Sequential Deliberation

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ABSTRACT. We present a dynamic model of sequential information acquisition by a heterogeneous committee. At each date agents decide whether to continue deliberation, generating costly information, or stop and take a binding vote yielding a decision. For homogeneous committees, the model is a reinterpretation of the classic Wald (1947) sequential testing of statistical hypotheses. In heterogeneous committees, the resources spent on deliberation depend on the committee's preference profile and on the rules governing the deliberation and decision-making processes. Three main insights emerge from our analysis and match an array of stylized facts on jury decision making. First, more heterogeneity of preferences, more consensual deliberation rules, or more unanimous decision voting rules, lead to greater duration of deliberation and more accurate decisions. Second, balanced committees unanimously prefer to delegate deliberation power to a moderate chairman rather than be governed by a deliberation rule such as unanimity. Last, voting rules at the decision stage are inconsequential when either information collection is cheap or deliberation rules are consensual.

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1. Introduction

1.1 Overview

Juries, boards of directors, congressional and university committees, government agencies such as the FDA or the EPA, and many other committees spend time deliberating issues before reaching a decision or issuing a recommendation. This paper presents a simple model of deliberation that captures some key features of the deliberative process. The model allows us to discuss how these features affect the accuracy of decisions, the length of deliberation, and the degree of disagreement within committees.

Previous literature on deliberation has focused on asymmetric information among members of the deliberating group, on how this information asymmetry can impede effective decision making, and on how different voting rules interact with this information asymmetry.[1] We abstract from private information and concentrate on a simpler aspect of collective action that emerges when studying sequential deliberation; namely, how information collection responds to conflicting preferences. Our model is a natural extension of much of the analysis of individual decision making to group contexts. Indeed, an important aspect of individual decision making is the appropriate amount of information to acquire before making a decision. An individual must weigh the cost of information against the value of making more accurate decisions. A classic approach to this question, going back to Wald (1947a,b), is that of Bayesian sequential analysis. In this approach an individual acquires information sequentially, and at every stage evaluates whether he has sufficient information to make a decision: if he does, he stops and takes a decision; if he does not, he proceeds to acquire additional information. Formally, Wald showed that the optimal procedure involves a sequential likelihood ratio test, whereby intermediate values of the likelihood ratio require obtaining a new sample, while high (low) values of the likelihood ratio require stopping and taking one (or the other) decision.

Our paper provides an analysis of how collective action affects such information acquisition. We use the term deliberation for this process. In a deliberating committee, there are two types of decisions to make: *deliberation decisions* and *action decisions*. Deliberation decisions are about whether to keep deliberating in order to obtain additional information. Action decisions regard the choice to be taken at the end of deliberation. Deliberation is in service of action decisions since

[1] E.g., Austen-Smith and Feddersen (2005, 2006), Coughlan (2000), Gerardi and Yariv (2007, 2008), Meirowitz (2006), and Persico (2004).

the information that is obtained is supposed to allow more accurate action decisions. Our goal is to understand how the institutional environment, as represented by the rules for decision making, affects both the final choice that is made *and* the amount of information that is collected (and therefore the accuracy of decisions).

In our model, a committee deliberates in a manner that is analogous to the decision maker testing a hypothesis sequentially à la Wald: at every date, the committee evaluates its current information and decides on one of three actions: continue sampling–i.e., continue deliberating, or stop and take one of two decisions, acquittal or conviction. We depart from Wald by introducing heterogeneity of views among committee members on the appropriate standard of evidence in order to vote to convict. In this version, there can be disagreement at the deliberation *and* at the decision stage.[2] We show that heterogeneous committees also utilize an appropriately modified sequential likelihood ratio test *in equilibrium* and we then discuss the effects of the committee structure on the length of deliberation, the accuracy of decisions, and the welfare of the committee and of society at large.

Prior literature has focused on the effect of the voting rules that govern action decisions. In our setting, committees make *both* deliberation decisions and action decisions. In terms of modeling, we introduce a distinction between *deliberation rules* and *decision rules*. A deliberation rule governs the deliberation process and determines when information acquisition stops. A decision rule governs the vote over issues at the end of deliberation. We model the deliberation process as a threshold rule R_d such that deliberation ends as soon as R_d members of the committee vote to end deliberation. Decision rules are analogously captured by a rule R_v that describes the specific qualified majority required for reaching a decision.

A special case of a deliberation rule in a jury setting corresponds to repeated straw polls, with a final vote taken according to the decision rule once the outcome of the straw poll indicates that sufficient consensus has been achieved.[3] In general, deliberation rules may be quite different from decision rules. For instance, many committees, including boards of directors, university committees, etc. feature a committee chairman who has the same power as all the other members over action

[2] In Section 7.2, we consider an extension of our analysis in which disagreement is exclusively on the importance of the decision (or, equivalently, on the cost of information acquisition), and hence on the length of the deliberation process. In this version, committee members share preferences over decisions conditional on the information available, but disagree on how much information is required before making the decision. The emergent outcomes correspond to the original Wald solution for one of the agents, determined according to the cost distribution and the deliberation rule in place.

[3] The guides distributed to jurors in many U.S. courts indicate that this reflects the deliberation process suggested to juries, see Murphy and Boatright (1999a,b).

decisions, but has a special role to play (and more power) in deliberation decisions. The chairman case can be modeled as either a dictator at the deliberation stage, or, if the chairman has been chosen to represent the interest of the majority, as the median voter in the committee (captured by a simple majority threshold rule). Such a median voter would be decisive at the deliberation stage, but not necessarily at the decision stage, if the decision rule is a supermajority rule. The chairman example is a case in point for a common distinction between decision rules and deliberation rules. It illustrates a different approach within a committee toward the consensus required for making a decision, as opposed to the consensus required for acquiring more information. In fact, less consensus is often required to gather additional information than to make a final decision.[4] Admittedly, while decision rules are often quite precisely described – some issues requiring a majority vote, others requiring a supermajority or unanimity – deliberation rules are often vague. Despite this vagueness, threshold deliberation rules are a natural starting point for capturing modeling committees that have more inclusive deliberation protocols than others. Our results suggest that it is important to study the role of the deliberation process, and to understand how it functions.

Three main insights emerge from our analysis. First, we show that increased heterogeneity prolongs deliberation, thereby increasing the expected time to a decision, but also reducing the two probability of mistakes (convicting the innocent and acquitting the guilty). Increasing heterogeneity has two effects. A direct, mechanical, effect is that the individuals who are pivotal during deliberation become more extreme and therefore each prefer to extend deliberations when they are pivotal. An indirect, strategic, effect, however, implies that pivotal agents take into account the tendency for prolonged deliberation by their more extreme peers and are therefore willing to halt deliberation earlier than they would have as single-decision makers; this allows them to avoid excessively costly information collection. We call this strategic response to increased heterogeneity the *moderation effect*.

Our second main result pertains to welfare. We show that in a symmetric environment members of a committee would unanimously prefer to delegate deliberation power to a moderate chairman rather than be governed by a deliberation rule such as unanimity. Similarly, jurors prefer committees in which the pivotal agents are as homogeneous (and moderate) as possible. Therefore, while increased heterogeneity may be beneficial for society at large since it yields more accurate decisions, the jurors

[4] E.g., the chairman example, or the job market process in which less consensus is required to interview a job candidate, or to invite them for a campus visit, than to make them a job offer.

themselves prefer a more moderate (and unified) group of decision makers at the deliberation stage. It is important to note that this willingness to delegate to a moderate chair does not hold for the decision stage. This distinction between the desired composition of the pivotal agents at the decision and deliberation stage can explain the fact that in some environments there are individuals (such as chairs) who have a lot of power at the deliberation stage but have no special power at the decision stage (they just have one vote).

Our last set of results pertains to the impact of deliberation and decision rules. We show that under some assumptions deliberation rules are more "effective" (or powerful) than decision rules. For instance, in certain cases, decision rules are irrelevant, while deliberation rules affect the length of deliberation and accuracy of ultimate decisions. Furthermore, we show that, for a range of parameters for which the decision rule does have an effect, in contrast with Feddersen and Pesendorfer (1998), Persico (2004), and Austen-Smith and Feddersen (2006, 2007), unanimity leads to more informative outcomes than majority rule.

The formal analysis in this paper is potentially relevant for a variety of collective decision processes such as R&D, hiring decisions, FDA drug approval, and so on. However, we focus much of our discussion on juries. This is partly because, for concreteness, it is useful to have a running application in mind. Furthermore, there are three additional reasons why we believe that juries are an interesting application for our model. First, in juries, the deliberation process is clear-cut and circumscribed: there is a well-defined beginning and end of deliberation, the time it takes the jury to deliberate is measurable, and one single verdict is the typical outcome of such deliberation. Second, the jury setting allows us to contrast our analysis with much of the extant body of literature on deliberation that has focused predominantly on the jury context. Third, the empirical literature has documented some patterns of deliberation in juries that we attempt to explain with our model.

1.2 Literature Review

One strand of literature (appearing mostly in law and psychology scholarship) studies opinion formation by jurors. This is relevant for our model for two reasons. First, it establishes that the deliberation process is important in the formation of jurors' opinions. Second, some features of the opinion formation process seem to mirror the updating process assumed in our model.

In relation to our setting, that literature suggests the "fact finding" role of juries: for instance, Vidmar and Hans (2007) write: "Formally their [the jurors'] task is to engage in sound fact finding from the evidence produced at trial." This formal requirement appears to be partially consistent with practice in mock juries and with surveys of actual juries: a substantial fraction of deliberation appears to be devoted to a discussion of the facts. In fact, starting from Kalven and Zeisel (1966), numerous legal scholars have argued that juries do a good job at reaching an understanding of the facts. For instance, in a study of mock juries, Ellsworth (1989) writes: "In general, over the course of deliberation, jurors appear to focus more on the important facts and issues, come to a clearer understanding of them, and approach consensus on the facts." This literature also suggests the importance of the interaction between decision rules and deliberation protocols. For instance, Hastie, Penrod, and Pennington (1983) point out that the volume of discussion substantially increases with the decision rule ("by any measure of volume of discussion the stricter the verdict quorum requirement, the greater the volume of discussion produced during deliberation. This result follows from the differences in total deliberation time that are associated with the different decision rules," page 97). We note that the increase in discussion is associated with discussion of information ("when the content of deliberation was classified by functional type, most remarks were clearly motivated to communicate information concerning the case from one juror to the others, with about 75% of the remarks falling into this category," ibid.). We return to a more detailed review of the legal literature as it links with our results in Section 6.

In Economics, the past two decades have delivered a rich collection of work on committee decision making (see Li and Suen, 2009 for an extended survey). Our paper ties directly to several sets of studies.

In terms of jury decision making, our setup can naturally be contrasted with several papers emphasizing information aggregation within juries. Feddersen and Pesendorfer (1998) study a model in which jurors have private information about the guilt or innocence of the defendant. They show that unanimity leads to less informative outcomes than does simple majority in large juries. Persico (2004) studies a related model, but also allows for private information collection prior to voting. He characterizes the optimal voting rule and shows that unanimity leads to inferior information collection. Austen-Smith and Feddersen (2006) extend Feddersen and Pesendorfer (1998) in another way by allowing for a round of cheap-talk communication before voting. They show that unanimity leads to less communication and poorer information aggregation. Gerardi and Yariv (2007, 2008) depart from these papers by studying general communication protocols and analyzing the entire set of equilibrium outcomes. They show that the set of equilibrium outcomes is invariant to the voting rules, as long as they are non-unanimous. In fact, unanimous voting rules generate a subset of equilibrium outcomes. In contrast with this literature, we assume that all information in the jury is public: signals are observed by all jurors, and preferences are common knowledge. This assumption represents a sharp departure from this literature. We view our model as a natural alternative extreme benchmark that is useful for identifying the tensions that arise in collective choice when trading off information collection costs and decision accuracy. Our result that unanimity leads to more accurate outcomes contrasts with the observations of Feddersen and Pesendorfer (1998), Persico (2004), and Austen-Smith and Feddersen (2005, 2006).

Several studies have looked at the effects of sequential collection of information within groups. Albrecht, Anderson, and Vroman (2010) and Compte and Jehiel (2010) study how group search is affected by voting. Messner and Polborn (2012) study a two-period model where voters receive information over time about the desirability of an irreversible decision. The main message of that paper is that the optimal voting rule requires a supermajority. Bognar, Meyer-ter-Vehn, and Smith (2013) also study a model of dynamic deliberation, but with very different ingredients. In their model jurors have common preferences and private information about a payoff-relevant state. They assume that jurors sequentially exchange coarse messages. In their model there are many equilibria that can be ranked in terms of generated welfare. Surprisingly, longer conversations are better.[5] Strulovici (2010) analyzes a model of voting over experimentation. He shows that voting by heterogeneous voters, who are learning their preferences, leads to an inefficient level of experimentation. He then describes a voting rule that can restore efficiency. Moldovanu and Shi (2013) study a collective version of a stopping problem in which individual agents can assess the performance of alternatives on one of several dimensions. In their setting, when information is publicly transmitted, the volume of information collected, as well as the group's welfare, are shown to decrease with the variance of preferences in the group.

From a technical perspective, the starting point of our analysis is Wald (1947a,b), who pioneered the study of sequential testing, and provided a characterization of the optimal sequential test as a

[5]A related paper is that of Eso and Fong (2008), who study a dynamic cheap talk model with multiple senders, where the receiver can choose when to make her decision. They show that when the senders are all informed of the state of nature, a perfect Bayesian equilibrium exists with instantenous, full revelation, regardless of the size and direction of the senders' biases. Wilson (2012) considers exogenous costs for both sending messages and receiving them, and illustrates the dependence of effective communication on agents' quality of information and messaging costs.

sequential likelihood ratio test.[6] We briefly describe the most directly relevant result in section 3.1. In that respect, Chan and Suen (2013) and Henry and Ottaviani (2013) are probably the closest paper to ours. Chan and Suen use the Wald setting to study the impact of heterogeneous patience (or different individual information costs) among group members. In their setting, decisions to stop information collection and decisions regarding which alternative to pick are linked to one another, essentially corresponding to the case in which our deliberation and decision rules coincide. When the group is heterogeneous on two dimensions – information costs and individual preferences over alternatives, they show that one individual who experiences very high information costs can drive the group to make hasty (and rather uninformed) decisions. Ottaviani and Henry apply the framework to a study of the approval process when a firm conducting clinical trials needs approval from a regulator such as the FDA. They also allow for the possibility that the firm may misrepresent the evidence and show that it may not be optimal to forbid such misrepresentation. The focus of our analysis is different from these two papers since we focus on the impact of the deliberation and voting rules on obervable outcomes and welfare. The papers also differ from a technical perspective in that they assume learning takes place in continuous time while we assume discrete time.[7]

To conclude, in comparison to all of this existing work, the main contribution of the framework proposed in this paper is that it allows for an analysis of the interplay between deliberation rules and decision rules. We identify when each plays an important role for outcomes, and how collective consequences are affected by different aspects of the environment (deliberation costs, preference heterogeneity, etc.).

