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Nutritious Weight Loss Diet

Lineal algebra solves many of the problems of the real life. For example, some people are not happy with their weights, and they are worried about what diet they should follow. In our project, we show with a small-scale problem what people should do to lose weight easily and fast. With our table, we can get a system of equations or even better, we can achieve a vector equation which will result in an augmented matrix. Before of starting the diet, we can predict if the amount of nutrients and the ingredients, which we chose, are the correct ones or if they are not. In other words, we show that linear algebra can be used in a common life.

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Our project speaks about a nutritious weight-loss diet, which is based on the formula for the Cambridge Diet. This diet was very popular in 1980 and was used by millions of persons who wanted to lose weight quickly and substantially. This diet was developed after years of investigations, and its formula combines a precise balance of carbohydrate, high-quality protein, and fat, together with vitamins, minerals, trace elements, and electrolytes.

Dr. Howard led the group of scientists who developed the Cambridge Diet. He carried out a meticulous work to achieve the perfect amounts and proportions of nutrients. However, Dr. Howard had to incorporate a large variety of foodstuffs in his diet. Each foodstuff supplied several of the required ingredients, but the proportions were not the right ones. Some of the foodstuffs were nonfat milk, soy flour and whey. These three ingredients have some advantages and some disadvantages. For example, although the nonfat milk is a great source of protein, it has too much calcium. For this reason, the soy flour was used instead of the nonfat milk. Nevertheless, the soy flour has too much fat, so whey was used. Always that he tried a different foodstuff, it had an excess. The whey contains too much carbohydrate.

Our project deals with this problem but on a small scale. Our table has three of the ingredients in the diet, and it has also the amounts of certain nutrients supplied by 100 grams(g) of each ingredient. The table, which we use in our project to show the problem described in previous paraph, is the following:

Amounts(g) Supplied per 100 g of Ingredient				**Amounts (g) Supplied by Cambridge Diet in One Day**
Nutrient	**Nonfat milk**	**Soy flour**	**Whey**	
Protein	36	51	13	33
Carbohydrate	52	34	74	45
Fat	0	7	1.1	3

We are going to work with this table to see if it is possible to find some combination of nonfat milk, soy flour, and whey to provide the exact amounts of protein, carbohydrate, and fat supplied in one day. There exist two different approaches to solve this problem. First, we can derive equations for each nutrient; for example, the product of the X1 units of nonfat milk times the protein per unit of nonfat milk gives the amount of protein supplied by X1 units of nonfat milk. We must do the same with the other two ingredients, the soy flour and whey, and then, we need to add the three products together; this sum should be equal to the amount of protein that we need. The same calculations must to be done with the three nutrients which will give us three equations.

The second approach is the one that we chose because it is a more efficient. This method is like the previous one; however, in this case, we have a nutrient vector for each foodstuff. This fact produces that we only need to work with one vector equation instead of with three equations. In the vector equation, the scalar multiple X1*a1 is the amount of nutrients supplied by X1 units of nonfat milk. In this multiplication, a1 is the first column of the above table, which corresponds to the vector for the nonfat milk. a2 and a3 are the corresponding vectors for soy flour and whey, respectively, and b is the vector of the last column of the table, which corresponds with the total nutrients required. To conclude, X2*a2 and X3*a3 give the nutrients supplied where X2 is the units of soy flour and X3 is the units of whey. The final vector equation is X1*a1 + X2*a2 + X3*a3 = b. If we insert the correspondent values, we get the following vector equation,

$$X1 * \begin{bmatrix} 36 \\ 52 \\ 0 \end{bmatrix} + X2 * \begin{bmatrix} 51 \\ 34 \\ 7 \end{bmatrix} + X3 * \begin{bmatrix} 13 \\ 74 \\ 1.1 \end{bmatrix} = \begin{bmatrix} 33 \\ 45 \\ 3 \end{bmatrix}$$

Having the vector equation, we can get the values of X1, X2, and X3. First, we are going to pass from a vector equation to a matrix equation. Our matrix equation is,

$$\begin{bmatrix} 36 & 51 & 13 \\ 52 & 34 & 74 \\ 0 & 7 & 1.1 \end{bmatrix} * \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} = \begin{bmatrix} 33 \\ 45 \\ 3 \end{bmatrix}$$

Now, we must use the augmented matrix to find the echelon form, which will allow us to get X1, X2, and X3.

