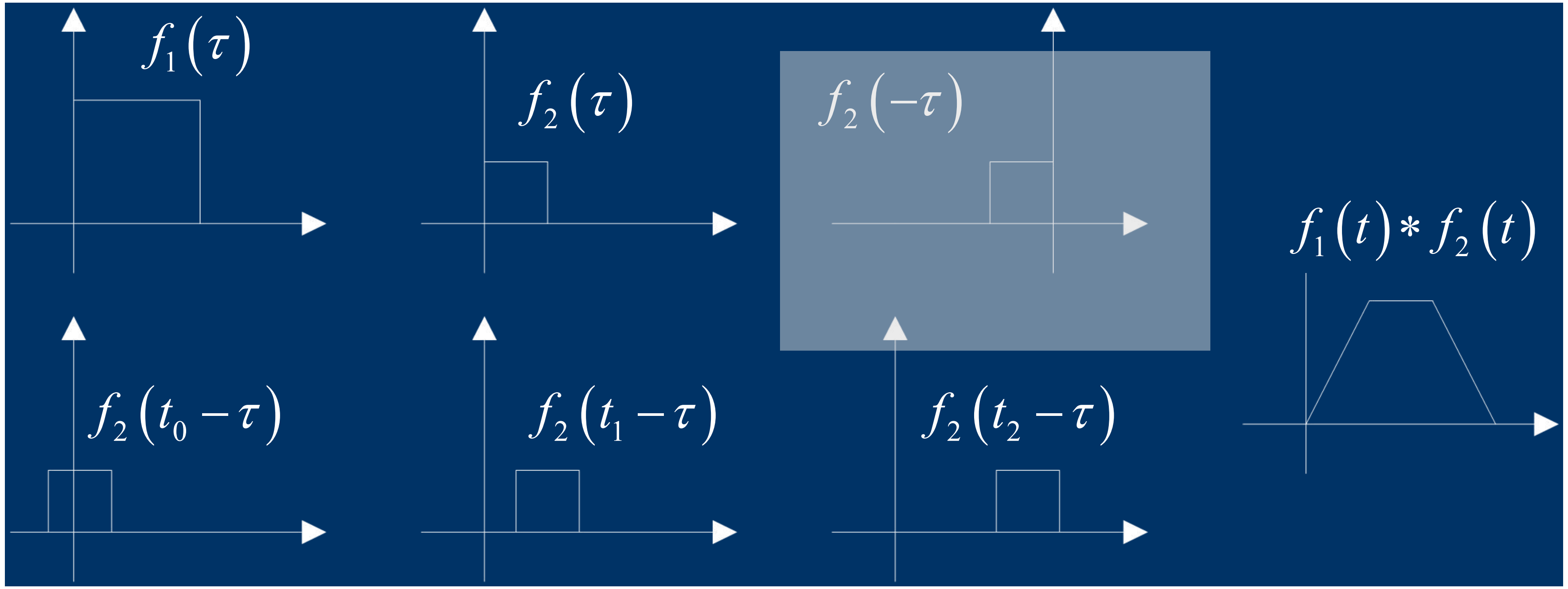
3、平移：反转后的信号随参变量t平移，得到$h(t-\tau)$;
（若$t>0$，则右向平移，若$t<0$，则左向平移）

