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Mathematics 4670

Solving System of Equation

1. For this first question, we have a Fortran code which solves systems of n linear equations in n variables. We tested it with one example of 5 linear equations in 5 variables.
My code is the following one:

```
implicit none
integer:: i, j, k, n
real, allocatable, dimension(:,:)::a
real, allocatable, dimension(:) :: x
real::m
n=5
allocate(a(n,n+1), x(n))
a(1,1)=4
a(1,2)=-2
a(1,3)=-1
a(1,4)=1
a(1,5)=2
a(1,6)=14

a(2,1)=1
a(2,2)=2
a(2,3)=2
a(2,4)=-1
a(2,5)=4
a(2,6)=14

a(3,1)=2
a(3,2)=-1
a(3,3)=4
a(3,4)=-2
a(3,5)=2
a(3,6)=-8

a(4,1)=1
a(4,2)=1
a(4,3)=1
a(4,4)=1
a(4,5)=1
a(4,6)=23
```

```
a(5,1)=6
a(5,2)=4
a(5,3)=1
a(5,4)=-6
a(5,5)=6
a(5,6)=-4
do k=1, (n-1)*(n-1), 1
do i=k+1, n, 1

m=a(i,k)/a(k,k)
do j=1, n+1,1
a(i, j)= a(i, j)-(m*a(k,j))
end do
end do

call solve(a, n, x)
end do

do i=0, n-1,1
write(*,*)a(1+i, 1), a(1+i,2), a(1+i,3), a(1+i, 4), a(1+i,5)
end do

do i=1,n,1
write(*,*)"x(", i, "):", x(i)
end do
end program


subroutine solve(a, n, x)
implicit none
integer:: i, j, n, k
real:: sum
real,dimension(n)::x
real, dimension(n, n+1) :: a

x(n)=a(n,n+1)/a(n,n)
do k=1, (n-1)*(n-1), 1
do i= (n-1), 1, -1
sum=0.0d0

do j=i+1, n, 1
sum=sum+(a(i,j)*x(j))
```

```
    end do

    x(i)=(a(i,n+1)-sum)/a(i,i)
    end do
    end do

    return
    end subroutine
```

My example is the following one.

4*X1 - 2*X2 - 1*X3 + 1*X4 + 2*X5 = 14

1*X1 + 2*X2 + 2*X3 – 1*X4 + 4*X5 = 14

2*X1 – 1*X2 + 4*X3 – 2*X4 + 2*X5 = -8

1*X1 + 1*X2 + 1*X3+ 1*X4 + 1*X5 = 23

6*X1 + 4*X2 + 1*X3 – 6*X4 + 6*X5 = -4

We can express the above equations as the same that the following,

4.00000	-2.00000	-1.00000	1.00000	2.00000
1.00000	2.00000	2.00000	-1.00000	4.00000
2.00000	-1.00000	4.00000	-2.00000	2.00000
1.00000	1.00000	1.00000	1.00000	1.00000
6.00000	4.00000	1.00000	-6.00000	6.00000

After we run the program, we get the following output

4.00000	-2.00000	-1.00000	1.00000	2.00000
0.00000	2.50000	2.25000	-1.25000	3.50000
0.00000	0.00000	4.50000	-2.50000	1.00000
0.00000	-5.960464E-08	-9.313226E-10	1.44444	-1.57778
0.00000	-1.329642E-07	8.546676E-08	-2.710228E-07	-12.6308

x(1): 4.00000

x(2): 6.00000

x(3): 2.00000

x(4): 10.0000

x(5): 1.00000

2. In the second question, we use the same code; however, we add extra. In the first question, if there is a zero in the diagonal, our problem breaks. We solve this problem by changing the order of the equation below the 0. I mean we put into the position where we get the 0, for example a(k,k), the number with largest possible absolute value, taken from a(k,k),a(k+1,k), … a(n, k). This is based on the idea that if zero is bad, far from zero is good. This method is called maximal row pivoting. The only way this can fail is if a(k,k) is equal 0 and all entries directly below that entry are also zero. This would flag the non-unique solution situation.
We tested this change with one example of 10 linear equations in 10 variables. My code is the following one:

```
implicit none
integer:: i, j, k, n, i0
real, allocatable, dimension(:,:)::a
real, allocatable, dimension(:) :: x
real::m
n=10

allocate(a(n,n+1), x(n))
a(1,1)=0
a(1,2)=5
a(1,3)=-6
a(1,4)=4
a(1,5)=1
a(1,6)=-8
a(1,7)=2
a(1,8)=-7
a(1,9)=3
a(1,10)=-3
a(1,11)=85

a(2,1)=-7
a(2,2)=-5
a(2,3)=-5
a(2,4)=3
a(2,5)=-3
a(2,6)=-2
a(2,7)=3
a(2,8)=-1
a(2,9)=2
a(2,10)=9
a(2,11)=-109
```