2. The Model

2.1 Setup

A jury of n individuals has to determine the fate of a defendant. There are two states: I (the defendant is innocent) and G (the defendant is guilty); and two decisions A (acquittal) and C (conviction). The prior probability of guilt is denoted by p_0.

All jurors want to make accurate decisions: convict the guilty and acquit the innocent. However, they differ in the importance they attribute to the two possible mistakes: convicting the innocent or

[6] Some precursors considered special cases, see Wald (1947a,b). See also De Groot (1970) for a simple modern exposition, and Moscarini and Smith (2001) for an extension that allows for richer sampling strategies.

[7] Moscarini and Smith (2001) consider a different extension of Wald's analysis, where they allow for simultaneous as well as sequential experimentation, and assume discounting and convex costs of sample size.

acquitting the guilty. Juror i's preferences are characterized by the following utility function:

$$u_i(A,I) = u_i(C,G) = 0, \quad u_i(A,G) = -(1-q_i), \quad u_i(C,I) = -q_i,$$

where q_i captures the juror's concern for convicting the innocent relative to that for acquitting the guilty. In a static model without information collection, q_i would be the threshold of reasonable doubt: the juror would want to convict if his posterior is higher than q_i, and acquit otherwise. Without loss of generality, we assume $q_1 \leq q_2 \leq ... \leq q_n$. We also assume that $q_1 > 0$ and $q_n < 1$ (otherwise at least one of the jurors would not be responsive to information collection).

In determining the verdict, the jury participates in two phases: *deliberation* and *decision making*. We assume that deliberation allows each juror to publicly acquire information about the guilt of the defendant. We formalize this collective information generation as follows. If the jurors still deliberate at time t, all observe the realization of the sequence of random variables $X_1, ..., X_t$, where $X_1, X_2, ...$ are independent and identically distributed conditional on the guilt or innocence of the defendant. Each random variable is drawn from an atomless distribution characterized by cumulative distribution functions $F_G(\cdot) \equiv F(\cdot|G)$ when the defendant is guilty and $F_I(\cdot) \equiv F(\cdot|I)$ when the defendant is innocent.[8]

The cost of deliberating an additional period is given by $k > 0$ per unit of time per agent. This is the cost of obtaining an additional signal (or the opportunity costs of time spent deliberating).

Each period, the jury decides whether to continue or stop deliberating using a threshold voting rule. Namely, at each period t, after having observed the history $X_1, ..., X_{t-1}$, each agent casts a vote whether to continue or stop information collection (in particular, at the outset, when $t = 1$, agents decide whether to collect the first signal based on the prior alone). Under *deliberation rule* $R_d = \lceil \frac{n}{2} \rceil, ..., n$, whenever at least R_d jurors choose to stop deliberating, the deliberation phase ends.[9]

Once deliberation comes to a halt, the decision phase takes place. The jury selects an alternative

[8] Allowing for atoms in the distribution would introduce some technical subtleties in the description of our results and their proofs without any qualitative differences.

[9] We concentrate on supermajoritarian deliberation rules, i.e., $R_d \geq n/2$. Our analysis could easily be extended to deliberation rules $R_d < n/2$. In fact, we use supermajoritarian deliberation rules simply as a way to capture succinctly the effective agenda setters during deliberation and highlight the interplay between these agents and those pivotal during the decision phase. Our analysis could allow for the specification of arbitrary agenda setters in the deliberation phase that do not come about through a vote (as is the case in settings in which, say, a committee chair determines when discussions should come to a halt).

by voting. Each juror can vote to acquit, a, or to convict, c. Under the *decision rule* $R_v = \lceil \frac{n}{2} \rceil, \ldots, n$, the alternative C is selected if and only if R_v or more jurors vote to convict, the alternative A is selected if and only if R_v or more jurors vote to acquit, and the jury is hung otherwise. We assume that when the jury is hung, A or C are determined by the flip of a fair coin.[10]

We denote posterior beliefs at time t by p_t. We restrict attention to strategies that depend only on posterior beliefs (and not on the history of prior votes). Therefore, a pure strategy is a pair (σ_d, σ_v), where the *deliberation strategy* is $\sigma_d : [0,1] \to \{\text{stop, continue}\}$ and the *voting strategy* is $\sigma_v : [0,1] \to \{a, c\}$.

2.2 Discussion of the model

The model is an extension of Wald (1947a,b) that allows the study of how collective action by heterogeneous agents affects information collection. In the model, longer deliberation corresponds to additional signals received by the committee. Our interpretation is that this is a reasonable shortcut for thinking about how deliberation helps jurors gain an understanding of the evidence presented at trial. Of course, in a jury setting, it could be claimed that no additional information is received by the jurors during deliberation. However, one instructive way to think of the trial may be the following: the pre-deliberation phase, where all the evidence is presented and discussed, is similar to a lecture given by a professor, and the deliberation phase is similar to a study group that looks through the notes taken during class to gain some further understanding of the topic. In fact, in the jury setting deliberation is rendered even more important by the fact that, in contrast to a lecture, the jury must sift through the mass of sometimes conflicting evidence presented by two opposing parties during the trial to figure out the relevance of different pieces of information, as well as the appropriate weight to attribute to these in establishing the guilt or innocence of the defendant. It is as if the lectures were held by two professors with opposite views on a policy, and the class had to figure out which arguments by the professors to take more seriously.

We note that the model fits a particular plausible protocol of deliberation in juries in which the jury conducts repeated straw polls until a sufficient consensus emerges. Indeed, this would correspond to the case in which $R_d = R_v$. For sufficiently low costs, the jury is never hung and whenever there

[10] The exact assumption we make about the payoff consequences of hung juries is for the most part inconsequential. Our initial analysis focuses on cases where hung juries do not occur (due to low costs of deliberation). Section 7.4 discusses the case of hung juries.

is an R_d-majority that votes to halt deliberations, there is then an $R_v(= R_d)$-majority that agrees on the verdict. Even when costs are so high that a hung jury may emerge, jurors can anticipate this outcome and it turns out that they will vote to halt deliberations immediately in a straw poll.[11] A different plausible protocol for many committees entails delegation to one of the members. If individual i chairs the deliberation process, then deliberation is governed by the optimal information collection for i, as characterized originally by Wald, which we soon describe formally (modified to take into account the possibility that the jury may hang when deliberation costs are too high). We evaluate the merits of such delegation in Section 4.

As mentioned in the introduction, there are many alternative applications that fit the model more literally, because actual additional signals are received as more information is collected. Specifically, in an R&D process, agents receive feedback about the likely success of specific avenues of research; in a drug approval process, the FDA can require additional clinical trials to be performed, and, in fact, the FDA approval process is explicitly designed to allow for several stages of testing; in hiring practices, follow-up interviews can be requested, or additional assessments of a job applicant can be performed; a board of directors can require additional due diligence before proceeding with a merger; and so on and so forth. In general, any scenario in which a group gathers information over time regarding two possible courses of action, with the status quo being chosen if the group cannot come to an agreement, shares features with our model.

Another restriction that we impose is that there are only two alternatives to choose from. This restriction is common to many voting models. It is not possible to obtain sharp characterizations in a model with more than two alternatives. However, we do consider a continuous action version of a simpler model (that illustrates the robustness of our basic results), which we discuss in Section 7.3.

Finally, in our model all disagreement among decision makers concerns the appropriate "threshold of reasonable doubt," the level of the posterior that is appropriate for a guilty/innocent verdict. Nonetheless, all jurors would agree on the appropriate action to take if they were fully informed. This is a reasonable assumption for modeling juries and perhaps drug trials. However, it is an extreme assumption for other applications, such as confirmation of supreme court justices, or hiring decisions, where there is preference heterogeneity even under full information. We believe that some

[11] We will discuss hung juries further in Section 7.4.

of the insights from our model are relevant also in these different contexts. Specifically, the model could be adapted to incorporate ideological heterogeneity.[12]

3. Preliminaries

3.1 Homogeneous Preferences

We start by considering the benchmark of a homogenous jury containing agents with the same preference parameter: $q_1 = ... = q_n \equiv q$. In that case, the agents all face the same objective in both phases of the process. Consequently, we focus on equilibria that emulate the single person decision by voting in unison during both the deliberation and decision phase.[13] Hence, the equilibria are the same as in the case in which $n = 1$, the case analyzed by Wald (1947a,b). From Wald, predictions are unique and can be characterized as follows:

Proposition 1 (Wald, 1947) *A unique equilibrium exists and is characterized by two thresholds: $p^a(q) \leq p^c(q)$ such that*

- *the agent stops information collection and acquits whenever $p_t \leq p^a$;*
- *the agent stops information collection and convicts whenever $p_t \geq p^c$;*
- *the agent continues information collection whenever $p_t \in (p^a, p^c)$.*

Since our analysis builds on Wald's analysis, we now illustrate the logic of this result, and discuss some additional properties of the solution.[14] Fix the information cost k. For any posterior probability p, denote by $V^0(p)$ the value function associated with stopping immediately at posterior p.

$$V^0(p) = \max\{-q(1-p), -(1-q)p\}. \tag{1}$$

Denote by $V^1(p)$ the value associated with continuing at least one more period, and $V(p)$ the overall

[12] One immediate way to extend the model is to allow for some jurors to convict (or acquit) no matter what the information is. All of our results can accommodate this case via a proper adjustment of the voting/deliberation thresholds for the remaining jurors.

[13] We discuss our equilibrium notion in Section 3.2 below.

[14] Much of this discussion is adapted from De Groot (1970).

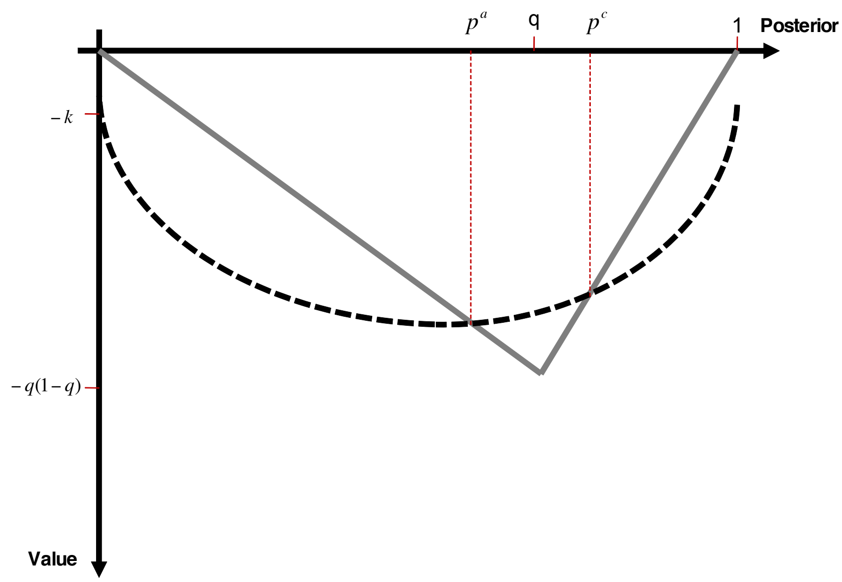

Figure 1: Homogeneous Groups – Existence and Uniqueness

value function for any posterior probability p.[15] It follows that

$$V(p) = \max \left\{ V^0(p), V^1(p) \right\}. \tag{2}$$

Note that $V(0) = V(1) = 0$, and therefore, $V^1(0) = V^1(1) = -k$. Standard arguments show that $V^1(p)$ is a convex function of p.[16] From linearity of $-(1-q)p$ and $-q(1-p)$, and the fact that their maximal value of 0 is achieved at $p = 0, 1$, respectively, it follows that there are two posterior probabilities (of the defendant's guilt), p^a and p^c, that define the stopping region, as in Figure 1.

When costs are high, they outweigh the benefits of information collection and stopping occurs immediately (in terms of Figure 1, when k is sufficiently high, the curve corresponding to the continuation payoff lies below that corresponding to the instantaneous utility from stopping). When costs are sufficiently low, there is an interior solution. Note that convexity of the value function assures the uniqueness of such an equilibrium.

[15] The continuation value $V^1(p)$ must equal the expectation of $V(p)$ with respect to the potential posteriors in the continuation, minus the cost k of continuation.

[16] Indeed, consider a scenario in which a juror is only told that with probability α, the probability that the defendant is guilty is given by p_1 and with probability $1-\alpha$, the probability that the defendant is guilty is given by p_2. In this case, she can guarantee $V^1(\alpha p_1 + (1-\alpha)p_2)$. In an alternative scenario, she is told whether p_1 or p_2 is realized. Then, with probability α, she can guarantee $V^1(p_1)$ and with probability $1-\alpha$ $V^1(p_2)$. Naturally, she can ignore the information provided to her, so in the second scenario she must be better off. Convexity follows.

3.2 Heterogeneous Preferences

We now move to the case where $q_1 \leq q_2 \leq ... \leq q_n$ (and some of the inequalities may be strict).[17]

There are many possible sources of multiplicity of equilibria in this environment: first, when voting rules are non-unanimous, voting games are known to display equilibria in weakly dominated strategies in which voters vote in consensus independently of their preferences because unilateral deviations cannot affect outcomes; second, there is a potential multiplicity linked to the infinite horizon nature of the game. Throughout the paper, to highlight the nature of the collective action problem relative to the Wald solution, we focus on equilibria satisfying two requirements:

1. Each juror i's strategy is characterized by two time-independent thresholds for the posterior, $p_i^a \leq p_i^c$. That is, each agent votes to stop deliberation whenever the posterior p_t (that the defendant is guilty) satisfies $p_t \leq p_i^a$ or $p_t \geq p_i^c$. In particular, strategies are Markov in the posterior p_t.

2. After every history, there is sincere voting at the deliberation stage and the decision stage. Sincere voting at the decision stage requires that, when the deliberation stage ends with a posterior of p, a juror of preference parameter q votes a (acquit) whenever $p < q$, and votes c (convict) whenever $p \geq q$.[18] Sincere voting at the deliberation stage implies the following. At each period, each juror takes other jurors' equilibrium behavior as given and therefore internalizes the regions of posteriors in which she can affect deliberation choices. The juror then calculates her (constrained) continuation values from carrying on with deliberations as well as the expected payoff from stopping and votes for the alternative that provides the higher payoffs.[19]

For homogeneous committees, the above requirements end-up selecting the Pareto-best equilib-

[17] In order to isolate the effects of preference heterogeneity on outcomes, we assume that deliberation costs are homogenous and fixed at $k > 0$. We discuss the case of heterogeneous costs in Section 7.2.

[18] Recall that a hung jury yields a payoff equivalent to that resulting from an equal probability of acquittal and conviction. In particular, whenever a juror prefers conviction over acquittal, she would also prefer conviction to a hung jury to acquittal. We break indifferences by requiring that the juror choose conviction when indifferent and acquittal when strictly preferring that to conviction. None of the results in the body of the paper are sensitive to this way of breaking indifferences.

