Event-Horizon Complementarity as Observer-Relative Facts: A Relational QM Analysis in Schwarzschild Spacetime

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Abstract

We revisit black-hole complementarity using Rovelli's relational quantum mechanics within the algebraic framework of quantum field theory in curved spacetime. For each observer, we construct a local von Neumann algebra as the inductive limit of open double-cone algebras contained strictly within their causal diamond, avoiding null-boundary ambiguities. Modeling late Hawking radiation as an entangled two-mode system in the Unruh state, we prove that no single observer's algebra contains both a late exterior mode and its interior partner. This reframes complementarity as compatibility of observer-relative facts rather than a global paradox. For a static exterior observer, we explicitly compute the accessible thermal entropy and derive an operational Holevo bound on retrievable information about the interior partner. Extensions to multimode systems and modular localization are discussed.

1 Introduction

Black-hole complementarity [7] posits that no single observer witnesses a violation of quantum unitarity, even though the global state may appear paradoxical from a God's-eye view. This principle attempts to resolve the information paradox by asserting that different observers have access to mutually incompatible descriptions of the same physical process.

We examine complementarity through Rovelli's Relational Quantum Mechanics (RQM) [2], which treats physical properties as existing only relative to observers. Using algebraic quantum field theory (AQFT) [1, 6], we formalize the notion of "what exists for whom" in Schwarzschild spacetime and prove a precise mathematical statement of operational complementarity.

The key insight is that apparent paradoxes dissolve when we abandon assumptions about observer-independent global descriptions. Instead of asking whether information is "really" destroyed, we ask what information is operationally accessible to each observer within their causal domain.

Notation and conventions

We work on the maximally extended Schwarzschild spacetime $(\mathcal{M}, g)$, set $c = \hbar = k_B = 1$, and use signature $(-, +, +, +)$. For an open, globally hyperbolic region $\mathcal{O} \subset \mathcal{M}$, $\mathcal{A}(\mathcal{O})$ denotes the von Neumann algebra generated by Weyl operators $W(f) = e^{i\phi(f)}$ with $\operatorname{supp} f \subset \mathcal{O}$. The net is isotonic and local [1, 6]. For a timelike worldline γ_O, the causal diamond is $D(O) := J^+(\gamma_O(\tau_1)) \cap J^-(\gamma_O(\tau_2))$. The Hawking temperature is $T_H = \kappa/(2\pi)$ with surface gravity $\kappa = 1/(4M)$. We denote von Neumann closures by $^{\text{w.o.t.}}$ and the Unruh state by ω_U.

2 Geometry, States, and Observer Algebras

2.1 Schwarzschild Background and Field State

Consider the maximally extended Schwarzschild spacetime $(\mathcal{M}, g)$ with mass parameter M. We focus on a free, massless scalar field ϕ in the Unruh state ω_U, which exhibits thermal flux at late times as seen by static exterior observers. The Haag–Kastler net assigns to every globally hyperbolic open region $\mathcal{O} \subset \mathcal{M}$ a von Neumann algebra $\mathcal{A}(\mathcal{O})$ generated by Weyl operators $W(f) = \exp(i\phi(f))$ where $\operatorname{supp} f \subset \mathcal{O}$. The net satisfies isotony, locality, and covariance.

2.2 Observer algebras via double-cone inductive limits

Let O be a timelike observer with worldline γ_O and causal diamond $D(O)$. To avoid ambiguities associated with null boundaries of $D(O)$, we define $\mathcal{A}_O$ as an *inductive limit of strictly interior, open double cones*. Specifically, choose an increasing net $\{\mathcal{O}_i\}_{i \in I}$ of open, relatively compact, globally hyperbolic double cones such that $\overline{\mathcal{O}_i} \subset D(O)$ for all i and $\bigcup_{i \in I} \mathcal{O}_i$ is dense in

$D(O)$ (in the manifold topology). Define

$$\mathcal{A}_O := \overline{\bigcup_{i \in I} \mathcal{A}(\mathcal{O}_i)}^{\text{w.o.t.}}. \tag{1}$$

By isotony, $\mathcal{A}(\mathcal{O}_i) \subset \mathcal{A}(\mathcal{O}_j)$ for $i \leq j$, so the union is an inductive system; the weak operator closure yields a von Neumann algebra. This construction is standard in AQFT for treating regions with null boundaries via interior approximants and is compatible with locality and the time-slice property [1, 3, 6]. All smearings $\phi(f)$ used below take $f \in C_c^\infty(\mathcal{O}_i)$ for some i, so no operator has support on $\partial D(O)$.

3 Two-mode Hawking model

Let $f_{\text{out}} \in C_c^\infty(\mathcal{M})$ be supported in an exterior neighborhood at late retarded time near $\mathscr{I}^+$ and $f_{\text{in}} \in C_c^\infty(\mathcal{M})$ supported strictly in region II. Define

$$a_{\text{out}} := \phi(g_{\text{out}})^+, \qquad a_{\text{in}} := \phi(g_{\text{in}})^+, \tag{2}$$

where $g_{\text{out}}, g_{\text{in}}$ are the positive-frequency solutions obtained from $f_{\text{out}}, f_{\text{in}}$ by the Klein–Gordon pairing. In the Unruh state ω_U, the restriction to the two-mode subspace is a two-mode squeezed (thermofield-double) form

$$|\Psi\rangle = (1-q)^{1/2} \sum_{n=0}^{\infty} q^{n/2} \, |n\rangle_{\text{out}} \otimes |n\rangle_{\text{in}}, \qquad q = e^{-\beta\omega}, \; \beta = T_H^{-1}. \tag{3}$$

This captures the essential entanglement of late Hawking pairs while remaining analytically tractable.