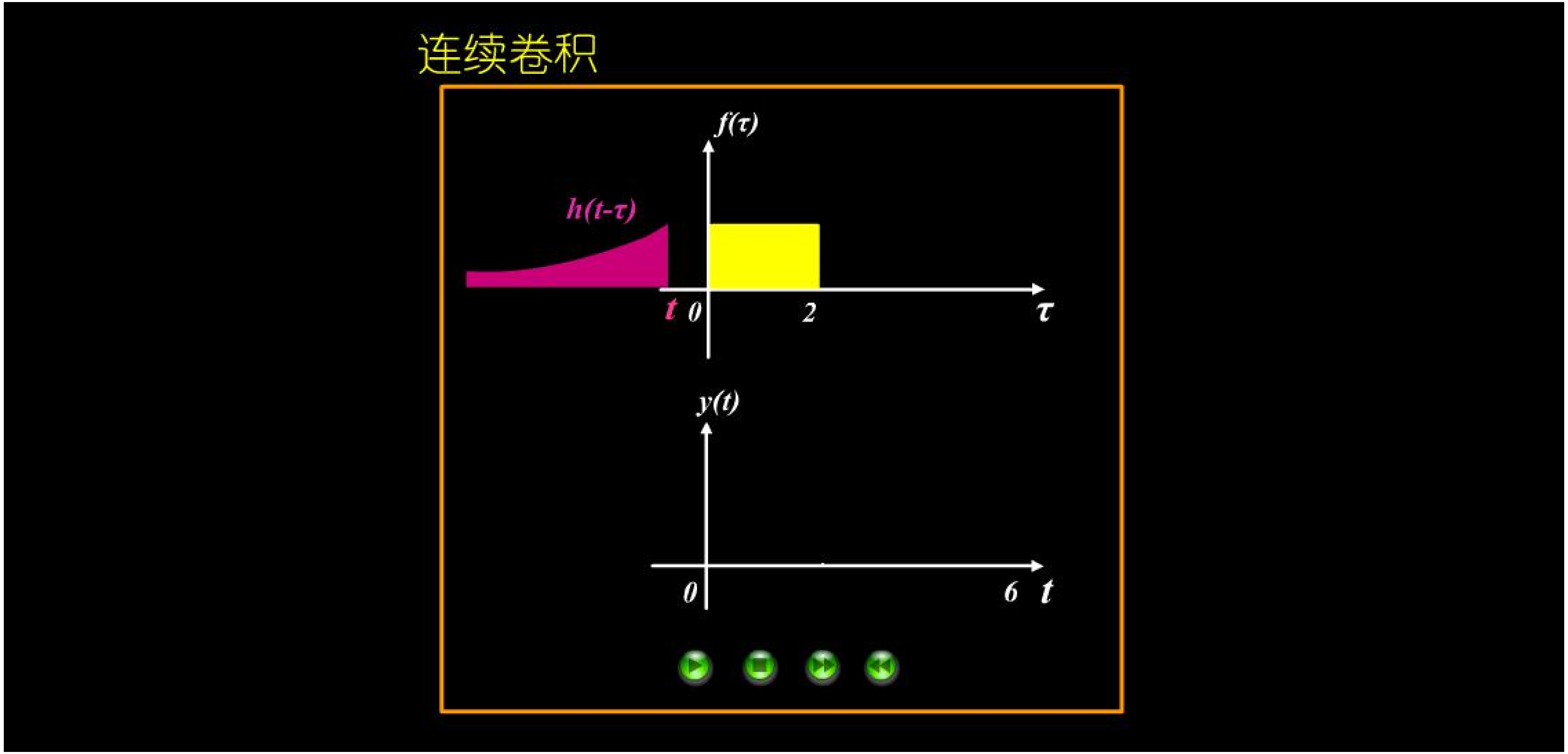
4、相乘：将$x(\tau)$与$h(t-\tau)$相乘;

5、积分：$x(\tau)$与$h(t-\tau)$乘积曲线下的面积即为t时刻的卷积值。

$f_1(t)=2[u(t)-u(t-2)],\quad f_2(t)=u(t)-u(t-1)$

求 $f_1(t)*f_2(t)$




连续卷积
f(τ)
h(t-τ)
t
0
2
τ
y(t)
0
6
t

03 卷积之性质

理解基础上善于应用

交换律

$$x(t) * h(t) = h(t) * x(t)$$

$$\begin{aligned} x(t) * h(t) &= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \\ &= \int_{\infty}^{-\infty} x(t-\lambda) h(\lambda) d(-\lambda) \\ &= \int_{-\infty}^{\infty} h(\lambda) x(t-\lambda) d\lambda = h(t) * x(t) \end{aligned}$$

