Networks and Experimental Economics

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Why Laboratory Experiments?

- Allow to control many aspects:
  - Vary networks, payoffs, information, node characteristics, etc.
- Allow to focus on particular layers of a network
- Allow for fairly cheap trials of new policies, advertising strategies, and so on

Two Approaches

- Impose the network or have the network form in the lab
- Elicit network in the population, study experimental tasks given the underlying network

Plan for today

- Networks in the lab:
  - Experiments on network formation
  - Experiments on network games
- Experiments given elicited network

Words of Warning

- Network experiments are in their inception
- So methodologies still being developed
- → Many existing experiments are interesting, but still have some issues

Imposing/Forming Networks in the Lab

Advantages:

- Can focus on one layer of the network
- No missing observations or errors
- Can study the impacts of different network architectures

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Disadvantages

- Inherently small networks
  - Most labs have 20-40 computers
  - Especially important if we study repeated interactions and incomplete information
  - Future: online platforms?

Network Formation in the Lab

In Search of Stars: Goeree, Riedl, and Ule, 2009

- Underlying Model based on Bala and Goyal (2000)
- Unilateral costly linking
- Do equilibrium networks emerge in the lab?

Underlying Model

- Collection of N agents who form a network
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- Collection of N agents who form a network
- Each agent j is associated with a value v_j
- At the outset, each agent decides which agents to connect to (simultaneously)
- Each link i creates costs her c_i
- Unilateral linking → if either i connects with j or j connects with i, they are connected

  [in contrast with pair-wise stability]

Underlying Model (2)

- The benefit of agent i from agent j depends on j's value v_j and d(i,j) (d(i,j)=∞ if i,j disconnected)

- Let n_i be the number of links i creates (≠ degree)

- Utility for i is:

$$U_i = \Sigma_{i \neq j} \, \delta^{d(i,j)-1} v_j - n_i c_i$$

where $0 \leq \delta \leq 1$

Information and Equilibria

Suppose $\{(v_i, c_i)\}$ drawn from some distribution

Two settings:

- **Complete Information:** each agent knows the full realization → Nash
- **Incomplete Information:** each agent i knows only own (v_i, c_i) → Bayesian Nash

Experimental Parameters

- N=6, parameters chosen to generate stars in equilibrium
- Values either normal or high, costs either normal or low
- $\delta = 3/4$

Table 1: Experimental parameters.

a) Linking costs and values of different agent types.

	cost per link made	value to other agents
normal agent	24	16
low cost agent	7	16
high value agent	24	32

b) Benefits (per agent accessed) from accessing different types of agents at different distances.

distance	1	2	3	4	5	∞
normal or low cost agent	16	12	9	7	5	0
high value agent	32	24	18	14	10	0

Treatments

	complete information	incomplete information
6o	BI (7)	-
5o + 1c	CI (6)	CN (6)
5o + 1v	VI (4)	VN (6)
4o + 1c + 1v	CVI (4)	CVN (6)

Note: Numbers of independent observations (i.e., groups) in parenthesis.
Abbreviations: o - normal agent, c - low-cost agent, v - high-value agent.

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Note: Numbers of independent observations (i.e., groups) in parenthesis.
Abbreviations: o - normal agent, c - low-cost agent, v - high-value agent.

In the lab, incomplete information means that the identities of others' types are unknown

Experimental Procedures

- 234 subjects at University of Amsterdam and Caltech
- Subjects in one treatment and one group of 6 for 30 rounds
- Subjects’ types and labels: “A”, “B”,… fixed throughout

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Experimental Procedures

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- Subjects in one treatment and one group of 6 for 30 rounds
- Subjects' types and labels: "A", "B",… fixed throughout
- Repeated game effects?
- Normal = "green", low-cost = "purple", high-value = "blue"
- Average payment of $25

Experimental Procedures

- After each round, subjects were informed of the network created, their benefits, costs, and payoffs
- Opportunities to learn others' types in incomplete information treatments…

Equilibrium Predictions

- Bayesian and Nash equilibrium predictions are all stars, “connected stars”, or the empty network

Equilibrium Predictions – Examples

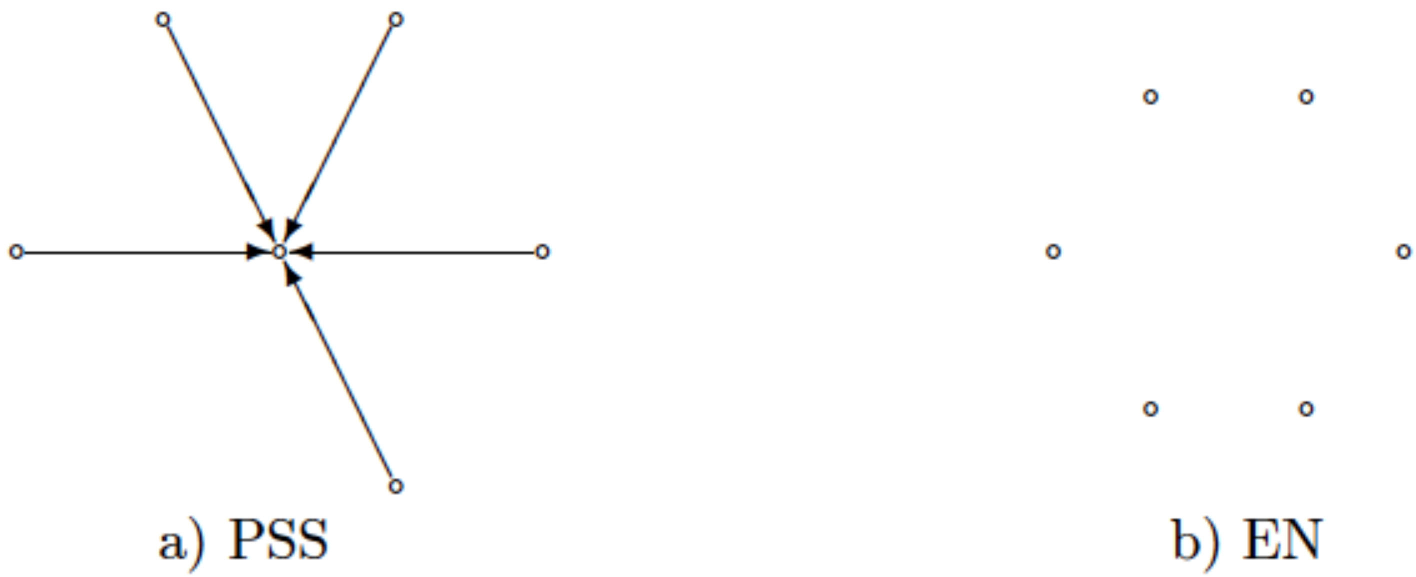

Figure 2: The (Bayesian) Nash networks for treatments BI and VN.

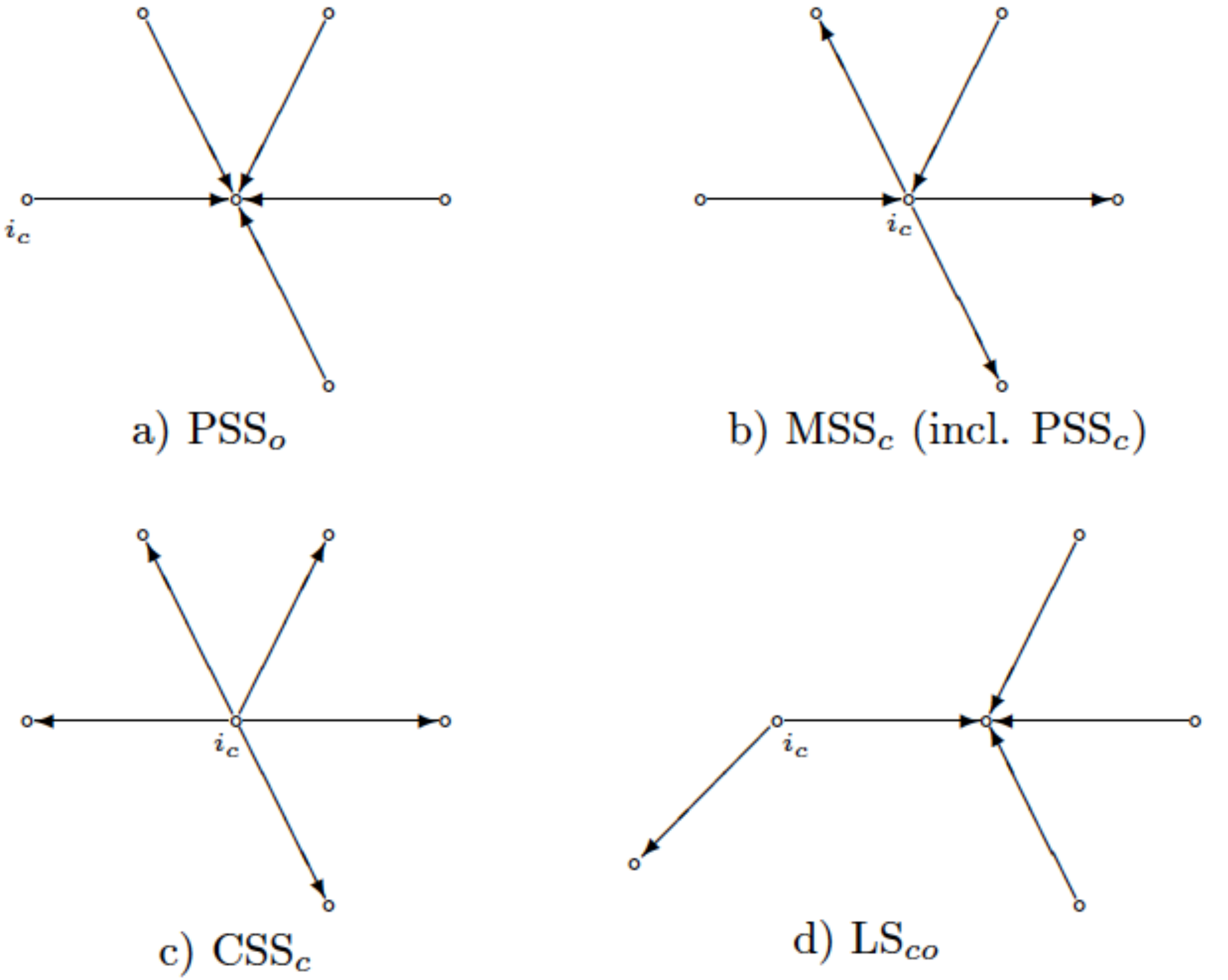

Figure 3: The (Bayesian) Nash networks for treatments CI, CN and CVN.

