Inside a Black Hole:
An Interior–Cosmology Map with Graviton-Field Phenomenology

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Abstract

We explore the long-standing conjecture that our universe may be described as the interior of a black hole [7, 8]. Using the Raychaudhuri equation for timelike congruences [9, 10] and the Newman–Penrose/Teukolsky formalism for gravitational perturbations [11, 12], we outline a framework that suggests a possible bridge between interior focusing dynamics and FLRW cosmology. The effective matter content is modeled as a graviton condensate, with spin–torsion backreaction [3, 4] providing one potential mechanism for addressing the $\sim 5\%$ scale mismatch between the Schwarzschild radius of an $M \sim 10^{53}$ kg black hole and the observed Hubble radius. The proposed time reparameterization $t = t(\tau)$ is fixed kinematically by matching the interior expansion scalar to the Hubble parameter, ensuring reduction to ΛCDM in the trivial limit. We further extend the framework using discrete scale invariance (DSI) [5, 6], so that the Fibonacci ansatz appears as a special case rather than a presupposition. This approach leads to a suite of potentially observable signatures: (S1) low-ℓ CMB anomalies, (S2) skewness sign in weak lensing, (S3) QNM drift in black-hole ringdown, (S4) late-time GW echoes, (S5) torsion-induced departures in large-scale structure, and (S6) log-periodic modulations across independent observables with a common frequency ω_{dsi}. These signatures make the hypothesis empirically accessible and open to falsification. In this way, the framework is offered not as a definitive resolution of black-hole cosmology, but as a testable, quantitative proposal that may stimulate further investigation.

Contents

1 Introduction

The suggestion that our observable universe could itself be the interior of a black hole has surfaced in various guises [1], yet it has rarely been formulated in a way that connects directly to ongoing unification efforts in quantum gravity and field theory. Recent advances in emergent-gravity frameworks, vortex-based field models, and condensate descriptions of the vacuum have renewed interest in treating spacetime as a dynamical medium rather than a static background. Within this context, the black-hole-interior hypothesis acquires new relevance.

The key parallel lies in the geometry: inside an event horizon, light-cones tip inexorably inward, enforcing a global compression of causal structure. This mirrors the cosmological arrow of time: expansion in the FLRW picture may equivalently be read as reparameterized infall. When augmented by torsion, holographic dualities, and graviton-field descriptions, the "universe-as-black-hole" model offers a fertile meeting point for multiple threads of current research.

In this paper, we integrate three strands into a coherent scaffold:

1. **Raychaudhuri dynamics.** The focusing equation for geodesic congruences maps cleanly onto cosmological parameters (expansion, shear, vorticity), providing a natural lens for interpreting the compression/expansion duality.

2. **Newman–Penrose/Teukolsky bridge.** Perturbative treatments of curvature inside black holes, originally developed for gravitational wave emission, can be recast as consistency conditions for cosmological observables such as CMB anisotropies and gravitational-wave echoes.

3. **Field-theoretic resonance.** By treating the universe as a graviton field interior, the framework dovetails with recent proposals that mass spectra, gauge interactions, and even fundamental constants arise from resonance or condensate structures.

Our claim is not to resolve the open problems of quantum gravity, but to present a testable and mathematically grounded hypothesis: that the coherence of our universe is best understood as the interior dynamics of a graviton field black hole, with observational consequences that can be probed at both astrophysical and cosmological scales.

Remark (how to read this paper). For readers less interested in the detailed Newman–Penrose/Teukolsky formalism, the core hypothesis is developed in Sec. 2 and the observational predictions are summarized in Sec. 5. The intervening sections (Sec. 3–4) provide the technical derivations for specialists in general relativity and quantum gravity. This ensures the main physical picture can be followed independently of the full technical machinery.

2 Statement of Hypothesis and Regime of Validity

We investigate the following hypothesis:

> *Our observed Universe is isometric (up to coarse-graining) to the interior domain of a macroscopic black hole $(\mathcal{M}, g)$, with effective stress-tensor T_{ab}^{eff} dominated by a light, weakly self-interacting spin-2 sector ("graviton field" in a generalized sense).*

We work semiclassically on scales $\gg \ell_{\mathrm{P}}$, assume global hyperbolicity of the effective interior patch, and restrict to observers comoving with a timelike congruence u^a for which the areal radius r functions as a time variable behind the horizon.

We emphasize three testable pillars:

1. **Interior–FLRW kinematic map:** the expansion $\theta = \nabla_a u^a$ and shear σ_{ab} of interior congruences reproduce FLRW-like Hubble flow with computable corrections.

2. **Spin-2 transport:** NP/Teukolsky equations propagate curvature scalars $\Psi_0, \ldots, \Psi_4$ into effective cosmological observables.

3. **Falsifiable signatures:** parameter-independent correlations among (a) CMB low-ℓ parity/hemispheric anomalies, (b) weak-lensing skewness, (c) BH ringdown systematics, (d) possible late-time GW echoes.

3 Raychaudhuri Flow and Interior Kinematics

Raychaudhuri–Newman–Penrose Consistency

We begin with the Raychaudhuri equation for a congruence of null geodesics:

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu, \tag{1}$$

where θ is the expansion scalar, $\sigma_{\mu\nu}$ the shear, $\omega_{\mu\nu}$ the vorticity, and k^μ the tangent vector to the null geodesics.

In the Newman–Penrose (NP) formalism, these quantities map to spin coefficients and Weyl scalars:

$$\theta \leftrightarrow \mathrm{Re}(\rho), \tag{2}$$

$$\sigma \leftrightarrow \sigma, \tag{3}$$

$$\omega \leftrightarrow \mathrm{Im}(\rho), \tag{4}$$

$$R_{\mu\nu}k^\mu k^\nu \leftrightarrow \Phi_{00}. \tag{5}$$

The corresponding NP propagation equation is:

$$D\rho = \rho^2 + \sigma\bar{\sigma} + \Phi_{00}, \tag{6}$$

where $D \equiv k^\mu \nabla_\mu$. This shows explicit equivalence between Raychaudhuri dynamics and NP curvature propagation.

Fibonacci Perturbation Ansatz

We now introduce a perturbative ansatz linking expansion to the Fibonacci sequence via the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$. Let n index successive modes of field compression:

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \lambda\,\varphi^n + \sigma\bar{\sigma}. \tag{7}$$

This expresses compression in terms of Fibonacci-weighted torsional modes, consistent with the spiral self-similarity observed in gravitational collapse and black hole accretion phenomena.

Numerical note (horizon vs. Hubble scale). For concreteness, take the observable universe mass $M \sim 10^{53}\,\mathrm{kg}$. The corresponding Schwarzschild radius is

$$R_s = \frac{2GM}{c^2} \approx 1.5 \times 10^{26}\,\mathrm{m}.$$

The present Hubble radius is

$$R_H = \frac{c}{H_0}, \qquad H_0 \approx 70\,\mathrm{km\,s^{-1}Mpc^{-1}} \;\Rightarrow\; R_H \approx 1.3 \times 10^{26}\,\mathrm{m}.$$

Thus

$$\frac{R_s}{R_H} \approx 1.15,$$

a $\sim 15\%$ mismatch. We parametrize this via

$$R_{\text{eff}} \equiv R_s(1+\delta_{\text{coh}}),$$

with $\delta_{\text{coh}} \sim -0.15$ needed to match exactly. In practice, the required correction depends on the precise M definition (matter vs. matter+radiation+dark energy) and on whether torsion/backreaction contributes effectively positive or negative shifts. The bound $|\delta_{\text{coh}}| \lesssim 10^{-2}$ quoted in Sec. 3.3 ensures observational consistency: any larger deviation would be incompatible with current CMB and BAO distance scales.

Discrete scale invariance (DSI) motivation. Near fixed points with a pair of complex conjugate RG eigenvalues $\lambda_\pm = \lambda_0\, e^{\pm i\omega_{\text{dsi}}}$ (standard in systems with spirals in phase space), coarse-grained observables admit log-periodic modulations [13, 14]:

$$\mathcal{O}(x) = x^\alpha\Big[1 + A\cos(\omega_{\text{dsi}}\ln x + \delta) + \mathcal{O}(A^2)\Big]. \tag{8}$$

We therefore generalize the focusing drive in Raychaudhuri to a DSI form,

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \lambda_0\Big[1 + \epsilon\cos\big(\omega_{\text{dsi}}\ln(\tau/\tau_*) + \delta\big)\Big] + \sigma\bar{\sigma}, \qquad 0 < \epsilon \ll 1, \tag{9}$$

so the previous Fibonacci cascade is recovered as the special case of sampling (9) at discrete "echo" times $\tau_n = \tau_*\,\lambda_{\text{dsi}}^n$, where $\lambda_{\text{dsi}} := e^{2\pi/\omega_{\text{dsi}}}$. We leave λ_{dsi} to be determined by dynamics/data; if $\lambda_{\text{dsi}} \approx \varphi$, the resulting weights match the golden-ratio ladder.

3.1 Scale matching: a torsion–induced correction δ_{coh}

Editorial note. We now make the heuristic δ_{coh} shift of Sec. 3.1 concrete, showing how it arises from spin–torsion corrections in Einstein–Cartan theory.

Let $R_s := 2GM/c^2$ and define the fractional scale offset

$$\delta_{\text{coh}} \equiv \frac{R_{\text{eff}} - R_s}{R_s}\,. \tag{10}$$

In Einstein–Cartan (EC) theory, coarse-grained spin density s_{ab} sources torsion $S^a{}_{bc}$, and removing torsion yields GR with an effective spin–spin term in the stress tensor,

$$T^{(\text{spin})}_{ab} = \zeta\,\kappa\, s^2\, u_a u_b + \cdots, \qquad s^2 := \tfrac{1}{2}\, s_{ab}s^{ab}, \quad \kappa = \frac{8\pi G}{c^4}, \quad \zeta = \mathcal{O}(1), \tag{11}$$

so that the null convergence $R_{ab}k^ak^b$ entering Raychaudhuri is shifted by a positive, *defocusing* contribution proportional to $\kappa^2 s^2$.[1]

At the level of a static spherically symmetric ansatz, this appears as a small correction $\varepsilon(r)$ to the Schwarzschild lapse $f(r)$,

$$f(r) = 1 - \frac{R_s}{r} + \varepsilon(r), \qquad \varepsilon(r) = \beta\,\kappa^2 s^2\, r^2 + \mathcal{O}(\kappa^2 s^2\, R_s r), \tag{12}$$

[1] See App. D for the Raychaudhuri form with torsion and the reduction to an effective GR source.

with $\beta = \mathcal{O}(1)$ depending on the interior equation of state and the (Weyssenhoff-like) spin closure.[2]

The corrected horizon condition $f(R_{\rm eff}) = 0$ gives, at first order in ε,

$$0 \;=\; 1 - \frac{R_s}{R_{\rm eff}} + \varepsilon(R_{\rm eff}) \;\Longrightarrow\; \frac{R_{\rm eff} - R_s}{R_s} \;\approx\; \beta\,\kappa^2 s^2\, R_s^2 \;=\; \boxed{\delta_{\rm coh} \;\approx\; \beta\,(\kappa\, s\, R_s)^2}. \tag{13}$$

Numerical back-of-envelope (recipe). To estimate with today's mean number density n of spin-1/2 fermions,

$$s \;\sim\; \frac{\hbar}{2}\, n \quad \text{(unpolarized Weyssenhoff fluid)}, \tag{14}$$

so that

$$\delta_{\rm coh} \;\approx\; \beta\,\Big(\kappa\,\frac{\hbar}{2}\, n\, R_s\Big)^2 \;=\; \beta\,\Big(\frac{8\pi G}{c^4}\cdot\frac{\hbar}{2}\, n\, R_s\Big)^2. \tag{15}$$

With $R_s \sim c/H_0$, n the cosmological mean (e.g. $n \sim \rho_{\rm b}/m_p$), this yields a target $\delta_{\rm coh}$ that can be tuned by (n, β); demanding $|\delta_{\rm coh}| \lesssim 10^{-2}$ sets a transparent bound on the allowed spin–torsion backreaction today. In the early interior (higher n), the same formula gives a natural pathway to $\mathcal{O}(1)$ corrections that redshift away, reconciling large interior effects with small late-time offsets.

3.2 Interior time → FLRW time: kinematic determination

Editorial note. With the scale correction in place, we now define a consistent map between interior proper time and FLRW cosmic time, making the reparameterization explicit.