[19] Behavior according to (stationary) thresholds as well as sincerity could be thought of as the consequence of the following refinement to Markov equilibria in our setting. Consider finite horizon truncations of our game and focus on subgame perfect equilibria that survive iterated elimination of weakly dominated strategies in which last-date strategies are eliminated first. The limits of such sequences of equilibria as the horizon becomes infinitely large correspond to individual threshold strategies satisfying our sincere voting restriction.

rium, which is equivalent to the single-person optimum. For readability purposes, and slightly abusing terminology, we will simply refer to the corresponding set as the set of 'equilibria' (or 'equilibrium' when unique).

We start by considering the case in which voting rules in the deliberation and decision stage coincide. That is, $R_d = R_v$. This allows us to focus on one set of pivotal agents, rather than consider potentially different pivotal agents at each stage of the decision-making process. It is also a case that is relevant for many applications: for example, it fits environments in which deliberation manifests as a sequence of (potentially implicit) straw polls (see Section 2.2). Later, we investigate the consequences of discordance between the two types of rules.

It turns out that there are two pivotal jurors, one determining the acquittal region, the other determining the conviction region. We now discuss in detail the problem faced by a juror.

From the point of view of a generic juror i, there are three possible cases:

(i) An R_d-majority of other jurors favors stopping. In this case deliberation stops, and juror i has no impact on the decision.

(ii) Strictly fewer than $R_d - 1$ other agents want to stop. In this case, deliberation must continue and juror i is in a constrained region in which deliberation continues regardless of her vote.

(iii) Exactly $R_d - 1$ other jurors want to stop. In this case juror i can affect the stopping decision.

Because of heterogeneity in preferences, agents disagree on their ideal thresholds for stopping deliberation, and therefore, the identity of the pivotal agent depends on the posterior.

The main complication, relative to the homogeneous jury setting, arises from the fact that the juror has no control over stopping in case (ii). This means that the juror's optimal action in region (iii) depends on the magnitude of the region of posteriors over which case (ii) arises. From the perspective of this juror, her problem is a constrained version of the original Wald problem, where she takes as given the fact that she cannot stop information collection in a certain region. We now analyze a specific version of such a constrained problem defined by two thresholds, $\underline{p} \leq \overline{p}$, such that the juror can only choose to stop and to acquit if $p \leq \underline{p}$, and to stop and convict if $p \geq \overline{p}$. For any $\underline{p}, \overline{p}$, we define the constrained value functions as follows (dropping the argument k). As before,

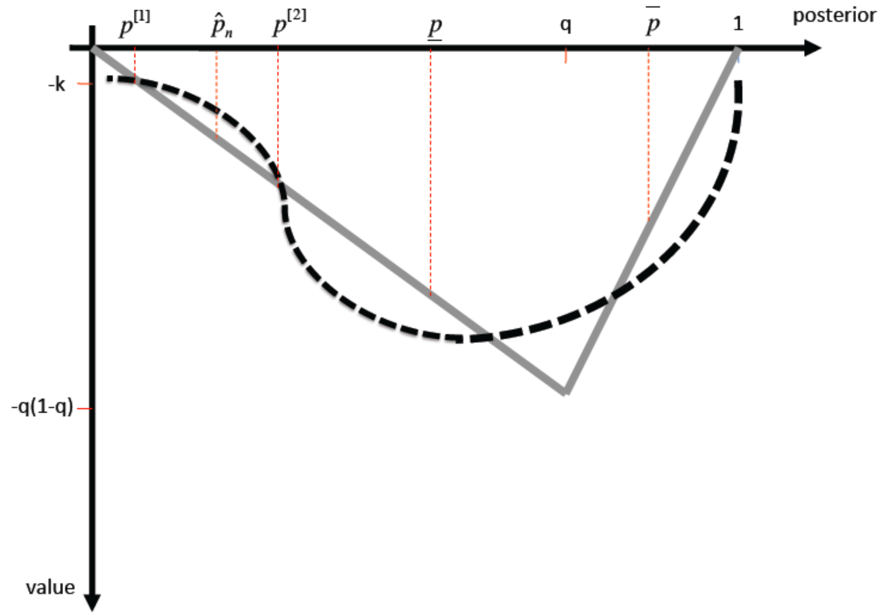

Figure 2: Non-Convexities of the Continuation Value Function

$V^1\left(p|\underline{p},\overline{p};q\right)$ is the value of continuing at least one period. The overall value function is given by:

$$V(p \mid \underline{p}, \overline{p}; q) = \begin{cases} \max\left\{V^1\left(p|\underline{p},\overline{p};q\right), -\left(1-q\right)p\right\} & p \leq \underline{p} \\ V^1\left(p|\underline{p},\overline{p};q\right) & \underline{p} < p < \overline{p} \\ \max\left\{V^1\left(p|\underline{p},\overline{p};q\right), -q\left(1-p\right)\right\} & p \geq \overline{p} \end{cases}. \tag{3}$$

The interpretation of this expression is the following: for $p \leq \underline{p}$, the juror chooses the best option between continuing deliberation and stopping to acquit. For $\underline{p} \leq p \leq \overline{p}$, the juror can only continue deliberation. For $p \geq \overline{p}$, the decision maker chooses the best option between continuing deliberation and stopping to convict.

In contrast with the unconstrained Wald problem, the value function for the constrained problem may not be convex**.** Non convexity may in principle lead to multiple intersections of the continuation value function with the stopping value as depicted in Figure 2. To clarify the meaning of this figure, note that, for any specific history of signals, i.e., for any given posterior p, there is a unique optimal action: either continue or stop and choose convict or acquit, depending on the region where p is. The multiplicity of thresholds depicted in Figure 2 mean that, depending on the posterior, there are multiple stopping regions, each of which lead to acquittal. However, we can show that under certain conditions, the case depicted in Figure 2 cannot arise, and hence, the constrained problem also has (as in the original Wald problem) a unique pair of thresholds identifying equilibrium information collection. The idea of our proof of this feature is the following. Suppose there are multiple thresholds for acquittal. This means that the continuation value intersects the straight line corresponding to the stopping value multiple times. Focus on one region, say to the left of the constrained region, in which it is above that straight line, as depicted in Figure 2. Now, consider a modified game in which each signal is less informative and less costly, but the overall information per unit cost is at least as high. This is conceptually similar to 'slicing time' into smaller intervals. The new (constrained) continuation value is then point-wise higher than the original one (since there is more flexibility during the information collection process, and there is at least as much information available per unit cost). Therefore, the continuation region we started out with expands. We can continue 'slicing time' in that manner until signals are such that if one starts with a posterior in the middle of this region, one signal will preserve the resulting new posterior within that region. Roughly, this means that the expected value of continuing from that point is lower, which leads to a contradiction. In Appendix B, we formalize the idea of 'slicing time' and describe the class of information structures for which there are unique best-responses. In what follows, we will assume our information structures satisfy the conditions required to guarantee uniqueness of best responses. The reader uninterested in details can focus for the rest of the paper on the case in which signals are drawn from normal distributions (with different means for the two different states, I and G) and sufficiently low accuracies.

A solution is therefore identified by two thresholds, which we denote by $\left(p^a\left(\underline{p},\overline{p};q\right),\ p^c\left(\underline{p},\overline{p};q\right)\right)$. For presentation simplicity, we slightly abuse notation and let $p^a\left(\overline{p};q\right) \equiv p^a\left(\overline{p},\overline{p};q\right); p^c\left(\underline{p};q\right) \equiv p^c\left(\underline{p},\underline{p};q\right)$. The threshold $p^a\left(\overline{p};q\right)$ corresponds to a constrained problem in which a juror can decide to stop for any posterior, but knows she can only decide to convict if $p \geq \overline{p}$ and can only decide to acquit if $p \leq \underline{p}$. Similarly for $p^c\left(\underline{p};q\right)$.

In the appendix we establish some key intuitive monotonicity properties of these thresholds. Say, for instance, that $p^a\left(\underline{p},\overline{p};q\right)$ is increasing in $\overline{p}$ and $p^c\left(\underline{p},\overline{p};q\right)$ is increasing in $\underline{p}$. Intuitively, as $\overline{p}$ increases, the problem the decision maker faces is more constrained, and so the continuation value decreases. Since we restricted our setting to situations in which the continuation value intersects the stopping-value line at most once to the left of the constrained region, when the continuation value decreases, the left intersection point increases (as would be the case were the continuation value in

Figure 1 decreased), implying that $p^a\left(\underline{p},\overline{p};q\right)$ increases. Similar arguments hold for changes in $\underline{p}$. It can also be shown that the continuation region shrinks (i.e., the thresholds move closer together) as the cost k increases. Indeed, k does not affect the payoffs from stopping and taking a decision (the lines $-(1-q)p$ and $-q\,(1-p)$ do not depend on k). However, as k increases, the continuation value $V^1\,(p)$ decreases point-wise. It follows that p^a increases, while p^c decreases, as k grows larger.[20]

The constrained problem is useful for characterizing the equilibrium corresponding to our underlying setting. Specifically, let us turn to the identity of the pivotal jurors; when posterior probabilities of guilt are low, it is the jurors who care most about the mistake of convicting the innocent that determine the decision. Whenever agent j prefers to continue deliberation, so does any agent $l > j$ who worries even more about innocent convictions. In particular, whenever juror q_{R_d} chooses to continue deliberation, or vote to convict, so will all jurors with $q > q_{R_d}$, and deliberation will carry on. In other words, when guilt posteriors are low, juror q_{R_d} is 'pivotal'. Similarly, whenever posterior probabilities of guilt are high, it is the jurors who worry most about guilty acquittals that determine decisions, the relevant pivotal juror being juror q_{n-R_d+1}. Therefore, as in the case of the median voter theorem, we can focus on a small number of decisive people, but in our model there are typically *two* decisive voters. Notice that whenever there is an odd number of jurors and the deliberation rule is simple majority, $q_{R_d} = q_{n-R_d+1}$ and there is a unique pivotal agent, the so-called median voter. It is super-majority rules (with continuation default when a super-majority consensus is not achieved) that lead to the emergence of two pivotal agents in our setting.

Consider a candidate equilibrium specified by p_* as the lower equilibrium threshold and by p^* as the upper equilibrium threshold. Proposition 2 below implies that the pivotal agent on the acquittal side is juror q_{R_d} and the pivotal agent on the conviction side is juror q_{n-R_d+1}.

Proposition 2 (Reduction to Two Juror Juries) *Assume $R_d = R_v$. An equilibrium exists. Furthermore,*

1. *Equilibrium thresholds for a jury composed of n jurors with preference parameters $q_1, q_2, ..., q_n$ are the same as equilibrium thresholds of a jury composed of two jurors with preference parameters q_{n-R_d+1}, q_{R_d} in which both deliberation and decision rules are unanimous.*

[20] This will be useful when we discuss the impact of the two types of voting rules for different cost regions in Section 5 as well as when we consider committees of heterogeneous costs in Section 7.2.

2. *Equilibrium thresholds are given by,* $p_* = p^a\left(p^*; q_{n-R_d+1}\right) \geq p^a\left(q_{n-R_d+1}\right)$*, and* $p^* = p^c\left(p_*; q_{R_d}\right) \leq p^c\left(q_{R_d}\right)$.

Part 1 of this Proposition simply reflects the ordering of jurors according to their concerns for convicting the guilty relative to acquitting the innocent and identifies the two 'pivotal' jurors.

Part 2 says that the equilibrium thresholds are determined by a pair of best responses of the two pivotal agents that can be derived through optimization of a particular version of the constrained decision problem with value function given in equation (3). Each of the agents takes one of the thresholds as given and optimally chooses the other one (that corresponds to the region she cares more about). As it turns out, only one threshold effectively constrains each pivotal juror. Indeed, consider for example, the juror R_d. The conviction threshold of the juror with preference parameter q_{n-R_d+1} is not a relevant constraint since that juror (with preference parameter q_{n-R_d+1}) would like to stop before the juror R_d (with preference parameter q_{R_d}) on the conviction side.

Finally, part 2 shows that collective action leads to a *moderation effect*: the two pivotal agents choose thresholds that are less extreme than those they would choose in their region of control were they in full control of the deliberation process (namely, the thresholds the Wald solution would prescribe for them). This is the consequence of the strategic impacts of collective deliberation. Consider, say, the juror $n - R_d + 1$ and the event in which the posterior $p_t \in (p^a(q_{n-R_d+1}), p^a(q_{R_d}))$, so that if she were by herself she would continue collecting information, while the other pivotal juror, juror R_d, by herself would not. Importantly, the continuation value for pursuing information collection is lowered by the existence of the other juror. Indeed, juror $n - R_d + 1$ knows that for high posteriors, juror R_d will demand longer deliberation than juror $n-R_d+1$ would like (in the analogous range $p_t \in (p^c(q_{n-R_d+1}), p^c(q_{R_d})))$. Thus, continuation values are lower, leading to equilibrium thresholds that are moderate relative to the most extreme individual thresholds $p^a(q_{n-R_d+1}), p^c(q_{R_d})$, as depicted in Figure 3 (when, for simplicity, we denote $\tilde{q}_1 = q_{n-R_d+1}$ and $\tilde{q}_2 = q_{R_d}$).[21]

4. Equilibrium Structure and Welfare in Symmetric Juries

We continue by focusing on the case $R_d = R_v$. Proposition 2 allows us to restrict attention to two jurors within the jury with preferences: $\tilde{q}_1 = q_{n-R_d+1}$ and $\tilde{q}_2 = q_{R_d}$. Assuming $\tilde{q}_1$ and $\tilde{q}_2$ are

[21] Part 2 of the Proposition is related to Strulovici (2010), who calls a similar phenomenon in an experimentation environment a "loss of control effect". It is also related to Chan and Suen (2013), who illustrate in a collective learning environment akin to ours that an impatient member can lead a more patient committee to hasten decisions.

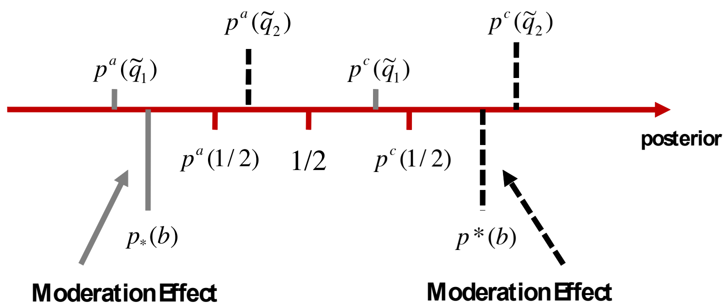

Figure 3: Equilibrium Spread and the Moderation Effect in Quasi-symmetric Juries

symmetric around $\frac{1}{2}$, i.e., $\tilde{q}_1 = \frac{1}{2} - b$ and $\tilde{q}_2 = \frac{1}{2} + b$ for some $b \in \left[\frac{1}{2}, 1\right]$, simplifies equilibrium characterization significantly, and is important for some of our results.