Equilibrium Predictions

- Bayesian and Nash equilibrium predictions are all stars, “connected stars,” or the empty network
- Multiplicity in every treatment
- Coordination game imposed on a network-formation game…

Results – Frequency of Equilibrium Outcomes

Treatment	Total	# obs.
BI	0.0%	210
CI	2.2%	180
CN	8.9%	180
VI	40.8%	120
VN	51.1%	180
CVI	33.3%	120
CVN	26.7%	180

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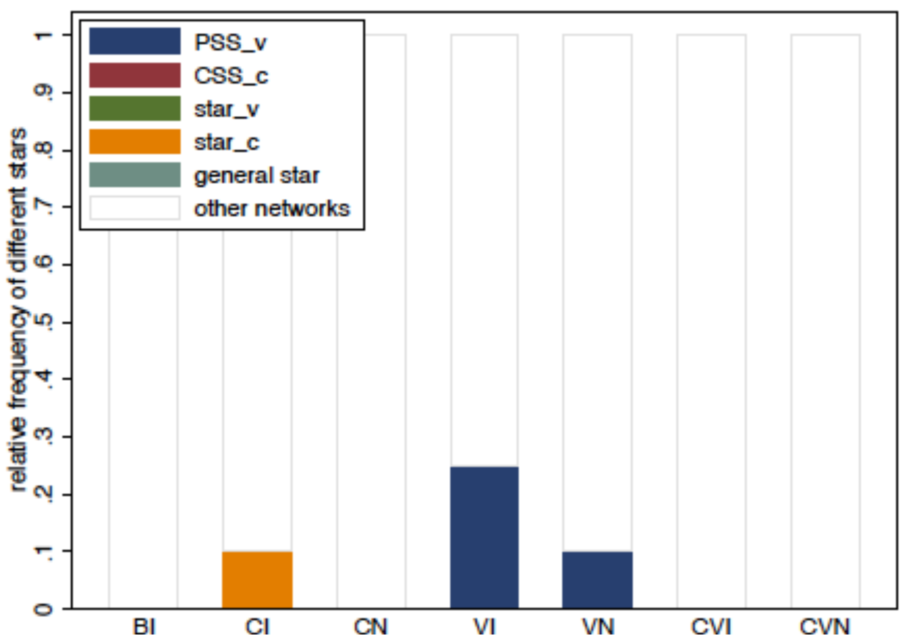
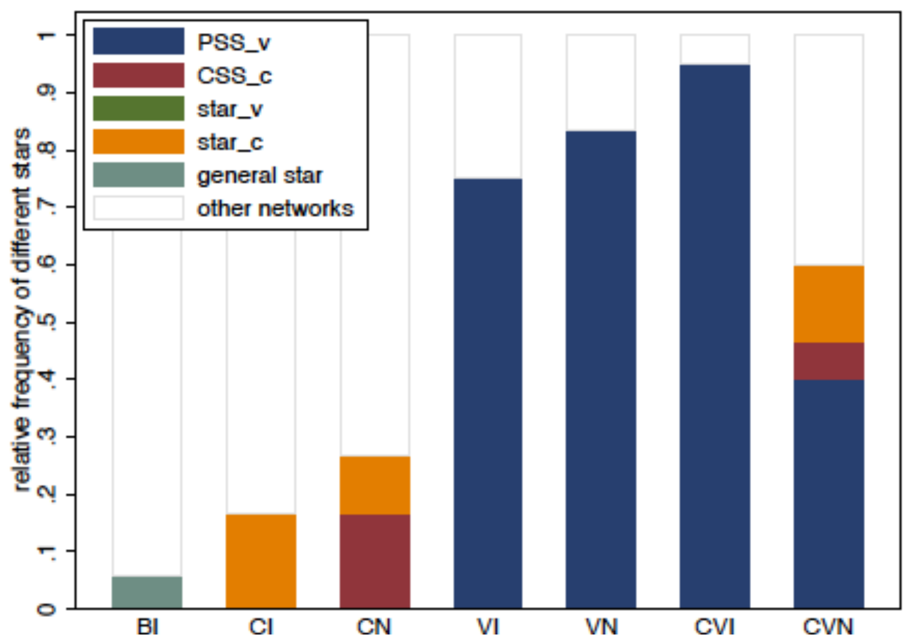
Equilibrium outcomes almost all stars with high-value agent in the center

Outcomes after Learning

- Frequency of equilibrium outcomes (stars) increases significantly in last five rounds when there is a **high-value agent**





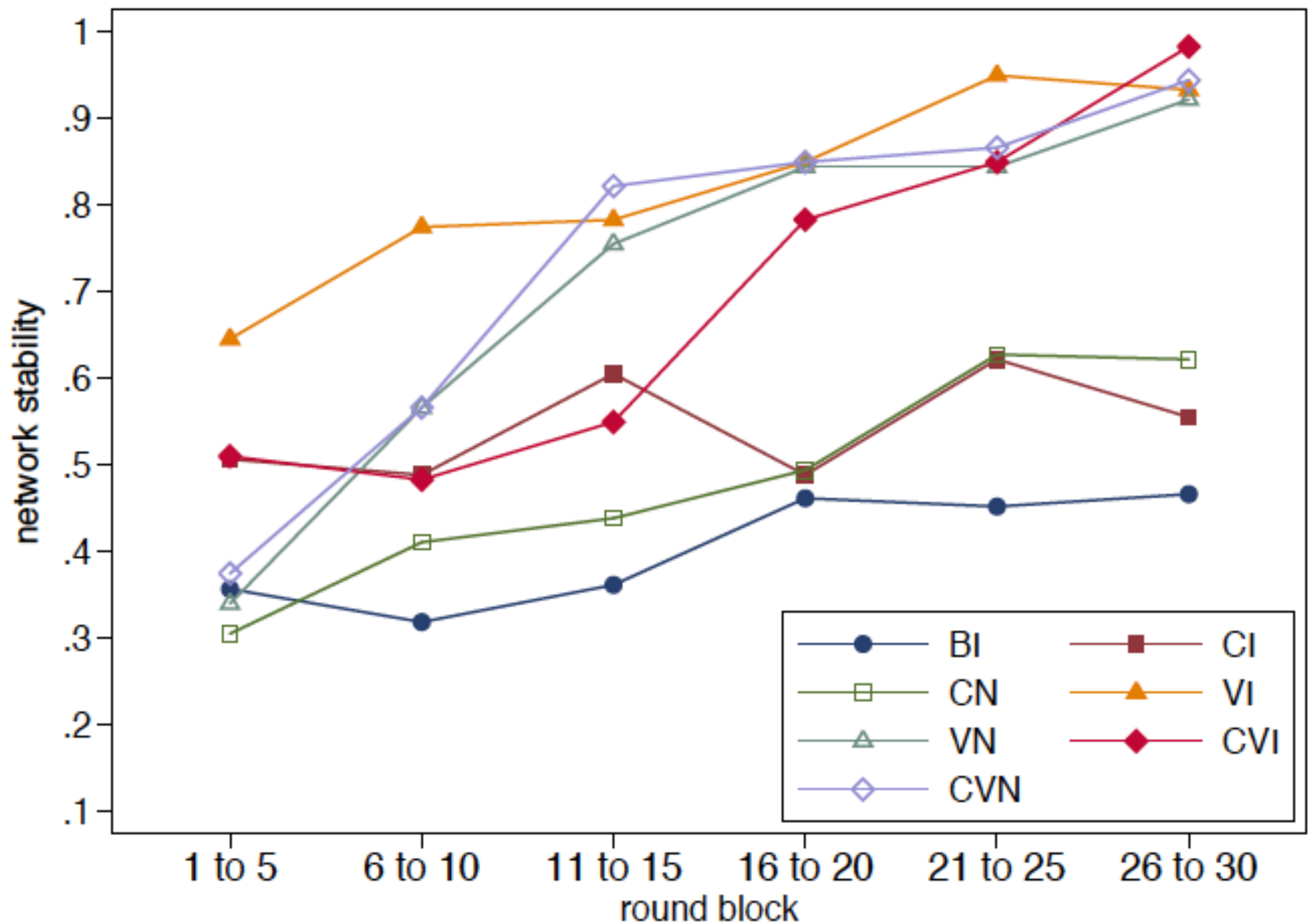
Outcomes after Learning

- Frequency of equilibrium outcomes (stars) increases significantly in last five rounds when there is a **high-value agent**

- Similar results for "almost stars," stars up to one link

“Stability of Networks”

- **Relative stability at t** – fraction of agents who do not change links between round t-1 and round t



network stability
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
1 to 5
6 to 10
11 to 15
16 to 20
21 to 25
26 to 30
round block
BI
CI
CN
VI
VN
CVI
CVN

Conclusions

- With homogeneous agents, equilibrium predictions fail completely
- Heterogeneity helps, at least for star-like equilibria in which heterogeneity generates a “focal” center
- But…

Other Work

- Several other papers studying unilateral connections (e.g., Callander and Plott, 2005; Falk and Kosfeld, 2005)
- A few papers studying experimentally bilateral connections with a particular linking protocol (e.g., Carrillo and Gaduh, 2016; Kirchsteiger et al., 2013; Pantz, 2006)

Future Work?

- Natural to experimentally study whether “free-form” protocols generate stable networks (a-la Jackson and Wolinsky)
- Akin to studies of the emergence of the core in GE and matching markets
- If observed outcomes are stable, may justify using stability to estimate preferences on observed networks

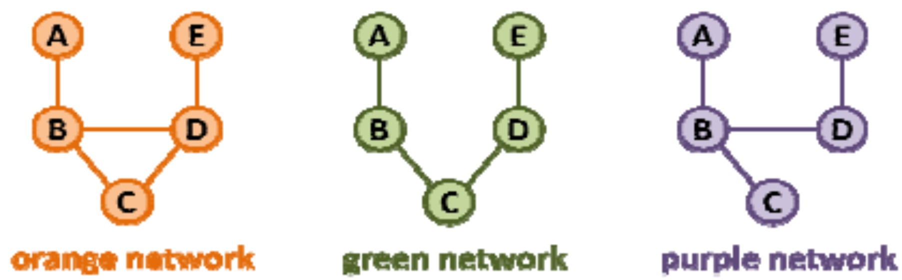
Experiments on Network Games

Equilibrium Selection in Experimental Games on Networks: Charness, Feri, Melendez-Jimenez, and Sutter, 2014

- Study experimentally the prediction of the network games model
- Vary:
  - **Payoffs**: complements or substitutes
  - **Information**: complete or private

Design - Networks

- Three different five person networks



- **Note:** The orange network has higher connectivity.