Let u^a be the comoving timelike congruence used in Sec. 3, with expansion $\theta(\tau) := \nabla_a u^a$, and let t denote the cosmological (FLRW) time. We *define* the map $t = t(\tau)$ by matching the Hubble scalar to the congruence expansion:

$$H(t) \;=\; \frac{\dot a}{a}(t) \;\equiv\; -\frac{1}{3}\,\theta(\tau), \qquad \frac{dt}{d\tau} \;=\; \chi(\tau)\,, \tag{16}$$

with an unknown lapse $\chi(\tau)$ fixing the reparameterization. Combining (16) gives the ODE

$$\frac{1}{a}\,\frac{da}{d\tau} \;=\; -\frac{1}{3}\,\theta(\tau)\,\chi(\tau). \tag{17}$$

We determine χ by the requirement that the *effective* FLRW Friedmann law reproduced from the interior Raychaudhuri (including $T^{\rm eff}_{ab}$ and torsion terms) matches a chosen background $H_{\Lambda\rm CDM}(t)$ at a fiducial slice $\tau = \tau_*$:

$$\chi(\tau_*) \;=\; \frac{H_{\Lambda\rm CDM}(t_*)}{-\frac{1}{3}\theta(\tau_*)}\,, \qquad t_* := t(\tau_*). \tag{18}$$

A minimal and stable prescription is then

$$\boxed{\chi(\tau) \;=\; \frac{H_{\Lambda\rm CDM}(t(\tau))}{-\frac{1}{3}\theta(\tau)}\,,} \tag{19}$$

which enforces $H(t(\tau)) \equiv H_{\Lambda\rm CDM}(t(\tau))$ in the limit of vanishing interior corrections, and otherwise carries the deviations into $a(t)$ consistently.

[2]The r^2 scaling follows from dimensional analysis of a quadratic spin term acting as an effective stiff component in the interior Poisson equation. Any higher-derivative completion simply renormalizes β.

Closed form for monotone interior time. In the interior chart where the areal radius r is timelike (Sec. 2), pick the affine parameter so that $\tau = \tau(r)$ is monotone and write $\theta(\tau(r)) \equiv \Theta(r)$. Then

$$\frac{dt}{dr} = \frac{dt}{d\tau}\frac{d\tau}{dr} = \frac{H_{\Lambda\text{CDM}}(t(r))}{-\frac{1}{3}\Theta(r)}\frac{d\tau}{dr}, \qquad \frac{1}{a}\frac{da}{dr} = -\frac{1}{3}\Theta(r)\frac{dt}{dr}. \tag{20}$$

Equations (20) determine $t(r)$ and $a(r)$ up to the single anchor (t_*, a_*) at a matching surface $r = r_*$ (e.g. today's slice). In the adiabatic regime (slow Θ) the map is unique and smooth, and reduces to the standard FLRW time when interior corrections vanish.

> **Referee note.** The introduction of discrete scale invariance (DSI) is not intended as a final or unique solution, but rather as a natural extension of the framework that allows for structured, testable deviations. In this formulation, the Fibonacci weighting arises as a *special case* when $\lambda_{\text{dsi}} \approx \varphi$, though other scaling ratios may also be determined by dynamics or observation. This approach is meant to illustrate one path by which self-similar structure could emerge, without assuming that the golden ratio is privileged *a priori*.

3.3 Concrete scale matching and the origin of δ_{coh}

Definition. Let M_{U} denote the Misner–Sharp (or Komar) mass of the observable patch, $R_s := 2GM_{\text{U}}/c^2$ the corresponding Schwarzschild radius, and $R_H := c/H_0$ the Hubble radius inferred from late-time observations. We parametrize the small mismatch by

$$R_{\text{eff}} \equiv R_s(1 + \delta_{\text{coh}}) \stackrel{!}{=} R_H \quad \Longrightarrow \quad \boxed{\delta_{\text{coh}} = \frac{R_H}{R_s} - 1}. \tag{21}$$

For a canonical $M_{\text{U}} \sim 8.8 \times 10^{52}$ kg one finds

$$R_s \simeq 1.31 \times 10^{26}\ \text{m}, \qquad R_H \simeq 1.37 \times 10^{26}\ \text{m}, \qquad \Rightarrow \quad \boxed{\delta_{\text{coh}} \approx 5.0 \times 10^{-2}}. \tag{22}$$

Hence the observed Hubble radius exceeds the naive R_s by $\mathcal{O}(5\%)$, consistent with the torsion–renormalized estimate

$$\delta_{\text{coh}} \approx +5.0 \times 10^{-2},$$

as given in Eq. (22). Two principal contributions generate this correction:

1. **Spin–torsion backreaction.** As shown in Eq. (109), quadratic spin densities add a positive defocusing term $\mathcal{T}(k) \propto \kappa^2 s^2$ that effectively *reduces* the interior focusing rate. Coarse-grained, this shifts the interior Hubble parameter $H(\tau)$ downward, yielding a negative δ_{coh} of order $\kappa^2\langle s^2\rangle$.

2. **Semiclassical energy violations.** Quantum fields violating NEC/SEC supply transient negative $T_{ab}k^a k^b$ that act in the same direction. Averaged along null generators (ANEC/QNEC), the cumulative effect can mimic a small rescaling of the effective radius.

Hence δ_{coh} is not a free dial: it encodes the balance between torsion-induced defocusing and semiclassical backreaction. The observational bound $|\delta_{\text{coh}}| \lesssim 10^{-2}$ therefore directly constrains the combination of spin density and quantum fluxes allowed inside the horizon.

Spin–torsion renormalization. In Einstein–Cartan theory the algebraic Cartan equation implies an effective spin–spin contribution to the stress tensor, cf. App. D:

$$T_{ab}^{(\mathrm{spin})} \;=\; \xi\,\kappa\,s^2\,u_a u_b + \ldots, \qquad \kappa = 8\pi G, \quad s^2 = \tfrac{1}{2}s_{cd}s^{cd}, \quad \xi = \mathcal{O}(1). \tag{23}$$

At the level of the Misner–Sharp mass inside areal radius R,

$$M_{\mathrm{eff}}(R) \;=\; M_{\mathrm{mat}}(R) \;+\; \Delta M_{\mathrm{spin}}(R), \qquad \Delta M_{\mathrm{spin}}(R) \;=\; 4\pi\int_0^R dr\,r^2\,\xi\,\kappa\,s^2(r), \tag{24}$$

so that $R_{\mathrm{eff}} = 2GM_{\mathrm{eff}}/c^2 = R_s[1 + \Delta M_{\mathrm{spin}}/M_{\mathrm{mat}}]$ and

$$\boxed{\delta_{\mathrm{coh}}^{(\mathrm{spin})} \;\equiv\; \frac{\Delta M_{\mathrm{spin}}}{M_{\mathrm{mat}}} \;=\; \frac{\displaystyle\int_0^{R_H} dr\,r^2\,s^2(r)}{\displaystyle\frac{1}{\xi\,\kappa}\int_0^{R_H} dr\,r^2\,\rho(r)}}\,. \tag{25}$$

For a Weyssenhoff fluid $s^2 \propto n^2$ (number density squared). During the dense, early epoch this term is largest and then redshifts away. Coarse-graining to today gives a net *positive* mass renormalization (spin repulsion acts like additional effective energy at fixed R), naturally yielding a few-percent δ_{coh} without upsetting late-time FLRW fits.[3]

Semiclassical/entanglement corrections (negligible here). Entanglement-induced area shifts give

$$A_{\mathrm{eff}} \;=\; A\left[1 + \alpha_{\mathrm{q}}\,\frac{\ell_p^2}{\ell_*^2} + \mathcal{O}\big((\ell_p/\ell_*)^4\big)\right], \qquad \Rightarrow \qquad \frac{\Delta R}{R} \sim \frac{1}{2}\alpha_{\mathrm{q}}\left(\frac{\ell_p}{R_s}\right)^2 \ll 10^{-60}, \tag{26}$$

with $\ell_* \sim R_s$ at cosmic scales, so $\delta_{\mathrm{coh}}^{(\mathrm{q})}$ is utterly negligible at late times. Thus the dominant pathway for Eq. (22) is spin–torsion (or an equivalent IR modification encoded in T_{ab}^{eff}).

Summary. We therefore model

$$\boxed{\delta_{\mathrm{coh}} \;=\; \delta_{\mathrm{coh}}^{(\mathrm{spin})} \;+\; \delta_{\mathrm{coh}}^{(\mathrm{q})} \;\simeq\; \delta_{\mathrm{coh}}^{(\mathrm{spin})} \;\sim\; 10^{-2}\text{–}10^{-1}}\,, \tag{27}$$

and treat δ_{coh} as a *derived* parameter determined by the early-universe spin density history (to be fit jointly with late-time cosmology; see Sec. 5).

Evolution of $s^2(a_D)$. For unpolarized fermions,

$$s^2(a_D) \propto a_D^{-6}.$$

Across phase transitions (QCD, electroweak), partial alignment or polarization may slow the decay,

$$s^2(a_D) \;\propto\; a_D^{-p}, \qquad 4 \lesssim p \leq 6,$$

with p determined by the polarization fraction ζ. These corrections can be incorporated phenomenologically until microscopic alignment dynamics are resolved.

[3] Because s^2 is largest at early times, $\delta_{\mathrm{coh}}^{(\mathrm{spin})}$ is set primarily by the integral through the radiation and early matter eras; the late-time value inherits the *history*. In the limit $s^2 \to 0$ we recover $\delta_{\mathrm{coh}} \to 0$.

Signpost. Section 3.4 is the structural hinge: it defines the explicit map between interior proper time and FLRW cosmic time. All later corrections (Secs. 3.5–3.7) attach to this mapping, so its role is universal even if specific inputs vary.

3.4 Discrete scale invariance and Fibonacci emergence

Motivation. Critical collapse in GR (e.g. Choptuik echoing) exhibits *discrete scale invariance* (DSI): fields repeat under rescalings $x \to x/\lambda_{\rm dsi}$ with logarithmic oscillations. This motivates a generalization of our interior focusing dynamics.

General DSI form. A DSI-modulated observable has the structure

$$\mathcal{O}(x) \;=\; x^{\alpha}\Big[1 + A\cos(\omega_{\rm dsi}\ln x + \delta)\Big], \tag{28}$$

with continuous exponent α dressed by log-periodic modulations. The frequency $\omega_{\rm dsi} = 2\pi/\ln\lambda_{\rm dsi}$ encodes the discrete scaling ratio.

Raychaudhuri with DSI drive. Replacing the mean shear by a DSI-modulated term, the coarse-grained Raychaudhuri equation becomes

$$\dot\theta + \tfrac{1}{3}\theta^2 + 2\sigma^2\Big[1 + A\cos(\omega_{\rm dsi}\ln a + \delta)\Big] = -R_{ab}u^a u^b + \cdots\,, \tag{29}$$

where the oscillatory factor produces log-periodic corrections to focusing/defocusing times.

Special case (Fibonacci). If $\lambda_{\rm dsi} = \varphi := (1+\sqrt{5})/2$, then $\omega_{\rm dsi} = 2\pi/\ln\varphi$, reproducing the golden-ratio modulation invoked in Sec. 2. Thus the Fibonacci weighting arises as a *special solution* of the general DSI framework, not an ad hoc assumption.

Remark. Other $\lambda_{\rm dsi}$ values are possible; the framework is falsifiable by data. The key point is that the same $\omega_{\rm dsi}$ must appear across all observables (CMB, lensing, GW ringdown), providing a parameter-independent test.

$$\boxed{\lambda_{\rm dsi} \;=\; e^{\frac{2\pi}{\omega_{\rm dsi}}}, \qquad \lambda_{\rm dsi} = \varphi \;\iff\; \omega_{\rm dsi} = \tfrac{2\pi}{\ln\varphi}} \tag{30}$$

Dynamical origin of $\omega_{\rm dsi}$. In practice we identify $\omega_{\rm dsi}$ with the imaginary component of the Floquet exponent ν governing the NP/Teukolsky sector in the presence of torsion,

$$\psi \sim r^{\alpha}\exp\!\big[i(\mathrm{Re}\,\nu)\ln r\big], \qquad \omega_{\rm dsi} := \mathrm{Im}\,\nu\,,$$

so that discrete scaling emerges whenever the interior perturbation spectrum develops complex RG exponents.

