Semiparametric Smooth Coefficient Quantile Estimation of the Production Profile

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Abstract

In this paper, quantile regression models are suggested as an alternative description of a production technology. The quantile of continuous order defines the production profile and the quantile-based individual technical efficiency relative to the quantile order. Quantile-based production frontier and efficiency are easy to derive and estimate and does not envelop all sample observation points. A quantile-based production frontier is more robust to extreme observations than DEA or FDH. Furthermore, quantile regression does not make a distribution assumption. It is more robust to the misspecification of error structure than DFA or SFA. In this paper, the quantile regression methods are extended to semiparametric smooth coefficient models. A local linear fitting scheme to estimate the smooth coefficients is proposed in the quantile framework. An empirical application of the model to the Taiwan manufacturing industry demonstrates the potential for the estimation of production technology and efficiency measures.

Keywords: Semiparametric smooth coefficient, quantile estimation, local linear, stochastic frontier

JEL classification: C14, C21, C67, D24

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1. Introduction

Over the last four decades, data envelopment analysis (DEA) and stochastic frontier analysis (SFA) have been widely used in productivity and efficiency studies to describe and estimate the production frontier, i.e., the most efficient production process. It is well known that DEA and SFA techniques are sensitive either to observation outliers or lack of robustness if the data generating process is misspecified. Recently, Aragon et al. (2005) and Daouia and Simar (2006, 2007) proposed a nonparametric data envelopment method based on quantile distribution to estimate the production frontier and efficiency. More specifically, let $X \in R_+^k$ be a set of k inputs used to produce a univariate output $Y \in R_+$, and the production process generates the sample observations through the joint distribution of (X, Y).[1] Define the conditional quantile function of order $\tau \in [0, 1]$ given $X \leq x$ as

$$q_Y(\tau|x) = inf\{y \geq 0|F(y|X \leq x) \geq \tau\}, \quad (1)$$

where $F(y|X \leq x)$ is the conditional distribution function of Y given $X \leq x$, the component-wise inequality. The conditional quantile $q_Y(\tau|x)$ is the output threshold exceeded by 100(1− τ)% of firms that use no more than x units of inputs. The production efficiency quantile of the firm with output at $q_Y(\tau|x)$ using x units of inputs is equal to τ as it produces more than 100 τ % of firms using no more than x units of input and produces less than 100(1− τ)% of firms. In the case in which the production technology is the support of the distribution of (X, Y), the conditional quantile function of order one, $\tau = 1$, corresponds to the free disposal hull (FDH) frontier. The conditional quantile function of continuous order τ thus provides the whole spectrum of the production process corresponding to different efficiency quantile levels. Estimating, for example, the 25%, 50%, and 75% conditional quantiles of the production distribution would give us better

[1] Daouia and Simar (2006, 2007) extended the production process to the case of multiple inputs and multiple outputs.

understanding of the production technology than simply the production frontier at the 100% obtained from DEA, FDH, or SFA.

Estimation of the production functions based on the conditional quantile function (1) is more robust to extreme observations than DEA since it does not envelop all sample observation points. It is more robust to the misspecification of error structure than SFA since it does not make distribution assumptions. However, estimation of the nonparametric conditional quantile function given in (1) suffers the curse of dimensionality problem. In this paper, we consider an alternative dimension-reduction conditional quantile model — smooth coefficient quantile regression (SQR, hereafter) models. More specifically, for a given level of inputs x, the production function, denoted as $\Phi(.)$, is characterized by the upper boundary of the support of the conditional distribution of the output Y given $X = x$ i.e.,

$$\Phi(x) = sup\{y \in R_+ | F(y|X = x\,) \leq 1\},$$

where $F(y|X = x)$ is the conditional distribution function of Y given $X = x$. This definition differs from the conditional distribution $F(y|X \leq x)$ defined in (1). The graph of $\Phi(x)$ defines the production frontier and the data generating process (DGP) Y given $X = x$ can alternatively be expressed as the familiar deterministic frontier regression

$$y = \Phi(x) + \epsilon, \tag{2}$$

where the one-side random error $\epsilon \leq 0$ represents the production (in)efficiency with the conditional distribution $F(\epsilon|X = x)$. The conditional quantile function of order $\tau \in [0, 1]$ of the output Y given $X = x$, $Q_Y(\tau|X = x)$, is defined as

$$\begin{aligned} Q_Y(\tau|x) &= inf\{y \geq 0 | F(y|X = x) \geq \tau\} \qquad (3) \\ &= \Phi(x) + Q_\epsilon(\tau|x) \end{aligned}$$

with $Q_\epsilon(\tau|x) \leq 0$, and $Q_\epsilon(1|x) = 0$. Thus $Q_Y(\tau|x)$ gives the whole profile of the production process corresponding to different quantile level, not just the frontier production process (2) only. Of course, $Q_Y(\tau|x) = \Phi(x)$ when $\tau = 1$.

To ease the "curse of dimensionality", in this paper the conditional quantile function $Q_Y(\tau|x)$ is assumed to take the following semiparametric form,

$$Q_Y(\tau|x) = \beta_0^\tau(z) + w'\beta_1^\tau(z), \tag{4}$$

where $x = (w', z')'$and the quantile coefficients $\left(\beta_0^\tau(z), \beta_1^\tau(z)\right)$ are nonparametric smooth function of z. The smoothing variable z might be a part of w or just another exogenous variable. The specification (4) is called smooth coefficient quantile regression (SQR).

The SQR is a natural generalization of many familiar forms in quantile regression and semiparametric regression models. For example, a cross-sectional production function where the right-hand-side variables in (4) are w = (labor, capital), and z is the firm's R&D input. When $\beta_0^\tau(z) = \beta_0^\tau + z\beta_2^\tau$, and $\beta_1^\tau(z) = \beta_1^\tau$, the quantile regression (4) would have assumed that the quantile coefficients are invariant to R&D, and the R&D variable can only shift the level of the τ-quantile frontier $Q_Y(\tau|x)$. In this case, the R&D variable is said to have a "neutral" effect on the production profile. In contract, the smooth coefficient quantile regression (4) allows R&D to affect the production profile "non-neutrally". Thus, both the marginal productivity of labor and the capital depend upon the firm's R&D. As a result, the returns to scale may also be a function of R&D. If there is no w in (4), it then reduces to the ordinary nonparametric quantile regression model. Daouia and Simar (2006, 2007) similarly generalize the conditional quantile function (1) by introducing environmental variables Z such that

$$q_Y(\tau|w, z) = inf\{y \geq 0|F(y|W \leq w, Z = z) \geq \tau\}$$

Again the above generalization is nonparametric as oppose to the semiparametric specification in (4) with the smooth coefficients as function of z.

The specification of the quantile function $Q_Y(\tau|x)$ in (3) or the semiparametric form in (4) differs from $q_Y(\tau|x)$ in (1) in that the conditional distribution $F(y|X)$ in the former is defined as conditional equality $X = x$ while in the latter is defined as conditional inequality $X \leq x$. In the context of frontier analysis, given a monotone nondecreasing assumption, $q_Y(\tau|x) = Q_Y(\tau|x)$ and converges to the full frontier as $\tau \rightarrow 1$. However, the quantile function (4) is arguably more informative, easy to interpret and estimate.

Though not in the framework of quantile regression, Li *et al.* (2002) proposed and applied the smooth coefficient regression to study the production function of China's nonmetal mineral manufacturing industry; Chou, Liu, and Huang (2004) examined the effect of national health insurance on the life-time consumption pattern in Taiwan; and Liu *et al*. (2014) studied the role of information technology (IT) in Solow's "productivity paradox". Although our interest in the SQR model lies in the context of production modeling, the smooth coefficient model (4) has been studied extensively in statistics. Honda (2004) applies the model to estimate the conditional medium; and Cai and Xu (2009) employ a local linear fitting method to estimate a dynamic quantile model.