结合律

$$\left[x(t)*h_1(t)\right]*h_2(t)=x(t)*\left[h_1(t)*h_2(t)\right]$$

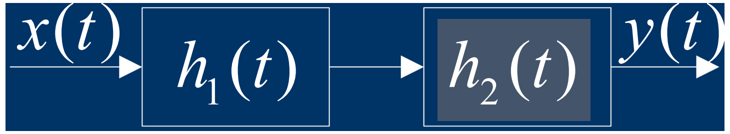
$$\begin{aligned}\left[x(t)*h_1(t)\right]*h_2(t)&=\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}x(\tau)h_1(\lambda-\tau)d\tau\right]h_2(t-\lambda)d\lambda\\&=\int_{-\infty}^{\infty}x(\tau)\left[\int_{-\infty}^{\infty}h_1(\lambda-\tau)h_2(t-\lambda)d\lambda\right]d\tau\\&=\int_{-\infty}^{\infty}x(\tau)\left[\int_{-\infty}^{\infty}h_1(\rho)h_2(t-\tau-\rho)d\rho\right]d\tau\\&=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau=x(t)*h(t)\end{aligned}$$

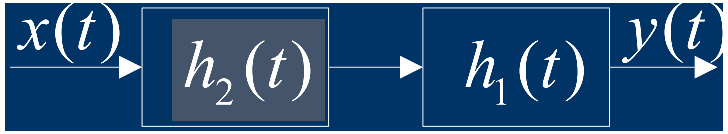
$$h(t-\tau)=\int_{-\infty}^{\infty}h_1(\rho)h_2(t-\tau-\rho)d\rho$$