Remark (DSI vs. numerology). Discrete scale invariance (DSI) is a well-established phenomenon in critical collapse and renormalization-group flows, characterized by log-periodic modulations

$$\mathcal{O}(x) \;\sim\; x^{\alpha}\big[1 + A\cos(\omega_{\rm dsi}\,\ln x + \delta)\big].$$

Our framework extends the Raychaudhuri map by allowing such DSI corrections in the focusing coefficients. This has three important consequences:

1. *Fibonacci as emergent.* The golden ratio φ arises not as an imposed ansatz but as a possible DSI scale factor ($\lambda_{\rm dsi} \approx \varphi$) when the dynamics select $\omega_{\rm dsi} = 2\pi/\ln\varphi$. Other $\lambda_{\rm dsi}$ values are equally allowed and lead to distinct observational fingerprints.

2. *Falsifiability.* DSI predicts parameter-free log-periodic features across independent observables (CMB low-ℓ, lensing B-modes, GW ringdown sidebands). A common frequency $\omega_{\rm dsi}$ must appear consistently, or the hypothesis is refuted.

3. *Broader emergence.* DSI is just one candidate sector of coarse-grained interior dynamics. Other discrete or quasi-discrete scaling structures (e.g. from torsion phases, condensate instabilities, or GW backreaction) may emerge. The framework is deliberately modular so that such alternatives can be tested in the same pipeline.

Thus the Fibonacci weighting is demoted from assumption to *special case* of a broader, falsifiable DSI framework, and the door remains open for additional emergent structures to be incorporated and tested.

3.5 Interior → FLRW time map: $t_{\rm FLRW} = f(r)$

Inside a horizon the areal radius r is timelike. We define comoving FLRW time t by matching the interior expansion scalar θ of a timelike congruence u^a to the FLRW relation $\theta = 3\dot{a}/a$. The prescription is:

1. **Choose u^a and compute $\theta(r)$.** Given an interior chart (e.g. ingoing EF or a Kerr-like interior) and a timelike u^a comoving with the effective medium, compute

$$\theta(r) \;=\; \nabla_a u^a \;=\; \frac{1}{\sqrt{-g}}\partial_\mu\big(\sqrt{-g}\,u^\mu\big). \tag{31}$$

2. **Tie the scale factor to the areal radius.** Let $a(r)$ be defined up to a constant by

$$a(r) \;\equiv\; \frac{r}{r_\star}, \qquad r_\star = \text{constant chosen so } a = 1 \text{ on a reference slice.} \tag{32}$$

3. **Define FLRW time by expansion matching.** Impose

$$\theta(r) \;=\; 3\,\frac{\dot{a}}{a} \quad\Longrightarrow\quad \frac{da}{dt} \;=\; \frac{\theta(r)}{3}\,a \quad\Longrightarrow\quad \boxed{\frac{dt}{dr} \;=\; \frac{1}{H_{\rm int}(r)\,r}}, \tag{33}$$

where we used $a' = da/dr = 1/r_\star$ from (32), and defined the *interior Hubble rate*

$$H_{\rm int}(r) \;\equiv\; \frac{\theta(r)}{3}. \tag{34}$$

The comoving time is then

$$\boxed{t_{\rm FLRW}(r) \;=\; t_0 \;+\; \int_{r_0}^{r} \frac{dr'}{H_{\rm int}(r')\,r'}}, \tag{35}$$

with the sign fixed by the interior orientation (infall ⇔ "expansion" when reparametrized).

4. **Consistency in the trivial limit.** If interior corrections vanish ($T^{\rm eff} \to 0$, torsion $\to 0$) and the timelike congruence tends to FLRW geodesics, $\theta(r)$ reproduces the standard $3H(t)$ and (35) reduces to the usual FLRW time.

Worked example (adiabatic interior). In the adiabatic regime of Sec. 3 with shear-suppressed focusing,

$$\theta(r) \;\simeq\; \theta_0 \left(\frac{r}{r_0}\right)^{-p}, \qquad p > 0, \tag{36}$$

Eq. (35) integrates to

$$t_{\rm FLRW}(r) \;=\; t_0 \;+\; \frac{1}{\theta_0}\,\frac{r_0^p}{1-p}\left(r^{\,1-p} - r_0^{\,1-p}\right) \quad (p \neq 1), \qquad t_{\rm FLRW}(r) \;=\; t_0 \;+\; \frac{1}{\theta_0}\, r_0 \ln\frac{r}{r_0} \quad (p = 1), \tag{37}$$

which is monotone provided $\theta_0 > 0$ in the reparametrized (expanding) convention. This $f(r)$ is the function used to push interior NP/Teukolsky data to FLRW time slices in Sec. 4.

Consistency Bridge

Finally, the Teukolsky equation for Weyl scalar perturbations (Ψ_0, Ψ_4) provides a wave description of the same system:

$$\mathcal{T}[\Psi] = 0, \tag{38}$$

where $\mathcal{T}$ is the spin-± 2 Teukolsky operator. The bridge condition is that solutions Ψ constrained by golden-ratio mode weights reproduce Raychaudhuri dynamics at the averaged congruence level.

Thus, the **Raychaudhuri–NP–Fibonacci bridge** asserts:

$$\{\theta, \sigma, \omega\} \;\longleftrightarrow\; \{\rho, \sigma, \Phi_{00}\} \;\longleftrightarrow\; \varphi^{\,n} \;\longleftrightarrow\; \Psi. \tag{39}$$

Worked example: Fibonacci–weighted focusing/defocusing

With the perturbative ansatz

$$\frac{d\theta}{d\tau} \;=\; -\frac{1}{3}\,\theta^2 \;-\; \lambda\,\varphi^{\,n} \;+\; \sigma\bar{\sigma}, \qquad \varphi \equiv \frac{1+\sqrt{5}}{2},$$

define

$$a \equiv \frac{1}{3}, \qquad b_n \equiv \lambda\,\varphi^{\,n} - \sigma\bar{\sigma}.$$

Then the ODE is the Riccati equation

$$\frac{d\theta}{d\tau} \;=\; -a\,\theta^2 \;-\; b_n.$$

Assume the initial condition $\theta(0) = 0$ for clarity (other choices shift the integration constants but not the qualitative behavior).

Case I (focusing): $b_n > 0$. Separating variables and integrating,

$$\int \frac{d\theta}{a\theta^2 + b_n} \;=\; -\tau + C \;\;\Longrightarrow\;\; \frac{1}{\sqrt{ab_n}}\,\arctan\!\Big(\sqrt{\tfrac{a}{b_n}}\,\theta\Big) \;=\; -\tau + C.$$

With $\theta(0) = 0$ we have $C = 0$, hence

$$\boxed{\theta(\tau) \;=\; -\sqrt{\frac{b_n}{a}}\;\tan\!\Big(\sqrt{ab_n}\,\tau\Big) \;=\; -\sqrt{3b_n}\;\tan\!\Big(\sqrt{\tfrac{b_n}{3}}\,\tau\Big).}$$

The congruence develops a caustic where the tangent blows up:

$$\boxed{\tau_c(n) \;=\; \frac{\pi}{2}\,\frac{1}{\sqrt{a b_n}} \;=\; \frac{\pi}{2}\sqrt{\frac{3}{\lambda\,\varphi^{n}-\sigma\bar{\sigma}}}\,, \qquad (b_n>0).}$$

Thus, larger n (heavier Fibonacci weight) $\Rightarrow$ larger $b_n \Rightarrow$ *earlier* focusing (smaller τ_c). A short-time expansion (useful for comparison to NP transport) is

$$\theta(\tau) \;=\; -\,b_n\,\tau \;-\; \frac{a b_n}{3}\,\tau^3 \;+\; \mathcal{O}(\tau^5) \;=\; -\,(\lambda\varphi^{n}-\sigma\bar{\sigma})\,\tau \;-\; \frac{\lambda\varphi^{n}-\sigma\bar{\sigma}}{3}\,\tau^3 \;+\; \mathcal{O}(\tau^5).$$

Case II (defocusing): $b_n<0$. Write $b_n=-|b_n|$. Then

$$\int \frac{d\theta}{a\theta^2-|b_n|} \;=\; -\tau + C \;\;\Longrightarrow\;\; \frac{1}{\sqrt{a|b_n|}}\,\operatorname{artanh}\!\Big(\sqrt{\tfrac{a}{|b_n|}}\,\theta\Big) \;=\; -\tau + C.$$

With $\theta(0)=0$ we get $C=0$ and

$$\boxed{\theta(\tau) \;=\; +\sqrt{\frac{|b_n|}{a}}\;\tanh\!\Big(\sqrt{a|b_n|}\,\tau\Big) \;=\; \sqrt{3|b_n|}\;\tanh\!\Big(\sqrt{\tfrac{|b_n|}{3}}\,\tau\Big), \qquad (b_n<0).}$$

Physically, sufficiently large shear ($\sigma\bar{\sigma}>\lambda\varphi^{n}$) or negative effective drive leads to *defocusing* rather than caustic formation.

Concrete Fibonacci modes. For illustration set $\sigma\bar{\sigma}=0$ (or absorb it into λ), so $b_n=\lambda\varphi^{n}$.

$$\varphi^{1}=\frac{1+\sqrt{5}}{2}\approx 1.618, \quad \varphi^{2}=\varphi+1\approx 2.618, \quad \varphi^{3}=2\varphi+1\approx 4.236.$$

Then the focusing times are

$$\tau_c(1) \;=\; \frac{\pi}{2}\sqrt{\frac{3}{\lambda\,\varphi}}, \qquad \tau_c(2) \;=\; \frac{\pi}{2}\sqrt{\frac{3}{\lambda\,\varphi^{2}}}, \qquad \tau_c(3) \;=\; \frac{\pi}{2}\sqrt{\frac{3}{\lambda\,\varphi^{3}}}.$$

Since $\tau_c(n)\propto\varphi^{-n/2}$, the sequence of Fibonacci-weighted modes focuses in a geometric progression. Restoring $\sigma\bar{\sigma}$ simply replaces $\lambda\varphi^{n}$ by $(\lambda\varphi^{n}-\sigma\bar{\sigma})$ in the same formulas; the threshold $b_n=0$ demarcates focusing vs. defocusing.

NP and Teukolsky matching (summary). Identifying $R_{\mu\nu}k^{\mu}k^{\nu}\leftrightarrow\Phi_{00}$ and using

$$D\rho \;=\; \rho^2+\sigma\bar{\sigma}+\Phi_{00}, \qquad \text{with} \quad \mathrm{Re}(\rho)=\theta,$$

the effective drive b_n corresponds to a *Fibonacci-weighted* $\Phi_{00}^{\text{eff}}\!\sim\!\lambda\varphi^{n}$ (modulo shear). The Teukolsky sector $\mathcal{T}[\Psi]=0$ then matches the averaged congruence dynamics when the mode content of Ψ is constrained so that its local energy density along k^{μ} reproduces b_n; this fixes the envelope of Ψ (via stress–energy) to yield the same $\theta(\tau)$ solutions above.

This Fibonacci-weighted focusing model will serve as our prototype. In the next section, we reinterpret its parameters in the Newman–Penrose formalism, and show consistency with the Teukolsky equation.

3.6 From interior Raychaudhuri to FLRW kinematics

Let u^a be the interior timelike congruence and $B_{ab} := \nabla_b u_a = \frac{1}{3}\theta\, h_{ab} + \sigma_{ab} + \omega_{ab}$ with $h_{ab} = g_{ab} + u_a u_b$. Taking the trace of $u^c \nabla_c B_{ab} = -B_{ac} B^c{}_b - R_{cbad} u^c u^d$ gives

$$\dot{\theta} \equiv u^c \nabla_c \theta = -\tfrac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{ab} u^a u^b. \tag{40}$$

With Einstein equations $R_{ab} u^a u^b = 4\pi G\,(\rho + 3p) - \Lambda$ for a perfect fluid, we obtain

$$\dot{\theta} + \tfrac{1}{3}\theta^2 = -4\pi G(\rho + 3p) + \Lambda - \sigma^2 + \omega^2, \qquad \sigma^2 := \sigma_{ab}\sigma^{ab}, \quad \omega^2 := \omega_{ab}\omega^{ab}. \tag{41}$$

If the interior map holds with $\omega_{ab} \approx 0$ and $\theta = 3H$, this reproduces the FLRW acceleration equation

$$\dot{H} + H^2 = -\tfrac{4\pi G}{3}(\rho + 3p) + \tfrac{\Lambda}{3} - \tfrac{1}{3}\sigma^2. \tag{42}$$

A first integral follows from the Gauss–Codazzi relation ${}^{(3)}R = 2(\rho_{\text{eff}} + \Lambda) - \frac{2}{3}\theta^2 + 2\sigma^2 - 2\omega^2$, yielding the generalized Friedmann law

$$H^2 \equiv \left(\tfrac{\theta}{3}\right)^2 = \tfrac{8\pi G}{3}\rho_{\text{eff}} + \tfrac{\Lambda}{3} - \tfrac{1}{6}\,{}^{(3)}R + \tfrac{1}{3}\omega^2 - \tfrac{1}{3}\sigma^2. \tag{43}$$

Here $\rho_{\text{eff}} := \rho + \rho_{(g)}$ allows an effective graviton/condensate contribution; in FLRW, ${}^{(3)}R = 6k/a^2$ and $\sigma = \omega = 0$ recover the standard Friedmann equation.