Definition (Symmetry in Juries) *We say the jury is quasi-symmetric with respect to $R_d = R_v$ whenever $q_{n-R_d+1} + q_{R_d} = 1$ and information is symmetric, i.e., $p_0 = \frac{1}{2}$ and there exists s_0 such that for any $s \geq 0$, $F_G(s_0 + s) = 1 - F_I(s_0 - s)$.*[22] *A jury is* symmetric *whenever it is quasi-symmetric with respect to all voting rules.*

When juries are quasi-symmetric, we focus on *symmetric threshold equilibria* corresponding to the relevant deliberation rule, ones in which both posterior thresholds are symmetric around 1/2 (i.e., equally distanced from 1/2). For the next results, we consider cases where the threshold equilibrium is non-trivial in that it entails some amount of information collection. This is the relevant case when k is not too high, but it excludes the case of hung juries that will be discussed later. As it turns out, in quasi-symmetric juries there is a unique prediction. This is established in the following lemma.

[22] We impose information symmetry in the form of symmetry around some point s_0 in the signal space of the functions F_I, F_G. These are designed to assure that whenever thresholds are symmetric, $p^a = 1 - p^c$, the distribution of posteriors conditional on acquittal is a mirror image of the distribution of posteriors conditional on conviction. In that sense, we could have imposed far weaker restrictions on the underlying information generation process. The one we use is chosen for presentation simplicity.

Lemma 1 (Quasi-symmetric Juries - Uniqueness) *Assume the jury is quasi-symmetric with respect to R_d. Then there exists a unique symmetric stationary threshold equilibrium* characterized by *thresholds $p_*(b) \leq 1/2 \leq p^*(b)$. Symmetry entails $p_*(b) + p^*(b) = 1$.*

As b increases, the juror with preference parameter $\tilde{q}_1$ is increasingly concerned about acquitting the guilty, while the juror with preference parameter $\tilde{q}_2$ is increasingly concerned about convicting the innocent. Equilibrium is then *spread*, as the following Proposition captures.

Proposition 3 (Equilibrium Spread) *(Spread) $p_*(b)$ is decreasing in b, $p^*(b)$ is increasing in b. Consequently, in symmetric juries, expected deliberation length and the accuracy of decisions increase with the deliberation and decision rules (both given by R_d).*

The Proposition implies that increased heterogeneity in the jury (manifested in a higher b) will reduce the two types of mistakes, and increase the expected time to a decision. This is consistent with the empirical observations of Sommers (2006), who used mock juries to test for the effects of racial heterogeneity on jury performance and found that heterogeneity was associated with longer deliberation and more accurate decisions. In a similar vein, Goeree and Yariv (2011) found increased preference heterogeneity to be associated with longer deliberation times in the laboratory.

The deliberation rule directly affects the spread or distance b between the two pivotal jurors. Indeed, the more demanding the deliberation rule (a higher $R_d = R_v$), the more extreme the pivotal jurors, and hence, the greater this distance: $q_{R_d} - q_{n-R_d+1}$ is increasing in R_d. The monotonicity of $p_*(b)$ and $p^*(b)$ then imply that indeed deliberation length and decision accuracy increase with R_d. This result highlights the qualitative difference between information that is collected privately and publicly in jury settings. Indeed. the result is in contrast with the results in Feddersen and Pesendorfer (1998), Persico (2004), and Austen-Smith and Feddersen (2005, 2006) – unanimity rule in our setting leads to more informative outcomes than majority rule.

The previous results would still hold if departures from symmetry are not too large. However, it is possible to construct examples where, absent symmetry the previous results do not hold. For instance, if q_{n-R_d+1} and q_{n-R_d} were close to one another, but q_{R_d+1} was much larger than q_{R_d}, then, the acquittal threshold would actually be larger with deliberation rule $R_d + 1$ than with rule R_d.[23]

[23] This follows from monotonicity of thresholds proved in the appendix.

Welfare effects of deliberation rules and decision rules depend on the perspective from which welfare is calculated. One point of view is to only consider the ex-ante welfare of the jury. Another is to include the benefit of accurate decisions for society at large, including individuals who do not directly bear the cost of deliberation.

From the perspective of the participating jurors, in any quasi-symmetric jury, we can assess the optimal spread of the pivotal agents. It turns out that the lowest possible spread is favored by *all* jurors, as captured by the following proposition.

Proposition 4 (Optimal Delegation) Assume that the prior is 1/2. *In a symmetric jury, jurors have unanimous preferences over deliberation rules: all jurors prefer pivotal agents with as little spread as possible or* $R_d = \lceil n/2 \rceil$.

Intuitively, recall expression (5) for a juror's utility. In a symmetric jury, thresholds are symmetric. Therefore, the two terms in (5) corresponding to the two types of mistakes are, in fact, a convex combination (via q_i) of an *identical* expected probability of mistake. It follows that the expected utility does not explicitly depend on q_i. In particular, all of the jurors gain the same level of expected utility as would a juror with preference parameter $\frac{1}{2}$ if she were to have the equilibrium thresholds imposed upon her.[24] However, note that a juror with preference parameter $\frac{1}{2}$ would prefer no spread at all ($b = 0$ in our notation above), as then she receives her optimal thresholds. Monotonicity then implies our result.[25]

Proposition 4 is particularly stark because of symmetry. However, the effect highlighted in this proposition is more general: in a large class of asymmetric juries, the most extreme jurors will not push for unanimity at the deliberation stage because unanimity means that deliberation is long on *both* sides of the prior, making the cost of deliberation too high from an ex-ante perspective to make it worth reducing the probability of mistakes further.

Proposition 4 takes an ex-ante perspective. *Prior* to deliberations, the jury would like to delegate deliberation decisions to a moderate individual. This perspective speaks to most applications in which the design of the committee's protocol of information collection is decided upon at the outset. Nonetheless, we note that once deliberations have commenced, symmetry breaks down as posteriors

[24] Unlike other results in the paper, this result hinges on the prior of both states being symmetric. We discuss relaxation of symmetry below.

[25] Proposition 4 hints at the possible effectiveness of deliberation taxes. Indeed, increasing the costs of deliberation would lead to shorter deliberation times, which may be preferrable to at least a fraction of the population.

may lean one way or another, and jurors may no longer be in consensus as to whom to delegate power to (despite equilibrium thresholds being stationary).

It is also useful to point to a contrast between decision rules and deliberation rules at this point. Proposition 4 provides a rationale for giving moderate jurors control over the deliberation phase of the trial. This could be done by the implementation of rules requiring a small majority (optimally, a simple majority rule) or by delegating the decision to a moderate chairman (optimally, with preference parameter $q = 1/2$). Nonetheless, at least some jurors may prefer a more stringent decision rule to determine the verdict. Indeed, no juror would willingly give up her voice in the decision phase and choose a decision rule that would end up excluding her. In particular, we cannot expect the same level of agreement on decision rules as the one achieved over deliberation rules.

5. Decision Rules and Deliberation Rules

We now consider a jury composed of n jurors of arbitrary preferences $q_1 \leq q_2 \leq ... \leq q_n$ and we allow for differences between deliberation and decision rules. When $R_d \neq R_v$, there are two sets of relevant pivotal agents: those pertaining to the deliberation stage and those pertaining to the decision stage.

In analogy to Proposition 2, during the decision stage, whenever juror j would prefer to convict if she were dictator, so would any juror $l < j$. Whenever juror j would prefer to acquit if she were dictator, so would any juror $l > j$. It follows that the jurors to focus on are those pivotal during deliberation: jurors R_d and $n - R_d + 1$, and those pivotal during the decision stage: jurors R_v and $n - R_v + 1$.

For sufficiently low costs, deliberation will lead to consensus on the decision to be taken: the jury will deliberate for an amount of time that is such that the only possible posteriors are sufficiently extreme that they are either to the left of q_1 or to the right of q_n. In particular, the rule R_v in the decision stage does not matter, and only the deliberation rule determines the length of the deliberation process.

There is another class of cases in which the decision rule R_v does not affect outcomes. Fix a deliberation rule R_d that is a strict super-majority. Then, any decision rule that is at most as consensual as R_d ($R_v \leq R_d$) leads to the same equilibrium outcomes. The reason is fairly mechanical: whenever there is a sufficient super-majority to halt deliberation, there must be at least the same super-majority at the decision stage. This is just the Wald logic: thresholds for stopping deliberation are inherently more demanding than the thresholds (preference parameters q_i) for taking a decision. Thus, equilibrium outcomes do not depend on R_v in this region.

The following proposition summarizes our discussion of the two cases in which the decision rule has no effect on final outcomes.

Proposition 5 (Decision Rule Irrelevance) *For any deliberation rule R_d,*

1. *(Restricted Irrelevance) For any decision rules R_v, $\tilde{R}_v \leq R_d$, the set of equilibrium outcomes corresponding to R_v and $\tilde{R}_v$ coincide.*

2. *(Irrelevance due to unanimous agreement: low costs) For any given preference profile, there exists a $\underline{k}$ such that, for $k \leq \underline{k}$, the rule R_v at the decision stage is irrelevant for equilibrium outcomes.*

Proposition 5 outlines two important cases in which the rules at the decision stage do not matter. In particular, these are cases in which neither the time to decision nor the relative probability of the two types of mistakes depend on the decision rule. This is consistent with the empirical and experimental evidence that we discuss later, which points to collective outcomes being insensitive to the decision rule in circumstances when communication is available (see, e.g., Baldwin and McConville, 1980, Devine et al., 2001, and Goeree and Yariv, 2011).

There are, however, cases in which the voting rule at the decision stage is important for outcomes. Before considering explicitly how decision rules can affect outcomes, we want to point out that whatever the consequence of decision rules is, it is easy to show that a shift to a more inclusive deliberation rule (keeping the decision rule fixed) induces longer deliberation than a mirror image shift of decision rules (keeping the deliberation rule fixed). Intuitively, this is a consequence of the fact that whenever deliberation occurs, it must be sufficient to potentially alter the minds of the jurors pivotal in the decision stage. In other words, the deliberation thresholds would envelope the preference parameters q_{n-R_v+1}, q_{R_v} of the pivotal jurors at the decision stage. Taken together with Proposition 5, this says that in some circumstances deliberation rules are more powerful than decision rules in driving the process of jury decision making and deliberation.

Consider now a case in which $R_v > R_d$ and there is a relatively high cost of deliberation k. Consider further a hypothetical situation in which the pivotal jurors at the deliberation stage (agents

$n - R_d + 1$ and R_d) ignore the fact that more extreme jurors (agents $n - R_v + 1$ and R_v) are pivotal at the decision stage. In this scenario the jury may end up hung because deliberation stops before a decision quorum is reached. Thus, in these cases, the pivotal jurors at the deliberation stage face a trade-off: they can either prolong deliberation to convince the pivotal jurors in the decision stage, or they can halt deliberations immediately. If the costs of deliberation are not too high, some additional deliberation may therefore be beneficial since it avoids a hung jury. Intuitively then, in these cases one should expect that unanimity rule at the decision stage will lead to longer deliberation and more accurate decisions than simple majority. We show below that this is true for juries with symmetric preferences (around 1/2).[26] We thus return to the case of quasi-symmetric juries.

It is easy to see that even when the decision rule R_v is more consensual than the deliberation rule R_d, $R_v \geq R_d$, symmetric equilibrium thresholds are still determined uniquely. Essentially, there are two cases to consider. First, when costs of deliberation are low, the equilibrium deliberation thresholds are sufficiently wide that the super-majority requirement at the decision stage is automatically met and exceeded. In the second case, with higher costs and $R_v > R_d$, the pivotal jurors at the deliberation stage would like to settle for deliberation thresholds $p_* > q_{n-R_v+1}$ and $p^* < q_{R_v}$. However, such thresholds would lead to a hung jury. In this scenario, the equilibrium deliberation thresholds are driven by the requirement to reach sufficient consensus at the decision stage so we obtain $p_* = q_{n-R_v+1}$ and $p^* = q_{R_v}$: deliberation continues until the moment at which the pivotal jurors at the decision stage are persuaded to join the required consensus. When costs are sufficiently high, achieving a consensus at the decision stage becomes too costly for the pivotal jurors at the deliberation stage and the unique equilibrium outcome involves no information collection and an immediate hung jury.

In the following proposition, we denote by $\left(p_*\left(R_d, R_v; k\right), p^*\left(R_d, R_v; k\right)\right)$ the unique symmetric equilibrium thresholds corresponding to deliberation rule R_d and decision rule $R_v \geq R_d$.[27]

Proposition 6 (Decision Rule Relevance: Inclusiveness Effect) *Consider a symmetric jury.*

For any deliberation rule R_d, consider two decision rules $\tilde{R}_v > R_v \geq R_d$ for which $q_{\tilde{R}_v} >$

[26] When juries are asymmetric, changing the decision rule or the deliberation rule may not lead to uniformly more accurate decisions because, for instance, more accurate acquittal decisions may come hand in hand with less accurate conviction decisions: it can be the case that, say, making the decision rule more extreme reduces the acquittal equilibrium threshold p_* but also reduces the equilibrium conviction threshold p^*.

[27] The previous analysis together with the construction in the proof of Proposition 6 illustrate that equilibria indeed take this form and are unique.

q_{R_v}. There exist $\underline{k}, \overline{k}$ such that, for $\underline{k} < k < \overline{k}$, the symmetric equilibrium thresholds satisfy $p_*\left(R_d, \tilde{R}_v; k\right) < p_*\left(R_d, R_v; k\right)$ and $p^*\left(R_d, \tilde{R}_v; k\right) > p^*\left(R_d, R_v; k\right)$: more inclusive decision rules induce longer deliberation.

Proposition 6 outlines a case where the larger the supermajority required for making a decision, the more information collection there is: the deliberation time and decision accuracy are greater under more consensual supermajority rules. The proposition pertains to moderate costs. Indeed, when costs are low, Proposition 5 illustrates that decision rules do not play any role since the amount of information collected is so large as to create a consensus at the decision stage. When costs are very high, any length of deliberation that may affect votes at the decision stage is prohibitively costly, and so individuals foresee a hung jury being formed and all prefer to forgo deliberation altogether.

This result provides a contrast between our characterization and those pertaining to private information collection, as in Feddersen and Pesendorfer (1998), Persico (2004), and Austen-Smith and Feddersen (2005, 2006). The intuition for this result can be understood by considering a special case. Consider a symmetric jury in which there is a median agent with preference parameter $q = \frac{1}{2}$. Assume also that R_d corresponds to simple majority. We contemplate the effect of moving from R_v = simple majority to R_v = unanimity. Suppose that costs of deliberation are sufficiently high that when R_v corresponds to a simple majority, it is not worthwhile for the median juror to deliberate long enough to reach consensus on the decision. Then, under unanimity, the median juror who is still pivotal in the deliberation process understands that, in order to reach a verdict, she cannot stop deliberation as early as when R_v = simple majority. In order to avoid a hung jury she must convince the extreme jurors to vote with everyone else. This requires longer deliberation. When costs are not too high, it is worthwhile to deliberate just long enough to obtain these jurors' votes on the decision. Of course, when the costs of deliberation become high enough, it is no longer optimal for the pivotal jurors to prolong deliberation. In this case, hung juries take place with positive probability.