Design - Payoffs

Each player can be *active* or *inactive*

Being active costs $c > 0$ (in the experiment, $c=1/2$)

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Design - Payoffs

Each player can be *active* or *inactive*

Being active costs $c > 0$ (in the experiment, $c=1/2$)

- **Strategic Substitutes:** A player earns 1 if either she or one of her neighbors is active, 0 otherwise (best-shot game)

- **Strategic Complements:** An inactive player earns 0, an active player earns $\alpha > 0$ times the number of neighbors that are active (in the experiment, $\alpha=1/3$)

Complete Information

- Know realized network and own position in the network

Incomplete Information

The probability of the orange network is *p*

The probability of the green and purple networks is $\frac{1-p}{2}$

In the experiments: *p=0.2* or *p=0.8*

Players are randomly (and uniformly) allocated to the five nodes of the realized network and know only their degree

A symmetric strategy profile is then $s = (s_1, s_2, s_3)$, where s_i is the probability that an agent of degree *i* is active

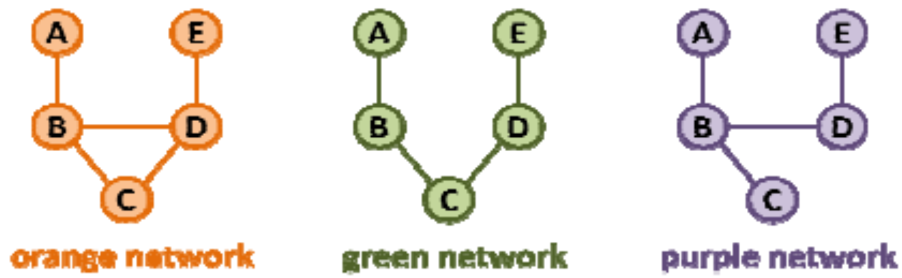
Theoretical Predictions – Summary

Table 1: Equilibria with complete information

	Network	Active nodes	Inactive nodes
Substitutes	Orange	A, C, E	B, D
		B, E	A, C, D
		A, D	B, C, E
	Green	A, C, E	B, D
		B, D	A, C, E
		B, E	A, C, D
		A, D	B, C, E
	Purple	A, C, D	B, E
		A, C, E	B, D
		B, E	A, C, D
Complements	Orange	B, C, D	A, E
		-	A, B, C, D, E
	Green	-	A, B, C, D, E
	Purple	-	A, B, C, D, E

[Focusing on pure equilibria]

Theoretical Predictions – Summary



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	Orange	B, E	A, C, D
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		A, C, E	B, D
		B, D	A, C, E
Substitutes	Green	B, E	A, C, D
		A, D	B, C, E
		A, C, D	B, E
	Purple	A, C, E	B, D
		B, E	A, C, D

Theoretical Predictions – Summary

Table 2: Equilibria with incomplete information

	Probability of g_O	Degree profile
Substitutes	0.2	(1, 0, 0)
	0.8	(1, 1, 0)
Complements	0.2	(0, 0, 0)
	0.8	(0, 0, 0) (0, 1, 1) (0, 0.65, 0.91)

Note: (x, y, z) represents the probability that participants with degree 1, 2, or 3, respectively, are active.

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Multiplicity → coordination game imposed on a network game

Experimental Implementation

- 240 students in the University of Innsbruck, Z-tree
- 12 sessions (20 participants) each
- 2 sessions (40 subjects) in each of the 6 treatments
- In each group, the 20 subjects were split into two matching groups of 10 each
- 40 periods (+5 trial periods)
- Random re-matching to groups of 5
- Average payoff = 16 Euros (+5 Euros show-up fee)

Experimental Feedback

At the end of each period:

- **Complete Information:** see neighbors' decisions and resulting payoffs

- **Incomplete Information:** see realized network, own position, number of active agents, and resulting payoffs

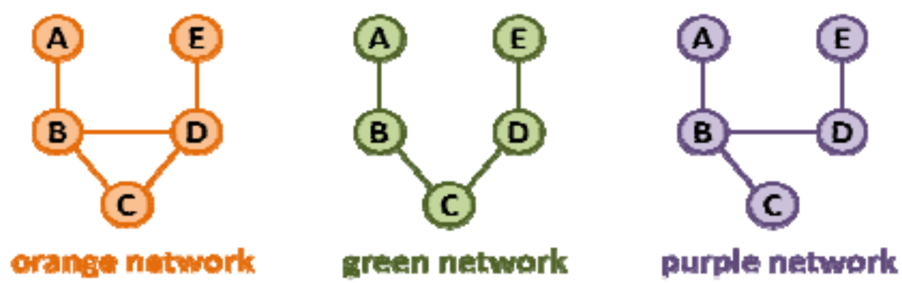
Results – Complete Information

Table 3: Frequencies (and relative frequencies, %) of choices by network and position – Complete information

		Orange		Green		Purple	
		Total Choices	Active (%)	Total Choices	Active (%)	Total Choices	Active (%)
Substitutes	A	93	88 (94.62)	105	96 (91.43)	122	113 (92.62)
	B	93	8 (8.60)	105	16 (15.24)	122	6 (4.92)
	C	93	63 (67.74)	105	70 (66.67)	122	115 (94.26)
	D	93	10 (10.75)	105	18 (17.14)	122	22 (18.03)
	E	93	85 (91.40)	105	99 (94.29)	122	112 (91.80)
	Total	465	254 (54.62)	525	299 (56.95)	610	368 (60.33)
Complements	A	114	4 (3.51)	105	1 (0.95)	101	1 (0.99)
	B	114	85 (74.56)	105	4 (3.81)	101	13 (12.87)
	C	114	83 (72.81)	105	11 (10.48)	101	1 (94.26)
	D	114	85 (74.56)	105	2 (1.90)	101	5 (18.03)
	E	114	6 (5.26)	105	1 (0.95)	101	1 (0.99)
	Total	570	263 (54.62)	525	19 (3.62)	505	21 (4.16)

Complete Information - Summary

- **Substitutes** – the equilibrium in which A, C, and E are active prevails. Least efficient, but most "robust to mistakes"



Complete Information - Summary

- **Substitutes** – the equilibrium in which A, C, and E are active prevails. Least efficient, but most “robust to mistakes”

- **Complements** – the unique equilibrium is played for the *Green* and *Purple* networks, the efficient one is played for the *Orange*

Results – Incomplete Information

Table 5: Frequencies (and relative frequencies, %) of choices by connectivity (p) and degree – Incomplete information

	Degree	$p = 0.2$ Total Choices	$p = 0.2$ Active (%)	$p = 0.8$ Total Choices	$p = 0.8$ Active (%)
Substitutes	1	771	731 (94.81)	676	628 (92.90)
	2	554	156 (28.16)	378	225 (59.52)
	3	275	3 (1.09)	546	55 (10.07)
	Total	1600	890 (55.63)	1600	908 (56.75)
Complements	1	763	15 (1.97)	681	12 (1.76)
	2	598	107 (17.89)	374	116 (31.02)
	3	239	106 (44.35)	546	278 (51.01)
	Total	1600	228 (14.25)	1600	908 (25.37)

Incomplete Information – Summary

- **Substitutes** – Subjects play consistently the unique equilibrium; The probability of being active decreases in degree and increases in connectivity

- **Complements** – With lower connectivity the modal play coincides with the unique equilibrium; The probability of being active increases in degree and connectivity

Deviation from Equilibrium

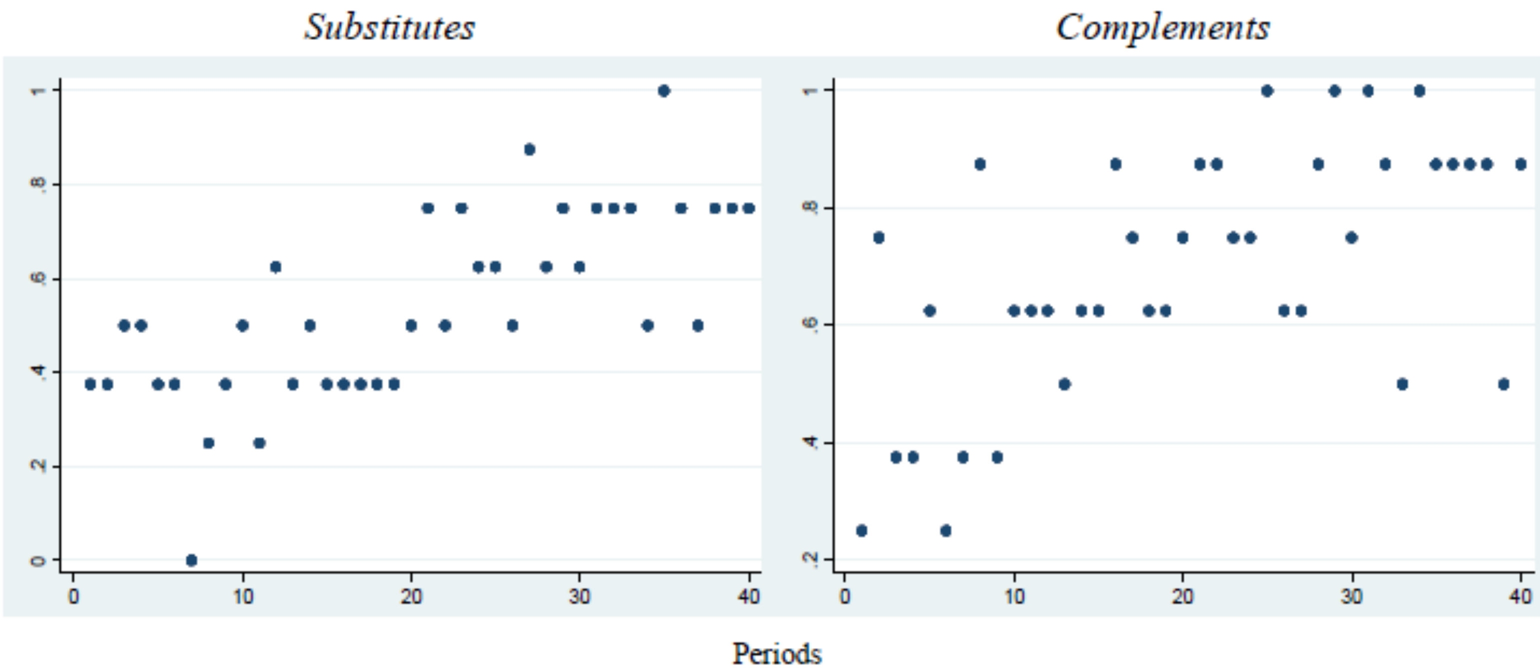
Table 7: Deviation rates in last 10 periods from primary equilibrium

	Substitutes						*Complements*					
	Position						Position					
	A	B	C	D	E	Avg.	A	B	C	D	E	Avg.
Orange	0.05	0.05	0.27	0.05	0.00	0.08	0.06	0.30	0.27	0.30	0.00	0.19
Green	0.07	0.07	0.20	0.10	0.03	0.09	0.00	0.00	0.00	0.00	0.00	0.00
Purple	0.04	0.04	0.00	0.07	0.07	0.04	0.00	0.00	0.00	0.00	0.00	0.00
	Degree						Degree					
	1		2		3	Avg.*	1		2		3	Avg.*
$p = 0.2$	0.02		0.27		0.00	0.10	0.00		0.04		0.10	0.03
$p = 0.8$	0.05		0.22		0.05	0.09	0.00		0.04		0.15	0.06

Notes: The average is calculated by weighting the rates with the number of observations in each cell.