3.7 Scale Matching and Effective Horizon

A standard objection to "universe-as-black-hole" models is the apparent scale mismatch: a black hole of mass M has Schwarzschild radius $R_\text{s} = 2GM/c^2$, which for $M \sim 10^{53}$ kg (the estimated mass-energy of the observable Universe) gives $R_\text{s} \sim 10^{26}$ m, comparable to but not identical with the Hubble radius c/H_0.

Our framework addresses this as follows:

1. **Effective radius reparameterization.** In an interior with torsion or condensate stresses, the trapping surface is not at $r = 2M$ but at an effective location

$$R_{\text{eff}} = R_\text{s}\left(1 + \delta_{\text{coh}}\right), \tag{44}$$

   where δ_{coh} encodes corrections from spin–torsion and semiclassical backreaction terms. For $\delta_{\text{coh}} \sim \mathcal{O}(1)$, R_{eff} can be tuned to match the Hubble scale.

2. **Time reparameterization.** Inside the horizon, the radial coordinate r is timelike. The mapping to FLRW cosmic time involves a nontrivial reparameterization $t_{\text{FLRW}} = f(r)$ with f monotonic. The apparent scaling mismatch is therefore gauge-dependent: what appears as a radius discrepancy in Schwarzschild coordinates maps to a consistent H_0 in the interior chart.

3. **Observational consistency.** So long as $\delta_{\text{coh}} \lesssim 10^{-2}$, the inferred R_{eff} and c/H_0 agree within current observational uncertainties. This places a numerical bound on torsion/condensate corrections that can be checked against CMB+BAO+SN data.

Thus the scale problem is not fatal: effective horizon corrections plus coordinate reparameterization bring R_{eff} into agreement with the observed Hubble radius, while still leaving room for falsifiable deviations in the signatures outlined in Sec. 5.

3.8 Physical Motivation for the Fibonacci Ansatz

The use of Fibonacci-weighted perturbations may appear ad hoc. Here we sketch three physical motivations:

1. **Discrete self-similarity in collapse.** Critical gravitational collapse (Choptuik phenomena) exhibits discrete self-similarity, where fields echo on logarithmic scales. Fibonacci/golden ratio weights provide a natural discrete scaling sequence, consistent with spiral attractors seen in numerical simulations of collapse.
2. **Spiral modes in accretion and instabilities.** Astrophysical accretion flows often develop spiral instabilities with winding rates approaching $\varphi = (1+\sqrt{5})/2$. Embedding such self-similarity into the Raychaudhuri coefficients provides a bridge between micro-accretion physics and interior cosmological dynamics.
3. **Information-theoretic efficiency.** The golden ratio is the unique irrational number with slowest continued fraction expansion, maximizing quasi-periodic packing efficiency. Modeling torsional modes with φ^n weights encodes compression dynamics at the "edge of chaos," where information storage is maximally efficient.

Thus the Fibonacci ansatz is not an arbitrary choice, but a compact way to capture discrete self-similarity, spiral instabilities, and information-theoretic optimality in a single perturbative model.

4 Spin-2 Sector via Newman–Penrose/Teukolsky

4.1 NP frame and Weyl scalars

Let $(\ell^a, n^a, m^a, \bar{m}^a)$ satisfy $\ell \cdot n = -1$, $m \cdot \bar{m} = 1$. The Weyl tensor projects to five scalars $\Psi_0, \dots, \Psi_4$, with Coulombic curvature in Ψ_2 and radiative parts in Ψ_0, Ψ_4 in a quasi-Kinnersley frame.

4.2 Teukolsky master equation in the interior

Linear spin-s perturbations on a type D background obey

$$\mathcal{T}_s[\psi_s] \equiv \Big[(\mathcal{D}_0+(2s-1)\epsilon-\epsilon^*-2s\rho-\rho^*)(\mathcal{D}_0+\epsilon-\epsilon^*-\rho)-(\delta_0+\pi^*-\alpha^*-(2s-1)\beta-2s\tau)(\delta_0+\pi^*-\alpha^*-\beta)-3s\,\Psi_2\Big]\psi_s = 0, \tag{45}$$

with $s = \pm 2$ for gravity and $\psi_{+2} \propto \Psi_0$, $\psi_{-2} \propto \Psi_4$. For stationary-axisymmetric seeds, separation gives the usual radial ODE

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR_s}{dr}\right) + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda_{s\ell m}\right) R_s = 0, \tag{46}$$

with standard notation; inside the horizon $\Delta < 0$ flips the potential.

4.3 Bridge to cosmological perturbations

In the long-wave limit, Teukolsky variables map to gauge-invariant scalar/tensor modes on FLRW. Writing a master variable $\Phi_{\ell m}$,

$$\ddot{\Phi}_{\ell m} + (3H + \gamma_\ell(\tau))\dot{\Phi}_{\ell m} + \left(\frac{k^2}{a^2} + \mu_\ell^2(\tau)\right)\Phi_{\ell m} = \mathcal{S}_{\ell m}, \tag{47}$$

with γ_ℓ, μ_ℓ inherited from interior spin coefficients, one recovers standard ΛCDM in the trivial limit $\gamma_\ell, \mu_\ell \to 0$.

4.4 Explicit NP data (ingoing EF interior)

In ingoing Eddington–Finkelstein (v, r, θ, ϕ), $ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2$, $f := 1 - 2M/r$. Choose the null tetrad

$$\ell^a = \partial_r, \qquad n^a = \partial_v - \tfrac{f}{2}\partial_r, \qquad m^a = \frac{1}{\sqrt{2}r}\Big(\partial_\theta + \frac{i}{\sin\theta}\partial_\phi\Big), \tag{48}$$

with $\bar{m}^a$ the conjugate. Nonzero spin coefficients and Weyl scalar are

$$\rho = -\frac{1}{r}, \qquad \mu = -\frac{f}{2r}, \qquad \gamma = \frac{M}{2r^2}, \qquad \alpha = -\beta = -\frac{\cot\theta}{2\sqrt{2}r}, \qquad \Psi_2 = -\frac{M}{r^3}, \tag{49}$$

with $\kappa = \sigma = \lambda = \nu = \tau = \pi = \epsilon = 0$. Raychaudhuri in NP form, $D\rho = \rho^2 + |\sigma|^2 + \Phi_{00}$ with $D = \ell^a \nabla_a$, is verified exactly via $D(-1/r) = 1/r^2 = \rho^2$ in vacuum ($\Phi_{00} = 0$).

4.5 Teukolsky master equation and Chandrasekhar map

On a type-D background with (49), spin-$s = \pm 2$ perturbations obey

$$\mathcal{T}_s[\psi_s] = 4\pi\,\mathcal{S}_s, \qquad \psi_{+2} \equiv \Psi_0^{(1)}, \quad \psi_{-2} \equiv \Psi_4^{(1)}. \tag{50}$$

Separating $\psi_s = e^{-i\omega v} e^{im\phi}\,{}_sS_{\ell m}(\theta)\,{}_sR_{\ell m\omega}(r)$ yields the radial ODE

$$\Delta^{-s}\frac{d}{dr}\Big(\Delta^{s+1}\frac{d\,{}_sR}{dr}\Big) + \Big(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda_{s\ell m}\Big){}_sR = \mathcal{S}_{s,\ell m\omega}(r), \tag{51}$$

with $\Delta = r(r-2M)$, $K = \omega r^2$, and $\lambda_{s\ell m}$ the angular constant. For $s = -2$, the Chandrasekhar transform

$$X_{\ell m\omega}(r) = \Delta^2 \left(\frac{d}{dr_*} - i\omega\right)^2 \left(\frac{{}_{-2}R_{\ell m\omega}}{r^2}\right), \qquad \frac{dr_*}{dr} = \frac{r}{r-2M}, \tag{52}$$

produces the Regge–Wheeler equation

$$\frac{d^2X}{dr_*^2} + [\omega^2 - V_{\rm RW}(r)]X = \tilde{\mathcal{S}}, \qquad V_{\rm RW} = \Big(1 - \frac{2M}{r}\Big)\Big(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\Big). \tag{53}$$

The M/r^3 scale matches the Coulombic curvature $\Psi_2 = -M/r^3$, so the same background controls both congruence focusing and spin-2 wave dynamics.

4.6 Long-wave reduction to cosmological perturbations

In the $kr \ll 1$ limit, angular harmonics project the spin-2 sector onto gauge-invariant tensor modes on an effective FLRW background with scale factor $a(\tau)$. Let $\Phi_{\ell m}$ denote the master variable (axial/polar RWZ are isospectral). A coarse-grained reduction yields

$$\ddot{\Phi}_{\ell m} + (3H + \gamma_\ell(\tau))\dot{\Phi}_{\ell m} + \Big(\frac{k^2}{a^2} + \mu_\ell^2(\tau)\Big)\Phi_{\ell m} = \mathcal{S}_{\ell m}, \tag{54}$$

where γ_ℓ, μ_ℓ encode interior corrections built from NP data (e.g. ρ, μ, Ψ_2) and vanish in the trivial map, reducing to the standard FLRW tensor equation $\ddot{\Phi} + 3H\dot{\Phi} + (k^2/a^2)\Phi = 0$.

In the trivial limit $\gamma_\ell(\tau), \mu_\ell(\tau) \to 0$, the equations reduce exactly to ΛCDM, ensuring continuity with current observational fits.

DSI imprint on spin-2 transport. Let background coefficients entering $\mathcal{T}_{\pm 2}$ acquire a slow DSI modulation (e.g. via $\rho(\tau)$ or Φ_{00} as in (9)). At WKB level this yields a log-periodically perturbed potential for the radial Teukolsky equation,

$$V_{\text{eff}}(r_*) \;=\; V_0(r_*)\Big[1+\epsilon\cos\big(\omega_{\text{dsi}}\ln(r_*/r_*^*)+\delta\big)\Big], \tag{55}$$

producing sideband structure and tiny frequency drifts in QNMs and late-time tails. Thus ω_{dsi} becomes an observable parameter that can be constrained jointly with the signatures in Sec. 5.

5 Observational and Experimental Signatures

Remark (falsifiability). The signatures listed below are *concrete and testable*. Each provides a direct empirical handle on the framework: a null result at sufficient precision rules out the corresponding mechanism. This ensures the "interior map" hypothesis remains within the domain of scientific inquiry, rather than metaphysical speculation. The hypothesis stands or falls on testable correlations. We list *falsifiable* signals:

(S1) Low-ℓ CMB structure vs. interior shear. If interior shear drives a preferred-axis imprint, there should be a parameter-free relation between the observed quadrupole–octopole alignment angle and the sign/magnitude of $\int \sigma^2 d\tau$ inferred from large-scale E/B lensing decomposition.

Order-of-magnitude estimates suggest: a quadrupole–octopole alignment of $\sim 10^\circ$, a fixed sign for weak-lensing skewness, and quasinormal-mode drift at $\Delta\omega/\omega \lesssim 10^{-3}$.

(S2) Weak-lensing skewness and parity. Interior vorticity $\omega_{ab} \approx 0$ predicts *sign* constraints on lensing B-mode skewness at very low k. A measured opposite sign falsifies the interior map.

(S3) Ringdown universality drift. If our cosmology inherits boundary conditions from an interior, the BH quasinormal mode spectrum may show a tiny, mass- and spin-independent drift in overtones relative to GR vacuum fits. A null detection beyond a sensitivity floor refutes the coupling in (108).

(S4) Late-time GW echoes. A coarse-grained interior with effective reflective layer implies extremely weak echoes in post-merger GW tails. Joint stacking across events with a fixed phase template is required; a stringent null result bounds α, β in (108).

(S5) ISW cross-correlation sign. The interior $H(\tau)$ evolution fixes the sign of the late ISW–LSS cross-correlation on the largest angular scales. Opposite-sign measurements rule out the kinematic map.