These results imply that interior equilibria *always* depend only on two jurors. When $R_v \leq R_d$, the jury outcome is equivalent to that of a jury composed of two jurors with preferences q_{R_d} and q_{n-R_d+1} (and unanimous deliberation and decision rules), while when $R_v > R_d$, any jury outcome entailing non-trivial deliberation is equivalent to that of a jury composed of two jurors with preferences q_{R_v} and q_{n-R_v+1} (and, again, unanimous deliberation and decision rules).

In practice (and outside our model), it may be the case that a change in the decision rule translates into a change in the deliberation rule; this can be called a *protocol effect.* In the presence of such a protocol effect, the decision rule has a clear impact. Indeed, when the deliberation and decision rules coincide, Proposition 2 holds, so that two pivotal jurors determine the outcome. The more demanding the decision rule, the more extreme these two jurors are. Consequently, Lemma 1 together with Proposition 2 imply that more stringent decision rules would correspond to longer deliberation and more accurate decisions.

6. Fit of the Model with Facts about Juries

Our analysis delivers a number of empirical implications. In this Section we discuss some evidence on juries documented in prior empirical literature. Some of this evidence speaks to the basic structure of the model and some is directly related to our results. This evidence suggests that our model is broadly consistent with a number of different patterns in the data. Furthermore, the model is a useful way to think about a broad category of features of the data.

Importance of deliberation and verdict patterns

Hannaford, Hans, Mott, and Musterman (2000) studied the timing of jury opinion formation. They used a special case study of a jury reform implemented in Arizona in 1995 that allowed for discussions during civil trial (Rule 39(f) of the Arizona Rules of Civil Procedure). Their data includes survey responses of $1,385$ jurors from 172 trials in four counties (accounting for a large majority of cases in Arizona) concerning when they formed their initial opinions, whether and when they changed their minds, and when they arrived at a resolution regarding the final outcome. Over 95% of jurors reported changing their mind at least once over the course of the trial and 15% reported changing their minds more than once during trial. Importantly, over 40% of jurors reported changing their minds during the final deliberations, suggesting that deliberation is a key component of opinion formation.

Hans (2007) studied jury deliberation by using surveys conducted by the National Center for State Courts (NCSC). Hans' data contains reports from close to $3,500$ jurors who had participated in felony trials in four large, urban courts. Figure 3 summarizes one key finding by Hans (2007). The figure groups each jury into five categories depending on the outcome of an initial straw poll. These go from "strongly favor innocent" where the great majority of jurors initially favored acquittal, to "leaning toward innocent," where a small majority initially favors acquittal, to "closely divided"

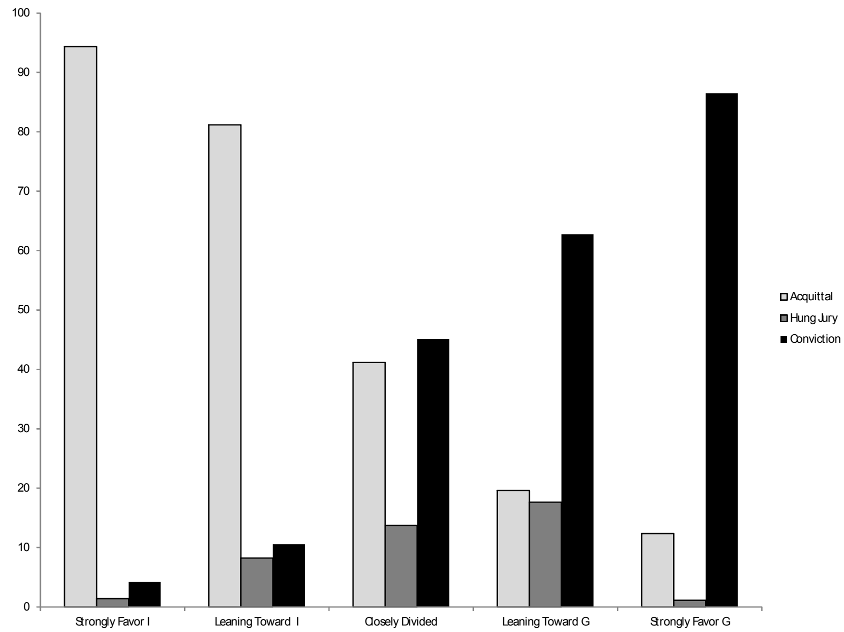

Figure 4: Distribution of Jury Outcomes (in %) According to Initial Jury Leanings (Hans, 2007)

where the jury is evenly split (5-7, 6-6 or 7-5), "leaning toward guilty," and, finally "strongly favor guilty." For each initial leaning of the jury, the figure describes the distribution of ultimate outcomes.

There are a number of interesting observations arising from this figure. First, the patterns of opinion change are consistent with collective information acquisition driven by a Bayesian updating process (as in our model). The most immediate observation is that when the initial vote in the jury strongly supports a particular outcome, that outcome is more likely to ultimately emerge. For instance, 77 of the 89 juries with strong majorities for guilt convicted the defendant. However, the ultimate outcome does change during deliberation: 11 of these juries 89 ended up acquitting the defendant, showing that in some cases, many jurors were persuaded during deliberation to switch their vote to acquittal.[28] Recall that the verdict had to be unanimous. Therefore, in all these 11 cases almost all jurors changed their mind during deliberation. Thus, the deliberation process did have a large effect in these juries.

It is also interesting to note that some features of this figure are not consistent with an alternative,

[28] On the other hand, 67 of the 71 juries with strong majorities for innocence in the straw poll acquitted the defendant, and 3 convicted.

These observations should be interpreted with some care, as initial polls within the jury sometimes take place after some amount of deliberations has already taken place. Thus, consensus may be overstated.

more "traditional" model of information gathering in which juries vote after receiving a fixed number of signals. For instance, consider the scenario in which the initial poll (the prior) strongly favors guilty. In this alternative model, the number of hung juries would be larger than the number acquittals: it would be much more likely that the jury would end up divided than that most jurors change their opinions. This is of course the opposite of what we see in the data. In contrast, in our model the amount of information collection responds to the state of uncertainty in which the jury finds itself, so, in this same scenario, should the jury find itself divided after a given number of signals, the jury would be likely to continue deliberation. In order to match all of the features of this figure we need to allow for some additional heterogeneity; otherwise, for instance, all closely divided juries would either all hang or they would all deliver a verdict.[29] If we add some initial draw of costs of deliberation that could for instance depend on the difficulty of the case, then our model is consistent with all features of this figure.[30]

Jury composition Increased heterogeneity has been found to increase quality and length of deliberation (see Sommers, 2006, who investigates how racial composition affects deliberation in mock juries, and Goeree and Yariv, 2011 who present experimental evidence that increased heterogeneity of exactly the payoff type that we discuss here increases deliberation length and accuracy of decisions). In our model, increased heterogeneity increases the length of deliberation since it makes the pivotal members at the deliberation stage more extreme, and therefore more definitive information is needed in order to stop deliberation. This translates immediately into longer deliberation and, in symmetric committees, more accurate decisions.

Effect on length of deliberation Devine et al. (2001) provides a meta-study of empirical jury research and reports that the decision rule affects length and quality of deliberation. On average, using mock juries, under unanimity verdicts tend to take as long as under majority. The quality, measured by legal experts, exhibits similar patterns – higher under unanimity than under majority.

In our model, the length of deliberation can be affected by the decision rule since pivotal members at the deliberation stage may, if not pivotal at the decision stage, prolong deliberation in order to convince the holdouts at the decision stage.

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[29] See Section 7 for a discussion of hung juries.

[30] We have simulated our model and it is easy to generate figures very much like this one.

Effects of the decision rule Baldwin and McConville (1980) studied a reform that was put in place in 1974 in England that allowed for majority verdicts in criminal trials, while prior to the reform unanimity was required. The vast majority of verdicts (311 out of 326 cases) were unanimous even after the reform, suggesting that the decision rule did not have much of an effect. Kalven and Zeisel (1966) report similar patterns for U.S. states that do not require unanimity for conviction: most verdicts are unanimous anyway.[31] Devine et al. (2001) report that in many mock jury studies there is no evidence that the decision rule has any effect on the verdict.[32] In contrast, Hastie, Penrod, and Pennington (1983) find that the volume of discussion substantially increases with the decision rule. In lab experiments, Goeree and Yariv (2011) find that, when subjects cannot talk before voting, the decision rule has an effect, whereas, when subjects can talk, the decision rule has very little effect.[33] Our model provides a possible explanation for the fact that in many circumstances the decision rule seems to have little effect. We show that, when costs of deliberation are sufficiently low, in equilibrium, deliberation always ends with unanimous decisions: whenever there is disagreement on the appropriate decision to take, members of the committee agree that it is worthwhile to continue deliberating.

7. Extensions

7.1. Simultaneous Deliberation. We now discuss a case in which the decision on the amount of information to be collected takes place in one shot and contrast this case with the sequential one considered up to now. When committee members are homogeneous, this is equivalent to the classic case of choosing the optimal sample size for the test of a binary hypothesis (see De Groot 1970, Chapter 11). We retain the jury language although the jury setting is no longer a natural application for this case. Nonetheless, there are many environments in which the size of the sample is determined at the outset. For example, drug companies decide on the sample size of patients at

[31] A caveat to this observation is that under simple majority, when a majority of jurors agrees, any other juror's vote cannot affect the final outcome. In particular, those jurors may vote against their private assessment to satisfy social pressures at no consequence to the defendant.

[32] However, Devine et al. also point out that the studies that present results on decision rules have some weaknesses such as small samples and little variation in verdicts.

[33] In the context of monetary policy, Blinder and Morgan (2008) look experimentally at two particular decision protocols (ones in which there is a designated leader, and ones in which there is not) and also find little differences in outcomes.

the beginning of many drug trials, academic departments decide on the number of outside recommendation letters at the start of most promotion processes, etc.[34]

In our version with heterogeneous jurors we need to specify some additional details of the model. A deliberation decision determines the sample size t. A sample of size t costs each juror kt. This is the only cost born by the committee. At time t, jurors observe the realization of the sequence of random variables $X_1, ..., X_t$, and they vote to acquit or convict according to a decision rule R_v just as in Section 2. Deliberation can be modeled in a number of ways but, for concreteness, we assume the following process. Deliberation takes place at no cost before the sample is drawn according to deliberation rule R_d. An index moves over discrete time starting from 1. At index τ, if jurors have not yet come to an agreement, then jurors vote on whether sample size τ is acceptable. If at least R_d jurors agree that the sample size is sufficient, then the deliberation process is over and a sample of size τ is drawn. If fewer than R_d jurors agree, then the index moves on to $\tau+1$. The process continues until an R_d majority is satisfied. This model would be identical to our sequential deliberation model if voters had to stop deliberation without seeing the realizations of the random variables. Given our result below that deliberation is always unanimous, the exact deliberation protocol is irrelevant for symmetric juries.[35] However, the model described above is easier to work with and is a closer match to the sequential deliberation model.[36]

Let p_t be the posterior if the deliberation process has yielded a sample size t. Then, at date t, a juror of type q votes to convict if $p_t \geq q$, and votes to acquit if $p_t < q$. If at least R_v votes are obtained, then a decision is reached. Otherwise we have a hung jury.[37]

Proposition 7 (Simultaneous Deliberation: Decision Rule Relevance) *In a symmetric jury, under simultaneous deliberation, jurors have common preferences over deliberation decisions.*

[34] It would, of course, be interesting to consider a mixed model, where decisions are partly sequential, partly simultaneous. In such a setting, at each date the commitee decides on a sample size, but the sample size can be augmented later. Moscarini and Smith (2001) provide an analysis of this problem for the single agent case.

[35] We could allow for a once and for all deliberation vote over the infinite set of alternatives corresponding to different sample sizes. We intentionally set this version of the model in the framework of binary decisions as it captures the same issues conceptually and allows us to naturally circumvent standard issues of equilibrium multiplicity that arise in voting models with more than two alternatives.

[36] As in the previous analysis, there are possible multiple equilibria. As before, we can think of a refinement where one considers finite truncations of the game. Equilibria surviving iterated elimination of weakly dominated strategies in the agent-form game would correspond to (timed) thresholds, and that sequence of thresholds converges to the thresholds we analyze here as the horizon of the game grows indefinitely.

[37] Recall that we maintain the assumption that in the event of a hung jury, the defendant is acquited or convicted with equal probability. The proposition would, in fact, hold as long as the ex-ante expectation of the costs of a hung jury are independent of q. For instance, a fixed cost of a hung jury independent of q would generate similar insights.

Therefore, the deliberation rule R_d is irrelevant. However, the decision rule matters: if $\widetilde{R}_v > R_v$, a jury voting under decision rule $\widetilde{R}_v$ chooses to collect more information.

The intuition for the irrelevance of the deliberation rule is related to the intuition of Proposition 4. Given that deliberation is simultaneous, jurors evaluate the optimal amount of information to be collected ex-ante, before seeing the realization of any signals. All jurors simply trade off increased accuracy against the cost of information collection independent of their preference parameters because increased information collection reduces mistakes of both types equally.

The intuition for the effect of the decision rule is the following. Under simultaneous deliberation, for any given amount of gathered information, a larger decision rule raises the probability of a hung jury. Acquiring additional information reduces the probability of this costly event.

This result is in sharp contrast with the results we obtained for the case of sequential deliberation. This is not very surprising given the very different nature of deliberation in the two scenarios.

The contrast between the consequences of simultaneous as opposed to sequential protocols is also familiar from the literature on search and auctions. Note also that, in contrast with the case of sequential deliberation, even with symmetry, the simultaneous scenario allows for the coexistence of significant information collection and hung juries.

There are a number of interesting additional comparisons that can be made between the sequential and simultaneous scenarios. First, there is a strong general effect leading to welfare being higher under sequential deliberation: welfare is obviously unambiguously higher in the sequential setting for the case of a single juror because there is more efficient use of information. Indeed, this was the original motivation behind Wald's analysis. By Proposition 4 we can also conclude that welfare from the point of view of the committee is higher in the sequential case under simple majority when juries are symmetric.

Another interesting comparison concerns the likelihood of consensual votes. For low costs, sequential deliberation leads to unanimous verdicts regardless of the decision rule (see Proposition 5). In contrast, simultaneous deliberation generates a positive probability of some disagreement for any decision rule, for any costs, because there are always positive probability sets of signals that are not very informative. For intermediate costs the comparison is less straightforward, but, for any fixed decision rule, simultaneous deliberation tends to generate more variation in consensus because in the case of sequential deliberation the vote is more likely to end at a quorum.