4 Operational complementarity

Proposition 1 (Operational complementarity)**.** *For any timelike observer O, the observer algebra $\mathcal{A}_O$ does not contain both a_{out} and a_{in} (equivalently, the local subalgebras generated by $\phi(f_{\text{out}})$ and $\phi(f_{\text{in}})$ are not both subalgebras of $\mathcal{A}_O$).*

Proof. Let R_{out} and R_{in} be open neighborhoods of $\text{supp}(f_{\text{out}})$ and $\text{supp}(f_{\text{in}})$, chosen so that $R_{\text{out}} \subset \text{I}$ (exterior) and $R_{\text{in}} \subset \text{II}$ (interior). Then $a_{\text{out}} \in \mathcal{A}(R_{\text{out}})$ and $a_{\text{in}} \in \mathcal{A}(R_{\text{in}})$ by construction.

Case 1 (static exterior observer). If O remains at fixed $r > 2M$, global hyperbolicity and the presence of the event horizon imply $D(O) \subset \mathrm{I}$; hence $R_{\text{in}} \not\subset D(O)$. By isotony of the net, $\mathcal{A}(R_{\text{in}}) \nsubseteq \mathcal{A}_O$, so $a_{\text{in}} \notin \mathcal{A}_O$.

Case 2 (infalling observer). If O crosses the horizon at proper time τ_c, then for any $\tau_2 > \tau_c$ the past set $J^-(\gamma_O(\tau_2))$ does not contain late-time exterior neighborhoods near $\mathscr{I}^+$: those regions are spacelike or to the future relative to the infaller once inside the horizon. Consequently, $R_{\text{out}} \not\subset D(O)$, and $\mathcal{A}(R_{\text{out}}) \nsubseteq \mathcal{A}_O$, so $a_{\text{out}} \notin \mathcal{A}_O$.

In both cases, at least one of $\{a_{\text{out}}, a_{\text{in}}\}$ lies outside $\mathcal{A}_O$, proving that no single observer algebra contains both. The argument uses only causal structure and isotony; it is independent of the two-mode truncation. □

Remark. A Penrose diagram makes the separation transparent: no causal diamond of a timelike curve can encompass both a late neighborhood of $\mathscr{I}^+$ in I and any subset of II. See Fig. 1.

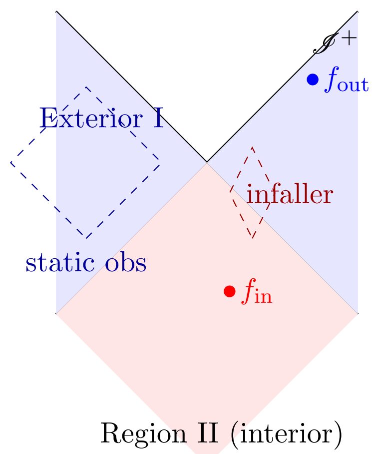

Figure 1: Figure 1: Penrose diagram of Schwarzschild spacetime showing causal diamonds for a static observer (blue) and an infalling observer (red). Neither causal diamond contains both supp(f_{out}) and supp(f_{in}).

5 Entropy and Information Bounds

For the static exterior observer, the reduced state is

$$\rho_{\text{out}} = (1-q) \sum_{n=0}^{\infty} q^n |n\rangle\langle n|, \quad q = e^{-\beta\omega}.$$

The thermal entropy is then

$$S(\rho_{\text{out}}) = -\sum_{n=0}^{\infty}(1-q)q^n\big[\ln(1-q) + n\ln q\big] = (\bar{n}+1)\ln(\bar{n}+1) - \bar{n}\ln\bar{n},$$

where

$$\bar{n} = \frac{1}{e^{\beta\omega}-1}.$$

For any ensemble encoding classical labels in the inaccessible partner mode, the accessible information satisfies the Holevo bound:

$$I_{\text{oper}}(O) \;\leq\; S(\rho_O),$$

where ρ_O is the reduced state relative to the observer algebra $\mathcal{A}_O$. Thus, the static observers operational information is strictly limited by the thermal entropy of the accessible exterior mode.

6 Discussion and Conclusion

The two-mode analysis illustrates the geometric principle underlying complementarity but has limitations. Real Hawking radiation involves a continuum of modes, and the idealized supports we used are somewhat artificial.

Several extensions merit investigation: multimode systems with realistic wavepacket decompositions, modular theory approaches using Tomita–Takesaki flow, dynamical spacetimes with formation and evaporation, and analogous statements in AdS black holes using holographic duality.

This perspective offers new insight into firewall arguments, which typically assume a global, observer-independent account of information flow. Within our framework, apparent contradictions between smooth horizon crossing and information preservation dissolve because they involve statements from incompatible observer algebras that cannot be simultaneously realized.

We note that a similar observer-centric reformulation of the information paradox has been developed by Tsenov [Tsenov, 2025], whose insights particularly the articulation of observer-relative information boundaries resonate with aspects of our own independent approach.

The central insightthat complementarity emerges from the algebraic structure of observer-relative factsremains robust under these generalizations. Information paradoxes dissolve when we abandon the assumption of global, observer-independent descriptions.

References

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