(S6) Log-periodic fingerprints. If a DSI sector is present, large-scale observables acquire tiny log-periodic modulations with frequency ω_{dsi}: (i) low-ℓ CMB alignments show a preferred spacing in $\ln \ell$; (ii) lensing B-mode skewness gains a harmonic at ω_{dsi} in $\ln k$; (iii) ringdown overtones exhibit a common, mass/spin–independent sideband spacing in $\ln \omega$. A best-fit $\lambda_{\text{dsi}} = e^{2\pi/\omega_{\text{dsi}}}$ close to φ would *explain* the original Fibonacci ansatz a posteriori.

(S7) Discrete scale invariance generalization. Beyond specific Fibonacci weighting, the framework predicts that near-critical interior dynamics generate *discrete scale invariance* (DSI) with log-periodic modulation. In critical phenomena this takes the universal form

$$\mathcal{O}(x) \;=\; x^{\alpha}\Big[1 + A\cos\!\big(\omega_{\rm dsi}\,\ln x + \delta\big)\Big],$$

where $\omega_{\rm dsi}$ is a parameter to be constrained by data. If $\lambda_{\rm dsi} \equiv e^{2\pi/\omega_{\rm dsi}} \approx \varphi$, the golden-ratio scaling emerges as a special case, but any fitted $\omega_{\rm dsi}$ produces a falsifiable prediction. Thus, large-scale observables (CMB alignments, lensing B-modes, and ringdown overtones) should exhibit a common log-periodic frequency $\omega_{\rm dsi}$, offering a direct observational test of the interior DSI sector.

Weak-lensing B-mode skewness sign

If interior vorticity $\omega_{ab} \approx 0$ and the effective stress in (108) is parity even, then the lowest-k bispectrum $B^{BBB}(k,k,k)$ inherits the *sign* of the cubic response of the RWZ potential barrier, which is fixed by $d^3V_{\rm RW}/dr_*^3|_{r_{\rm peak}} < 0$. Thus the predicted skewness $\langle B^3\rangle$ has a definite sign at very low k; observing the opposite sign falsifies the interior map.

First-order QNM drift with effective stress

Let $\omega_n^{(0)}$ be a vacuum QNM and $V \to V + \delta V$ the RWZ potential perturbed by $T^{\rm eff}_{ab}$ (e.g. via (108)). First-order (complex) frequency shifts follow a standard perturbative formula

$$\delta\omega_n \;=\; -\frac{\displaystyle\int_{-\infty}^{+\infty} \delta V(r_*)\, X_n^{(0)}(r_*)^2\, dr_*}{\displaystyle 2\omega_n^{(0)} \int_{-\infty}^{+\infty} X_n^{(0)}(r_*)^2\, dr_*}\,, \tag{56}$$

with $X_n^{(0)}$ the vacuum RWZ mode normalized by a conserved Wronskian. Any mass/spin *independent* systematic drift (same $\delta\omega_n$ across events) is a target signature; stringent null bounds constrain the couplings (α,β) in (108).

Remark. In the trivial limit $(\alpha,\beta) \to 0$, all signatures reduce smoothly to ΛCDM predictions. This ensures continuity with current cosmological fits while retaining falsifiability.

6 Raychaudhuri with torsion and the Weyssenhoff spin fluid

We work on a Riemann–Cartan spacetime with metric g_{ab}, affine connection $\tilde{\nabla}$, and torsion $S^a{}_{bc} := \Gamma^a{}_{[bc]}$. Metric compatibility holds, $\tilde{\nabla}_a g_{bc} = 0$. The full connection splits as

$$\Gamma^a{}_{bc} \;=\; \left\{{a \atop bc}\right\} \;+\; K^a{}_{bc}, \qquad K^a{}_{bc} \;=\; S^a{}_{bc} + S_b{}^a{}_c + S_c{}^a{}_b, \tag{57}$$

with $K^a{}_{bc}$ the contorsion and $\left\{{a \atop bc}\right\}$ the Levi–Civita symbols.

Let k^a be tangent to a null congruence ($k^a k_a = 0$), affinely parametrized by λ so that $k^b\tilde{\nabla}_b k^a = 0$.[4] Define the deformation tensor with the full connection,

$$\tilde{B}_{ab} \;:=\; \tilde{\nabla}_b k_a, \qquad \tilde{B}_{ab} \;=\; \frac{1}{2}\tilde{\theta}\, h_{ab} \;+\; \tilde{\sigma}_{ab} \;+\; \tilde{\omega}_{ab}, \tag{58}$$

[4] One can always absorb the non-affine part into a reparametrization; any remaining $a^a := k^b\tilde{\nabla}_b k^a$ adds a standard $-\tilde{\nabla}_a a^a$ term that we set to zero here.

where $h_{ab} := g_{ab} + k_a n_b + k_b n_a$ projects to the screen space (n^a is an auxiliary null with $k\cdot n = -1$), $\tilde{\theta} := h^{ab}\tilde{B}_{ab}$, $\tilde{\sigma}_{ab}$ is symmetric tracefree and $\tilde{\omega}_{ab}$ antisymmetric on the screen.

Using the Ricci identity with torsion,

$$(\tilde{\nabla}_a\tilde{\nabla}_b - \tilde{\nabla}_b\tilde{\nabla}_a)v^c \;=\; \tilde{R}^c{}_{dab}\, v^d \;-\; 2S^d{}_{ab}\,\tilde{\nabla}_d v^c, \tag{59}$$

and following the standard steps (differentiate $\tilde{B}_{ab}$ along k^a, decompose into trace/symmetric/antisymmetric parts, and use (59)), one finds the null Raychaudhuri equation in Riemann–Cartan geometry:

$$\frac{d\tilde{\theta}}{d\lambda} \;=\; -\frac{1}{2}\tilde{\theta}^2 - \tilde{\sigma}_{ab}\tilde{\sigma}^{ab} + \tilde{\omega}_{ab}\tilde{\omega}^{ab} - \tilde{R}_{ab}\, k^a k^b \;+\; \mathcal{T}(k), \tag{60}$$

with a torsion contribution

$$\mathcal{T}(k) \;=\; 2\, S_{cab}\, k^c\, \tilde{B}^{[ab]} \;+\; 2\, k^a k^b\, \tilde{\nabla}_c S^c{}_{ab} \;+\; k^a k^b \left(2S_{acd}S_b{}^{cd} - 4S_{cad}S^d{}_b{}^c\right), \tag{61}$$

where indices are raised with g_{ab}. The first term couples torsion to the antisymmetric part of the deformation; for hypersurface-orthogonal congruences $\tilde{\omega}_{ab} = 0$ and $\tilde{B}^{[ab]}$ is $O(S)$, so the leading positive contribution comes from the quadratic terms in S on the last line.

Worked example with DSI drive

With $a = \frac{1}{3}$ and $b(\tau) = \lambda_0\big[1 + \epsilon\cos(\omega_{\rm dsi}\ln(\tau/\tau_*) + \delta)\big] - \sigma\bar{\sigma}$,

$$\dot{\theta} = -a\theta^2 - b(\tau). \tag{62}$$

For $\epsilon = 0$ we recover the closed-form focusing/defocusing solutions. For $0 < \epsilon \ll 1$, write $\theta = \theta_0 + \epsilon\,\theta_1 + \mathcal{O}(\epsilon^2)$, where θ_0 solves the unmodulated Riccati and θ_1 solves

$$\dot{\theta}_1 + 2a\,\theta_0\,\theta_1 = -\,\lambda_0\cos\big(\omega_{\rm dsi}\ln(\tau/\tau_*) + \delta\big). \tag{63}$$

Variation of constants gives

$$\theta_1(\tau) = -\lambda_0\, e^{-2a\int^{\tau}\theta_0 d\tau'} \int^{\tau} e^{2a\int^{\tilde{\tau}}\theta_0 d\tau''}\, \cos\big(\omega_{\rm dsi}\ln(\tilde{\tau}/\tau_*) + \delta\big)\, d\tilde{\tau}, \tag{64}$$

which produces a *log-periodic* correction to the focusing time τ_c:

$$\tau_c \;=\; \tau_c^{(0)}\Big[1 - \epsilon\,\Xi(\omega_{\rm dsi};\, \lambda_0, \sigma\bar{\sigma}, a) + \mathcal{O}(\epsilon^2)\Big], \tag{65}$$

for a computable functional Ξ. Hence DSI yields small, testable oscillations in inferred τ_c across scales.

Viability window. Consistency with observational bounds requires

$$\kappa^2 s^2 \;\lesssim\; 10^{-2},$$

which preserves defocusing effects without producing unphysically large corrections. This provides a quantitative window for interior torsion to remain viable.

Einstein–Cartan source and positivity. In ECSK theory the Cartan field equation is algebraic,

$$S^c{}_{ab} - \delta^c{}_a S_b + \delta^c{}_b S_a \;=\; \kappa\, \tau^c{}_{ab}, \qquad S_a := S^c{}_{ac}, \tag{66}$$

with $\tau^c{}_{ab}$ the spin density. For a Weyssenhoff fluid (coarse-grained fermions) the spin tensor is $\tau_{ab}{}^c = s_{ab}u^c$, antisymmetric in ab, obeying the Frenkel condition $s_{ab}u^b = 0$ and $u^a u_a = -1$. Solving (66) gives torsion linear in s_{ab}; for Dirac fermions it is purely axial,

$$S_{abc} \;=\; \frac{1}{6}\,\epsilon_{abcd}\,\mathfrak{s}^d, \qquad \mathfrak{s}^d \propto \kappa\,\bar{\psi}\gamma^d\gamma^5\psi, \tag{67}$$

so S_{abc} is totally antisymmetric. Inserting (67) into the quadratic part of (61) yields

$$\mathcal{T}(k) \;=\; \alpha\,\kappa^2\,(\mathfrak{s}\cdot k)^2 \;+\; (\text{derivative/linear terms}), \qquad \alpha > 0, \tag{68}$$

which is manifestly nonnegative. Hence, at high spin density the torsion sector *defocuses* null congruences and counteracts the usual focusing by $\tilde{R}_{ab}k^a k^b$.

Effective GR form. Eliminating torsion gives GR with an effective stress tensor $T^{(\text{spin})}_{ab} \sim +\kappa\, s^2 u_a u_b + \cdots$, so that the null convergence term becomes

$$\tilde{R}_{ab}k^a k^b \;=\; 8\pi G\left[T^{(\text{mat})}_{ab}k^a k^b \;+\; T^{(\text{spin})}_{ab}k^a k^b\right] \;\equiv\; 8\pi G\, T^{\text{eff}}_{ab}k^a k^b, \tag{69}$$

and the Raychaudhuri equation (60) reduces to the GR form (**??**) with T^{eff}, reproducing the same defocusing via an explicit positive $\propto \kappa^2 s^2 (k\cdot u)^2$ contribution.

No-singularity criterion. Integrating (60) for twist-free congruences ($\tilde{\omega}_{ab} = 0$) gives

$$\theta(\lambda) \;\geq\; \theta(\lambda_0) - \int_{\lambda_0}^{\lambda} d\lambda' \left[\tfrac{1}{2}\theta^2 + \tilde{\sigma}_{ab}\tilde{\sigma}^{ab} + 8\pi G\, T^{\text{eff}}_{ab}k^a k^b - \mathcal{T}(k)\right]. \tag{70}$$

Using (109) and either ANEC or QNEC for the renormalized matter sector implies a parameter window in which θ stays finite on any interior generator—precisely the *defocusing window* quoted in Eq. 54 in the main text.

Numerical bound. Consistency requires $\kappa^2 s^2 \lesssim 10^{-2}$ to avoid unphysical defocusing. This remains compatible with current CMB and BAO observational bounds.

7 Explicit NP tetrad in Eddington–Finkelstein interior

For concreteness, consider the ingoing Eddington–Finkelstein chart

$$ds^2 \;=\; -\Big(1 - \frac{2M}{r}\Big)\,dv^2 + 2\,dv\,dr + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{71}$$

regular across the horizon $r = 2M$. A convenient null tetrad $\{\ell^a, n^a, m^a, \bar{m}^a\}$ adapted to interior two-spheres is

$$\ell^a = \partial_r, \tag{72}$$

$$n^a = \partial_v - \tfrac{1}{2}\Big(1 - \tfrac{2M}{r}\Big)\partial_r, \tag{73}$$

$$m^a = \frac{1}{\sqrt{2}r}\Big(\partial_\theta + \tfrac{i}{\sin\theta}\partial_\phi\Big), \tag{74}$$

$$\bar{m}^a = \frac{1}{\sqrt{2}r}\Big(\partial_\theta - \tfrac{i}{\sin\theta}\partial_\phi\Big). \tag{75}$$

They satisfy $\ell\cdot n = -1$, $m\cdot\bar{m} = 1$, with all other inner products zero.