Learning Equilibrium – Complete Information

Figure 3: Relative frequency of equilibrium play across period by treatment – Complete information



[[y-axis: frequency of groups in which all members are coordinated on an equilibrium]]

Learning Equilibrium – Incomplete Information

Figure 5: Relative frequencies of equilibrium play across periods by game and connectivity – Incomplete information

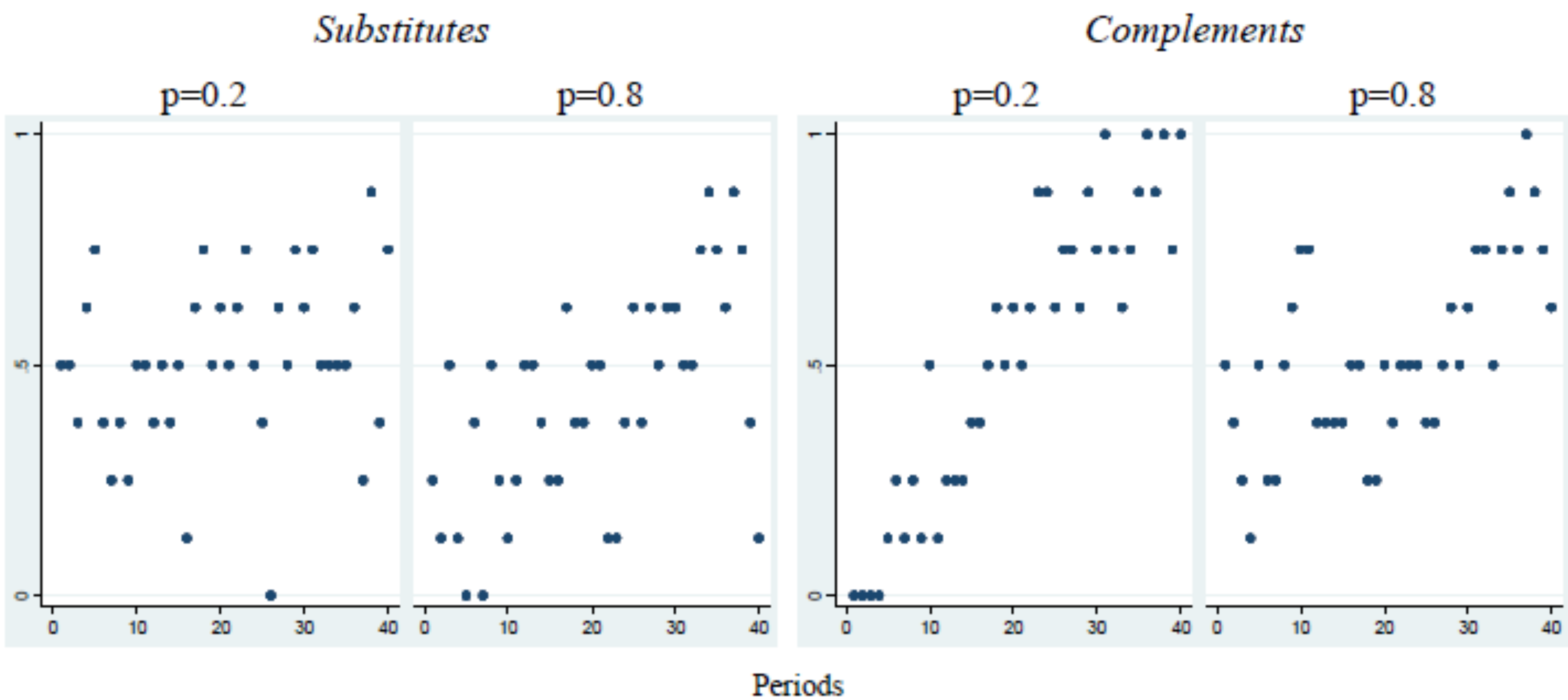
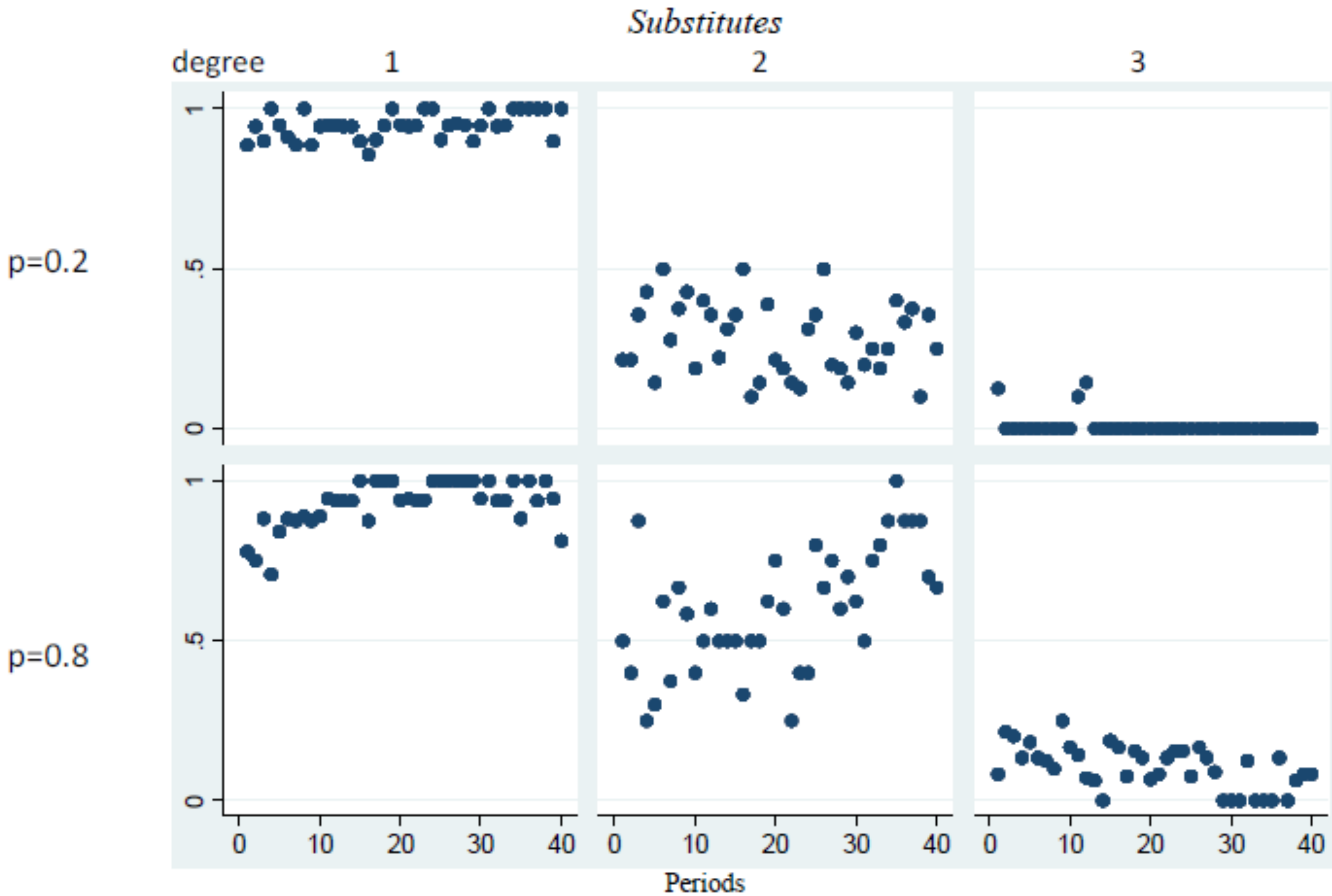


Figure 4: Relative frequencies of choices by degree, games, and connectivity (*p*) – Incomplete information