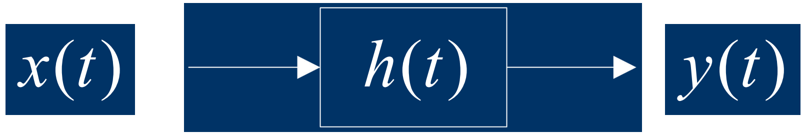
$$\Downarrow$$

$$h(t)=\int_{-\infty}^{\infty}h_1(\rho)h_2(t-\rho)d\rho=h_1(t)*h_2(t)$$

交换律与结合律的意义



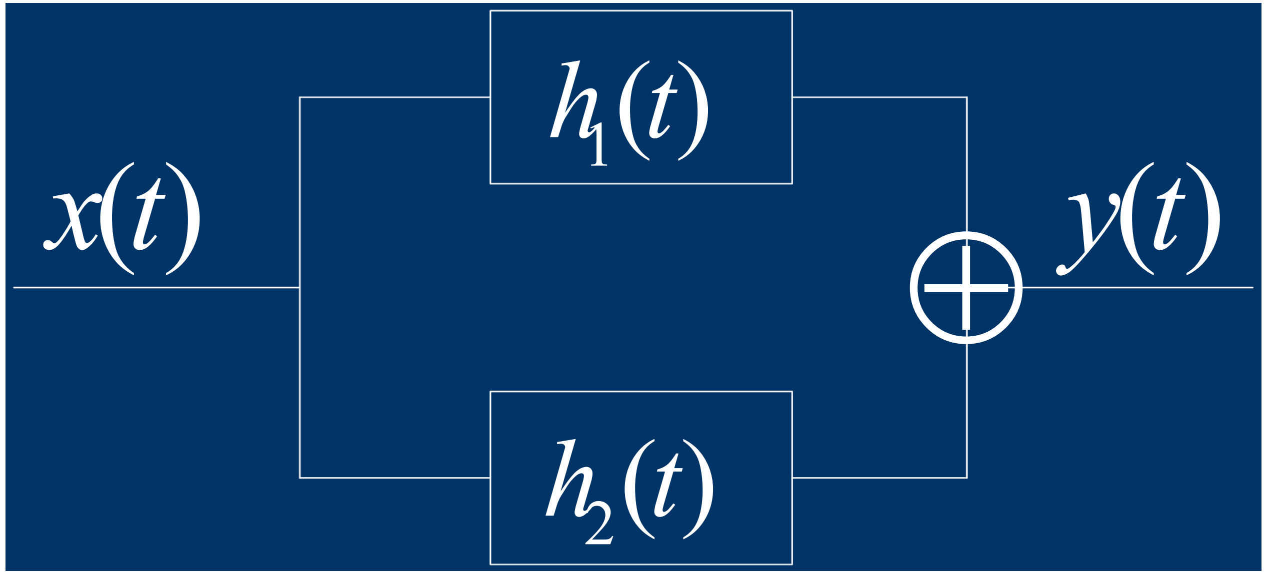




- 串联系统的冲激响应，等于各子系统冲激响应之卷积
- 串联系统与子系统次序无关

$$x(t)*\left[h_1(t)+h_2(t)\right]=x(t)*h_1(t)+x(t)*h_2(t)$$

$$\begin{aligned}x(t)*\left[h_1(t)+h_2(t)\right]&=\int_{-\infty}^{\infty}x(\tau)\left[h_1(t-\tau)+h_2(t-\tau)\right]d\tau\\&=\int_{-\infty}^{\infty}x(\tau)h_1(t-\tau)d\tau+\int_{-\infty}^{\infty}x(\tau)h_2(t-\tau)d\tau\\&=x(t)*h_1(t)+x(t)*h_2(t)\end{aligned}$$



并联系统等效单位冲激响应：$h(t)=h_1(t)+h_2(t)+\cdots$

一个并联系统的冲激响应等于各个子系统冲激响应之和

$$\frac{d}{dt}\left[x(t)*h(t)\right]=\frac{dx(t)}{dt}*h(t)=x(t)*\frac{dh(t)}{dt}$$

$$\begin{aligned}\frac{d}{dt}\left[x(t)*h(t)\right]&=\frac{d}{dt}\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau\\&=\int_{-\infty}^{\infty}x(\tau)\frac{d}{dt}h(t-\tau)d\tau\\&=x(t)*\frac{d}{dt}h(t)\end{aligned}$$

$$\int_{-\infty}^{t}\left[x(\lambda)*h(\lambda)\right]d\lambda=\left[\int_{-\infty}^{t}x(\lambda)d\lambda\right]*h(t)$$
$$=x(t)*\left[\int_{-\infty}^{t}h(\lambda)d\lambda\right]$$

$$\int_{-\infty}^{t}\left[x(\lambda)*h(\lambda)\right]d\lambda=\int_{-\infty}^{t}\left[\int_{-\infty}^{\infty}x(\tau)\,h(\lambda-\tau)\,d\tau\right]d\lambda$$
$$=\int_{-\infty}^{\infty}x(\tau)\left[\int_{-\infty}^{t}h(\lambda-\tau)\,d\lambda\right]d\tau$$
$$=\int_{-\infty}^{\infty}x(\tau)\left[\int_{-\infty}^{t-\tau}h(\lambda)\,d\lambda\right]d\tau$$
$$=x(t)*\left[\int_{-\infty}^{t}h(\lambda)\,d\lambda\right]$$

$$y(t)=x(t)*h(t)$$
$$=\frac{dx(t)}{dt}*\left[\int_{-\infty}^{t}h(\lambda)d\lambda\right]$$
$$=\left[\int_{-\infty}^{t}x(\lambda)d\lambda\right]*\frac{dh(t)}{dt}$$

$x(-\infty)=0$或$h_{-1}(\infty)=0$

$h(-\infty)=0$或$x_{-1}(\infty)=0$

信号与冲激函数的卷积

$$
\begin{aligned}
x(t)*\delta(t) &= \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau \\
&= \int_{-\infty}^{\infty} x(\tau)\delta(\tau-t)\,d\tau \\
&= x(t)
\end{aligned}
$$

信号与冲激函数的卷积-**时移**

$$x(t)*\delta(t-t_0)=\int_{-\infty}^{\infty}x(\tau)\delta(t-t_0-\tau)d\tau$$
$$=x(t-t_0)$$

$$x(t-t_1)*\delta(t-t_2)=\int_{-\infty}^{\infty}x(\tau-t_1)\delta(t-\tau-t_2)d\tau$$
$$=x(t-t_1-t_2)$$