The remainder of the paper is organized as follows. In section 2, a local linear quantile estimator is briefly discussed. Section 3 applies the SQR model to the production data from the manufacturing industry in Taiwan. Concluding remarks are given in section 4.

2. The smooth coefficient quantile regression model

2.1 The local linear approach

The conditional quantile function (3) can equivalently be expressed as

$$\underset{Q_Y(\tau|x)}{\operatorname{argmin}} E\{\rho_\tau(y - Q_Y(\tau|x))\} = E\{\rho_\tau(\epsilon - Q_\epsilon(\tau|x))\}$$

where $\rho_\tau(v) = v\big(\tau - I(v < 0)\big)$ is the loss function and $I(v)$ is the indicator function. The second equality is due to the fact that $\{y - Q_Y(\tau|x)\} = \{\epsilon - Q_\epsilon(\tau|x)\}$ from (2) and (3). Given n observations, $(y_i, w_i, z_i) \in R_+^1 \times R_+^{k-1} \times R_+^1$, $i = 1,2,\dots,n$, and assume a linear quantile model, $Q_Y(\tau|x) = \beta_0^\tau + w_i'\beta_1^\tau + z_i\beta_2^\tau$, Koenker and Bassett (1978) show that $Q_Y(\tau|x)$ can be estimated by minimizing

$$\min_{\beta^\tau}\{\textstyle\sum_{i=1}^n \rho_\tau(y_i - \beta_0^\tau - w_i'\beta_1^\tau - z_i\beta_2^\tau)\}, \qquad (5)$$

As a special case, when τ = ½, the minimization yields the least absolute deviation (LAD) estimator, which is the estimation of the median regression. In this paper, the software STATA 13.1 is used to estimate β^τ.

To estimate the SQR model (4), we follow Cai and Xu (2009) and apply the local linear approach to approximate the unknown quantile coefficient functions $\big(\beta_0^\tau(z), \beta_1^\tau(z)\big)$. In other words, we assume the coefficient $\beta_j^\tau(z)$, where j = 0, 1, can be locally approximated by a linear function at a point z_0. For simplicity, the superscript τ of the coefficient is omitted, and the coefficient is approximated linearly as

$$\beta_j(z) \approx \beta_j(z_0) + \beta_j^{(1)}(z_0)(z - z_0),$$

where $\beta_j^{(1)}(z_0)$ is the first-order derivative evaluated at z_0. For notational simplicity, let $W = (1, w')'$, $b_0 = (\beta_0(z), \beta_1(z)')'$, $b_1 = \left(\beta_0^{(1)}(z), \beta_1^{(1)}(z)'\right)'$ and

$$B(z) = b_0 + b_1(z - z_0), \qquad (6)$$

where both b_0 and b_1 are $k \times 1$ vectors. The quantile function (4) is then approximated as

$$Q_Y(\tau|W_i, z_i) \approx W_i'B(z_i) \qquad (7)$$

The local linear estimator for the SQR of the τ^{th} order is the solution of the following locally weighted loss function,

$$\min_{b_0,b_1}\left\{\textstyle\sum_{i=1}^n \rho_\tau\big(y_i - W_i'B(z_i)\big)K_h(z_i - z_0)\right\}, \qquad (8)$$

where $K_h(v) = K(v/h)/h$ is a kernel function and h is the smoothing parameter. By varying the expansion point z_0, we obtain a series of estimates for b_0 and b_1 that corresponds to the quantile coefficient functions, $\beta_0(z)$ and $\beta_1(z)$, and the respective derivatives, $\beta_0^{(1)}(z)$ and $\beta_1^{(1)}(z)$. We refer the estimator obtained from (8) the local linear quantile (LLQ) estimator.

Define

$$\mu_j = \int z^j K(z)dz,\ j = 0,1,2,\ldots$$

$$v_j = \int z^j K^2(z)dz,\ j = 0,1,2,\ldots$$

$$\Omega(z_0) = E(W_i W_i' | z_i = z_0),$$

$$\Omega^*(z_0) = E\left[W_i W_i' f_{y|w,z}\big(Q_Y(\tau|W_i, z_i)\big) | z_i = z_0\right], \text{ and}$$

$$\Sigma(z_0) = \Omega^*(z_0)^{-1}\Omega(z_0)\Omega^*(z_0)^{-1}/f_z(z_0),$$

where $f_{y|w,z}(y)$ is the conditional pdf of y given w and z, and $f_z(z)$ denotes the marginal pdf of z. It follows from (6) that, under some regularity conditions[2] and the bandwidth is such that $h \to 0$ and $nh \to \infty$, the LLQ estimator for b_0 at the point z_0 is

$$\hat{b}_0 = \hat{B}(z_0), \tag{9}$$

where $\hat{B}(z) = \hat{b}_0 + \hat{b}_1(z - z_0)$; and $\hat{b}_0$ has the asymptotic variance

$$\text{AVar}(\hat{b}_0) = \frac{\tau(1-\tau)v_0}{nh}\Sigma(z_0), \tag{10}$$

The slope estimator for b_1 at the point z_0 is the first-order derivative of $\hat{B}(z)$ at z_0 i.e.,

$$\hat{b}_1 = \hat{B}^{(1)}(z_0) \tag{11}$$

with the asymptotic variance[3]

$$\text{AVar}(\hat{b}_1) = \frac{\tau(1-\tau)v_2}{nh^3}\Sigma(z_0). \tag{12}$$

Let r be a nonrandom vector, then it follows from (12) that the estimator for the linear combination of $r'b_1$ is $r'\hat{b}_1$, which has the asymptotic variance

[2] See Cai and Xu (2009) for more discussion about the regularity conditions and the derivation of the asymptotic variance.

[3] See the appendix of Cai and Xu (2009) for the statistical properties of $\hat{b}_1$.

$$\mathrm{AVar}(r'b_1) = \frac{\tau(1-\tau)v_2}{nh^3} r'\Sigma(z_0)r. \tag{13}$$

Computation of the asymptotic variance of $\hat{b}_0$ and $\hat{b}_1$ is deferred to the next section.

2.2 Empirical issues

In this section, we discuss some empirical issues when implementing the local linear quantile approach. For a given bandwidth, the computation of (8) can be easily implemented in the aforementioned STATA software with the weight function $K_h(.)$. Therefore, the first issue is to choose an ideal bandwidth. The second issue regards the statistical inference and, more specifically, the estimator of $\Sigma(z_0)$ in (10), (12) and (13) for the asymptotic standard errors of the LLQ estimators $\hat{b}_0$, $\hat{b}_1$ and $r'\hat{b}_1$.

Cai and Xu (2009) suggest the following bias-corrected AIC for the bandwidth selection,

$$\mathrm{AIC}(h) = \ln(\hat{\sigma}_\tau^2) + 2(\hat{p}_h + 1)/[n - (\hat{p}_h + 2)], \tag{14}$$

where

$$\hat{\sigma}_\tau^2 = \frac{1}{n}\sum_{j=1}^{n} \rho_\tau\left(y_j - W_j'\hat{B}(z_j)\right) \text{ and}$$

$$\hat{p}_h = (nh)^{-1}K(0)\sum_{j=1}^{n} W_j'\left[\hat{f}_z(z_j)\hat{\Omega}^*(z_j)\right]^{-1} W_j.$$

Here, $\hat{\sigma}_\tau^2$ is similar to the mean squared error in the least square setting and plays the role of a measure for the goodness of fit. p_h is the nonparametric version of degrees of freedom (also called the effective number of parameters), which is approximated by the trace of the quasi-projection matrix of the local linear estimators.