$$\begin{bmatrix} 36 & 51 & 13 & 33 \\ 52 & 34 & 74 & 45 \\ 0 & 7 & 1.1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 36 & 51 & 13 & 33 \\ 52 & 34 & 74 & 45 \\ 0 & 7 & 1.1 & 3 \end{bmatrix} \quad \text{R2 – 52/36 *R1} \rightarrow \text{R2}$$

$$\begin{bmatrix} 36 & 51 & 13 & 33 \\ 0 & -1428/36 & 1988/36 & -96/36 \\ 0 & 7 & 1.1 & 3 \end{bmatrix} \quad \text{R3 + (7*36)/1428*R2} \rightarrow \text{R3}$$

$$\begin{bmatrix} 36 & 51 & 13 & 33 \\ 0 & -1428/36 & 1988/36 & -96/36 \\ 0 & 0 & 15486.8/1428 & 3612/1428 \end{bmatrix}$$

In the next step, we are going to pass from fraction to decimals to be able to check if our echelon form matrix is the same one that our program gives us.

$$\begin{bmatrix} 36 & 51 & 13 & 33 \\ 0 & -39.6667 & 55.2222 & -2.66667 \\ 0 & 0 & 10.8451 & 2.52941 \end{bmatrix}$$

From here, we can solve the three equations

36*X1 + 51*X2 +13*X3 = 33 → 36 * X1 =33 – (51 * 0.391921) – (13 * 0.233231) = 9.980026 → X1 = 9.980026/36 = 0.277223 → <u>X1 = 0.277223</u>

-1428/36 * X2 + 1988/36*X3 = -96/36 → -1428/36 * X2 + 1988/36 * 0.233231 = -96/36 →X2=0.391921

15486.8/1428 * X3 = $3612/1428$ →X3 = 3612/15486.8 = 0.233231 → X3 = = 0.233231

To make sure that our hand calculates are correct, we created a program with give us the echelon form and the X1, X2, and X3 values by only using the original augmented matrix.

```
    36.0000          51.0000          13.0000          33.0000
    9.536743E-07    -39.6667          55.2222         -2.66667
    1.682955E-07    2.749410E-07      10.8451          2.52941

x1=      0.277223
x2=      0.391921
x3=      0.233231
```

We can see that both the hand calculations and the program gives the same answers. By our results, we know that to get the desired amounts of protein, carbohydrate and fat, the diet requires 0.277223 units of nonfat milk, 0.391921 units of soy flour, and 0.233231 units of whey. Also, the echelon form matrix is the same although the program does not give 0, instead it gives us small numbers. For example, 9.536743E-07.

The most important detail about the values of X1, X2, and X3 is their sign. They must be nonnegative to make possible to find a combination of nonfat milk, soy flour, and whey. For instant, it is not possible to use -0.277223 units of nonfat milk.

To summary, we can say that the diet construction problem leads to X1*a1 + X2*a2 + X3*a3 = b because the amount of nutrients can be written as a scalar multiple of a vector. In this equation, X1 represents the units of nonfat milk, X2 is the units of soy flour, and X3 shows the units of whey. a1, a2, and a3 represent the nutrients per X1, X2, and X3, respectively. Also, X1, X2, X3 are scalars, and a1, a2,a3 are vectors. From the previous linear equation, we get an augmented matrix. Our goal is to get the values for X1, X2 and X3, so we reduce to the echelon form, and we solve the three equations to get the values.

On the other hand, we decided to add an additional nutrient to the nutrients listed in the above table; the new nutrient is the calcium. The Cambridge Diet supplies 0.8g of calcium per day. The amounts of calcium per units supplied by the three ingredients in the Cambridge Diet are: 1.26 g from nonfat milk, 0.19 g from soy flour, and 0.8 g from whey. In addition to incorporate a new nutrient, we also include another ingredient in the diet mixture which was the soy protein. This ingredient provided 80 g of protein, 0 g of carbohydrate, 3.4 g of fat, and 18 g of calcium. After including all the previous values to our table, it looks like the following one,

Amounts(g) Supplied per 100 g of Ingredient					Amounts (g) Supplied by Cambridge Diet in One Day
Nutrient	**Nonfat milk**	**Soy flour**	**Whey**	**Soy Protein**	
Protein	36	51	13	80	33
Carbohydrate	52	34	74	0	45
Fat	0	7	1.1	3.4	3
Calcium	1.26	0.19	0.8	0.18	0.8

Now, the relevant equation is X1*a1 + X2*a2 + X3*a3 + X4*a4= b where X4 represents the units of soy protein, and the a4 represents the nutrients per X4. Including the values from the previous table in the vector equation, we get the following,

$$X1 * \begin{bmatrix} 36 \\ 52 \\ 0 \\ 1.26 \end{bmatrix} + X2 * \begin{bmatrix} 51 \\ 34 \\ 7 \\ 0.19 \end{bmatrix} + X3 * \begin{bmatrix} 13 \\ 74 \\ 1.1 \\ 0.8 \end{bmatrix} + X4 * \begin{bmatrix} 80 \\ 0 \\ 3.4 \\ 0.18 \end{bmatrix} = \begin{bmatrix} 33 \\ 45 \\ 3 \\ 0.8 \end{bmatrix}$$