7.2. Heterogeneous Deliberation Costs. Suppose now that jurors differ in the costs that are imposed upon them through deliberation (e.g., if costs are linked with the time away from work, variance in wages may translate to variance in deliberation costs). Formally, in order to assess the effects of cost heterogeneity, we assume that all jurors share the same preference parameter q, but juror i's deliberation cost is given by k_i, where without loss of generality $k_1 \geq k_2 \geq ... \geq k_n$.

Because jurors share the same q, the decision rule R_v does not affect outcomes since for any given posterior the jurors all agree on the optimal action to be taken. The deliberation rule, however, does have an effect. Whenever agent j wants to stop information collection, so does any agent experiencing higher costs ($l < j$). It follows that the pivotal juror during deliberation is the R_d'th juror so that a jury with rule R_d chooses thresholds of a homogeneous committee with costs k_{R_d} . Hence, deliberation length and accuracy of decisions increase with the decision rule R_d. This implies that a designer who values the benefit of accurate decisions because it benefits society at large and would would be inclined to choose as demanding a deliberation rule as possible. In general though, the welfare optimal deliberation rule depends on the distribution of waiting costs in the relevant population.

7.3. Civil Juries. We now consider an example of a related model, where a jury must take a decision from a continuum of possible choices. This can be interpreted as a model of a civil jury choosing the amount of damages to award a plaintiff. The jury is uncertain about the true level of damages D. The prior distribution over these true damages is given by a normal distribution $N(\mu, \frac{1}{p_0})$ with mean μ and precision p_0.

Suppose we allow a limited degree of heterogeneity among jurors that, as above, is given by how costly it is for them to continue collecting information (in the current setting, this will be tantamount to allowing heterogeneity in how strongly each juror desires to make the correct decision). Specifically, assume the payoffs for each juror if true damages are D and the jury awards Q are given by

$$U(D, Q) = -\alpha(D - Q)^2,$$

where $\alpha > 0$.

The jury deliberates as in the model presented in Section 2: at each deliberation date they observe a new signal X_t at cost k. The signals $X_1, X_2, ...$ are conditionally independent and identically distributed. In fact, we assume that X_t is normal with mean D and precision p_X (i.e., for all t, $X_t \sim N(D, \frac{1}{p_X})$). The jury stops deliberating if at least R_d jurors vote to stop, otherwise it continues. Note that, given the assumption about payoffs, once deliberation has ended, the jury is unanimous about the optimal decision. Thus, all disagreements arise in deliberation choices.

The optimal choice if the jury stops deliberating at t is the conditional expectation of D (equivalently, X_{t+1}) given the prior history. As is well known, in this normal quadratic setting, this conditional expectation takes a convenient form:

$$\mathbb{E}[X_{t+1}|X_1 = x_1, \ldots, X_t = x_t] = \frac{p_0\mu + p_X \sum_{s=1}^{t} x_s}{p_0 + tp_X}.$$

Thus, the payoff to a juror experiencing a cost k when the jury stops at time t is given by

$$-\frac{\alpha}{p_t} - kt = -\frac{\alpha}{p_0 + tp_X} - kt \tag{4}$$

Note that this payoff is independent of the realizations of $X_1, ...X_t$ so, in this setting, in contrast with our prior analysis, there is no difference between sequential and simultaneous deliberation.

From equation (4) we can immediately conclude that jurors with lower costs k want to stop later and we obtain a similar result to our original one concerning the effects of deliberation rules: The accuracy of damage awards is higher under more consensual deliberation rules. This is in line with the empirical observations reported in Hans (2001).

Note that, in equation (4), increasing the cost k has similar effects to lowering the preference parameter α. In particular, as mentioned above, cost heterogeneity plays a similar role to preference heterogeneity (when manifested through heterogeneous parameters α in the jury).

7.4. Incomplete Information, Stationarity, and Hung Juries. Throughout the paper, we have assumed that jurors' preferences are commonly known. This assumption allowed us to focus on stationary strategies and extract the main tensions between the deliberation and decision phases. Nonetheless, a natural extension to our model is to the case in which jurors have some incomplete information about the learning process at hand, either due to preferences that are not commonly known or due to the informativeness of the collective signals not being fully transparent. In such environments, the process of deliberation confounds two learning processes: regarding the guilt of the defendant, and regarding the prevailing characteristics of the jury (distribution of preferences or signal informativeness). In particular, the deliberation phase is inherently non-stationary.

While the analysis of such a model requires some novel techniques and goes beyond the scope of the current paper, we view it as especially important for explaining the patterns identified by the empirical literature regarding hung juries. Indeed, in such a model, it is conceivable that the longer deliberation goes on, the more likely it is that jurors are "high strung" or that the process is not very informative, and that agreement is likely to take a longer time than was initially estimated. Under certain additional conditions, this modification is likely to deliver that hung juries deliberate longer.[38] This would be in line with the evidence provided by Kalven and Zeisel (1966) and Hans (2003), who find that hung juries deliberate a significantly longer time than juries that deliver a verdict.[39] Furthermore, the tensions between costly information collection and decision accuracy, that are the driving force of our results, would persist in such a setting.

In reality juries cannot unilaterally declare themselves hung. If they attempt to do so too early the judge will instruct them to continue deliberations. Our model can easily accommodate this case, by introducing a constraint of a minimal length of deliberation before the jury can declare itself hung. The results are not significantly affected if we add this constraint.

[38] An alternative assumption that we suspect would lead to a similar conclusion is that deliberation costs are increasing in time.

[39] In our baseline setting of complete information, a robust consequence is that even in cases of asymmetries, hung juries deliberate for a shorter time than juries that deliver a verdict. The reason is that agents can anticipate when a hung jury is likely to occur and thereby not invest in any information collection whatsoever.

8. Appendix A – Main Proofs

Let $T(p^a, p^c)$ denote the stopping time associated with thresholds p^a and p^c, $p^a \leq p^c$. That is, $T(p^a, p^c) = \inf\{t : p_t \leq p^a \text{ or } p_t \geq p^c\}$. The probability of convicting the innocent is then given by $\Pr(p_T \geq p^c | I; p^a, p^c)$ and the probability of acquitting the guilty is given by $\Pr(p_T \leq p^a | G; p^a, p^c)$. For future reference, it is also useful to note that expected utility for a juror of preference parameter q can be expressed as:

$$U(q; p^a, p^c) = \begin{array}{l} -\frac{1}{2}\left[q \Pr(p_T \geq p^c | I; p^a, p^c) + k\mathbb{E}\left(T(p^a, p^c)|I\right)\right] \\ -\frac{1}{2}\left[(1-q)\Pr(p_T \leq p^a | G; p^a, p^c) + k\mathbb{E}\left(T(p^a, p^c)|G\right)\right], \end{array} \tag{5}$$

where $\mathbb{E}\left(T(p^a, p^c)|G\right)$ and $\mathbb{E}\left(T(p^a, p^c)|I\right)$ correspond to the expected deliberation time when the defendant is either innocent or guilty, respectively.[40]

For presentational simplicity, we assume that the signal distributions F_G and F_I are sufficiently well-behaved so that $\Pr(p_T \leq p^x | Y; p^a, p^c)$ and $\mathbb{E}\left(T(p^a, p^c)|Y\right)$ are twice continuously differentiable with respect to p^a and p^c, for $x = a, c$ and $Y = G, I$. This allows us to use calculus techniques to identify properties of the solution. The assumption is satisfied for many commonly used signal distributions. For instance, it holds if F_G and F_I are normal distributions.

We call a sequential problem *regular* if it satisfies $\frac{\partial \Pr(p_T \leq p^a|G;p^a,p^c)}{\partial p^a}, \frac{\partial \Pr(p_T \leq p^a|G;p^a,p^c)}{\partial p^c} \geq 0$ and $\frac{\partial \Pr(p_T \geq p^c|I;p^a,p^c)}{\partial p^a}, \frac{\partial \Pr(p_T \geq p^c|I;p^a,p^c)}{\partial p^c} \leq 0$. Intuitively, regularity implies that the higher p^a is, i.e., the easier it is to get an acquittal, the more likely it is to get acquittals of guilty defendants and the less likely it is to get convictions of innocent defendants. Analogously for changes in p^c. We will assume regularity throughout. Regularity is satisfied for many information generation processes, in particular normal processes with sufficiently low accuracies assumed in the body of the paper.[41]

We now define a 'one-sided' best response – fixing the conviction threshold at $p^c = \overline{p}$, and assuming no constraint on the choice of acquittal threshold, $\underline{p} = \overline{p}$, the utility maximizing threshold for acquittal is given by $\tilde{p}^a(p^c; q)$. Similarly, fixing the acquittal threshold at $p^a = \underline{p}$, and assuming no constraint on the choice of conviction threshold, $\underline{p} = \overline{p}$, the utility maximizing threshold for conviction is given by $\tilde{p}^c(p^a; q)$. There is an important difference between $p^a(\overline{p}; q)$ and $\tilde{p}^a(p^c; q)$: for the latter

[40] These are well-defined, see De Groot (1970).

[41] A continuous-time version of our environment, in which information follows a Brownian motion with a state-dependent drift, trivially satisfies regularity (since posteriors at the end of deliberation always coincide with one of the thresholds). This of course implies that a large class of discrete-time processes satisfy regularity if the time between periods is short. Alternatively, if signals are binary (indicating the true state with some probability, as in Feddersen and Pesendorfer, 1998 and many follow-up papers), the corresponding sequential problem is regular.

the conviction threshold is actually fixed at p^c, whereas for the former, it is only constrained to be at least $\overline{p}$. We start with the following lemma that summarizes some of the important technical features of these functions that will be useful for our analysis. This lemma assumes uniqueness of best responses that we show in Lemma A2 that appears in Appendix B, as it requires some additional notation. As in the body of the paper, the reader who is only interested in the basic characteristics of our problem can focus on normal signal distributions with sufficiently low accuracies and skip the discussion of Appendix B.

Lemma A1 (Properties of Constrained Problem)

1. *$p^a\left(\underline{p}, \overline{p}; q\right)$ is non decreasing in $\overline{p}$ and $p^c\left(\underline{p}, \overline{p}; q\right)$ is non decreasing in $\underline{p}$.*

2. For all p, $\tilde{p}^a(p; q) \geq \tilde{p}^a(p^c\left(q\right); q) = p^a\left(q\right)$, $\tilde{p}^c(p; q) \leq \tilde{p}^c(p^a\left(q\right); q) = p^c\left(q\right)$.

3. *For any (p^a, p^c), $\tilde{p}^a(p^c; q)$ and $\tilde{p}^c(p^a; q)$ are increasing in q.*

Proof of Lemma A1

Part 1. For any $p, \underline{p}, \overline{p}$,

$$\max\left\{V^1(p \mid \underline{p}, \overline{p}; q), -q\left(1-p\right)\right\} \geq V^1(p \mid \underline{p}, \overline{p}; q).$$

Consider $\overline{p}_1 > \overline{p}_2$. From (3) it follows that for any $\underline{p}$, $V^1(p \mid \underline{p}, \overline{p}_1; q) \leq V^1(p \mid \underline{p}, \overline{p}_2; q)$. There are two cases to consider. In the first case, $p^a\left(\overline{p}_2; q\right) < \underline{p}$, so that the solution is given by the intersection between the line $-(1-q)p$, which is decreasing in p (see Figure 1), and $V^1(p \mid \underline{p}, \overline{p}_2; q)$ (that is unique, according to Lemma A2 in Appendix B below). If $p^a\left(\underline{p}, \overline{p}_2; q\right) = \underline{p}$, monotonicity follows. Suppose then that $p^a\left(\overline{p}_1; q\right) < \underline{p}$. Since $V^1(p \mid \underline{p}, \overline{p}_1; q) \leq V^1(p \mid \underline{p}, \overline{p}_2; q)$, the intersection point with $V^1(p \mid \underline{p}, \overline{p}_1; q)$ must be (weakly) higher than that with $V^1(p \mid \underline{p}, \overline{p}_2; q)$, implying that $p^a\left(\overline{p}_2; q\right) \leq p^a\left(\overline{p}_1; q\right)$ as needed. In the second case, $p^a\left(\overline{p}_2; q\right) = \underline{p}$, and since $p^a\left(\overline{p}_1; q\right) \leq \underline{p}$, monotonicity follows. Monotonicity of $p^c\left(\underline{p}; q\right)$ follows analogously.

Part 2. Because $p^a\left(q\right), p^c\left(q\right)$ are the thresholds for the unconstrained optimal (Wald) solution, for all p, we must have $V^1(p \mid p^c = p^c\left(q\right); q) \geq V^1(p \mid p^c = p'; q)$. Therefore, by the same reasoning as in part 1 of this Lemma, $\tilde{p}^a(p'; q) \geq \tilde{p}^a(p^c\left(q\right); q) = p^a\left(q\right)$. Analogous steps prove that $\tilde{p}^c(p'; q) \leq \tilde{p}^c(p^a\left(q\right); q) = p^c\left(q\right)$

Part 3. Notice that

$$\begin{aligned}\frac{\partial U(q;p^a,p^c)}{\partial p^a} = & -\frac{1}{2}\left[q\frac{\partial \Pr(p_T \geq p^c|I;p^a,p^c)}{\partial p^a} + k\frac{\partial \mathbb{E}(T(p^a,p^c)|I)}{\partial p^a}\right] \\ & -\frac{1}{2}\left[(1-q)\frac{\partial \Pr(p_T \leq p^a|G;p^a,p^c)}{\partial p^a} + k\frac{\partial \mathbb{E}(T(p^a,p^c)|G)}{\partial p^a}\right]\end{aligned}$$

and, therefore:

$$\frac{\partial^2 U}{\partial q \partial p^a} = -\frac{1}{2}\frac{\partial \Pr(p_T \geq p^c|I;p^a,p^c)}{\partial p^a} + \frac{1}{2}\frac{\partial \Pr(p_T \leq p^a|G;p^a,p^c)}{\partial p^a}$$

Since the sequential problem is regular, $\frac{\partial \Pr(p_T \leq p^a|G;p^a,p^c)}{\partial p^a} \geq 0$ and $\frac{\partial \Pr(p_T \geq p^c|I;p^a,p^c)}{\partial p^a} \leq 0$. Thus, $\frac{\partial^2 U}{\partial q \partial p^a} \geq 0$. Topkis' Theorem (see, e.g., Topkis, 1998), ensures that $\tilde{p}^a(p^c;q)$ increases in q. Similar arguments hold for $\tilde{p}^c(p^a;q)$. ■

Proof of Proposition 2

Step 1. Let us first assume that the acquittal threshold is given by p_* and let us identify the pivotal juror for conviction and her one-sided best response. By Lemma A1, the one-sided best responses $\widetilde{p}^c(p_*;q)$ are increasing in q. Thus, given p_*, if a juror with preference parameter q_i is willing to stop and convict, so is a juror with preference parameter q_{i-1}. Conversely, if a juror with preference parameter q_i is not willing to stop and convict, neither is a juror with preference parameter q_{i+1}. Thus, for a deliberation rule R_d, the pivotal juror for determining the conviction threshold is juror q_{R_d}, and her deliberation threshold is given by $\widetilde{p}^c(p_*;q_{R_d})$. Similarly, when the conviction threshold is fixed at p^*, the pivotal juror for determining the acquittal threshold is the juror with preference parameter q_{n-R_d+1} and his deliberation threshold is given by $\widetilde{p}^a(p^*;q_{n-R_d+1})$. We have therefore determined that, given a pair of candidate equilibrium thresholds p_* and p^*, and if we focus on the one-sided best responses, the two pivotal jurors for determining these best responses are given, respectively, by those of preference parameters q_{R_d} and q_{n-R_d+1}.

