

Homework 1: AutoCalib!

Camera Calibration

Wenyan Li
Email: wenyanyi@umd.edu

Abstract—Camera calibration is the process of estimating camera intrinsics, distortion coefficients, and camera extrinsics using images of a special calibration pattern. This project implements camera calibration by estimating the aforementioned parameters with the pipeline addressed in Ref. [1].

I. INTRODUCTION

In this project, the camera calibration procedure follows the one suggested in Ref. [1]:

- 1) Images under thirteen different orientations are taken of the model plane with pattern attached.
- 2) Feature points are then detected in the images by using the InitCalibration() Function.
- 3) Camera intrinsic matrix K and extrinsic matrix $[R \ t]$ is estimated.
- 4) Camera intrinsic parameters are refined by computing and minimizing the geometric error using maximum likelihood estimation and taking radial distortion into account.

II. PARAMETER ESTIMATION AND DISCUSSION

Since the parameter estimation method provided in Ref. is quite clear and easy to follow, repeating the steps to calculate camera intrinsic parameters can be verbose. There are several issues that I noticed during implementing the pipeline may be worth discussing:

The first is that the calculated 6d vector

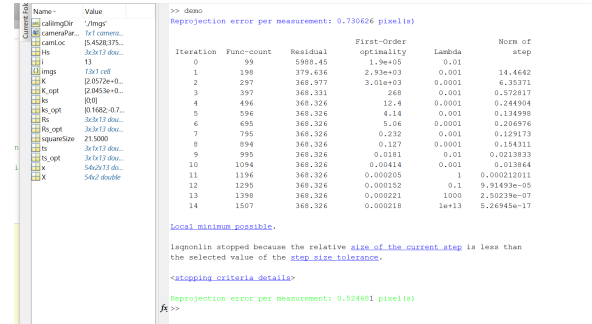
$$b = [B_{11} \ B_{12} \ B_{22} \ B_{13} \ B_{23} \ B_{33}]^T \quad (1)$$

according to Eq.(5) and Eq.(6) in Ref. [1] can be very small that if you output vector b in the matlab terminal the first three elements become zeros, which implies that several intrinsic parameters can become zeros and that is not the result we desired. However, by checking the workspace, it can be noticed that the elements in b are just very small (10^{-4} to 10^{-7}) and not exactly equal to zero. By preserving the real value of the elements in b (though small), the estimation of K is not effected. But if the precision is changed during our estimation, the result can be totally different (I'm still kinda confused why the output vector b is so small).

The second problem is that when computing the linear equations $Vb = 0$, I failed when trying to use the nullspace function $\text{null}(V)$, but solving the equations using SVD works out fine and gives the equivalent output vectors.

III. NON-LINEAR GEOMETRIC ERROR MINIMIZATION

In this part, it's also quite straightforward to minimize the geometric error and refine the parameters by following the tutorial provided and Section 3.3 in Ref. Since we neglected the distortion for the initial estimates, I was too careless to incorporate the two-parameter distortion model when implementing the error minimization function at first. It thus turns out that the error blows up to more than 0.8px after minimization (i.e. the minimization result turns out to be even worse) if neglecting the distortion in the projection of the points in the image. After I fixed the problem, the final minimized error returns to the normal value (around 0.5 px), which is 0.524681 (clearly improved after minimization as shown in Fig. 1). Hence, I see the importance of the corrected two-parameter distortion model in optimizing our non-linear error function. The output camera positions are shown in Fig. 2.



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IV. CONCLUSION

In this homework, the camera calibration pipeline is understood and implemented. Exclusive intrinsic parameters of the camera are estimated including the focal lengths, principal points, and the radial distortion parameters, while the skew parameter s is neglected. The estimation has a reasonable geometric error around 0.5 px and relative problems are discussed during the pipeline implementation.

REFERENCES

- [1] Zhengyou Zhang. A flexible new technique for camera calibration. *IEEE Transactions on pattern analysis and machine intelligence*, 22(11):1330-1334, 2000