To implement (14) for the bandwidth selection, it is necessary to estimate $f_z(z_j)\Omega^*(z_j)$ and choose a pilot bandwidth. The pilot bandwidth can be chosen via $h = c\boldsymbol{z}_{sd}n^{-1/5}$, where c is a constant, n is the sample size and $\boldsymbol{z}_{sd}$ is the sample standard deviation of z. Two consistent estimators for $f_z(z_j)\Omega(z_j)$ and $f_z(z_j)\Omega^*(z_j)$ are suggested by Cai and Xu (2009). The former, $f_z(z_j)\Omega(z_j)$, can be estimated by

$$\widehat{\Omega}_{n,0}(z_j) = \frac{1}{n}\sum_{i=1}^{n} W_i W_i' K_h(z_i - z_j); \tag{15}$$

and the latter $f_z(z_j)\Omega^*(z_j)$ can be estimated by

$$\widehat{\Omega}_{n,1}(z_j) = \frac{1}{n}\sum_{i=1}^{n} f_{y|w,z}\left(Q_Y(\tau|W_i, z_i)\right) W_i W_i' K_h(z_i - z_j), \tag{16}$$

where $f_{y|w,z}\left(Q_Y(\tau|W,z)\right)$ can be replaced by its kernel density estimator[4]

$$\hat{f}_{y|w,z}\left(\hat{Q}_Y(\tau|W,z)\right) = \frac{\sum_{i=1}^{n} K_{h_2}(W_i - W, z_i - z) K_{h_1}\left(Y_i - \hat{Q}_Y(\tau|W_i, z_i)\right)}{\sum_{i=1}^{n} K_{h_2}(W_i - W,\ z_i - z)}. \tag{17}$$

In equation (17), $K_{h_1}(v) = K(v)/h_1$, $K_{h_2}(v_1, \dots, v_k) = \prod_{l=1}^{k} K\left(v_l/h_{2,l}\right)/h_{2,l}$ is the product kernel, $h_2 = (h_{2,1}, \dots, h_{2,k})$ denotes the bandwidth vector, and $\hat{Q}_Y(\tau|W_i, z_i) = W_i'\hat{B}(z_i)$. For the empirical study, one may follow the suggestion of Silverman (1986) using the default bandwidth $1.06\sigma_v n^{-1/5}$ for equations (15)-(17), where σ_v denotes the standard error of the variable in the kernel function $K(\cdot)$. More specifically, if the kernel function $K_h(v)$ is one-dimension, as specified as in (15) and (16) and the second term of the nominator on (17), the default bandwidth is $h = 1.06\sigma_v n^{-1/5}$. For the multivariate product kernel function $K_{h_2}(v_1, \dots, v_k)$ in (17), the l^{th} element of the bandwidth vector is $h_{2,l} = 1.06\sigma_{v_l} n^{-1/5}$, where $l = 1, \dots, k$, the bandwidth $h_{2,l}$ may vary with it standard deviation σ_{v_l}. With the estimator for $f_z(z_j)\Omega^*(z_j)$ given in (16), we may apply (14) as the criterion for choosing the optimal bandwidth.

Given the optimal bandwidth, it remains to find the estimator for the asymptotic variance $\Sigma(z_j)$ in (11) and (12). Recall that $\Sigma(z_j)$ is defined as

$$\Sigma(z_j) = \Omega^*(z_j)^{-1}\Omega(z_j)\Omega^*(z_j)^{-1}/f_z(z_j),$$

[4] See Fan, Yao and Tong (1996).

which suggests that $\Sigma(z_j)$ can be estimated using (15) and (16). Therefore, the estimator for $\Sigma(z_j)$ is

$$\Sigma(z_j) = \widehat{\Omega}_{n,1}(z_j)^{-1}\widehat{\Omega}_{n,0}(z_j)\widehat{\Omega}_{n,1}(z_j)^{-1}, \tag{18}$$

where $\widehat{\Omega}_{n,1}(z_j)$ and $\widehat{\Omega}_{n,0}(z_j)$ are given in (15) and (16).

Below we briefly state the estimation procedure under the Gaussian kernel specification, i.e., $K_h(v) = K(v/h)/h$ and $K(v) = e^{-0.5v^2}/\sqrt{2\pi}$, which implies that $v_0 = 0.5/\sqrt{\pi}$, $v_2 = 0.25/\sqrt{\pi}$, and $\mu_2 = 1$.

Step 1: Given a pilot bandwidth of the form $h_0 = c\boldsymbol{z}_{sd}n^{-1/5}$, one may estimate (8). With the estimate $\hat{B}(z)$ at $z = z_j$, we can then obtain $\hat{Q}_Y(\tau|W_j, z_j)$ by (7), $\widehat{\Omega}_{n,1}(z_j)$ by (16), and also compute $\hat{p}_h$, $\hat{\sigma}_\tau^2$ and $\mathrm{AIC}(h)$ by (14).

Step 2: Repeat Step 1 for different values of $h_0's$ and obtain the optimal bandwidth

$$h_{opt} = arg\min_h AIC(h).$$

Step 3: Given h_{opt}, we then reestimate (8) and compute $\hat{\Sigma}(z) = \widehat{\Omega}_{n,1}(z)^{-1}\widehat{\Omega}_{n,0}(z)\widehat{\Omega}_{n,1}(z)^{-1}$ using (18). The estimators for the asymptotic variance of $\hat{b}_0$ and $\hat{b}_1$ in (10) and (12) at $z = z_0$ are

$$\widehat{\mathrm{AVar}}(\hat{b}_0) = \left(\frac{\tau(1-\tau)}{2\sqrt{\pi}nh_{opt}}\right)\hat{\Sigma}(z_0) \text{ and } \widehat{\mathrm{AVar}}(\hat{b}_1) = \left(\frac{\tau(1-\tau)}{4\sqrt{\pi}nh_{opt}^3}\right)\hat{\Sigma}(z_0),$$

respectively.

2.3 Measurement of the technical efficiency

In output-orientation, the τ-quantile function (3) corresponds to the τ-quantile frontier,

$$Q_Y(\tau|x) = inf\{y \geq 0|F(y|x) \geq \tau\}$$

where $x = (w', z')'$. Define the Shephard's output distance function of the τ-quantile for an observed (x, y) as

$$\lambda(x, y|\tau) = sup\{\lambda \geq 0|F(y/\lambda\,|x) \geq \tau\}. \tag{19}$$

The τ-quantile output distance function, $\lambda(x, y|\tau)$, is the expansion (if $\lambda < 1$) or reduction (if $\lambda > 1$) of the observed output level y that a firm should produce in order to outperform other firms with the same level of input level x with probability τ. The corresponding Debreu-Farrell τ-quantile output technical efficiency is

$$TE^{\tau} \equiv \lambda(x, y|\tau) = y/Q_Y(\tau|x) \tag{20}$$

Clearly when $\tau = 1$, the quantile frontier $Q_Y(\tau = 1|x)$ corresponds to the traditional deterministic frontier model with $TE^{\tau} = \lambda(x, y|\tau) \leq 1$. Thus, $Q_Y(\tau|x)$ measures the profile of a production process and TE^{τ} measures the associated efficiency level relative to the τ-quantile frontier. A firm's efficiency needs not to compare its production with the absolute potential best firm, the quantile frontier of order 1, $Q_Y(\tau = 1|x)$, but rather with, say, the 90 percentile firm's potential production using the same inputs. However, when a firm's performance is compared to the τ-quantile frontier $Q_Y(\tau < 1|x)$, its technical efficiency may be larger than 1, i.e., $TE^{\tau} = \lambda(x, y|\tau) > 1$. Those firms with the technical efficiency scores greater than 1 are called super-efficient firms at the τ-quantile frontier level.[5]