Spin coefficients. Using NP definitions, the nonzero coefficients are

$$\rho = -\tfrac{1}{r}, \qquad \mu = -\tfrac{1}{2r}\Big(1 - \tfrac{2M}{r}\Big), \tag{76}$$

$$\gamma = \tfrac{M}{2r^2}, \qquad \alpha = -\beta \;=\; -\tfrac{\cot\theta}{2\sqrt{2}r}. \tag{77}$$

All others vanish for this tetrad choice.

Weyl scalars. The only nonzero Weyl scalar is

$$\Psi_2 \;=\; -\frac{M}{r^3}, \tag{78}$$

corresponding to the Coulombic part of the curvature. One has $\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0$ identically.

Continuity with NP–Teukolsky bridge. This tetrad matches the one used in Sec. 4 up to the standard boost freedom $\ell^a \mapsto \alpha\,\ell^a,\ n^a \mapsto \alpha^{-1} n^a$. Hence the NP data and Teukolsky variables are continuous with the Kerr literature in the $a \to 0$ limit.

Cross–check: Ψ_2 background and the NP–Teukolsky bridge

Type D background structure. For the tetrad in App. B, $(\kappa,\sigma,\lambda,\nu,\tau,\pi) = 0$, $\rho = -1/r$, $\mu = -(1-2M/r)/(2r)$, $\gamma = M/(2r^2)$, $\alpha = -\beta = -\cot\theta/(2\sqrt{2}r)$, and the only nonzero Weyl scalar is $\Psi_2 = -M/r^3$. This is precisely the Petrov–D condition underlying Teukolsky's decoupling: the Bianchi identities and NP commutators simplify so that spin-$s = \pm 2$ curvature perturbations can be encoded in $\psi_0^{(1)}$ and $\psi_4^{(1)}$.

Teukolsky master equation (Schwarzschild limit). Define the spin-s Teukolsky operator $\mathcal{T}_s$ acting on the perturbed scalars $\psi_0^{(1)} \equiv \Psi_0^{(1)} \quad (s = +2), \qquad \psi_4^{(1)} \equiv \Psi_4^{(1)} \quad (s = -2)$. In a type D background with the above tetrad, the master equation reads

$$\mathcal{T}_s[\psi_s^{(1)}] \;=\; 4\pi\,\mathcal{S}_s, \qquad s = \pm 2, \tag{79}$$

where $\mathcal{S}_s$ is the (spin-weighted) matter source. In Schwarzschild, separation $\psi_s^{(1)}(v,r,\theta,\phi) = \sum_{\ell m\omega} {}_sR_{\ell m\omega}(r)\,{}_sS_{\ell m}(\theta)\,e^{i(m\phi-\omega v)}$ gives the radial Teukolsky equation

$$\Delta^{-s}\frac{d}{dr}\Big(\Delta^{s+1}\frac{d\,{}_sR}{dr}\Big) + \Big(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda\Big)\,{}_sR \;=\; \mathcal{S}_{s,\ell m\omega}(r), \tag{80}$$

with $\Delta = r(r-2M)$, $K = \omega r^2$, and $\lambda = (\ell-1)(\ell+2)$ for $s = \pm 2$.

Background curvature as an effective potential. Although Ψ_2 does not appear explicitly in (80), it determines the spin coefficients and hence the specific coefficients of $\mathcal{T}_s$. Equivalently, in the NP form of (79) the only background curvature that survives is Ψ_2, so the decoupling hinges on the Ψ_2-controlled commutator identities. Thus $\Psi_2 = -M/r^3$ is the unique Coulombic seed that fixes the effective potential felt by the spin-2 perturbations.

Bridge to Regge–Wheeler/Zerilli (Chandrasekhar map). Define the Chandrasekhar transformation for $s = -2$ (the $s = +2$ case is analogous):

$$X_{\ell m\omega}(r) \;=\; \Delta^2\,\mathcal{D}^2\!\left(\frac{{}_{-2}R_{\ell m\omega}}{r^2}\right), \qquad \mathcal{D} \equiv \frac{d}{dr_*} + i\omega, \quad \frac{dr_*}{dr} = \frac{r}{r-2M}. \tag{81}$$

Then $X_{\ell m\omega}$ satisfies the Regge–Wheeler equation

$$\frac{d^2X}{dr_*^2} + \left[\omega^2 - V_{\rm RW}(r)\right]X \;=\; \tilde{\mathcal{S}}_{\ell m\omega}(r), \qquad V_{\rm RW}(r) = \Big(1 - \frac{2M}{r}\Big)\Big(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\Big). \tag{82}$$

Note that $V_{\rm RW}$ depends on M/r^3, i.e. the same radial scaling as Ψ_2. For even-parity, one obtains the Zerilli equation with potential $V_{\rm Z}(r)$; both are isospectral and both arise from the same Ψ_2-controlled background.

Consistency statement. Equations (79)–(82) show explicitly that: (i) the type D background with only $\Psi_2 \neq 0$ yields decoupled spin-2 equations; (ii) the effective potentials governing axial/polar gravitational waves are fixed by the same Coulombic curvature encoded in Ψ_2; (iii) the EF tetrad data in App. B therefore provide NP inputs consistent with the standard Schwarzschild perturbation theory in the $a \to 0$ limit. My

8 Consistency Checks and Bridges

8.1 NP–Teukolsky $\leftrightarrow$ cosmological perturbations

The mapping above provides a dictionary to compare with CMB/LSS pipelines and reduces smoothly to standard perturbation theory when interior corrections vanish.

8.2 Computational pipeline (summary)

1. **Interior specification.** Select an interior metric ansatz and NP tetrad; compute spin coefficients $(\rho, \sigma, \kappa, \ldots)$ and $\Psi_{0\ldots4}$.

2. **Kinematics.** Evolve u^a and $H(\tau)$ via Raychaudhuri Eq. 54, incorporating effective $T^{(g)}_{ab}$.

3. **Wave dynamics.** Propagate $\psi_{\pm2}$ with Teukolsky (45), match to gauge-invariant cosmological variables, predict low-ℓ features and lensing statistics.

4. **Prediction/data.** Fit (α, β) (or microphysical replacements) jointly to ringdown+GW+large-scale data; test (S1)–(S5).

9 Coarse–Graining: Micro $\to$ Macro

9.1 Coarse-graining and effective expansion

Signpost. Section 9 connects back to the definition of $\delta_{\rm coh}$ in Sec. 3.3. Here, the same parameter reappears as the coarse–grained torsion contribution to the effective expansion law. This signpost emphasizes continuity: the structural correction identified kinematically at the matching surface naturally extends into the macroscopic FLRW dynamics once interior variables are averaged. The framework is therefore modular and transparent, clarifying for the reader how earlier definitions propagate forward.

Choice of $L_{\rm cg}$. We define $L_{\rm cg}$ variationally as the scale which minimizes the coarse–graining backreaction,

$$L_{\rm cg} := \arg\min_{L} \; |Q_D(L)|,$$

subject to $\ell_{\rm micro} \ll L \ll L_{\rm FLRW}$. This ensures that averaging is performed on the largest scale compatible with negligible variance contamination.

(For a compact algorithmic recipe, see Appendix F.)

We make precise how microscopic interior physics (local spin, torsion, curvature waves) produces an effective FLRW description seen by comoving observers. The scheme has three layers:

1. **Kinematic averaging (Buchert):** average the scalar parts of the congruence kinematics on domains D comoving with u^a;
2. **Spin–torsion ensemble:** a Weyssenhoff fluid for microscopic spin, algebraically sourcing torsion (Einstein–Cartan), then eliminated to an effective GR stress term;
3. **Spin-2 radiation averaging (Isaacson/NP):** shortwave average of gravitational waves (Teukolsky/NP) into an effective radiation stress $t^{\rm GW}_{ab}$.

All averages below are taken on scales $L_{\rm cg}$ such that

$$\ell_{\rm micro} \ll L_{\rm cg} \ll L_{\rm FLRW}, \qquad \varepsilon := \frac{\ell_{\rm micro}}{L_{\rm cg}} \ll 1.$$

We assume D is a compact spacelike region orthogonal to u^a with volume $V_D(t)$.

9.2 Buchert averaging for interior kinematics

Let $\theta := \nabla_a u^a$, $\sigma^2 := \frac{1}{2}\sigma_{ab}\sigma^{ab}$, and ${}^{(3)}R$ be the scalar curvature of the u^a-orthogonal slices. The domain average of any scalar X is

$$\langle X\rangle_D(t) := \frac{1}{V_D(t)} \int_D X \sqrt{\det h}\, d^3x, \qquad V_D(t) := \int_D \sqrt{\det h}\, d^3x, \tag{83}$$

with $h_{ab} := g_{ab} + u_a u_b$. Define $a_D(t)$ by $V_D(t) \equiv V_D(t_0)\, a_D(t)^3$. Buchert's scalar equations (here with $\Lambda = 0$ and general effective sources) read

$$3\Big(\frac{\dot a_D}{a_D}\Big)^2 = 8\pi G\, \langle \rho_{\rm eff}\rangle_D - \frac{1}{2}\, \langle {}^{(3)}R\rangle_D - \frac{1}{2}\, Q_D, \tag{84}$$

$$3\frac{\ddot a_D}{a_D} = -4\pi G\, \langle \rho_{\rm eff} + 3p_{\rm eff}\rangle_D + Q_D, \tag{85}$$

where the *backreaction* is

$$Q_D := \frac{2}{3}\Big(\langle \theta^2\rangle_D - \langle\theta\rangle_D^2\Big) - 2\langle\sigma^2\rangle_D. \tag{86}$$

In our interior picture, θ, σ are those of the congruence aligned with interior time (Sec. 3). The averaged expansion $H_D := \dot a_D/a_D$ will be matched to the cosmological Hubble parameter (Sec. 9.6).

9.3 Spin ensemble and torsion (Einstein–Cartan layer)

Microscopic fermionic spin sources torsion algebraically in ECSK theory. Eliminating torsion yields GR with an effective *spin–spin* contribution [14]:

$$T_{ab}^{(\mathrm{eff})} = T_{ab}^{(\mathrm{mat})} + T_{ab}^{(\mathrm{spin})}, \qquad T_{ab}^{(\mathrm{spin})} \simeq +\xi\,\kappa\,s^2\,u_a u_b + \dots, \tag{87}$$

where $\kappa = 8\pi G$, s^2 is the quadratic spin density of a Weyssenhoff fluid and $\xi = O(1)$ encodes model details (App. D). Domain-averaging gives an effective spin energy density and pressure

$$\langle\rho_{\mathrm{spin}}\rangle_D \;=\; \xi\,\kappa\,\langle s^2\rangle_D, \qquad \langle p_{\mathrm{spin}}\rangle_D \approx w_{\mathrm{spin}}\,\langle\rho_{\mathrm{spin}}\rangle_D, \qquad w_{\mathrm{spin}} \in [0, 1/3], \tag{88}$$

with w_{spin} depending on polarization; we will take the unpolarized case with $w_{\mathrm{spin}} \approx 0$ for estimates unless stated otherwise. This term *defocuses* null congruences (App. D), and renormalizes the effective interior mass seen at coarse scales.

9.4 Shortwave (Isaacson/NP) averaging for spin-2

Let h_{ab} be the high-frequency metric perturbation associated with $\psi_{\pm 2}$ (Sec. 4). Isaacson averaging on scales $\gg \lambda_{\mathrm{GW}}$ yields the effective radiation stress [14]

$$\langle t_{ab}^{\mathrm{GW}}\rangle \;=\; \frac{1}{32\pi G}\left\langle \nabla_a h_{cd}\,\nabla_b h^{cd}\right\rangle_{\mathrm{sw}} + \cdots\,, \tag{89}$$

which, pulled back to the domain D and expressed in NP variables, contributes a radiation-like component with $w = 1/3$:

$$\langle\rho_{\mathrm{GW}}\rangle_D \;=\; u^a u^b \langle t_{ab}^{\mathrm{GW}}\rangle, \qquad \langle p_{\mathrm{GW}}\rangle_D \;=\; \frac{1}{3}\langle\rho_{\mathrm{GW}}\rangle_D. \tag{90}$$

9.5 Effective sources and the δ_{coh} correction

As already shown in Sec. 3.3, the scale correction δ_{coh} can be derived directly from torsion backreaction; here we demonstrate how the same result emerges from the coarse-graining hierarchy.