若：$x(t)*h(t)=y(t)$

则：$x(t-t_1)*h(t-t_2)=y(t-t_1-t_2)$

信号与阶跃函数的卷积

$$x(t)*u(t)=x(t)*\int_{-\infty}^{t}\delta(\tau)d\tau$$

$$=\int_{-\infty}^{t}x(\tau)d\tau*\delta(t)$$

$$=\int_{-\infty}^{t}x(\tau)d\tau$$

$$u(t)*u(t)=tu(t)$$

例

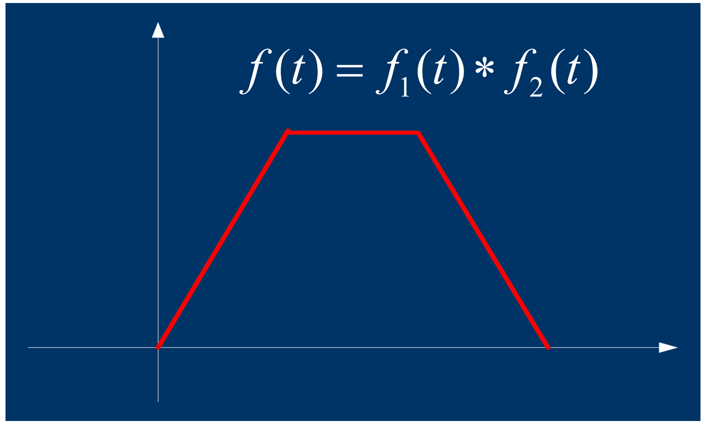
$f_1(t)=2[u(t)-u(t-2)],\quad f_2(t)=u(t)-u(t-1)$

求 $f(t)=f_1(t)*f_2(t)$

$$2[u(t)-u(t-2)]*[u(t)-u(t-1)]$$
$$=2[u(t)*u(t)-u(t)*u(t-1)-u(t-2)*u(t)+u(t-2)*u(t-1)]$$

$$=2[tu(t)-(t-1)u(t-1)-(t-2)u(t-2)+(t-3)u(t-3)]$$

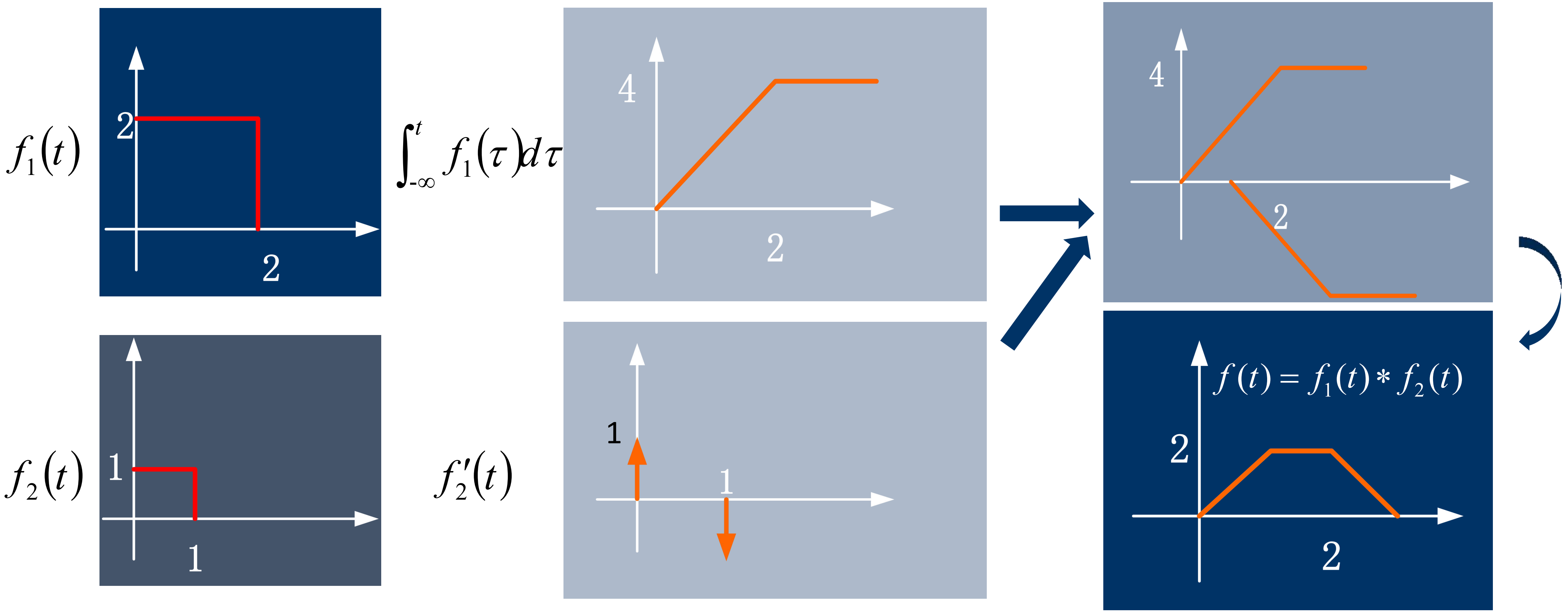
$$\begin{cases} t\le 0 & f(t)=0 \\ 0<t\le 1 & f(t)=2t \\ 1<t\le 2 & f(t)=2 \\ 2<t\le 3 & f(t)=6-2t \\ 3<t & f(t)=0 \end{cases}$$