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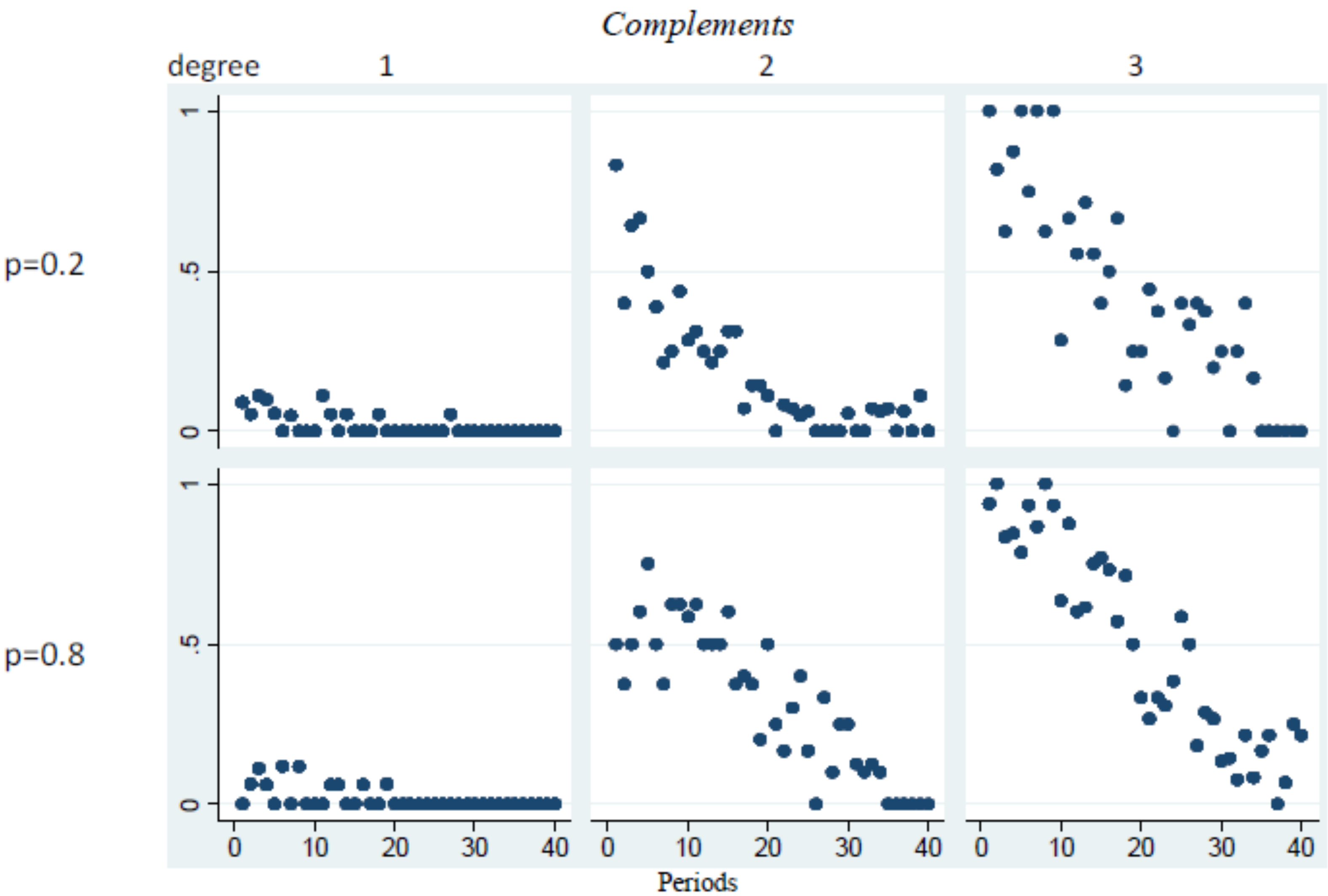


Some Comments

- Best responses to experimental play?
- Decisions based on indifferences
- Difficult to coordinate – resort to more "robust" outcomes? (Communication…)
- Small networks – Does incomplete information make sense? Reputation effects?

Other Work

Several experiments on specific games:

- Bargaining on networks (Agranov and Elliott, 2016)
- Solving difficult problems by local incentives (Dworkin and Kearns, 2015)
- Learning on Networks (Choi, Gale, and Kariv, 2005)

Challenges of Experimental Games on Networks

- Dealing with multiplicity of equilibria

- In the lab, small networks
  - At odds with incomplete information
  - Number of possible networks gets large quickly

Challenges of Experimental Games on Networks

- Problems are complex and it is hard to disentangle:
  - Use of simple heuristics
  - Misunderstanding of strategic incentives
  - Misunderstanding of the game

- Certain known "experimental" biases (e.g., altruism, egalitarian motives, etc.)
  - What is the reference group?
  - Need more control treatments

Experiments on Elicited Networks

- Some experiments rely on elicited organic network

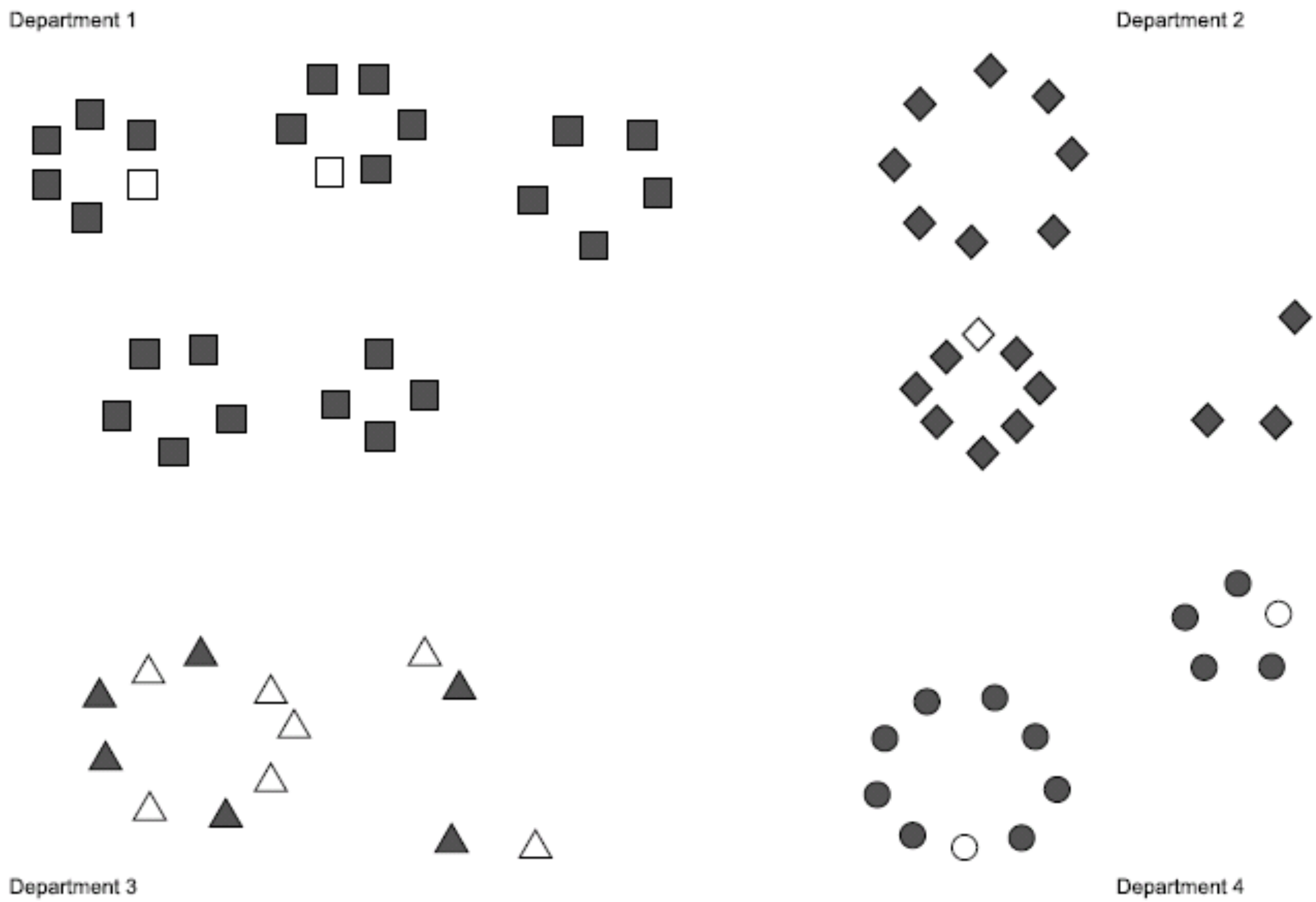
- Use network information to design incentivized tasks for participants

Subject Populations

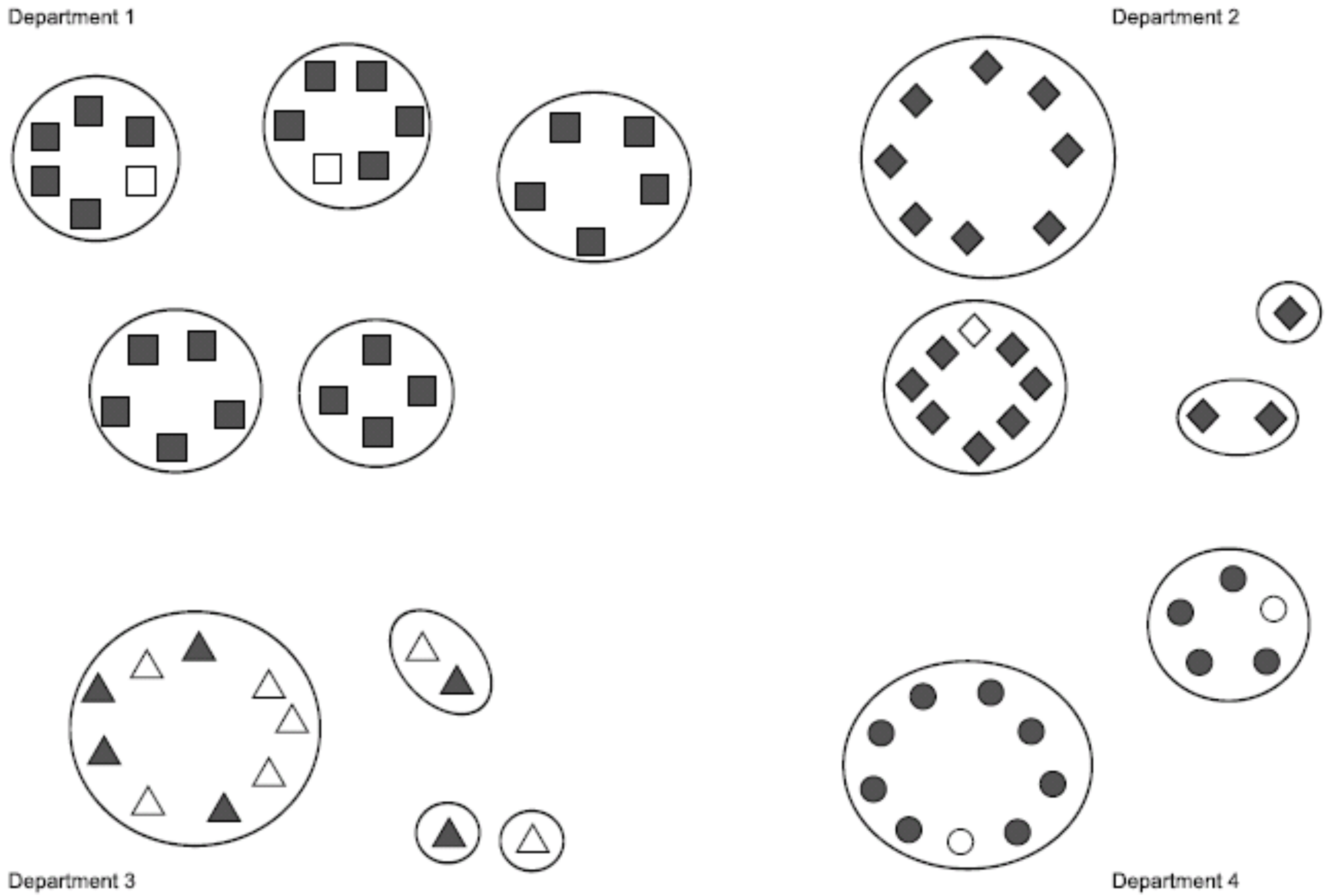
- Many adult populations are problematic:
  - Online network platforms such as Facebook, Instagram, etc. – what does a link mean?
  - Multiple layers: social, professional, geographical, etc.