Occasionally the quantile function is estimated with the monotone transformation of the output *Y*, say, the logarithmic function $ln\,Y$ in the Cobb-Douglas specification. We specify the quantile function in $ln\,Y$ as

$$Q_{ln\,Y}(\tau|x) = \beta_0^{\tau}(z) + (ln\,w)'\beta_1^{z}(z)$$

Since, for any monotone transformation $h(.)$,

$$Q_{h(Y)}(\tau|x) = h\big(Q_Y(\tau|x)\big)$$

we have the estimate of the quantile function in Y as the inverse function of $h(.)$, i.e.,

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[5] The existence of super-efficiency in the τ-quantile model differs from the super-efficiency in the traditional DEA model. In the τ-quantile model, $TE^{\tau} > 1$ is due to $\tau < 1$, while in the DEA model, it is due to the exclusion of own DEA as peer in forming the frontier.

$$Q_Y(\tau|x) = exp\big(Q_{ln\,Y}(\tau|x)\big) = exp\big(\beta_0^\tau(z) + (ln\,w)'\beta_1^z(z)\big).$$

The corresponding τ-quantile output technical efficiency is computed as

$$TE^\tau = y/Q_Y(\tau|x) = y/\Big(exp\big(\beta_0^\tau(z) + (ln\,w)'\beta_1^z(z)\big)\Big).$$

3. An application

Over the last two decades, most industrial countries have made a significant investment in information technology (IT). Some have argued, however, that the IT investment does not necessarily result in productivity growth. Solow (1987) called this phenomenon the "productivity paradox." In this section, we apply the smooth coefficient conditional quantile model to examine the paradox by estimating the production profile and the production efficiency quantile of the manufacturing industry in Taiwan. Data were obtained from the 1165 samples of firm-level survey conducted by the Directorate General of Budget, Accounting and Statistics (DGBAS) in 1991.

The output variable (Y) is the value-added output defined as the total output sales net of intermediate inputs. The inputs are labor (Lab) which is measured in the number of employees; information technology (IT) capital; and the non-IT capital (NonIT) which is obtained by subtracting the IT capital from the net fixed capital. Two models are specified and estimated a linear conditional quantile model and a smooth coefficient conditional quantile model. The linear conditional quantile model is specified in the logarithmic form as

$$Q_{lnY}(\tau|x) = \beta_0^\tau + \beta_1^\tau\,ln\,Lab + \beta_2^\tau\,ln\,NonIT + \beta_3^\tau\,ln\,IT, \tag{21}$$

where x = (lnLab, lnNonIT, lnIT) is the logarithmic input vector. The productivity paradox implies the non-positive coefficient, $\beta_3^\tau \leq 0$. Estimating the profile of the coefficient β_3^τ over τ will empirically reveal if the productivity paradox occurs uniformly in the output distribution.

A more general smooth coefficient conditional quantile model, with coefficients being a function of IT, is specified as

$$Q_{lnY}(\tau|x) = \beta_0^\tau(ln\, IT) + \beta_1^\tau(ln\, IT)\, ln\, Lab + \beta_2^\tau\, (ln\, IT) ln\, NonIT. \tag{22}$$

The specification implies that the role of IT in production is not simply a factor of production through the coefficient function $\beta_0^\tau(ln\, IT)$. It also serves as a factor in augmenting labor and non-IT productivity through coefficient functions $\beta_1^\tau(ln\, IT)$ and $\beta_2^\tau(ln\, IT)$. The spillover impact of IT on productivity varies with τ at a different level of the τ th-frontier in the production profile. The elasticity of the IT productivity, directly or indirectly, is the derivative,

$$\frac{\partial Q_{lnY}(\tau|x)}{\partial\, ln\, IT} = \frac{\partial \beta_0^\tau(ln\, IT)}{\partial\, ln\, IT} + \frac{\partial \beta_1^\tau(ln\, IT)}{\beta_1^\tau(ln\, IT)} \times ln\, Lab + \frac{\partial \beta_2^\tau(ln\, IT)}{\beta_1^\tau(ln\, IT)} \times ln\, NonIT \tag{23}$$

For the purposes of comparison, a conventional stochastic frontier analysis (SFA) model is also specified and estimated,

$$\ln Y = \beta_0 + \beta_1\, ln\, Lab + \beta_2\, ln\, NonIT + \beta_3\, ln\, IT + u + v, \tag{24}$$

where one-side non-positive random errors u and v are assumed to be distributed respectively as $u \sim N^-(0, \sigma_u^2)$ and $v \sim N(0, \sigma_u^2)$. A firm-specific production technical efficiency of the SFA specification is computed based on Battese and Coelli (1988),

$$TE_{SFA}(x, y) = E(exp(u)|u + v). \tag{25}$$

Figure 1 plots the estimated coefficient of the linear quantile model (21). Each plot depicts a coefficient β^τ as function of τ for each independent variable. The solid line represents the estimates with the 95% pointwise confidence interval shown in the shaded gray area. The horizontal line in each plot depicts the SFA estimate of the regression. The two horizontal dashed lines represent the 95% confidence interval for the SFA estimate.

The upward trend of the intercept coefficient β_0^τ in Figure 1 indicates, as expected, that the production surface of more efficient firms shifts upward closer to the order one quantile $Q_Y(1|x)$.

Other panels estimate the input elasticity or the input share of output at various quantile τ levels. The labor elasticity is estimated around 0.52 according to the SFA estimate in Fig.1b (see also the bottom row of Table 1), but, as is clear from the results, it is much larger in the lower end of the conditional output distribution and much smaller in the upper tail of the distribution. Thus, the impact of labor input is much larger at the lower τ^{th}-frontier than in the upper τ^{th}-frontier. Firms at the first quartile (τ = 0.25) frontier $Q_Y(0.25|x)$ have a labor share at 64% ($\beta_1^{0.25} = 0.6434$) compared to 47% ($\beta_1^{0.75} = 0.4730$) for firms at the third quartile frontier $Q_{lnY}(0.75|x)$. On the other hand, as shown in the fourth panel, firms in the upper tail of output distribution tend to be associated with a higher IT share at 16% ($\beta_3^{0.75} = 0.1658$) versus 8% ($\beta_3^{0.25} = 0.0869$) for the lower tail firms. The quantile estimates seems to indicate that the firms at the upper tail of the output distribution tend to substitute IT for labor in production in the manufacturing industry. Evidently, the contribution of IT and its share in production become much more significant for firms at the upper tail of the output distribution. This empirical evidence thus suggests the absence of the IT productivity paradox in the manufacturing industry in Taiwan.

The estimated effect of non-IT input on productivity presents an interesting phenomenon. The non-IT input appears to have a uniform elasticity around 0.38 over the middle range of the output distribution. The disparity of the impact of non-IT investment on productivity occurs only at both tails of the efficiency distribution. Highly efficient firms have a somewhat higher elasticity than firms in the lower tail of efficiency distribution.

Table 1 tabulates the quantile estimates of some selected τ values for LQ and SQR models. The estimates and numbers inside parentheses under each β_j^τ heading are the estimated coefficients and the bootstrapped standard errors of the LQ model (21). For comparison, the last row of the table shows the estimated coefficients and the standard errors from the SFA model. The horizontal lines in each panel in Figure 1 are these SFA estimates. Except for the intercept coefficient, the stochastic frontier estimates of the slope coefficient are very close to the estimates of the median quantile model at $\tau = 0.50$. The estimated SFA production function resembles a parallel upward shifting of the production function of the median efficient firms.