a(3,1)=-3
a(3,2)=5
a(3,3)=-4
a(3,4)=-2
a(3,5)=2
a(3,6)=-1
a(3,7)=4
a(3,8)=7
a(3,9)=-6
a(3,10)=-7
a(3,11)=201

a(4,1)=-2
a(4,2)=-4
a(4,3)=5
a(4,4)=8
a(4,5)=-7
a(4,6)=-1
a(4,7)=2
a(4,8)=-6
a(4,9)=-5
a(4,10)=-4
a(4,11)=-62

a(5,1)=-8
a(5,2)=-8
a(5,3)=2
a(5,4)=-2
a(5,5)=1
a(5,6)=-5
a(5,7)=1
a(5,8)=-5
a(5,9)=5
a(5,10)=-1
a(5,11)=-37

a(6,1)=5
a(6,2)=6
a(6,3)=7
a(6,4)=-5
a(6,5)=-6
a(6,6)=4

a(6,7)=2
a(6,8)=4
a(6,9)=-9
a(6,10)=5
a(6,11)=-27

a(7,1)=6
a(7,2)=-4
a(7,3)=9
a(7,4)=8
a(7,5)=6
a(7,6)=2
a(7,7)=-5
a(7,8)=6
a(7,9)=-5
a(7,10)=-3
a(7,11)=38

a(8,1)=-3
a(8,2)=5
a(8,3)=-6
a(8,4)=-2
a(8,5)=8
a(8,6)=-4
a(8,7)=-3
a(8,8)=4
a(8,9)=-8
a(8,10)=-5
a(8,11)=258

a(9,1)=-5
a(9,2)=7
a(9,3)=-2
a(9,4)=-4
a(9,5)=-8
a(9,6)=4
a(9,7)=-9
a(9,8)=-5
a(9,9)=-6
a(9,10)=-3
a(9,11)=77

a(10,1)=-6

```
a(10,2)=-1
a(10,3)=-4
a(10,4)=-5
a(10,5)=-8
a(10,6)=8
a(10,7)=8
a(10,8)=-5
a(10,9)=-7
a(10,10)=4
a(10,11)=-101



do k=1, (n-1)*(n-1), 1
do i=k+1, n, 1

if(k==(i-1))then
if(a(i-1,k)==0.0) then
call order(a,n,k)
end if
end if


m=a(i,k)/a(k,k)
do j=1, n+1,1

a(i, j)= a(i, j)-(m*a(k,j))
end do
end do
end do


call solve(a, n, x)
do i0=0, n-1,1
write(*,*)a(1+i0, 1), a(1+i0,2), a(1+i0,3), a(1+i0, 4), a(1+i0, 5)
end do
print*," "
do i0=0, n-1,1
write(*,*)a(1+i0, 6), a(1+i0,7), a(1+i0,8), a(1+i0, 9), a(1+i0,10)
end do

do i=1,n,1
write(*,*)"x(", i, "):", x(i)
```

```
end do
end program

subroutine order(a, n, k)
implicit none
integer:: m, k, n, i, j, bigm, i0
real, dimension(n, n+1) :: a
real, dimension(n) :: temp
real:: big
do m=k,n,1
if(big<ABS(a(m,k))) then
big=ABS(a(m,k))
bigm=m
end if
end do

if(bigm /= m) then
 do i0=k,n+2,1
temp(i0)=a(k,i0)
a(k,i0)=a(bigm, i0)
a(bigm, i0)=temp(i0)
end do
end if
return
end subroutine

subroutine solve(a, n, x)
implicit none
integer:: i, j, n, k
real:: sum
real,dimension(n)::x
real, dimension(n, n+1) :: a

x(n)=a(n,n+1)/a(n,n)

do k=1, (n-1)*(n-1), 1
do i= (n-1), 1, -1
sum=0.0d0

do j=i+1, n, 1
sum=sum+(a(i,j)*x(j))
end do
```

x(i)=(a(i,n+1)-sum)/a(i,i)
end do
end do
return
end subroutine

My example is the following one.