Step 2. We now show that a pair of mutual one-sided best responses exists. One-sided best responses are derived as a maximization of $U(q;p^a,p^c)$ with respect to one parameter (either p^a or p^c), holding all other parameters fixed. Since our assumptions assure that $U(q;p^a,p^c)$ is well-behaved, it follows from the Theorem of the Maximum that $\widetilde{p}^a(p;q_{n-R_d+1})$ and $\widetilde{p}^c(p;q_{R_d})$ are continuous in p. Let $\phi(p) \equiv \widetilde{p}^c(\widetilde{p}^a(p;q_{n-R_d+1});q_{R_d}) - p$. The function ϕ is continuous. We now show that there exist p_l and p_h such that $\phi(p_l) \leq 0$ and $\phi(p_h) \geq 0$.

Consider first the Wald thresholds $p^a(q_{n-R_d+1}), p^c(q_{n-R_d+1})$. By optimality of these thresholds, $p^a(q_{n-R_d+1}) = \widetilde{p}^a(p^c(q_{n-R_d+1}); q_{n-R_d+1})$ and $p^c(q_{n-R_d+1}) = \widetilde{p}^c(p^a(q_{n-R_d+1}); q_{n-R_d+1})$. Furthermore, by part 3 of Lemma A1, $\widetilde{p}^c(p^a(q_{n-R_d+1}); q_{R_d}) \geq \widetilde{p}^c(p^a(q_{n-R_d+1}); q_{n-R_d+1})$. Therefore,

$$\phi(p^c(q_{n-R_d+1})) = \widetilde{p}^c(\widetilde{p}^a(p^c(q_{n-R_d+1}); q_{n-R_d+1}); q_{R_d}) - p^c(q_{n-R_d+1}) \geq 0$$

Consider now the Wald thresholds $p^a(q_{R_d}), p^c(q_{R_d})$.

$$\phi(p^c(q_{R_d})) = \widetilde{p}^c(\widetilde{p}^a(p^c(q_{R_d}); q_{n-R_d+1}); q_{R_d}) - p^c(q_{R_d}) \leq 0$$

because $\widetilde{p}^a(p^c(q_{R_d}); q_{n-R_d+1}) \leq \widetilde{p}^a(p^c(q_{R_d}); q_{R_d}) = p^a(q_{R_d})$ by part 3 of Lemma A1 and therefore, by part 2 of Lemma A1 and optimality of the Wald solution, $\widetilde{p}^c(\widetilde{p}^a(p^c(q_{R_d}); q_{n-R_d+1}); q_{R_d}) \leq p^c(q_{R_d}) = \widetilde{p}^c(p^a(q_{R_d}); q_{R_d})$.

From the Intermediate Value Theorem, it follows that there exists p^* such that $\phi(p^*) = 0$. Thus, $p_* = \widetilde{p}^a(p^*; q_{n-R_d+1})$ and $p^* = \widetilde{p}^c(p_*; q_{R_d})$ constitute mutual (one-sided) best responses as above.

By construction, such a p must lie in an interval between $p^c(q_{n-R_d+1})$ $p^c(q_{R_d})$. By part 2 of Lemma A1, $p^* \leq p^c(q_{R_d})$. But then it cannot be the case that $p^* < p^c(q_{n-R_d+1})$, otherwise the interval would be empty. Thus, $p^* \in [p^c(q_{n-R_d+1}), p^c(q_{R_d})]$. By similar reasoning, if we define $\psi(p) = \widetilde{p}^a(\widetilde{p}^c(p; q_{R_d}); q_{n-R_d+1}) - p$ we can establish that $p_* \in [p^a(q_{n-R_d+1}), p^a(q_{R_d})]$.

Step 3. Consider for a moment the case of a jury composed only of jurors q_{n-R_d+1} and q_{R_d} when the deliberation rule is unanimity. In order to conclude that any such pair of mutual (one-sided) best responses (p_*, p^*) constitutes an equilibrium, we now need to show that the one-sided best responses are full best responses. Specifically, juror q_{R_d} could choose $p^a < p_*$. However, since this juror is not constrained on the conviction side, this would imply that the optimal choice for this juror is his pair of Wald thresholds $p^a(q_{R_d}), p^c(q_{R_d})$. However, from step 2 we know that $p_* \leq p^a(q_{R_d})$. Therefore, this juror's one-sided best responses must be full best responses, either because they are the Wald thresholds or because he is constrained. This reasoning immediately extends to the full n-juror committee: as in the two-juror case, each juror must be constrained on (at least) one of the thresholds: $q_i < q_{R_d}$ are constrained on the conviction side and $q_i > q_{n-R_d+1}$ are constrained on the acquittal side. This implies the monotonicity described above. ■

Proof of Lemma 1

Suppose there are two symmetric threshold equilibria: (p_*, p^*) and $(\tilde{p}_*, \tilde{p}^*)$. Suppose $p_* < \tilde{p}_*$. From the characterization of equilibrium in Proposition 2 and the monotonicity with respect to the constrained problem's thresholds given by Lemma A1, it must be the case that $p^* \leq \tilde{p}^*$. This would imply $\tilde{p}_* + \tilde{p}^* > p_* + p^* = 1$, in contradiction to the equilibrium $(\tilde{p}_*, \tilde{p}^*)$ being symmetric. ■

Proof of Proposition 3

Assume by way of contradiction that, for $b > b'$, the symmetric equilibria corresponding to b and b' satisfy $p_*(b') < p_*(b)$ and $p^*(b') > p^*(b)$. From the equilibrium characterization of Proposition 2 and the monotonicity attributes implied by Lemma A1, the best response p^a to $p^*(b)$ for the juror with preferences $\frac{1}{2} - b'$ must be such that $p^a \leq p_*(b') < p_*(b)$. But since $p_*(b)$ is a best response to $p^*(b)$ for the juror with preferences $\frac{1}{2} - b$, this violates monotonicity of best responses in q (Lemma A1). ■

Proof of Proposition 4

Consider the expression for juror payoffs in equation (5). In a quasi-symmetric jury, equilibrium thresholds are symmetric and so $p^*(R_d) = 1 - p_*(R_d)$, and the probability of either mistake is identical:

$$\Pr(p_T \geq p^*(R_d) | I; p_*(R_d), p^*(R_d)) = \Pr(p_T \leq p_*(R_d) | G; p_*(R_d), p^*(R_d)).$$

Furthermore, the expected time for conviction and acquittal coincide:

$$\mathbb{E}(T(p_*(R_d), p^*(R_d)) | I) = \mathbb{E}(T(p_*(R_d), p^*(R_d)) | G).$$

Slightly abusing notation by dropping the conditioning on the two thresholds, we can write (5) as:

$$\begin{aligned}
U(q; R_d) &= -\tfrac{1}{2}\left[q \Pr(p_T \geq p^*(R_d) | I; p_*(R_d), p^*(R_d)) + k\mathbb{E}(T(p_*(R_d), p^*(R_d)) | I)\right] \\
&\quad -\tfrac{1}{2}\left[(1-q)\Pr(p_T \leq p_*(R_d) | G; p_*(R_d), p^*(R_d)) + k\mathbb{E}(T(p_*(R_d), p^*(R_d)) | G)\right] \\
&= -\tfrac{1}{2}\Pr(p_T \geq p^*(R_d) | I; p_*(R_d), p^*(R_d)) - k\mathbb{E}(T(p_*(R_d), p^*(R_d)) | I).
\end{aligned}$$

This expression shows that preferences over deliberation rules are independent of q. To show that all jurors prefer the least inclusive deliberation rule, suppose first that two jurors with preference $q = 1/2$ existed (symmetry entails there being an even number of jurors having such a preference). For $R_d = n/2$, deliberation would precede just as it would with a dictator of preference parameter $q = 1/2$. Since all other jurors share their preferences with agents with such neutral preferences, $R_d = n/2$ would be optimal for all jurors. Using the notation of Section 4 (with $b = 0$ for a neutral juror), we denote by $p_*(0)$ and $p^*(0)$ the resulting equilibrium thresholds. From the construction of the equilibrium (Proposition 2), if the lower threshold were constrained to be $p_{**} < p_*(0)$, an agent with preference parameter $q = 1/2$ would prefer to halt deliberation for any posterior $p > p^*(0)$,the analogue holding for the upper threshold. From Proposition 3, it follows directly that preferences would therefore be monotonically decreasing in R_d for a juror of preferences $q = 1/2$ and therefore for all jurors. ■

Proof of Proposition 5

Part 1. Consider an equilibrium $p_*(R_d, R_v)$, $p^*(R_d, R_v)$ and any $R_v \leq R_d$. The pivotal jurors at the deliberation stage are $q_{n-R_d+1} \leq q_{n-R_v+1}$ and $q_{R_d} \geq q_{R_v}$. By the construction of optimal thresholds, $p_*(R_d, R_v) \leq q_{n-R_d+1}$ and $p^*(R_d, R_v) \geq q_{R_d}$. Consider any posterior p such that, if reached, in equilibrium deliberation terminates: we have $p \leq p_*(R_d, R_v) \leq q_{n-R_d+1} \leq q_{n-R_v+1}$ and $p \geq p^*(R_d, R_v) \geq q_{R_d} \geq q_{R_v}$ so that whenever there is a quorum for stopping deliberation there is also a quorum for taking a decision. Thus, the binding thresholds are the deliberation thresholds.

Part 2. It is clear that, as k decreases to zero, for any $q \in (0, 1)$, $p^a(p^*; q)$ converges to zero and $p^c(p_*; q)$ converges to one. Thus, p_* and p^* must also converge to zero and one respectively. This implies that, for any R_d, for low enough k, $p_* < q_1$ and $p^* > q_n$. This in turn means that no matter what R_v is, when deliberation stops, there is unanimity in the decision. ■

Proof of Proposition 6

Consider first $R_v = R_d$. Recall the discussion following Lemma 1 illustrating that the optimal thresholds of the constrained problem shrink (i.e., the acquittal threshold increases and the conviction threshold decreases), as k grows larger.

For $\tilde{R}_v > R_v = R_d$, let $\underline{k}$ be such that $p_*(R_d, R_v; \underline{k}) = q_{n-\tilde{R}_v+1}$, $p^*(R_d, R_v; \underline{k}) = q_{\tilde{R}_v}$.[42] When the cost is $\underline{k}$, the pivotal voters under rule $\tilde{R}_v$ are just indifferent between voting to acquit and voting to convict. This, along with the monotonicity of thresholds with respect to k implies that, with cost $k \leq \underline{k}$, the equilibrium outcome under R_d, R_v would be also be an equilibrium outcome under $R_d, \tilde{R}_v$. However, for $k > \underline{k}$, thresholds under R_d, R_v move inwards: $p_*(R_d, R_v; k) > p_*(R_d, R_v; \underline{k})$ and $p^*(R_d, R_v; k) < p^*(R_d, R_v; \underline{k})$. This means that for $k > \underline{k}$, the equilibrium outcome under R_d, R_v induces a hung jury with positive probability under $\tilde{R}_v$. For instance, after a history that reaches $\hat{p}$ such that $p^*(R_d, R_v; k) < \hat{p} < q_{\tilde{R}_v}$, the jury would be hung under $\tilde{R}_v$.

Now assume that $k = \underline{k} + \varepsilon$. We claim that, for ε sufficiently small, the unique symmetric equilibrium under rules $R_d, \tilde{R}_v > R_v$ involves extending deliberation thresholds just enough to avoid a hung jury. In fact, equilibrium for such k is characterized by two thresholds $p_*\left(R_d, \tilde{R}_v; k\right) = q_{n-\tilde{R}_v+1}$, $p^*\left(R_d, \tilde{R}_v; k\right) = q_{\tilde{R}_v}$. To see this, first note that the jurors with preference parameters q_{n-R_d+1}, q_{R_d} are still pivotal at the deliberation stage. Any conviction threshold $p^* < q_{\tilde{R}_v}$ (or acquittal threshold $p_* > q_{n-\tilde{R}_v+1}$) would induce a hung jury with some probability, which is not optimal for k sufficiently close to $\underline{k}$. Indeed, note that, by definition, for the pivotal jurors, at $k = \underline{k}$ the continuation value is exactly equal to the value of stopping. The value of stopping at $p = q_{\tilde{R}_v}$ for juror q_{R_d} is $-q_{R_d}\left(1 - q_{\tilde{R}_v}\right)$. The value of a hung jury at the same posterior and for the same juror is lower than the value of stopping: $-\frac{1}{2}\left[q_{R_d}\left(1 - q_{\tilde{R}_v}\right) + (1 - q_{R_d})\, q_{\tilde{R}_v}\right] < -q_{R_d}\left(1 - q_{\tilde{R}_v}\right)$ because $q_{R_d} < q_{\tilde{R}_v}$ and $q_{R_d} > \frac{1}{2}$. By continuity, when k is sufficiently close to $\underline{k}$, and p is sufficiently close to $q_{\tilde{R}_v}$, the value of continuing is larger than the value of a hung jury. Thus, it is not optimal to stop before $q_{\tilde{R}_v}$. Since it is clearly suboptimal for these jurors to continue beyond $q_{\tilde{R}_v}$, the optimal threshold in this case is $q_{\tilde{R}_v}$. A similar argument holds for the acquittal threshold. Since the construction follows for any $\tilde{R}_v > R_d$ and sufficiently small costs, the result follows. ■

[42] The existence of such a $\underline{k}$ follows from the Intermediate Value Theorem, noticing that as costs approach 0, more and more information would be acquired and equilibrium thresholds would expand without limit; As information costs become prohibitively high, less and less information is acquired and equilibrium thresholds approach the prior of 1/2.Uniqueness follows since interior equilibria are strictly monotonic in k (from the arguments of Section 3).