The coefficient estimates for the selected τ's under the SQR model are given under the heading $\beta_j^\tau(\ln IT)$ in Table 1. These estimates are based on the optimal bandwidths chosen by the criterion in (14) and shown in the last column. Since these estimates are functions of the logarithm of IT, Table 1 shows the sample mean and the 5th and 95th percentiles shown in brackets of the 1165 firm-specific estimates. The SQR estimates of the slope coefficients, $\beta_1^\tau(\ln IT)$ and $\beta_2^\tau(\ln IT)$, are directly comparable to the LQ estimates, β_1^τ and β_2^τ. The estimates are the labor and non-IT input elasticity respectively. While the sample averages of the slope coefficients are very close in magnitude to the estimates from the LQ model, the individual firm's input elasticities, $\beta_1^\tau(\ln IT)$ and $\beta_2^\tau(\ln IT)$, show patterns that distinctly depart from the constant input elasticities, β_1^τ and β_2^τ, which are implied by the LQ model.

Figure 2 show the graphs of $\beta_j^\tau(\ln IT)$ for j = 0, 1, 2 and τ = 0.5, 0.75, 0.95. To keep the graphs simple, we only provide the 95 % confidence interval (CI) for each $\beta_j^\tau(\ln IT)$ at $\tau = 0.95$, where the CI is based on the standard errors estimated from equation (10). The upper-right panel in Figure 2 shows the input elasticity of labor $\beta_1^\tau(\ln IT)$ as a function of $(\ln IT)$. For the $Q_{lnY}(0.95|x)$ frontier, the labor elasticity $\beta_1^\tau(\ln IT)$ is an almost monotonic increase of IT. For the $Q_{lnY}(0.75|x)$ and $Q_{lnY}(0.5|x)$ frontiers, the elasticities for the SQR model are shown to be U-shaped. This finding is in sharp contrast to the LQ model estimates $\beta_1^{0.75} = 0.4730$ and $\beta_1^{0.50} = 0.5555$. This empirical evidence of the non-neutral impact of IT on labor productivity is consistent with the rising trend of the wage rate in the high-tech industries, which is known as the phenomenon of the "skill-biased technological change." On the other hand, the non-IT elasticity, $\beta_2^\tau(\ln IT)$, shows the opposite pattern of an inverted U-shape for the upper tail firms at $\tau = 0.95$ and

0.75, and monotonic increasing of IT for the median firms. This outcome also is in sharp contrast to the LQ model where the estimates are constant at $\beta_1^{0.95} = 0.4191$ and $\beta_1^{0.75} = 0.3970$. Since the intercept is not identifiable in the smooth coefficient function, the estimate $\beta_0^\tau(\ln IT)$ may be compared to the estimate $\beta_0^\tau + \beta_3^\tau \times \ln IT$, which is linear in *ln*IT. The upper-left panel in Figure 2 shows that $\beta_0^\tau(\ln IT)$ exhibits a non-linear pattern at all τ levels.

Figure 3 shows the quantile estimates of the returns to scale (RTS) for the SFA, LQ and SQR models, labeled as RTS_SFA, RTS_LQ and RTS_SQR in the graph, respectively. In the LQ model, the returns to scale are the sum of the slope coefficients, $RTS^\tau = \beta_1^\tau + \beta_2^\tau + \beta_3^\tau$ for each quantile τ. Overall, the manufacturing industry is subject to increasing returns to scale. However, it has the downward trend over the distribution of the firm's efficiency level with a small hump around τ = 0.70. The horizontal line shows the SFA estimate of the returns to scale (RTS_SFA) is 1.0511, which is again very close to the estimate from the median regression where RTS_LQ at τ = 0.5 is 1.0534. For the SQR model, the returns to scale is estimated by

$$RTS^\tau(\ln IT) = \frac{\partial Q_{lnY}(\tau|x)}{\partial \ln IT} + \frac{\partial Q_{lnY}(\tau|x)}{\partial \ln Lab} + \frac{\partial Q_{lnY}(\tau|x)}{\partial \ln NonIT},$$

$$= \frac{\partial \beta_0^\tau(\ln IT)}{\partial \ln IT} + \frac{\partial \beta_1^\tau(\ln IT)}{\partial \ln IT} \ln Lab + \frac{\partial \beta_2^\tau(\ln IT)}{\partial \ln IT} \ln NonIT + \beta_1^\tau(\ln IT) + \beta_2^\tau(\ln IT) \qquad (26)$$

For a given τ, the line, RTS_SQR, shows the sample average of $RTS^\tau(\ln IT)$ over *ln*IT. The manufacturing industry is subject to increasing returns to scale for $\tau < 0.92$. However, as in the case of the estimates from the linear quantile model, it has the downward trend over the output distribution at all quantile levels.

Figure 4 shows the estimate of the elasticity of the IT productivity, i.e., the elasticity $\frac{\partial Q_{lnY}(\tau|x)}{\partial\, ln\, IT}$ given in (23). Consistent with the estimate from the LQ model, the estimate from the SQR model also suggests a significant impact of IT on output, particularly for the upper tail firms.

Both model specifications confirm the absence of the IT productivity paradox in the manufacturing industry in Taiwan.

Figure 5 shows the empirical distribution of the estimated technical efficiencies under LQ, SQR, SFA, and DEA, labeled as $TE_{LQ}^{0.95}$, $TE_{SQR}^{0.95}$, TE_{SFA} and TE_{DEA}, respectively. The efficiency distribution is slightly skewed with a mean at $TE_{LQ}^{0.95}$ = 0.4709. This quantile estimate of the mean efficiency is less than the SFA estimate of the mean efficiency TE_{SFA} = 0.6655. In fact, the SFA estimate is much closer to the quantile estimates at $TE_{LQ}^{0.9}$ = 0.5873 and $TE_{SQR}^{0.9}$= 0.5919, the distance measure of efficiency relative to the τ = 0.90 quantile output. This outcome is consistent with the empirical finding that the SFA estimate of the variance ratio, $\frac{\hat{\sigma}_v^2}{\hat{\sigma}_u^2+\hat{\sigma}_v^2} = 0.425$, indicates that a large percentage of the deviations from the frontiers are attributed to random noise rather than to inefficiency. Thus, the SFA approach tends to adjust the efficiency measure upward after purging the random noise.

The descriptive statistics of TE_{SFA}, TE_{DEA}, TE_{LQ}^{τ}, and TE_{SQR}^{τ} at selected quantiles are summarized in Table 2; and both of their correlation coefficients and the rank correlation coefficients are listed in Table 3. The estimates for technical efficiency index for LQ and SQR are quite similar and have high correlation coefficients at the selected quantiles as shown by Tables 2 and 3; however, they are significantly different from the TE estimates under SFA. Moreover, the correlation coefficients between TE_{LQ}^{τ} and TE_{SFA} (TE_{SQR}^{τ} and TE_{SFA}) have a decreasing trend as the quantile τ increases. These findings are also consistent with Figure 5.

On the other hand, the quantile efficiency estimate is shown to be much larger than the DEA estimate at 0.234 under the variable returns to scale technology, which is closer to the quantile estimate at $TE_{SQR}^{0.99} = 0.2843$. This result is expected, as the DEA estimate of frontier is equivalent to the order one quantile output.

4. Conclusion and Remarks

In this paper quantile regression models are suggested as an alternative description of a production technology. The quantile modeling of production technology can be very fruitful since the technique enables one to estimate the entire production profile at different points of the production distribution, not just the frontier production of the most efficient firms, as with data envelopment analysis, free disposal hull, or stochastic frontier analysis. The quantile estimation is more robust to extreme observations since it does not envelop all sample observation points, and is more robust to the misspecification of error structure since it does not make a distribution assumption. The paper proposes a smooth coefficient quantile model for studying a general quantile regression with varying coefficients. Because the smooth coefficient functions are unspecified, the model provides a more flexible alternative to a linear quantile model. Furthermore, since the smooth coefficient quantile model is semiparametric, the usual "curse of dimensionality" problem is not as severe as in the case of a full nonparametric quantile-based model.