Baccara, Imrohorogly, Wilson, Yariv (2012)

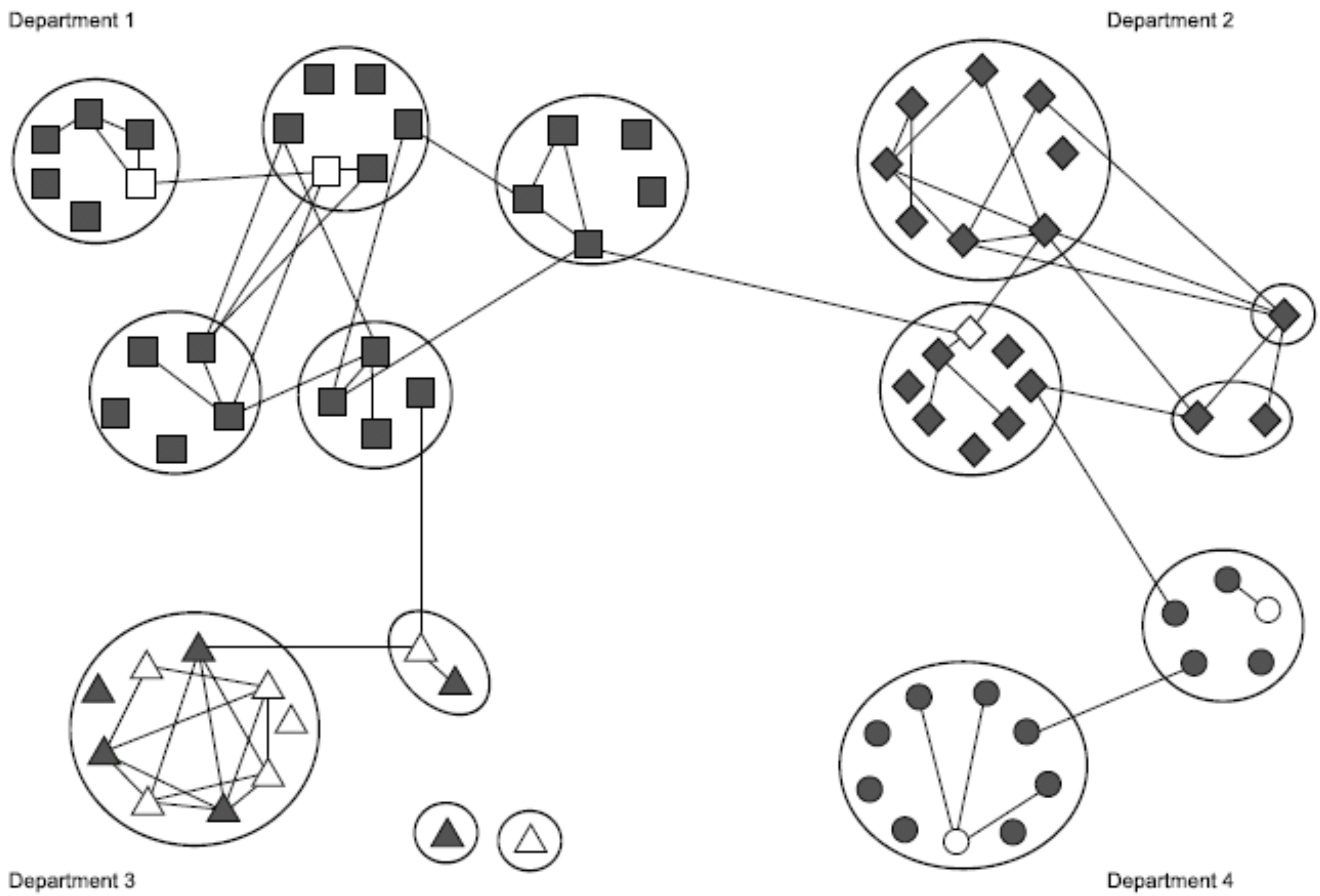
Department



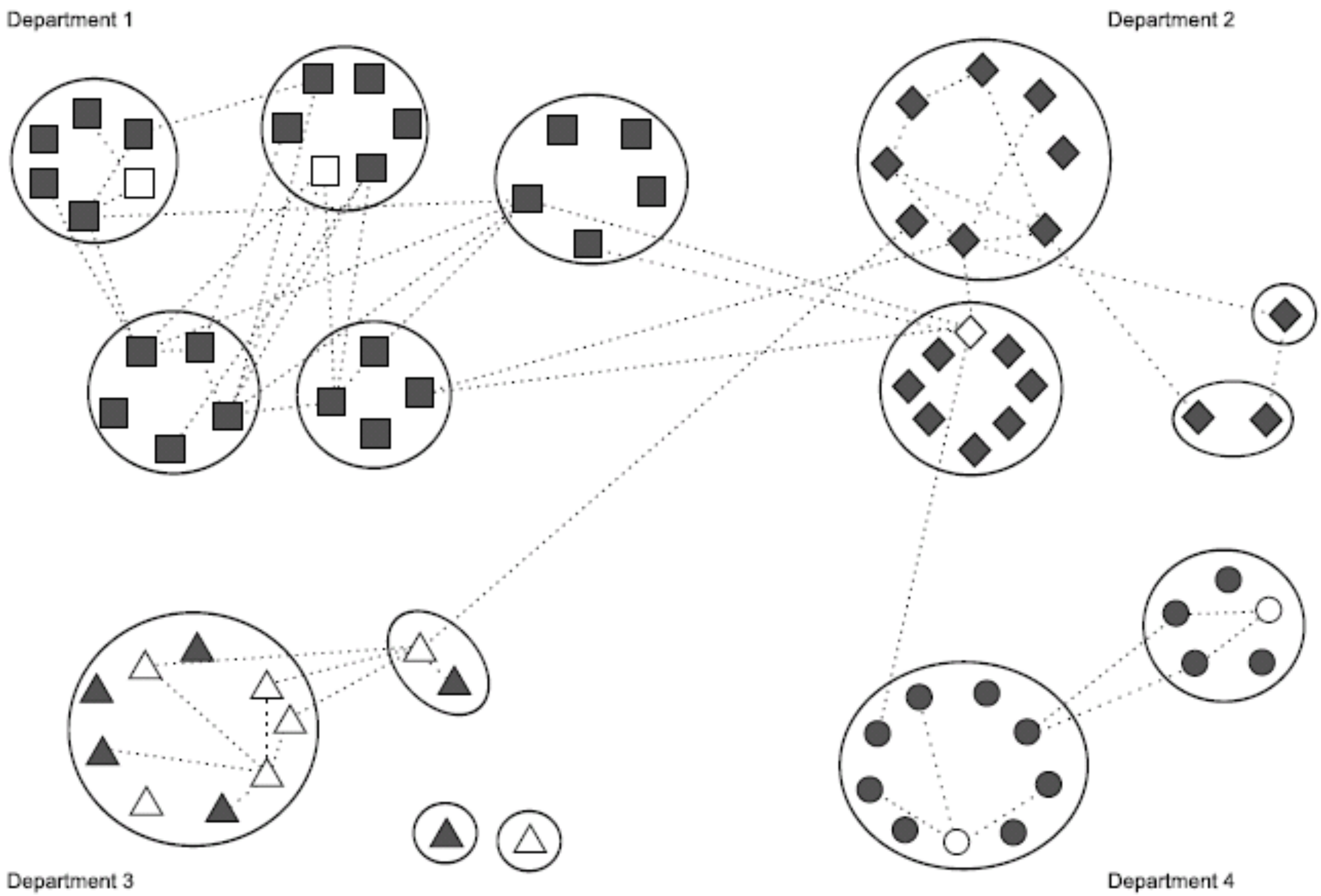
Research field



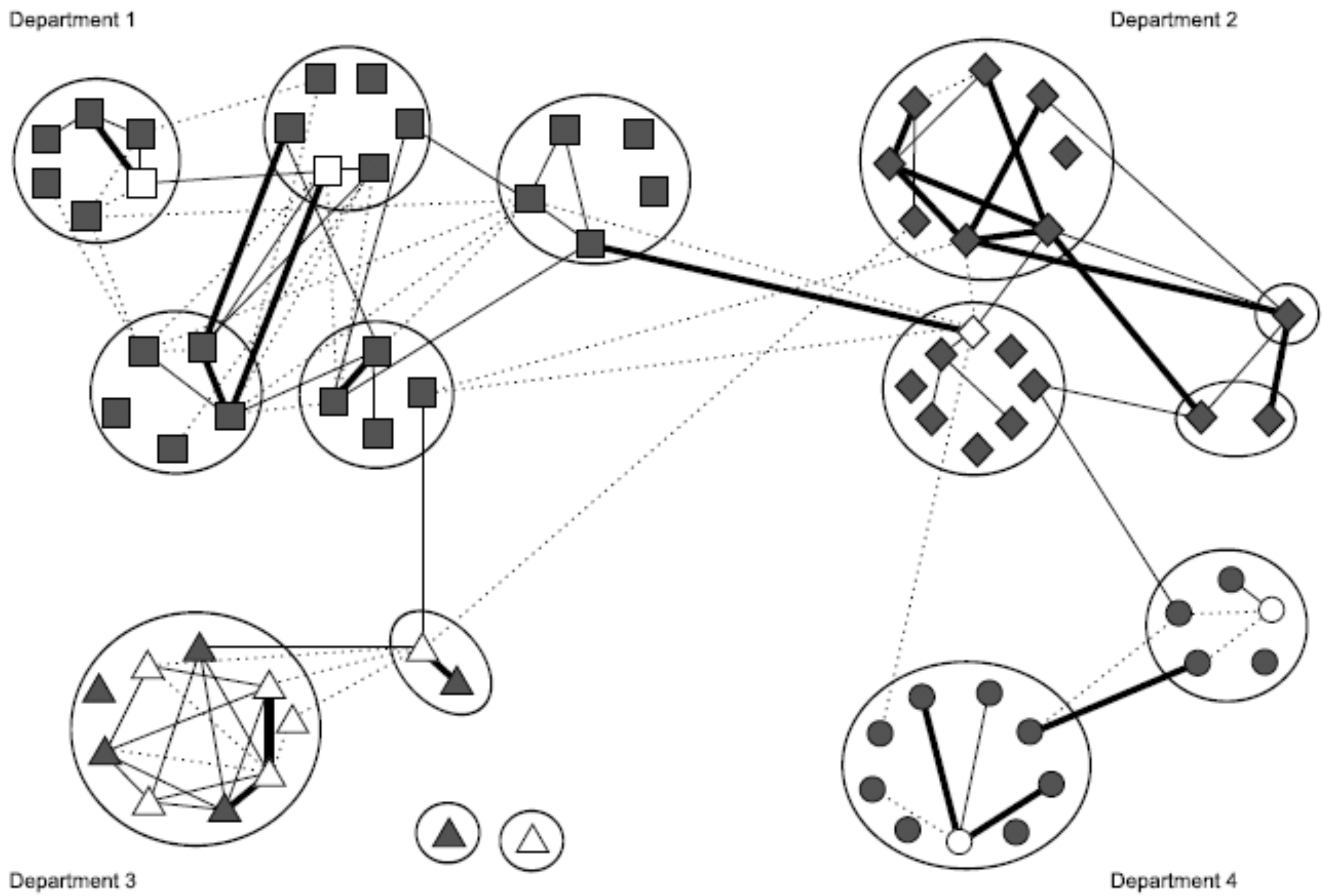
Coauthorships



Friendships



Composite



Subject Populations

- Many adult populations are problematic:
  - Online network platforms such as Facebook, Instagram, etc. – what does a link mean?
  - Multiple layers: social, professional, geographical, etc.
  - Network can be endogenous given strategic interactions

- One solution: use special populations
  - Children in school
  - Undergraduates in a university
  - Villages in developing countries

A Note on Network Eliciation

- Many studies – 60-70% links are bilateral
- Many elicitations ask for k closest friends
- Suppose k=5. If I have three friends, one of whom has 10 friends, they are likely not to reciprocate…
- We still need techniques for correcting for measurement error:
  - Due to sampling of a sub-population (Chandrasekhar and Lewis, 2011)
  - Due to misreporting
  - Due to truncation through elicitation

Altruism through Social Networks

The 1/d Law of Giving: Goeree, McConnell, Mitchell, Tromp, and Yariv, 2010

Our Goal

The 1/d Law of Giving: Goeree, McConnell, Mitchell, Tromp, and Yariv, 2010

- **Field Data:** Map friendship network and individual characteristics of teenagers in an all-girls school in Pasadena

- **Experiments:** Series of dictator games, design using elicited network structure

Experimental Design - Survey (1)

- **Population:** 5th to 12th graders at Westridge, an all girls school in Pasadena
- Surveys conducted in January, 2006, experiments conducted with 5th and 6th graders in April, 2006
- **Age group advantage:** Network likely to be self contained