0*X1 + 5*X2 - 6*X3 + 4*X4 + 1*X5 – 8 *X6 + 2*X7 – 7*X8 + 3*X9 – 3 *X10 = 85

-7*X1 -5*X2 -5*X3 +3*X4 -3 *X5 -2*X6 + 3*X7 -1*X8 + 2*X9 + 9*X10 = -109

-3*X1 + 5*X2 - 4*X3 -2*X4 + 2*X5 -1*X6 + 4*X7 + 7*X8 - 6*X9 -7 *X10 = 201

-2*X1 -4*X2 + 5*X3 + 8*X4 - 7 *X5 - 1*X6 + 2*X7 -6*X8 -5*X9 - 4*X10 = -62

-8*X1 -8*X2 +2*X3 -2*X4 + 1*X5 -5*X6 + 1*X7 -5*X8 + 5*X9 -1*X10 = -37

5*X1 +6*X2 +7*X3 -5*X4 -6*X5 +4*X6 + 2*X7 +4*X8 -9*X9 + 5*X10 = -27

6*X1 -4*X2 +9*X3 +8*X4 +6 *X5 +2*X6 -5*X7 +6*X8 -5*X9 -3*X10 = 38

-3*X1 +5*X2 -6*X3 -2*X4 +8 *X5 -4*X6 -3*X7 +4*X8 -8*X9 -5*X10 = 258

-5*X1 +7*X2 -2*X3 -4*X4 -8 *X5 +4*X6 -9*X7 -5*X8 -6*X9 -3*X10 = 77

-6*X1 -1*X2 -4*X3 -5*X4 -8 *X5 +8*X6 + 8*X7 -5*X8 -7*X9 + 4*X10 = -101

We can express the above equations as the same that the following,

0.00000	5.00000	-6.00000	4.00000	1.00000
-7.00000	-5.00000	-5.00000	3.00000	-3.00000
-3.00000	5.00000	-4.00000	-2.00000	2.00000
-2.00000	-4.00000	5.00000	8.00000	-7.00000
-8.00000	-8.00000	2.00000	-2.00000	1.00000
5.00000	6.00000	7.00000	-5.00000	-6.00000
6.00000	-4.00000	9.00000	8.00000	6.00000
-3.00000	5.00000	-6.00000	-2.00000	8.00000
-5.00000	7.00000	-2.00000	-4.00000	-8.00000
-6.00000	-1.00000	-4.00000	-5.00000	-8.00000

-8.00000	2.00000	-7.00000	3.00000	-3.00000
-2.00000	3.00000	-1.00000	2.00000	9.00000
-1.00000	4.00000	7.00000	-6.00000	-7.00000
-1.00000	2.00000	-6.00000	-5.00000	-4.00000
-5.00000	1.00000	-5.00000	5.00000	-1.00000
4.00000	2.00000	4.00000	-9.00000	5.00000
2.00000	-5.00000	6.00000	-5.00000	-3.00000
-4.00000	-3.00000	4.00000	-8.00000	-5.00000
4.00000	-9.00000	-5.00000	-6.00000	-3.00000
8.00000	8.00000	-5.00000	-7.00000	4.00000

After we run the program, we get the following output

-8.00000	-8.00000	2.00000	-2.00000	1.00000
0.00000	2.00000	-6.75000	4.75000	-3.87500
0.00000	0.00000	22.2500	-20.2500	17.1250
0.00000	0.00000	-3.911555E-08	11.2022	-9.39326
0.00000	0.00000	-5.254266E-08	-2.430841E-08	4.01329
0.00000	0.00000	-1.464582E-08	1.397800E-08	-3.577259E-08
0.00000	0.00000	-5.824406E-08	1.086742E-07	-2.912714E-07
0.00000	0.00000	-5.194217E-07	2.445273E-07	-3.715342E-07
0.00000	0.00000	1.982546E-06	-4.225124E-08	-7.852131E-07
0.00000	0.00000	5.971300E-07	-9.912916E-10	-6.088426E-07

-5.00000	1.00000	-5.00000	5.00000	-1.00000
2.37500	2.12500	3.37500	-2.37500	9.87500
-8.62500	-4.87500	-4.62500	1.62500	-46.1250
1.75281	3.38202	-1.84270	-8.46067	1.46067

-10.0384	-1.54037	-12.8443	9.67076	-5.40697
-22.9896	-0.604998	-32.4647	21.8333	8.80456
-1.279275E-07	2.64423	11.9482	-5.29700	25.8410
-2.840636E-07	1.454623E-07	21.2912	-16.5345	58.3521
4.021240E-07	-5.923781E-07	-6.778814E-07	24.5564	-4.68117
3.765645E-07	-2.498016E-07	4.332476E-07	-6.854107E-07	36.3973

x(1): -11.2701

x(2): 15.6723

x(3): 3.04452

x(4): 2.51214

x(5): 11.8870

x(6): -1.41405

x(7): -0.381361

x(8): 2.63446

x(9): -2.96792

x(10): -6.58846