A smooth coefficient quantile model is applied to estimate the production profile and the production efficiency quantile of the manufacturing industry in Taiwan. The empirical findings show that the contribution of information technology input in production becomes much more significant for highly technically efficient firms. There is a significant evidence of the spill-over, non-neutral impact of information technology on labor productivity. The empirical evidence suggests the absence of Solow's productivity paradox in the manufacturing industry in Taiwan.

Conditional quantile model and the smooth coefficient quantile regression proposed in this paper are useful econometric tools for estimating production profile and the associate production efficiency. Despite the capability of empirically estimating the profile of τ-quantile frontiers and τ-quantile efficiencies for $\tau \in [0, 1]$, the quantie approach to efficiency measures may suffer two kinds of awkward problems of quantile crossing in interpreting efficiency.[6] In theory the quantile function $Q_Y(\tau|x)$ is a monotone increasing function of τ so that $Q_Y(\tau_1|x) > Q_Y(\tau_2|x)$ for $\tau_1 > \tau_2$. It is expected that a firm's measured τ-quantile efficiency is lower at τ_1 than at τ_2, i.e., $TE^{\tau_1} < TE^{\tau_2}$. Empirically, the first kind of the quantile crossing occurs when the estimated quantile functions cross over at some observed points in *x* resulted in a firm is more technically efficient at higher quantile τ_1 than at lower quantile τ_2. Although this kind of crossing phenomenon typically only occurs in an outlying region of the input space *x*, it is nevertheless an undesirable phenomenon for empirical applications and interpretation of the quantile frontiers and quantile efficiencies. In our empirical application to the manufacturing industry in Taiwan using the 1165 firm-level data in Section 3, the crossing occurs in less than 1% of the data points. Although crossing problem is well known in literature, but no simple or general solution generally exists. Takeuchi et al. (2005) proposed to impose non-crossing constraints on the data points; Wu and Liu (2009) suggested a stepwise method by fitting the quantile sequentially to ensure non-crossing; Bondell, Reich and Wang (2010) proposed to estimate the quantile simultaneously by imposing the non-crossing restriction. However, none of these proposed methods extends to the semiparametric smooth coefficient quantile specification. This issue certainly deserves future exploration and studies.

Empirically the quantie approach to efficiency measures may suffer a second-kind problem of quantile crossing. For two firms' observations at $(y_{i,} x_i)$ and $(y_{j,} x_j)$, if $Q_Y(\tau_1|x_j)/Q_Y(\tau_1|x_i) > y_j/y_i > Q_Y(\tau_2|x_j)/Q_Y(\tau_2|x_i)$, then firms i's production is more technical efficient than firm j's production at τ_1; however, the reverse is true at τ_2, where $1 > \tau_1 > \tau_2 > 0$. That is, $TE_i^{\tau_1} > TE_j^{\tau_1}$ *and* $TE_i^{\tau_2} < TE_j^{\tau_2}$ if the condition holds in the observations. The occurrence of such conflicting results may weaken the properness of the quantile approach to rank the firm's technical

[6] We appreciate an anonymous referee in raising the empirical issue of crossing problem.

efficiency. How often do the apparent conflicting results occur in our empirical application to the manufacturing industry in Taiwan? We use the Kendall's tau, the concordance measure over the quantiles, to check for the conflicting results. For any two firms *i* and *j*, the two pairs $(TE_i^{\tau}, TE_j^{\tau})$ and $(TE_i^{\tau'}, TE_j^{\tau'})$ are concordant if $\left(TE_i^{\tau} - TE_j^{\tau}\right)\left(TE_i^{\tau'} - TE_j^{\tau'}\right) > 0$, and disconcordant if $\left(TE_i^{\tau} - TE_j^{\tau}\right)\left(TE_i^{\tau'} - TE_j^{\tau'}\right) < 0$. The Kendall's tau between firms *i* and *j* is defined as

$$\text{Kendall's tau}_{ij} = \Pr\left(\left(TE_i^{\tau} - TE_j^{\tau}\right)\left(TE_i^{\tau'} - TE_j^{\tau'}\right) > 0\right)$$
$$-\Pr\left(\left(TE_i^{\tau} - TE_j^{\tau}\right)\left(TE_i^{\tau'} - TE_j^{\tau'}\right) < 0\right),$$

which measures the difference between the probability of concordance and that of discordance for the pairs $(TE_i^{\tau}, TE_j^{\tau})$ and $(TE_i^{\tau'}, TE_j^{\tau'})$ over the quantiles. The $\text{Kendall's tau}_{ij}$ has a value bounded between -1 and 1; and the following Kendall's sample tau is its unbiased estimator[7]

$$\text{Kendall's sample tau}_{ij} = \frac{2}{H(H-1)} \sum\sum_{\tau<\tau'} I\left(\left(TE_i^{\tau} - TE_j^{\tau}\right)\left(TE_i^{\tau'} - TE_j^{\tau'}\right) > 0\right),$$

where *H* denotes the number of quantile points being evaluated. For each paired firms, *i* and *j*, we pick the quantiles τ = 0.05, 0.1,0.15, 0.2,….,0.90, 0.95, 0.98, 0.99, so there are totally 21 quantile points (*H*=21) used to compute the Kendall's sample tau. With 1165 firms in the sample, there are C_2^{1165} = 678,030 combinations for all possible pairs of firms. We compute the $\text{Kendall's sample tau}_{ij}$ for each pair of firms, and obtain the averaged Kendall's sample tau for the C_2^{1165} combinations to be 0.98982, which is very closed to 1. This implies that most of the predicted TEs are concordant and it indicates that the conflicting result is not serious in the empirical application to the manufacturing industry in Taiwan. Nevertheless, the empirical issue of the quantile crossing of second kind deserves further exploration and studies.

7 See Cherubini et al. (2004), page 99.

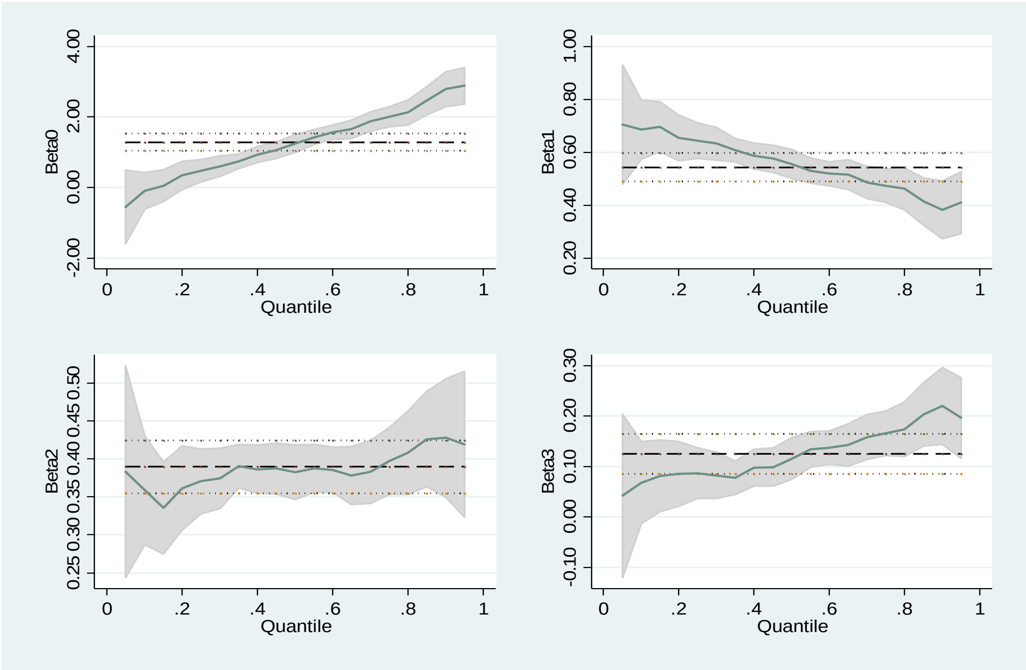


Figure 1. Estimates of the Linear Quantile Regression (21)