The matrix equation is,

$$\begin{bmatrix} 36 & 51 & 13 & 80 \\ 52 & 34 & 74 & 0 \\ 0 & 7 & 1.1 & 3.4 \\ 1.26 & 0.19 & 0.8 & 0.18 \end{bmatrix} * \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix} = \begin{bmatrix} 33 \\ 45 \\ 3 \\ 0.8 \end{bmatrix}$$

We have set up a matrix equation whose solution determines the amounts of nonfat milk, soy flour, whey, and isolated soy protein necessary to supply the precise amounts of protein, carbohydrate, fat, and calcium in the Cambridge Diet. The variables of this equation, X1, X2, X3, and X4, represent the number of units of nonfat milk, soy flour, whey, and isolated soy protein. In this case, the augmented matrix is

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 52 & 34 & 74 & 0 & 45 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 1.26 & 0.19 & 0.8 & 0.18 & 0.8 \end{bmatrix}$$

Now, we must use the above augmented matrix to find the echelon form, which will allow us to know X1, X2, X3, and X4.

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 52 & 34 & 74 & 0 & 45 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 1.26 & 0.19 & 0.8 & 0.18 & 0.8 \end{bmatrix}$$

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 52 & 34 & 74 & 0 & 45 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 1.26 & 0.19 & 0.8 & 0.18 & 0.8 \end{bmatrix}$$ R2 <–>R3

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 52 & 34 & 74 & 0 & 45 \\ 1.26 & 0.19 & 0.8 & 0.18 & 0.8 \end{bmatrix}$$ R3 – 52/36 *R1→R3

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 0 & -1428 & 1988 & -4160 & -96 \\ 1.26 & 0.19 & 0.8 & 0.18 & 0.8 \end{bmatrix}$$ R4 – 1.26/36 *R1→R4

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 0 & -1428 & 1988 & -4160 & -96 \\ 0 & -57.42 & 12.42 & -94.32 & -12.78 \end{bmatrix}$$ R4 + 57.42/7 * R2→ R4

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 0 & -1428 & 1988 & -4160 & -96 \\ 0 & 0 & 150.102 & -465.012 & 82.8 \end{bmatrix}$$ R3 + 1428/7 * R2→ R3

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 0 & 0 & 15486.8 & -24264.8 & 3612 \\ 0 & 0 & 150.102 & -465.012 & 82.8 \end{bmatrix}$$ R4 – 150.102/15486.8 * R3→ R4

$$\begin{bmatrix} 36 & 51 & 13 & 80 & 33 \\ 0 & 7 & 1.1 & 3.4 & 3 \\ 0 & 0 & 15486.8 & -24264.8 & 3612 \\ 0 & 0 & 0 & -3559352.832 & 740138.616 \end{bmatrix}$$

From here, we can solve the four equations

36*X1 + 51*X2 + 13*X3 + 80*X4 = 33 → 36*X1 + 51*0.544119 + 13*(-0.0925737) + 80*(-0.207942) = 33 →<u>X1=0.641354</u>

7*X2 + 1.1*X3 + 3.4*X4 = 3 → 7*X2 + 1.1*(-0.0925737) + 3.4*(-0.207942) = 3 → <u>X2 = 0.544119</u>

15486.8*X3 – 24264.8*X4 = 3612 → 15486.8*X3 – 24264.8*(-0.207942) = 3612 →
<u>X3 = -0.0925737</u>

-3559352.832*X4 = 740138.616 →<u>X4 =-0.207942</u>

To check that our calculations are right, we modified our previous program, and we get the echelon form and the X1, X2, X3, and X4 values for this new augmented matrix.

```
   36.0000          51.0000          13.0000          80.0000
  9.536743E-07     -39.6667          55.2222         -115.556
  1.682955E-07     2.749410E-07      10.8451          -16.9922
 -2.414442E-08     2.848537E-08    -5.085261E-08    -0.912029

x1=      0.641354
x2=      0.544119
x3=    -9.257372E-02
x4=    -0.207942
```

We can see that both the hand calculations and the program gives the same answers. However, we get two negative values to the variables X3 and X4. As we say before, it is not possible to make a mixture with negative amounts, so it is the reason why we can say that the solution for this system is not possible. For instant, it is not possible to use -0.207942 units of isolated soy protein.

We used two similar codes in our project. We include both below. They are written in Fortran, and the way to run them is using FTN95 Command Prompt.