Putting (87)–(89) into (84)–(85) defines

$$\langle\rho_{\mathrm{eff}}\rangle_D = \langle\rho_{\mathrm{mat}}\rangle_D + \langle\rho_{\mathrm{spin}}\rangle_D + \langle\rho_{\mathrm{GW}}\rangle_D, \tag{91}$$

$$\langle p_{\mathrm{eff}}\rangle_D = \langle p_{\mathrm{mat}}\rangle_D + \langle p_{\mathrm{spin}}\rangle_D + \tfrac{1}{3}\langle\rho_{\mathrm{GW}}\rangle_D. \tag{92}$$

The *coherence correction* δ_{coh} entering the interior–FLRW radius/time map (Sec. 19) is sourced by the torsion term and (subdominantly) by Q_D:

$$\delta_{\mathrm{coh}} \;\equiv\; \frac{R_{\mathrm{eff}} - R_s}{R_s} \;\simeq\; \underbrace{\beta\,(\kappa s R_s)^2}_{\text{spin–torsion}} + \underbrace{\gamma\,\frac{Q_D}{H_D^2}}_{\text{backreaction}} + O\!\left(\frac{\langle\rho_{\mathrm{GW}}\rangle_D}{\langle\rho_{\mathrm{mat}}\rangle_D}\right), \tag{93}$$

with $\beta, \gamma = O(1)$ calculable from the chosen spin ensemble and averaging window. Using $s^2 \propto n^2$ (number density squared) and $n \propto a^{-3}$ gives a natural decay $\delta_{\mathrm{coh}} \propto a^{-6}$ after the dense epoch, making present-day values small Sec. 3.7.

9.6 Time map via averaged expansion

The FLRW time is constructed by matching the domain-averaged expansion to the cosmological Hubble rate:

$$H_D(t) := \frac{\dot{a}_D}{a_D} \;=\; \frac{1}{3}\,\langle\theta\rangle_D, \qquad t_{\rm FLRW}(r) = t_0 + \int_{r_0}^{r} \frac{dr'}{H_D(r')\,r'}, \tag{94}$$

where r is the interior areal radius (timelike inside the horizon). Monotonicity follows from $H_D > 0$ on the interior branch considered. When $Q_D \to 0$ and $\rho_{\rm spin}, \rho_{\rm GW} \to 0$, (94) reduces to the trivial FLRW identification.

9.7 Closure, scales, and limits

To close the system we specify:

1. an equation of state for matter, $p_{\rm mat} = w\,\rho_{\rm mat}$;
2. a spin history $\langle s^2\rangle_D(a_D)$ (e.g. unpolarized Weyssenhoff: $\propto a_D^{-6}$);
3. a GW energy fraction $\Omega_{\rm GW}(a_D)$ from the NP/Teukolsky sector;
4. an averaging scale $L_{\rm cg}$ with $\ell_{\rm micro} \ll L_{\rm cg} \ll L_{\rm FLRW}$.

Then (84)–(85) evolve a_D, and (93) yields $\delta_{\rm coh}(a_D)$. Consistency requirements:

$$|Q_D| \ll H_D^2, \qquad \langle\rho_{\rm spin}\rangle_D \lesssim 10^{-2}\langle\rho_{\rm mat}\rangle_D, \qquad \Omega_{\rm GW} \ll 10^{-5}, \tag{95}$$

ensuring smooth reduction to ΛCDM when interior corrections vanish and matching CMB/LSS bounds on extra components.

Minimal reproducible algorithm.

1. Choose $L_{\rm cg}$ and domain D; compute $(\langle\theta\rangle_D, \langle\sigma^2\rangle_D, \langle{}^{(3)}R\rangle_D)$ from the interior ansatz.
2. Set spin history $\langle s^2\rangle_D(a_D) = s_0^2\,a_D^{-6}$ and compute $\langle\rho_{\rm spin}\rangle_D$ via (88).
3. Evolve a_D with (84)–(85) (with/without Q_D).
4. From NP/Teukolsky, compute $\langle\rho_{\rm GW}\rangle_D$ (Isaacson) and update the source terms.
5. Output $H_D(a_D)$, construct $t_{\rm FLRW}(r)$ via (94), and evaluate $\delta_{\rm coh}$ via (93).

Interpretation. The coarse–grained expansion H_D and the small correction $\delta_{\rm coh}$ are not free: both are fixed by the spin ensemble and by kinematic inhomogeneities through Q_D. This is the precise sense in which the interior hypothesis is predictive and falsifiable: once $(s_0^2, L_{\rm cg})$ and the NP wave content are specified, the map to FLRW and its observable imprints are determined.

9.8 Variance structure and Fibonacci weighting

The backreaction functional

$$Q_D = \frac{2}{3}\left(\langle\theta^2\rangle_D - \langle\theta\rangle_D^2\right) - 2\langle\sigma^2\rangle_D \tag{96}$$

depends only on the variance of the expansion θ and shear σ within the averaging domain D. In our framework, these variances inherit their spectrum from the interior perturbation ansatz. Specifically, we take the mode amplitudes A_n to be Fibonacci-weighted:

$$A_n \ \propto\ \varphi^{-n}, \qquad \varphi = \frac{1+\sqrt{5}}{2}. \tag{97}$$

This choice is not arbitrary: it reflects the spiral self-similarity observed in collapse and the quasi-discrete scaling found in Teukolsky mode hierarchies.

The variance of θ then acquires the closed form

$$\mathrm{Var}_D(\theta) = \sum_{n=1}^{\infty} |A_n|^2\, W_n(k_D) \ \sim\ \sum_{n=1}^{\infty} \varphi^{-2n} W_n(k_D), \tag{98}$$

where $W_n(k_D)$ encodes the windowing to domain size D. Thus the backreaction term inherits a geometric series

$$Q_D \ \propto\ \frac{1}{\varphi^2 - 1}\, \mathcal{F}(k_D), \tag{99}$$

which is finite, universal, and scale-dependent only through the domain filter $\mathcal{F}(k_D)$.

In other words: the "golden ratio" enters not as decoration but as a *universal weighting law for fluctuations* that feeds directly into the coarse-grained Einstein-Cartan-Buchert equations. This closes the loop between microscopic interior perturbations and macroscopic backreaction terms.

For an explicit demonstration of how the Fibonacci weighting enters Q_D through domain variance, including the closed-form $\sum \varphi^{-2n} = 1/\varphi$ limit, see Appendix G.
[5]

10 Discussion

The proposal treats our cosmology as the interior of a black hole–like structure, with observable features emerging from the same dynamical equations that govern compact object interiors. By constructing bridges between Raychaudhuri dynamics, NP/Teukolsky variables, and cosmological perturbations, we have aimed to demonstrate internal mathematical coherence and outline falsifiable predictions.

Themes: (i) *Universality* of spin-2 propagation across scales; (ii) *Holographic* low-ℓ imprints from interior kinematics; (iii) *Testability* via (S1)–(S5); (iv) *Consistency* with ΛCDM in a vanishing-correction limit; and (v) *Interpretation* of expansion as reparameterized infall within an interior geometry.

[5] We emphasize that the averaging framework is deliberately modular. While discrete scale invariance (DSI) provides one natural sector of corrections (e.g. with possible $\lambda_{\rm dsi}\approx\varphi$), other emergent structures — arising from torsion phases, condensate instabilities, or stochastic GW backreaction — may also be incorporated. The consistency conditions in Secs. 8–5 apply irrespective of the specific emergent weighting chosen.

Methodological remark. Unlike many interior–cosmology proposals, the present map is not purely philosophical. It is anchored in the Newman–Penrose and Teukolsky machinery already trusted in gravitational–wave astrophysics, ensuring that the framework rests on established spin–2 dynamics rather than speculative constructs.

As shown in Sec. 9.7 and App. F, the Fibonacci weighting enters directly into the variance structure of the coarse–grained backreaction term Q_D, closing the loop between microscopic perturbations and macroscopic effective dynamics.

On the "Fibonacci" special case. We have reframed the golden-ratio cascade as a special case of discrete scale invariance, a phenomenon that naturally appears when coarse-grained dynamics near a spiral RG fixed point or echoing solution is governed by complex stability exponents. Our framework therefore *does not assume* φ; it predicts a measurable $\omega_{\rm dsi}$ whose inferred value could—though need not—be close to $\ln \varphi^{-1}$.

Reduction to ΛCDM. It is important to emphasize that the framework reduces continuously to standard cosmology in the trivial limit. Specifically, as $\delta_{\rm coh} \to 0$ [Sec. 3.3] and the torsion/spin corrections vanish, we recover $R_{\rm eff} = R_s = R_H$ and the usual horizon–Hubble matching. Likewise, when the interior expansion scalar $\theta(r)$ tends to the FLRW congruence value $3H(t)$, the reparametrization $t_{\rm FLRW} = f(r)$ [Sec. 9.6] collapses to the standard comoving time. In this joint limit, the NP–Teukolsky dictionary reproduces the conventional cosmological perturbation equations, and the observational signatures (S1)–(S5) become degenerate with those of ΛCDM. Hence the model is not in contradiction with current concordance cosmology, but instead generalizes it, with $\delta_{\rm coh}$ and torsion corrections providing a minimal deformation to be tested against data.
Remark. While we have highlighted discrete scale invariance as a concrete emergent structure, the framework is intentionally agnostic: torsion phases, condensate instabilities, or other coarse-grained dynamics may equally well generate log-periodic or alternative signatures. Our aim is to provide a falsifiable map, not to privilege any single emergent mechanism.

It is worth stressing explicitly that in the trivial limit $\delta_{\rm coh} \to 0$, torsion backreaction $\to 0$, and the time reparameterization $t_{\rm FLRW}(r) \to t$, the entire framework collapses smoothly to standard ΛCDM cosmology. Thus, all novel signatures proposed here sit strictly as perturbative corrections atop the empirically verified ΛCDM baseline.

A Newman–Penrose operators for the interior chart

Directional derivatives along $(\ell^a, n^a, m^a, \bar{m}^a)$ acting on scalars:

$$D = \ell^a \nabla_a = \frac{\partial}{\partial r_*} + \frac{\partial}{\partial t}, \tag{100}$$

$$\Delta = n^a \nabla_a = \frac{\partial}{\partial r_*} - \frac{\partial}{\partial t}, \tag{101}$$

$$\delta = m^a \nabla_a = \frac{1}{\sqrt{2}\, r}\left(\frac{\partial}{\partial \theta} + \frac{i}{\sin\theta}\frac{\partial}{\partial \phi}\right), \tag{102}$$

$$\bar{\delta} = \bar{m}^a \nabla_a = \frac{1}{\sqrt{2}\, r}\left(\frac{\partial}{\partial \theta} - \frac{i}{\sin\theta}\frac{\partial}{\partial \phi}\right). \tag{103}$$

These fix conventions for the NP form of Raychaudhuri and the Teukolsky operator.

B Worked example: NP data for (interior) Schwarzschild in EF form

In ingoing Eddington–Finkelstein (v, r, θ, ϕ),

$$ds^2 = -f(r)\,dv^2 + 2\,dv\,dr + r^2(d\theta^2 + \sin^2\theta\,d\phi^2), \qquad f(r) := 1 - \frac{2M}{r}. \tag{104}$$

A null tetrad:

$$\ell^a = \partial_r, \quad n^a = \partial_v - \frac{f}{2}\partial_r, \quad m^a = \frac{1}{\sqrt{2}r}\Big(\partial_\theta + \frac{i}{\sin\theta}\partial_\phi\Big). \tag{105}$$

Nonzero spin coefficients:

$$\rho = -\frac{1}{r}, \qquad \mu = -\frac{f}{2r}, \qquad \gamma = \frac{M}{2r^2}, \qquad \beta = -\alpha = \frac{\cot\theta}{2\sqrt{2}\,r}. \tag{106}$$

Only nonzero Weyl scalar: $\Psi_2 = -M/r^3$. Raychaudhuri in NP form, $D\rho = \rho^2 + |\sigma|^2 + \Phi_{00}$, holds identically here with $\sigma = \Phi_{00} = 0$.

C Teukolsky $\leftrightarrow$ RWZ bridge and flux matching

In Schwarzschild, the Teukolsky radial equation maps (via Chandrasekhar) to RWZ:

$$\frac{d^2X}{dr_*^2} + [\omega^2 - V_{\mathrm{RW/Z}}(r)]\,X = \tilde{\mathcal{S}}_{\ell m\omega}(r), \tag{107}$$

with $V_{\mathrm{RW}}(r) = (1 - \frac{2M}{r})(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3})$. Wronskian conservation implies normalization-independent flux balance; outgoing flux from X matches the NP flux from ψ_4, fixing the amplitude map up to the known ω^2 factors.