Participation

- Surveys and experiments conducted during class time: 77% participation in survey for the entire school (373 girls)

- 95% participation in survey for those that took part in experiment (5^{th} and 6^{th} grades)

- High participation of 5^{th} and 6^{th} grades in the experiments: only 5% absent

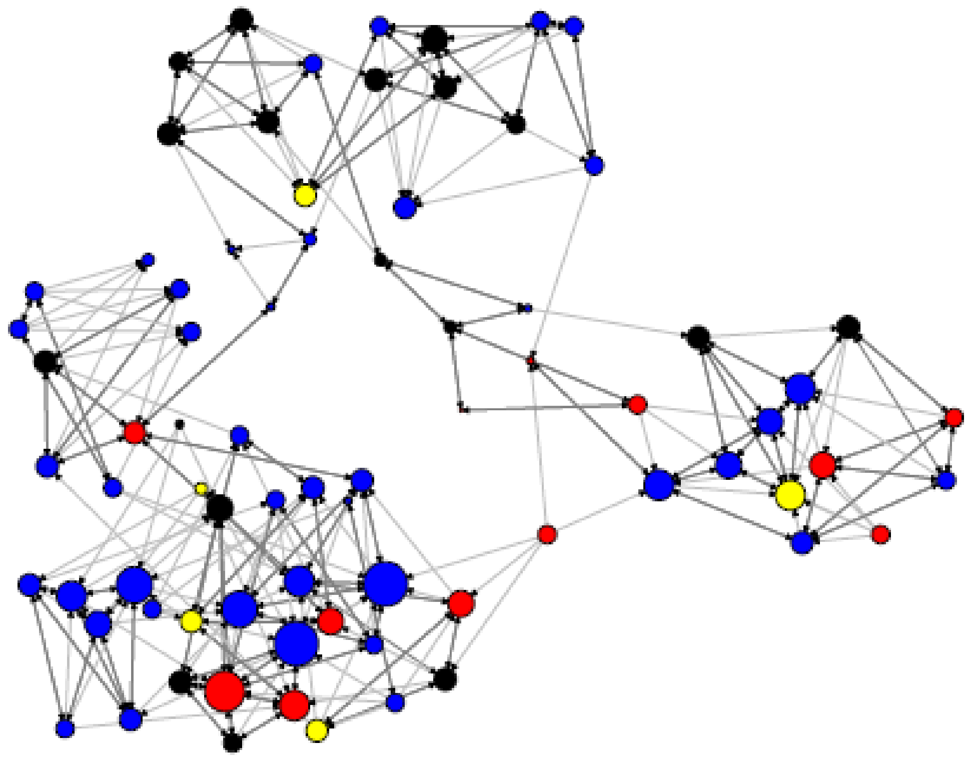
Experimental Design - Survey (2)

- **Demographic Characteristics:** height, age, number of siblings
- **Psychological Characteristics:** personality traits (confident, shy, extroverted, optimistic)
- **Physical Characteristics:** physical appearance (hair color, eye color, braces)
- **Network Characteristics:** Students name 5 friends, how much time they spend with each friend, how much time they spend with others

Summary – Survey Findings

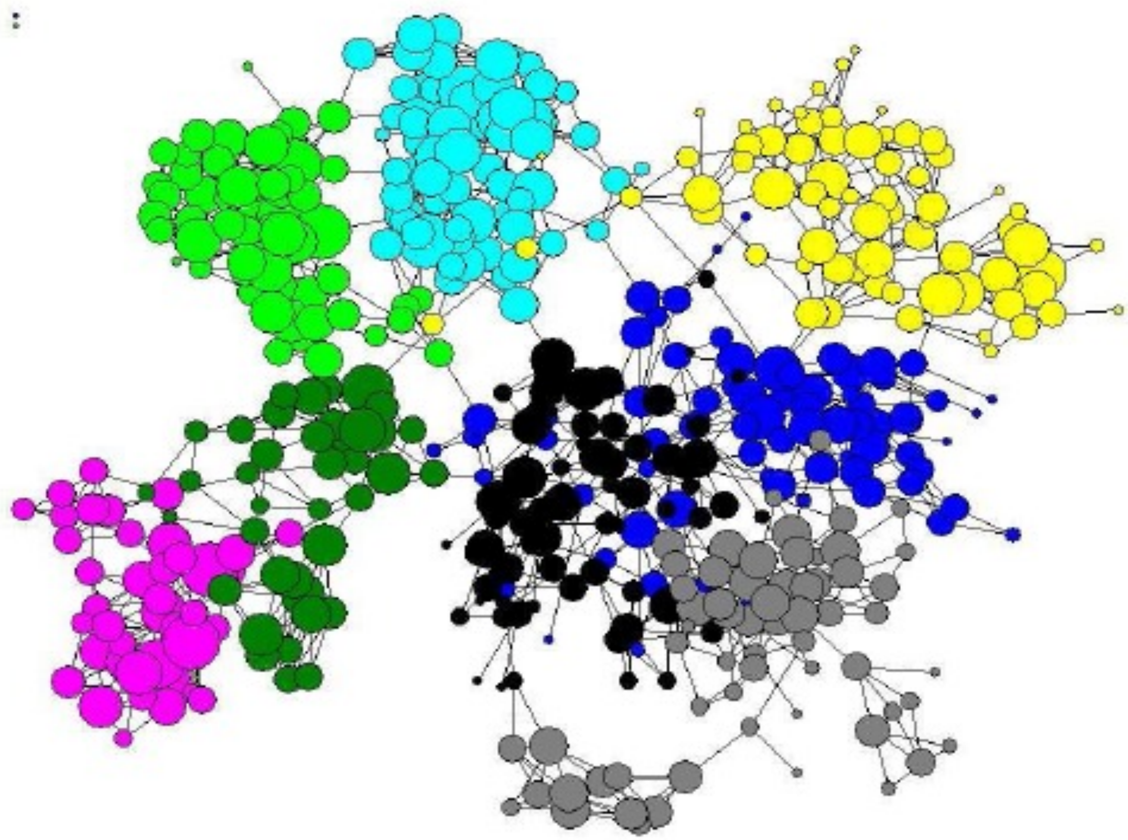
- 63.4% of links are two-way links
- Subjects reported anywhere from 2 to 5 friends, average of 4.4

5^{th} and 6^{th} Graders



Legend: blue-Caucasian, black-Asian, red-Mixed, and yellow-Other, thin lines-one-way links, thick lines-two-way links, size of node represents degree

Entire School



Legend: Colors represent grade, size of node represents degree

Experimental Design

- Follows Leider, Mobius, Quoc-Anh, and Rosenblat (2004)
- Students make 10 consecutive decisions, dividing $6 between themselves and another student
- **Randomly chosen recipients:** 3 first degree, 3 second-degree, and 4 with distance at least 4
- One decision implemented at random using a 10-sided die
- Earnings distributed the next day with additional unannounced $2

Dictator Experiments

Decision sheet (#1 out 10): Hello dictator's name! Please choose how you want to divide $6 between you and recipient's name.