The first example uses the following code,

```
implicit none
integer:: i, j, k, n, i0
real, allocatable, dimension(:,:)::a
real, allocatable, dimension(:) :: x
real::m
n=3
allocate(a(n,n+1), x(n))
a(1,1)=36
a(1,2)=51
a(1,3)=13
a(1,4)=33

a(2,1)=52
a(2,2)=34
a(2,3)=74
a(2,4)=45

a(3,1)=0
a(3,2)=7
a(3,3)=1.1
```

```
a(3,4)=3

do k=1, (n-1)*(n-1), 1
do i=k+1, n, 1
if(k==(i-1))then
if(a(i-1,k)==0.0) then
call order(a,n,k)
end if
end if

m=a(i,k)/a(k,k)
do j=1, n+1,1
a(i, j)= a(i, j)-(m*a(k,j))
end do
end do
end do

call solve(a, n, x)
do i0=0, n-1,1
write(*,*)a(1+i0, 1), a(1+i0,2), a(1+i0,3), a(1+i0,4)
end do
print*," "
print*,"x1= ", x(1)
print*,"x2= ", x(2)
print*,"x3= ", x(3)
end program

subroutine order(a, n, k)
implicit none
integer:: m, k, n, bigm, i0
real, dimension(n, n+1) :: a
real, dimension(n) :: temp
real:: big
do m=k,n,1
if(big<ABS(a(m,k))) then
big=ABS(a(m,k))
bigm=m
end if
end do

if(bigm /= m) then
 do i0=k,n+2,1
temp(i0)=a(k,i0)
```

```
a(k,i0)=a(bigm, i0)
a(bigm, i0)=temp(i0)
end do
end if
return
end subroutine
subroutine solve(a, n, x)
implicit none
integer:: i, j, n, k
real:: sum
real,dimension(n)::x
real, dimension(n, n+1) :: a

x(n)=a(n,n+1)/a(n,n)

do k=1, (n-1)*(n-1), 1
do i= (n-1), 1, -1
sum=0.0d0

do j=i+1, n, 1
sum=sum+(a(i,j)*x(j))
end do

x(i)=(a(i,n+1)-sum)/a(i,i)
end do
end do
return
end subroutine
```

The second exercise uses the next code,

```
implicit none
integer:: i, j, k, n, i0
real, allocatable, dimension(:,:)::a
real, allocatable, dimension(:) :: x
real::m
n=4
allocate(a(n,n+1), x(n))
a(1,1)=36
a(1,2)=51
a(1,3)=13
a(1,4)=80
a(1,5)=33
```

```
a(2,1)=52
a(2,2)=34
a(2,3)=74
a(2,4)=0
a(2,5)=45

a(3,1)=0
a(3,2)=7
a(3,3)=1.1
a(3,4)=3.4
a(3,5)=3

a(4,1)=1.26
a(4,2)=0.19
a(4,3)=0.8
a(4,4)=0.18
a(4,5)=0.8

do k=1, (n-1)*(n-1), 1
do i=k+1, n, 1
if(k==(i-1))then
if(a(i-1,k)==0.0) then
call order(a,n,k)
end if
end if

m=a(i,k)/a(k,k)
do j=1, n+1,1

a(i, j)= a(i, j)-(m*a(k,j))
end do
end do
end do

call solve(a, n, x)

do i0=0, n-1,1
write(*,*)a(1+i0, 1), a(1+i0,2), a(1+i0,3), a(1+i0,4)
end do

print*," "
print*,"x1= ", x(1)
print*,"x2= ", x(2)
```

```
print*,"x3= ", x(3)
print*,"x4= ", x(4)
end program


subroutine order(a, n, k)
implicit none
integer:: m, k, n, bigm, i0
real, dimension(n, n+1) :: a
real, dimension(n) :: temp
real:: big
do m=k,n,1
if(big<ABS(a(m,k))) then
big=ABS(a(m,k))
bigm=m
end if
end do

if(bigm /= m) then
 do i0=k,n+2,1
temp(i0)=a(k,i0)
a(k,i0)=a(bigm, i0)
a(bigm, i0)=temp(i0)
end do
end if
return
end subroutine

subroutine solve(a, n, x)
implicit none
integer:: i, j, n, k
real:: sum
real,dimension(n)::x
real, dimension(n, n+1) :: a
x(n)=a(n,n+1)/a(n,n)

do k=1, (n-1)*(n-1), 1
do i= (n-1), 1, -1
sum=0.0d0

do j=i+1, n, 1
sum=sum+(a(i,j)*x(j))
end do
```

```
x(i)=(a(i,n+1)-sum)/a(i,i)
end do
end do
return
end subroutine
```

Work Cited

C.Lay, *David. Linear Algebra and Its Applications*. Pearson.