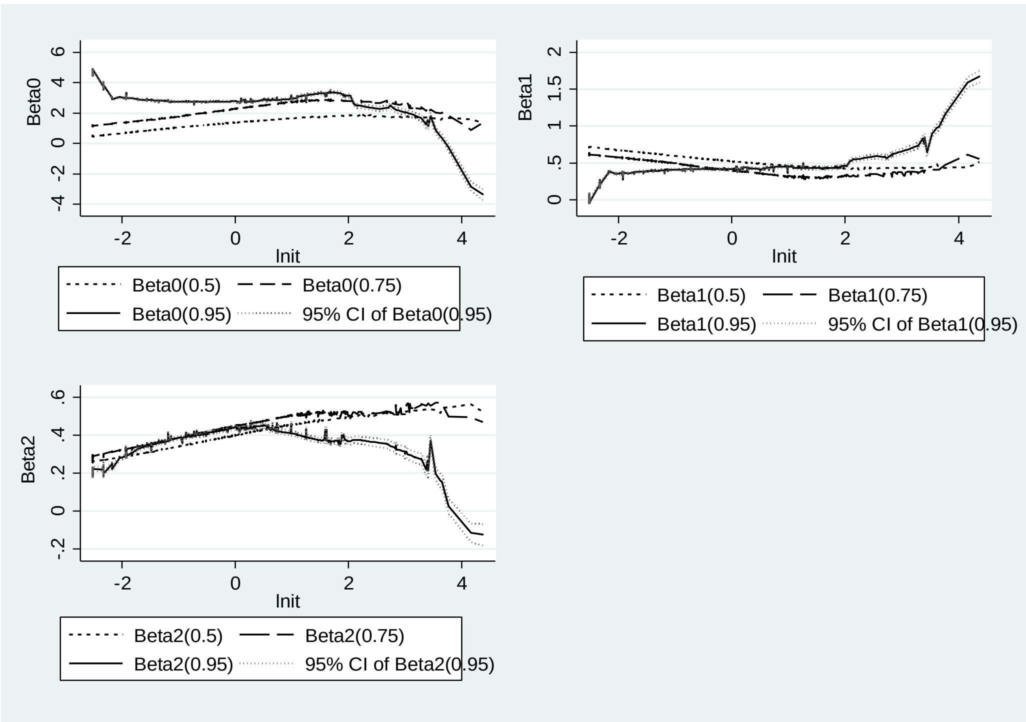


Figure 2. Estimates of the Smooth Coefficient Quantile Regression (22)

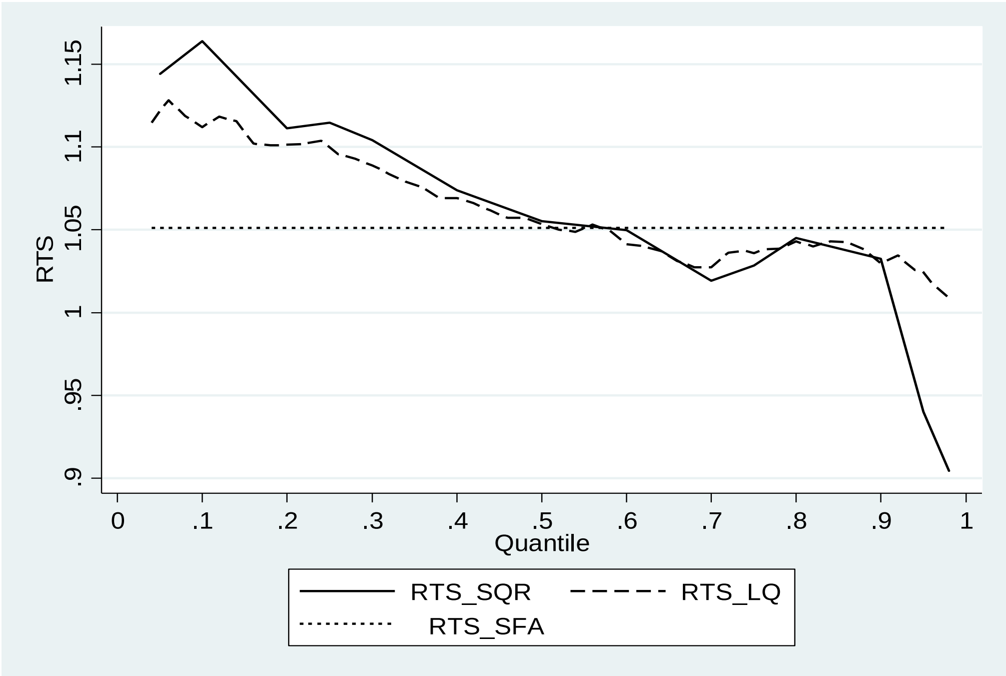


Figure 3. Returns to Scale of SFA, LQ and SQR

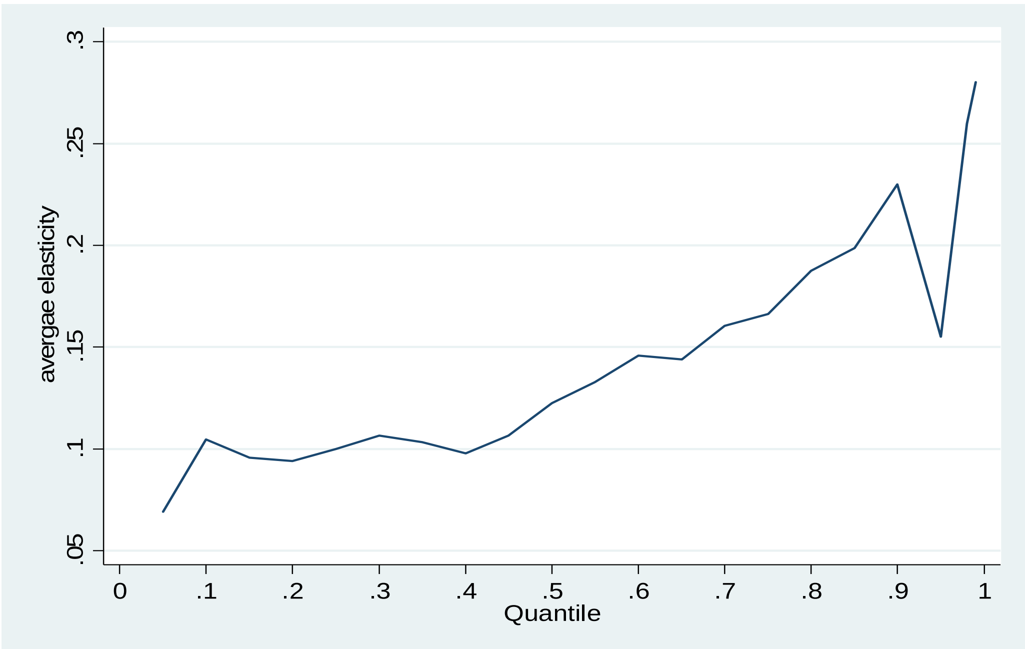
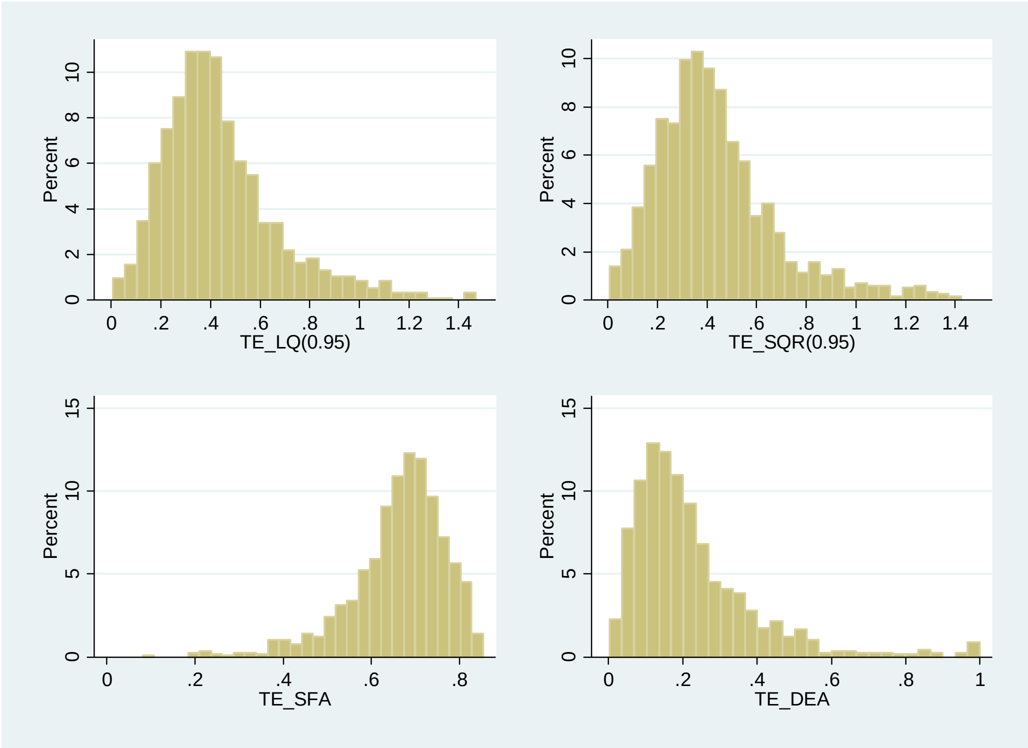