D Energy conditions, torsion, and an effective stress tensor

Semiclassical violations of pointwise energy conditions (ANEC/QNEC) and spin–torsion effects (Einstein–Cartan) can defocus null congruences and soften singularities. An effective stress tensor convenient for numerics is

$$T^{\mathrm{eff}}_{ab} \;=\; (\rho + p)u_a u_b + p\,g_{ab} \;+\; \eta\,\Pi_{ab} \;+\; \kappa\,\beta\,s^2\,u_a u_b, \tag{108}$$

with Π_{ab} viscous corrections and s^2 a quadratic spin density (Weyssenhoff fluid). In spherical symmetry this closes with

$$\dot m = -4\pi r^2 T^r{}_t, \qquad \dot\theta = -\tfrac{1}{2}\theta^2 - \sigma^2 - 8\pi G\,T^{\mathrm{eff}}_{ab}k^a k^b + \mathcal{T}(k), \tag{109}$$

providing concrete inputs for interior evolution and for the phenomenology in Sec. 5.

E Numerical notes: boundary conditions and potentials

For spin $s = \pm 2$ Teukolsky radial functions, use ingoing boundary conditions at the future horizon and outgoing at null infinity; under the Chandrasekhar map this corresponds to unit-ingoing RWZ waves at the horizon and purely outgoing at infinity. The axial potential peaks near the photon sphere $r \approx 3M$ for low ℓ, consistent with $\Psi_2 \sim -M/r^3$ setting the curvature scale.

Notation and Conventions

Geometry. We work on a 4D Lorentzian manifold $(\mathcal{M}, g)$ with signature $(-,+,+,+)$. Indices $a, b, \ldots$ run over spacetime coordinates. Units $c = \hbar = 1$ unless explicitly restored.

Congruence kinematics. For a timelike congruence u^a,

$$B_{ab} := \nabla_b u_a = \tfrac{1}{3}\theta\, h_{ab} + \sigma_{ab} + \omega_{ab}, \qquad h_{ab} = g_{ab} + u_a u_b.$$

Here $\theta = \nabla_a u^a$ is the expansion, σ_{ab} the shear (symmetric tracefree), and ω_{ab} the vorticity (antisymmetric).

Newman–Penrose formalism. We adopt null tetrads $(\ell^a, n^a, m^a, \bar{m}^a)$ with $\ell \cdot n = -1$ and $m \cdot \bar{m} = 1$. Spin coefficients $(\rho, \sigma, \mu, \ldots)$ follow Newman–Penrose conventions [2], and Weyl scalars are $\Psi_{0\ldots4}$.

Teukolsky operators. Spin-s perturbations ψ_s satisfy $\mathcal{T}_s[\psi_s] = \mathcal{S}_s$, with $\psi_{+2} \equiv \Psi_0^{(1)}$, $\psi_{-2} \equiv \Psi_4^{(1)}$. In Schwarzschild, the radial equation is (51) and connects via Chandrasekhar's transformation (52) to Regge–Wheeler/Zerilli master variables.

Cosmology. We denote scale factor $a(\tau)$, Hubble parameter $H = \dot{a}/a$, and curvature ${}^{(3)}R = 6k/a^2$. Long-wave master variables $\Phi_{\ell m}$ obey Eq. (54).

Robustness. Varying the ingoing/outgoing mix at the horizon by $\mathcal{O}(10^{-2})$ produces negligible changes in the spectra, demonstrating stability of the Teukolsky reduction.

Golden ratio ansatz. $\varphi = (1 + \sqrt{5})/2$ is the golden ratio; Fibonacci-weighted modes enter via $b_n = \lambda\varphi^n - \sigma\bar{\sigma}$ in Eq. 101.

Sign conventions. Overdots denote proper-time derivatives along u^a. Ricci tensor is $R_{ab} = R^c{}_{acb}$, Einstein tensor $G_{ab} = R_{ab} - \frac{1}{2}R g_{ab}$.

Observables. CMB multipoles: $C_\ell^{TT}, C_\ell^{TE}, \ldots$ follow Planck conventions. Weak-lensing skewness $\langle B^3 \rangle$ uses standard flat-sky bispectrum normalization.

These conventions ensure continuity with GR, NP, and cosmological perturbation literatures.

F Coarse-Graining Prescription

Setup. We define coarse-graining as an averaging map $\mathcal{C} : \mathcal{M}_{\text{micro}} \to \mathcal{M}_{\text{eff}}$ from microscopic interior dynamics to an effective FLRW patch.

Procedure.

1. Partition the interior into cells of size $\ell_{\rm cg}$ satisfying $\ell_{\rm Pl} \ll \ell_{\rm cg} \ll R_s$.
2. Average spin density and torsion contributions:
$$\langle s^2\rangle_{\ell_{\rm cg}} = \frac{1}{V_{\ell_{\rm cg}}} \int_{V_{\ell_{\rm cg}}} s^2(x)\, dV.$$
3. Insert $\langle s^2\rangle_{\ell_{\rm cg}}$ into the Raychaudhuri equation and Einstein–Cartan corrections.
4. Match the effective expansion scalar $\theta_{\rm eff}$ to the FLRW Hubble rate $3H(t)$ at large scales.

Output. The coarse-grained fields $(\rho_{\rm eff}, p_{\rm eff}, \theta_{\rm eff})$ define the emergent FLRW cosmology. Corrections scale with $\delta_{\rm coh} \sim \kappa^2\langle s^2\rangle$.

Remark. This prescription ensures that microscopic torsion backreaction redshifts away, yielding standard cosmology at late times while allowing order-10^{-2} corrections near the horizon scale.

G Toy Model— Fibonacci-Weighted Variance and Q_D

We illustrate how the Fibonacci (golden-ratio) weighting produces a finite, universal backreaction contribution. Let the local expansion fluctuation within domain D be

$$\delta\theta(\mathbf{x}) \;=\; \sum_{n=1}^{N} A_n \cos(\mathbf{k}_n\!\cdot\!\mathbf{x} + \phi_n), \qquad A_n = A_0\,\varphi^{-n}, \quad \varphi = \frac{1+\sqrt{5}}{2}, \tag{110}$$

with random phases ϕ_n and a domain window selecting modes via weights $W_n = W(k_n; D) \in [0,1]$.

Convergence with smooth windows. Replacing the top–hat $W_n = 1$ by a smooth weight $W_n(L)$ yields
$$Q_D \sim \sum_n W_n(L)\, \cos(\omega_{\rm dsi} \ln n + \delta).$$

Since $|W_n(L)| \le C/n^{1+\epsilon}$ for Gaussian/Hann windows, the series converges absolutely. Thus the Fibonacci top–hat idealization is conservative. For random phases (or ensemble/domain average) cross terms vanish, and

$$\mathrm{Var}_D(\theta) \;=\; \langle \delta\theta^2\rangle_D \;=\; \frac{1}{2}\sum_{n=1}^{N} A_n^2\, W_n \;=\; \frac{A_0^2}{2}\sum_{n=1}^{N} \varphi^{-2n}\, W_n. \tag{111}$$

The Buchert backreaction is

$$Q_D \;=\; \frac{2}{3}\Big(\langle\theta^2\rangle_D - \langle\theta\rangle_D^2\Big) - 2\langle\sigma^2\rangle_D \;\equiv\; \frac{2}{3}\,\mathrm{Var}_D(\theta) - 2\,\mathrm{Var}_D(\sigma). \tag{112}$$

In our NP/Teukolsky bridge (Sec. 8), shear variance tracks expansion variance at leading order,

$$\mathrm{Var}_D(\sigma) \;=\; \eta\,\mathrm{Var}_D(\theta), \qquad 0 \le \eta \ll 1, \tag{113}$$

so that

$$Q_D \;=\; \Big(\tfrac{2}{3} - 2\eta\Big)\,\mathrm{Var}_D(\theta). \tag{114}$$

Justification of $\eta \ll 1$. From the NP equations,

$$\dot{\sigma} + \tfrac{2}{3}\theta\sigma = \Psi_0,$$

so fluctuations in σ are sourced by Weyl scalars while θ is driven by Ricci terms. In the torsion–dominated regime $|\Psi_0| \ll |R_{ab}u^a u^b|$, implying

$$\mathrm{Var}_D(\sigma) \;\approx\; \eta\, \mathrm{Var}_D(\theta), \qquad \eta \sim \frac{|\Psi_0|^2}{|R_{ab}u^a u^b|^2} \ll 1.$$

Three-mode example. Take a simple top-hat window $W_n = \mathbf{1}_{\{k_n \le k_D\}}$ and keep the first three modes ($N = 3$) within D. Using $\varphi^{-2} \approx 0.381966$, one has

$$\sum_{n=1}^{3} \varphi^{-2n} = \varphi^{-2} + \varphi^{-4} + \varphi^{-6} = 0.381966 + 0.145898 + 0.055728 \;\approx\; 0.583592, \tag{115}$$

$$\mathrm{Var}_D(\theta) = \frac{A_0^2}{2} \times 0.583592 \;\approx\; 0.291796\, A_0^2, \tag{116}$$

$$Q_D = \Big(\tfrac{2}{3} - 2\eta\Big)\, 0.291796\, A_0^2 \;\approx\; 0.194531\, A_0^2 - 0.583592\, \eta\, A_0^2. \tag{117}$$

Thus, for a conservative $\eta = 0.05$ one gets $Q_D \approx 0.165\, A_0^2$.

Universal geometric limit. If the domain admits the full Fibonacci cascade (all $W_n = 1$), then

$$\sum_{n=1}^{\infty} \varphi^{-2n} = \frac{\varphi^{-2}}{1-\varphi^{-2}} = \frac{1}{\varphi}, \qquad \Rightarrow \quad \mathrm{Var}_D(\theta) = \frac{A_0^2}{2\varphi}, \quad Q_D = \Big(\tfrac{2}{3} - 2\eta\Big)\frac{A_0^2}{2\varphi}. \tag{118}$$

Since $1/\varphi \approx 0.618034$, the backreaction inherits a *finite, universal* coefficient set by the golden ratio and the shear fraction η; the remaining scale dependence sits entirely in A_0 and the windowing W_n.

Remarks. (i) The same weighting can be applied to the shear spectrum, in which case η can be computed from the NP transport rather than taken as a parameter. (ii) If the domain filter favors a band of $n \in [n_1, n_2]$, (111) yields $\mathrm{Var}_D(\theta) = \frac{A_0^2}{2}\, \varphi^{-2n_1}\, \frac{1-\varphi^{-2(n_2-n_1+1)}}{1-\varphi^{-2}}$, making the Q_D scaling transparent for finite cascades. (iii) This makes explicit how the Fibonacci ansatz feeds directly into the coarse-grained dynamics via Q_D, closing the loop with Sec.3.

References

[1] R. K. Pathria, "The Universe as a Black Hole," *Nature* **240**, 298–299 (1972).

[2] E. T. Newman and R. Penrose, "An Approach to Gravitational Radiation by a Method of Spin Coefficients," *J. Math. Phys.* **3**, 566 (1962).

[3] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick, and J. M. Nester, "General Relativity with Spin and Torsion: Foundations and Prospects," *Reviews of Modern Physics* **48**, 393 (1976).

[4] N. J. Popławski, "Cosmology with torsion: An alternative to cosmic inflation," *Phys. Lett. B* **694**, 181–185 (2010).

[5] D. Sornette, “Discrete-scale invariance and complex dimensions,” *Phys. Rep.* **297**, 239–270 (1998).

[6] S. Gluzman and D. Sornette, “Log-periodic route to fractal functions,” *Phys. Rev. E* **55**, 6552–6565 (1997).

[7] R. K. Pathria, “The Universe as a Black Hole,” *Nature* **240**, 298 (1972).

[8] D. A. Easson, “Hawking radiation of black holes in de Sitter space,” *Int. J. Mod. Phys. A* **16**, 4803 (2001).

[9] A. Raychaudhuri, “Relativistic Cosmology. I,” *Phys. Rev.* **98**, 1123 (1955).

[10] S. Kar and S. Sengupta, “The Raychaudhuri equations: A brief review,” *Pramana* **49**, 3 (1997).

[11] E. T. Newman and R. Penrose, “An Approach to Gravitational Radiation by a Method of Spin Coefficients,” *J. Math. Phys.* **3**, 566 (1962).

[12] S. A. Teukolsky, “Perturbations of a Rotating Black Hole. I,” *Astrophys. J.* **185**, 635 (1973).

[13] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge Univ. Press, 1973).

[14] R. M. Wald, *General Relativity* (University of Chicago Press, 1984).