Proof of Proposition 7

Recall that a hung jury leads to equal probabilities of conviction and acquittal. Given rules R_d and R_v, the payoff to a juror with preference q when the information sample is of size t is then given by:

$$\begin{aligned} U(R_d, R_v; q) &= -q\left(1 - \mathbb{E}\left(p_t | p_t \geq q_{R_v}\right)\right)\Pr\left(p_t \geq q_{R_v}\right) \\ &\quad - (1-q)\,\mathbb{E}\left(p_t | p_t < q_{n-R_v+1}\right)\left(\Pr\left(p_t < q_{n-R_v+1}\right)\right) \\ &\quad + \frac{1}{2}\mathbb{E}\left(-q(1-p_t) - (1-q)p_t | q_{n-R_v+1} < p_t < q_{R_v}\right)\left(\Pr\left(q_{n-R_v+1} < p_t < q_{R_v}\right)\right) - kt. \end{aligned}$$

With symmetric juries, $q_{n-R_v+1} = 1 - q_{R_v}$, $\mathbb{E}\left(p_t | p_t < q_{n-R_v+1}\right) = 1 - \mathbb{E}\left(p_t | p_t \geq q_{R_v}\right)$, and $\Pr\left(p_t < q_{n-R_v+1}\right) = \Pr\left(p_t \geq q_{R_v}\right)$. Furthermore, symmetry implies that

$$\frac{1}{2}\mathbb{E}\left(-q(1-p_t) - (1-q)p_t | q_{n-R_v+1} < p_t < q_{R_v}\right) = -\frac{1}{4}$$

is independent of q.[43] Therefore,

$$U(R_d, R_v; q) = -\mathbb{E}\left(p_t | p_t < q_{R_v}\right)\Pr\left(p_t < q_{n-R_v+1}\right) - \frac{1}{4}\left(\Pr\left(q_{n-R_v+1} < p_t < q_{R_v}\right)\right) - kt$$

is also independent of q. It follows that jurors are unanimous in their deliberation votes, implying that the deliberation rule R_d is irrelevant. However, the decision rule R_v does matter: a larger R_v raises the probability of a hung jury. This feeds back into the optimal sample size (for the unanimous jurors). In particular, a more inclusive decision rule implies more information collection. ■

[43] Indeed, note that

$$\Pr(p_t < \frac{1}{2} | q_{n-R_v+1} < p_t < q_{R_v}) = \Pr(p_t > \frac{1}{2} | q_{n-R_v+1} < p_t < q) = \frac{1}{2}.$$

Furthermore, for any p_t, $q_{n-R_v+1} < p_t < q_{R_v}$, it must be that $q_{n-R_v+1} < 1 - p_t < q_{R_v}$ and

$$-q(1-p_t) - (1-q)p_t - q(1-(1-p_t)) - (1-q)(1-p_t) = 1.$$

9. Appendix B – Uniqueness of Best Responses

We now turn to discuss general conditions under which best responses in the constrained optimization problem are unique.

Definition ((m,ε)- and ε-refinements) *For any distributions (F_G, F_I) and (F'_G, F'_I) of signals, we say that (F_G, F_I) is an (m,ε) refinement of (F'_G, F'_I) if:*

1. *Any collection of m i.i.d. draws from (F_G, F_I) is more Blackwell informative than one draw from (F'_G, F'_I); and*
2. *For any prior p that the defendant is guilty, and any draw S from (F_G, F_I), $\Pr(|\Pr(G \mid S = s) - p| < \varepsilon) > 1 - \varepsilon$.*

We say that the game defined by (F_G, F_I) is an $\varepsilon-$refinement of the game defined by (F'_G, F'_I) with signal costs k if there exists m such that (F_G, F_I) is an (m,ε) refinement of (F'_G, F'_I), and the cost of each signal in the game defined by (F_G, F_I) is k/m.

In words, (F_G, F_I) corresponds to a "weak splitting" of the information contained in (F'_G, F'_I) into m independent and identically distributed signals, so that m signals drawn from (F_G, F_I) contain more information than one signal drawn from (F'_G, F'_I) (condition 1). Furthermore, each signal drawn from (F_G, F_I) is informationally small, in that the likelihood that the posterior, after observing a signal drawn from (F_G, F_I), is very different from the prior exceeds a certain ε is smaller than ε (condition 2).

There are several simple examples of (m,ε) refinements. For instance, when all signals are drawn from normal distributions, a refinement of signals simply corresponds to an appropriate reduction in precision of each individual signal. Alternatively, if (F_G, F_I) corresponds to a Bernoulli draw with very low accuracy (namely, one generating a signal corresponding to the underlying state with a probability only slightly higher than $\frac{1}{2}$, and the signal corresponding to the other state with the complementary probability), then whenever (F'_G, F'_I) correspond to m independent draws as such, (F_G, F_I) is an (m,ε) refinement of (F'_G, F'_I).[44]

[44] One way to construct (m,ε) refinements is by considering infinitely divisible random variables, ones that can be described as the sum of any arbitrary number of independent and identically distributed random variables. While this notion is more restrictive than our refinement notion, it is an old and well-studied one in the statistics literature, going back to de Finetti in 1929 and developed in the thirties by Kolmogorov, Levy and Khintchine (see Mainardi and Rogosin, 2006 and references therein).

An ε-refinement of a game intuitively corresponds to a game in which information is "split" but the per-unit cost of information is comparable.

Lemma A2 (Unique Best Responses) *There exists an $\bar{\varepsilon}$ such that for all $\varepsilon < \bar{\varepsilon}$, an $\varepsilon-$refinement of the original game entails unique best responses.*

Claim *$V(p \mid \underline{p}, \bar{p})$ is continuous in p in each of the three regions $p \leq \underline{p}$, $\underline{p} < p < \overline{p}$, and $p \geq \overline{p}$.*

Proof of Claim. Notice first that the value function must satisfy:

$$\begin{aligned} &V(p \mid \underline{p}, \bar{p}) = -p(1-q) \text{ for all } p \leq k/(1-q)\text{; and} \\ &V(p \mid \underline{p}, \bar{p}) = -(1-p)q \text{ for all } p \geq 1-k/q. \end{aligned} \tag{6}$$

Notice first that if $\underline{p} \leq k/(1-q)$ and $\overline{p} \geq 1-k/q$, a best response entails immediate stopping when possible, and the claim follows immediately. We will therefore assume that $\underline{p} > k/(1-q)$ and $\overline{p} < 1-k/q$ (the case in which immediate stopping occurs only on one side follows the same lines of proof described in what follows).

Denote the class of functions with non-positive image on $[0,1]$ satisfying the restrictions specified in (6) on the value function by Δ. Endow Δ with the sup norm d_∞ and consider the operator $\Psi : \Delta \to \Delta$ defined by:

$$\Psi(g)(p) = \begin{cases} \max\left\{\mathbb{E}_{F_G,F_I} g(p' \mid s, p) - k, -(1-q)\,p\right\} & p \leq \underline{p} \\ \mathbb{E}_{F_G,F_I} g(p' \mid s, p) & \underline{p} < p < \overline{p} \\ \max\left\{\mathbb{E}_{F_G,F_I} g(p' \mid s) - k, -q\,(1-p)\right\} & p \geq \overline{p} \end{cases}.$$

We now show that Ψ is a contraction mapping over Δ. Indeed, let $\pi_1 = \min_p \Pr_{F_G,F_I}(\Pr(G \mid s, p) \leq k/(1-q))$ and $\pi_2 = \min_p \Pr_{F_G,F_I}(\Pr(G \mid s, p) \geq 1-k/q)$. Assume first that signals have are sufficiently informative 'in the tails' so that $\pi_1 + \pi_2 > 0$ (this is true for any normal signal however low the accuracy). That is, regardless of the prior, there is a strictly positive probability (bounded from below) that extremal posteriors, will be reached. From the restriction on Δ, recall that for any $g, h \in \Delta$, for any p satisfying $p \leq k/(1-q)$ or $p \geq 1-k/q$, $g(p) = h(p)$. It follows that there is at least a probability of $\pi_1 + \pi_2$ that any two functions $g, h \in \Delta$ will coincide over a posterior generated from any prior. Furthermore, for any p and for any pair of functions even outside Δ,

$\sup_p \{\|E(g|p) - E(h|p)\|\} \leq \sup_p \{\|g(p) - h(p)\|\}$. Therefore, for any $g, h \in \Delta$,

$$d_\infty(\Psi(g), \Psi(h)) \leq (1 - \pi_1 - \pi_2) d_\infty(g, h).$$

From the Banach Fixed Point Theorem (or Contraction Mapping Theorem), it then follows that there is a unique fixed point of Ψ in Δ and, furthermore, for any $g_0 \in \Delta$, the sequence $g_{n+1} = \Psi(g_n)$, starting at g_0, converges to the fixed point in the metric d_∞.

It therefore suffices to show that Ψ preserves continuity in the regions $p < \underline{p}$, $\underline{p} < p < \overline{p}$, and $p > \overline{p}$ within Δ.

Indeed, suppose g is continuous. Conditional expectations are continuous, and so is the maximum of two continuous functions. Therefore, $\Psi(g)$ is continuous within each of the three regions and the result follows.

Now assume that the distribution does not have sufficiently informative tails so that $\pi_1 + \pi_2 = 0$. In particular, for some p, some posteriors are unreachable. In this case the argument above can be adapted by using the fact that there exists an m such that the m-iteration of the operator Ψ is a contraction. This is immediate because for any informative signal process and for any $p_1 > p$, and $p_2 < p$, there is an m such that there is positive probability that the posterior will travel above p_1 and below p_2 within m periods. In fact, since for any p, as m grows, posteriors become arbitrarily close to the extremes (of 0 or 1) with arbitrarily high probability, there exists a finite m such that the probability that m signals lead to a posterior beyond the thresholds $k/(1-q)$ and $1 - k/q$ is uniformly bounded above 0. Since the contraction mapping theorem still holds if the m-fold operator is a contraction (see Ok, 2007), the previous argument also works in this case.

From now on we focus on $\varepsilon-$refinements of the game such that we can guarantee that there is a unique acquittal threshold and a unique conviction threshold. This is tantamount to assuming that the time intervals between the arrival of potential signals is sufficiently small.[45]

Proof of Lemma A2

Suppose that for given constraints there are multiple optimal thresholds for the deliberation game. Figure 2 depicts such a setting in which there are two stopping regions encompassing one continuation

[45] In the limit, as ε approaches 0, the setting essentially becomes one of continuous time, which often serves as an underlying assumption, see Chan and Swen (2013).

region to the left of the constrained continuation region. Consider the left-most continuation region and call the two thresholds $p^{[1]}$ and $p^{[2]}$.

Notice that for any ε-refinement of the game, the corresponding continuation value is pointwise higher than that corresponding to the underlying game. Indeed, in the ε-refined game the agent has more flexibility as to when to stop, and overall at least as much information per unit cost. In particular, if we denote the corresponding thresholds of the left-most continuation region by $p_\varepsilon^{[1]}$ and $p_\varepsilon^{[2]}$, we have that $p_\varepsilon^{[1]} \leq p^{[1]}$, while $p_\varepsilon^{[2]} \geq p^{[2]}$ for all ε.

Consider any sequence of ε-refinements. For any ε, denote the corresponding value function by $V_\varepsilon(p \mid \underline{p}, \bar{p})$ and let $\widehat{p}_\varepsilon \in \arg\max_{p \in \left[p_\varepsilon^{[1]}, p_\varepsilon^{[2]}\right]} V_\varepsilon(p \mid \underline{p}, \bar{p}) - V^0(p)$. Consider a convergent sequence $\{\widehat{p}_{\varepsilon_i}\}_i$, where $\lim_{i\to\infty} \varepsilon_i = 0$.

There are then two cases to consider:

1. $\lim_{i\to\infty} \widehat{p}_{\varepsilon_i} < \liminf_{i\to\infty} p_{\varepsilon_i}^{[2]}$. By construction, for any ε, for sufficiently large i, the probability of leaving the continuation region with one signal in the ε_i-refined game is lower than ε. In particular, for sufficiently large i, $\mathbb{E}V_{\varepsilon_i}(p' | \widehat{p}_{\varepsilon_i}) < V(\widehat{p}_{\varepsilon_i})$, in contradiction.

2. $\lim_{i\to\infty} \widehat{p}_{\varepsilon_i} = \liminf_{i\to\infty} p_{\varepsilon_i}^{[2]}$. From the claim, all value functions are continuous. Therefore, for any $\varepsilon > 0$, there exists a sequence $\{p_{\varepsilon_i}\}$ and $\delta > 0$ such that $p_{\varepsilon_i} = \widehat{p}_{\varepsilon_i} - \delta$ is in the continuation region and for sufficiently high i, $\left|V_{\varepsilon_i}(p_{\varepsilon_i} \mid \underline{p}, \bar{p}) - V_{\varepsilon_i}(\widehat{p}_{\varepsilon_i} \mid \underline{p}, \bar{p})\right| < \varepsilon$. Furthermore, for any $\varepsilon^* > 0$, and for sufficiently high i, the probability of leaving the continuation region starting from p_{ε_i} is lower than ε^*. It follows then, that for sufficiently large i, $\mathbb{E}V_{\varepsilon_i}(p' | p_{\varepsilon_i}) < V(p_{\varepsilon_i})$. That is, for any $\delta > 0$ small enough, for sufficiently high i, $\mathbb{E}V_{\varepsilon_i}(p' | \widehat{p}_{\varepsilon_i} - \delta) < V(\widehat{p}_{\varepsilon_i} - \delta)$, and $\mathbb{E}V_{\varepsilon_i}(p' | \widehat{p}_{\varepsilon_i}) \leq V(\widehat{p}_{\varepsilon_i})$, which is in contradiction.

It follows that for sufficiently small ε, any ε-refinement of the game will entail a unique intersection of the continuation value in the region in which $p < \underline{p}$. Similar arguments hold for intersections in the region $p > \bar{p}$.

We now need to ensure that uniqueness holds for any constraint (i.e., any action of the other player). Indeed, notice that for any preference parameter q, there exists $\bar{p}(q)$ such that for any ε-refinement of the game, and for any $\underline{p}$, the agent prefers to stop for $p > \bar{p}(q)$. Similarly, for any preference parameter q, there exists $\underline{p}(q)$ such that for any ε-refinement of the game, and for any $\bar{p}$, the agent prefers to stop for $p < \underline{p}(q)$. Consider then the closed interval

$$\Lambda = [\min\{\underline{p}(q_1), ..., \underline{p}(q_n)\}, \max\{\bar{p}(q_1), ..., \bar{p}(q_n)\}].$$

For each $\underline{p}, \bar{p} \in \Lambda$,and any q_i, there exists $\varepsilon(\underline{p}, \bar{p}, q_i)$ such that for all $\varepsilon < \varepsilon(\underline{p}, \bar{p}, q_i)$,the ε-refinement entails a unique choice of thresholds for the agent of preference parameter q_i. Consider then

$$\varepsilon^* = \min_{i, (\underline{p}, \bar{p}) \in \Lambda} \varepsilon(\underline{p}, \bar{p}, q_i).$$

Then, for any $\varepsilon < \varepsilon^*$, any ε-refinement of the game will entail a unique threshold, ■

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