Figure 4. Smooth coefficient quantile estimates of average elasticity of IT



Note: Few extreme predicted $TE_{LQ}^{0.95}$ and $TE_{SQR}^{0.95}$ larger than 1.5 are excluded in the histogram.

Figure 5. Distribution of $TE_{LQ}^{0.95}$ $TE_{SQR}^{0.95}$, TE_{SFA} and TE_{DEA}

Table 1. Estimates of the β coefficients under SFA, LQ and SQR

τ	Constant		ln Lab		ln NonIT		ln IT	Optimal Bandwidth
	β_0^τ	$\beta_0^\tau(\ln IT)$	β_1^τ	$\beta_1^\tau(\ln IT)$	β_2^τ	$\beta_2^\tau(\ln IT)$	β_3^τ	
0.98	3.9957 (0.6128)[a]	[3.4296] 3.9614 [4.7949][b]	0.2923 (0.1123)	[0.0960] 0.2679 [0.4530]	0.3729 (0.0440)	[0.2987] 0.3768 [0.4475]	0.3434 (0.0760)	1.5648
0.95	2.8841 (0.3234)	[2.4841] 2.8784 [3.3564]	0.4089 (0.0564)	[0.3372] 0.4082 [0.5498]	0.4191 (0.0404)	[0.2232] 0.3768 [0.4471]	0.1965 (0.0483)	0.8242
0.90	2.7881 (0.2484)	[1.7520] 2.6646 [3.4975]	0.3825 (0.0464)	[0.2918] 0.4122 [0.5940]	0.4275 (0.0279)	[0.2443] 0.3903 [0.4780]	0.2199 (0.0408)	1.5969
0.75	2.0030 (0.1595)	[1.2472] 2.0572 [2.8033]	0.4730 (0.0380)	[0.3080] 0.4405 [0.5927]	0.3970 (0.0253)	[0.3102] 0.4217 [0.5202]	0.1658 (0.0235)	1.5326
0.50	1.2472 (0.1497)	[0.5988] 1.2216 [1.7566]	0.5554 (0.0299)	[0.4326] 0.5531 [0.6885]	0.3824 (0.0210)	[0.2735] 0.3796 [0.4989]	0.1155 (0.0201)	1.1462
0.25	0.4700 (0.1694)	[-0.2339] 0.4547 [1.2063]	0.6434 (0.0362)	[0.4554] 0.6464 [0.7917]	0.3704 (0.0276)	[0.2946] 0.3684 [0.5444]	0.0869 (0.0298)	0.9208
0.15	0.0538 (0.2076)	[-0.4656] 0.1257 [0.8817]	0.6961 (0.0434)	[0.4649] 0.6704 [0.8106]	0.3354 (0.0311)	[0.2510] 0.3554 [0.5443]	0.0812 (0.0355)	1.9189
0.05	-0.5567 (0.3500)	[-1.8446] -0.5321 [0.3231]	0.7039 (0.0889)	[0.4995] 0.7018 [0.9351]	0.3832 (0.0591)	[0.3023] 0.3730 [0.5104]	0.0419 (0.0581)	0.7598
SFA	1.8550 (0.1417)		0.5221 (0.0271)		0.3920 (0.0173)		0.1370 (0.0203)	

Note: a. Numbers in parenthesis are the bootstrapped standard errors under 1000 replications based on the linear quantile model.

b. Numbers in brackets are the 5^{th} and 95^{th} percentiles of the estimates based on the smooth coefficient quantile model.

Table 2. Estimates of the technical efficiency indices for SFA, LQ and SQR models

	TE_{LQ}^{τ}				TE_{SQR}^{τ}			
τ	mean	s.d.	min	max	mean	s.d.	min	max
0.5	1.2043	1.2313	0.0180	32.1696	1.2172	1.4044	0.0187	39.6572
0.75	0.8528	0.7910	0.0126	19.1545	0.8550	0.8678	0.0128	22.9080
0.8	0.7785	0.7275	0.0115	17.7579	0.7882	0.8182	0.0117	21.9691
0.9	0.5873	0.5120	0.0085	11.5305	0.5919	0.6202	0.0084	16.8358
0.95	0.4709	0.4147	0.0069	9.4823	0.4733	0.5194	0.0067	14.2277
0.98	0.3517	0.2773	0.0047	4.8178	0.3477	0.3210	0.0044	7.0880
0.99	0.2802	0.2104	0.0040	2.9036	0.2843	0.2200	0.0038	3.4855
TE_{SFA}	0.6655	0.1111	0.0819	0.9273				
TE_{DEA}	0.2340	0.1846	0.003	1				

Table 3. Correlation coefficients between the technical efficiency indices for SFA, LQ and SQR models

τ	TE_{SFA} and TE_{LQ}^{τ}		TE_{SFA} and TE_{SQR}^{τ}		TE_{LQ}^{τ} and TE_{SQR}^{τ}	
	Corr.	Rank Corr.	Corr.	Rank Corr.	Corr.	Rank Corr.
0.5	0.5496	0.9987	0.4906	0.9875	0.9913	0.9889
0.75	0.5949	0.9969	0.5453	0.9825	0.9903	0.9871
0.8	0.5908	0.9959	0.5337	0.9787	0.9886	0.9857
0.9	0.6240	0.9802	0.5263	0.9670	0.9728	0.9841
0.95	0.6198	0.9871	0.5134	0.9495	0.9656	0.9646
0.98	0.6699	0.9197	0.5878	0.8849	0.9637	0.9677
0.99	0.6925	0.8922	0.6764	0.8628	0.9679	0.9854

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