卷积性质-应用

例 $f_1(t)=2[u(t)-u(t-2)],\quad f_2(t)=u(t)-u(t-1)$

求 $f(t)=f_1(t)*f_2(t)$

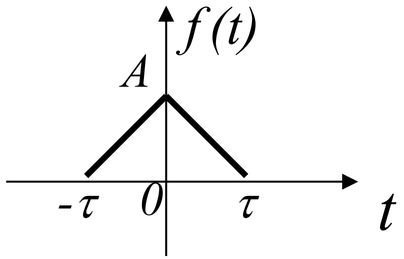


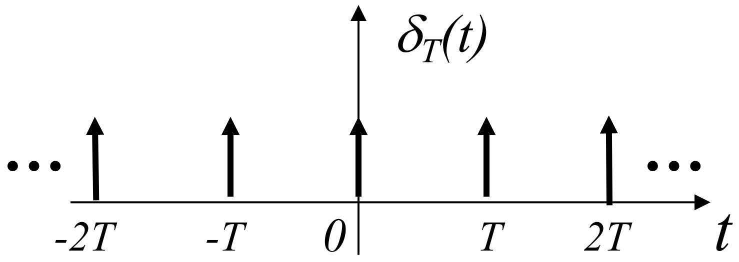
例

$\delta_T(t)$为周期为T的周期性单位冲激函数序列

$$\delta_T(t)=\sum_{k=-\infty}^{\infty}\delta(t+kT)=\delta(t)+\delta(t+T)+\delta(t+2T)+\cdots+\delta(t+kT)+\cdots$$

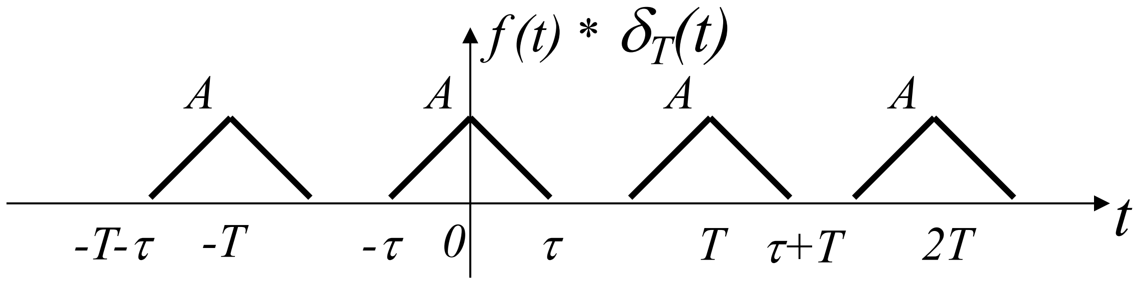
$f(t)$如图所示，试求$f(t)*\delta_T(t)$





$$f(t)*\delta(t)=\int_{-\infty}^{\infty}f(\tau)\delta(t-\tau)d\tau$$

$$f(t)*\delta(t-T)=f(t-T)$$



作业

2.4 (a)(c)

2.5 (b)(d)