Amount for You	Amount for Recipient's Name
$___	$___

Please make sure that the amounts in the boxes add up to **$6.**

Strategic Reciprocity

Design reduces likelihood of strategic reciprocity (either through ex-post favors or punishments)

- Randomness in selection of friends and others
- Random selection of one of ten decisions
- Acting as both dictator and recipient

Results – Explaining Offer Amounts using Attribute Data

- **Dictator Attributes:** height, race, shy, popular
- **Recipient Attributes:** shy, popular
- **Interactions between Dictator and Recipient Attributes:** height, race, confidence

$$\boldsymbol{y_{ij}} = \beta_0 + \beta_1 \text{ Dictator Attributes} + \beta_2 \text{ Recipient Attributes} +$$

$$+ \beta_3 \text{ Interactions} + \varepsilon_{\boldsymbol{ij}}$$

Variable Share of 6$	Coefficient (Standard Error)
	Model 1
height	-0.002
	(0.004)
asian	0.057
	(0.039)
shy	-0.037**
	(0.018)
shy_recipient	-0.010
	(0.01)
popular	-0.011
	(0.009)
popular_recipient	0.010**
	(0.005)
samerace	-0.014
	(0.024)
sameheight	0.005
	(0.021)
sameconf	0.028
	(0.022)
closeness	
betweenness	
power	
Constant	0.324***
	(0.020)
Observations	629
R-squared	0.05

Robust standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%

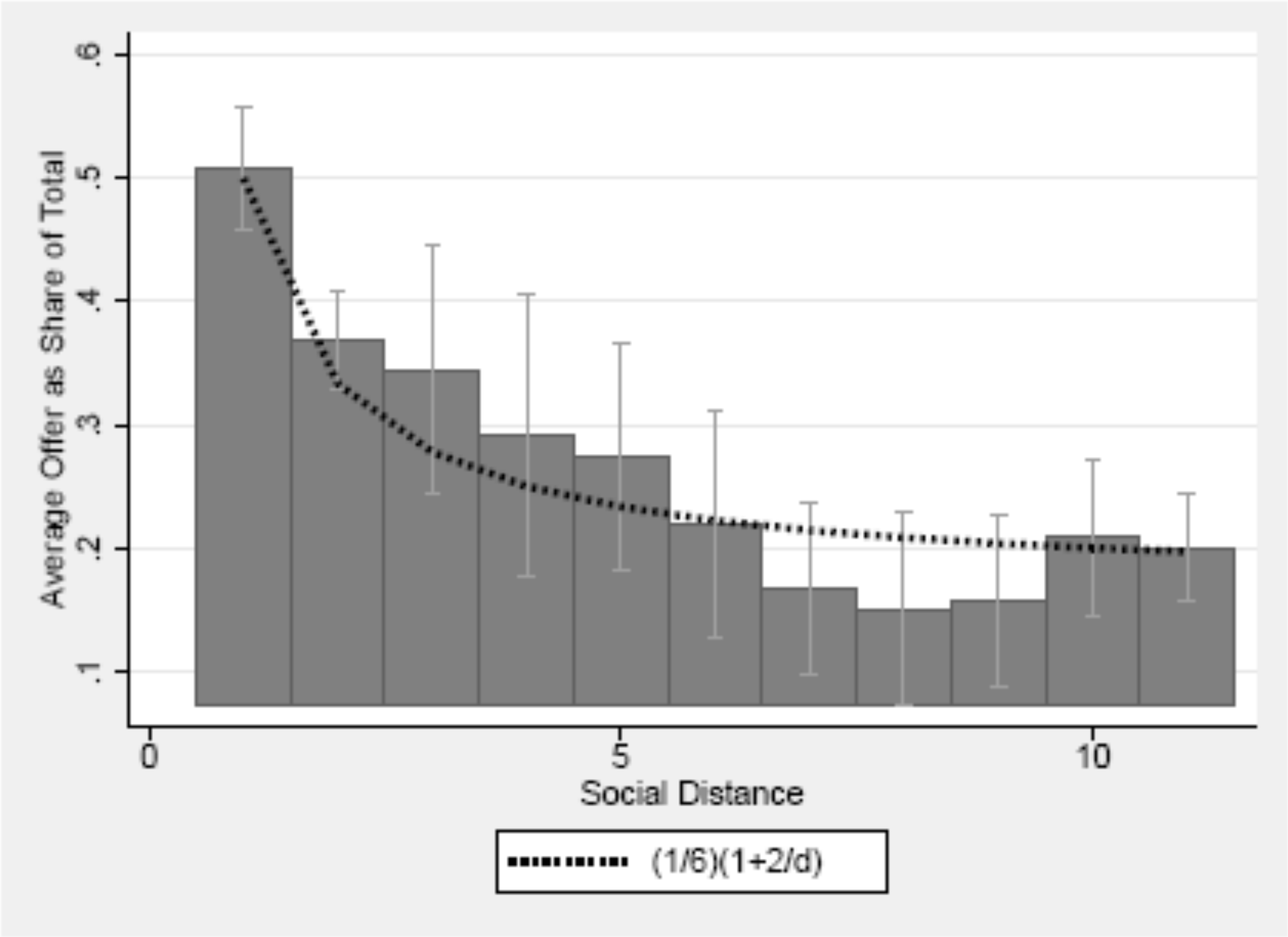
Evidence: Attribute Data

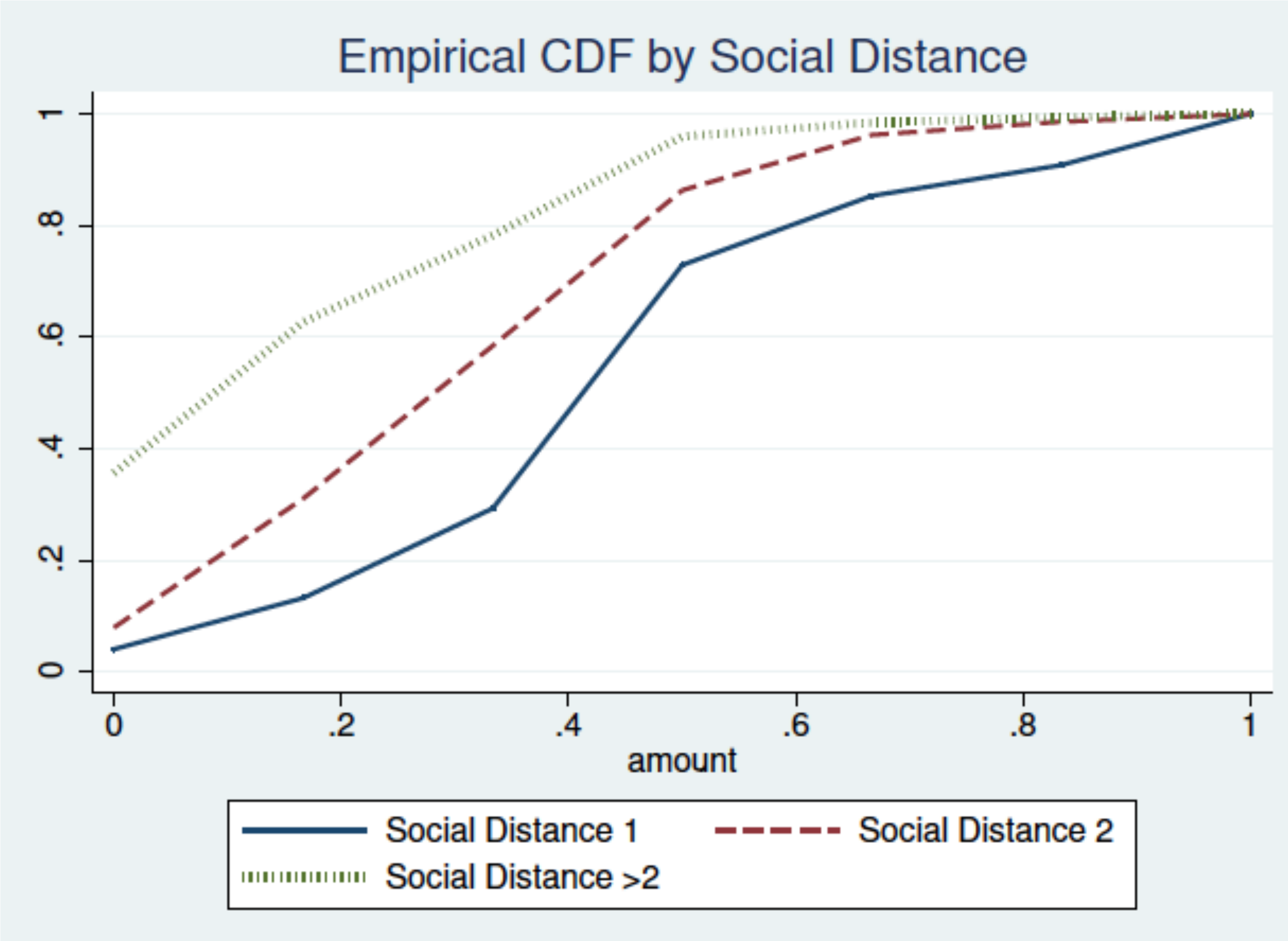
- Individual characteristics explain little, only a measure of the student's deviation from mean "shyness" is significant at the 5% level

- Popularity (the in-degree of the subject) is a significant predictor of receiving more: "popularity premium"

- The model has poor fit, R-squared is only 0.05

Distance Pattern




Empirical CDF by Social Distance
0
.2
.4
.6
.8
1
0
.2
.4
.6
.8
1
amount
Social Distance 1
Social Distance 2
Social Distance >2

Explaining Offer Amounts by Network Structure

- Distance effects: δd_{ij}^{γ}

- **Network Controls:** betweeness, closeness (sum of inverse distance to all others), power (Bonacich centrality)

Variable Share of 6$	Coefficient (Standard Error)	
	Model 1	Model 2
delta (δ)		0.359***
		(0.041)
gamma (γ)		-0.843***
		(0.151)
height	-0.002	-0.003
	(0.004)	(0.004)
asian	0.057	0.039
	(0.039)	(0.040)
shy	-0.037**	-0.036**
	(0.018)	(0.018)
shy_recipient	-0.010	-0.001
	(0.01)	(0.009)
popular	-0.011	-0.017
	(0.009)	(0.012)
popular_recipient	0.010**	-0.004
	(0.005)	(0.005)
samerace	-0.014	-0.023
	(0.024)	(0.022)
sameheight	0.005	-0.007
	(0.021)	(0.018)
sameconf	0.028	0.008
	(0.022)	(0.019)
closeness		-0.011
		(0.009)
betweenness		0.003
		(0.002)
power		-0.002
		(0.007)
Constant	0.324***	0.154***
	(0.020)	(0.038)
Observations	629	629
R-squared	0.05	0.28

Robust standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%

Explaining Offer Amounts by Social Distance

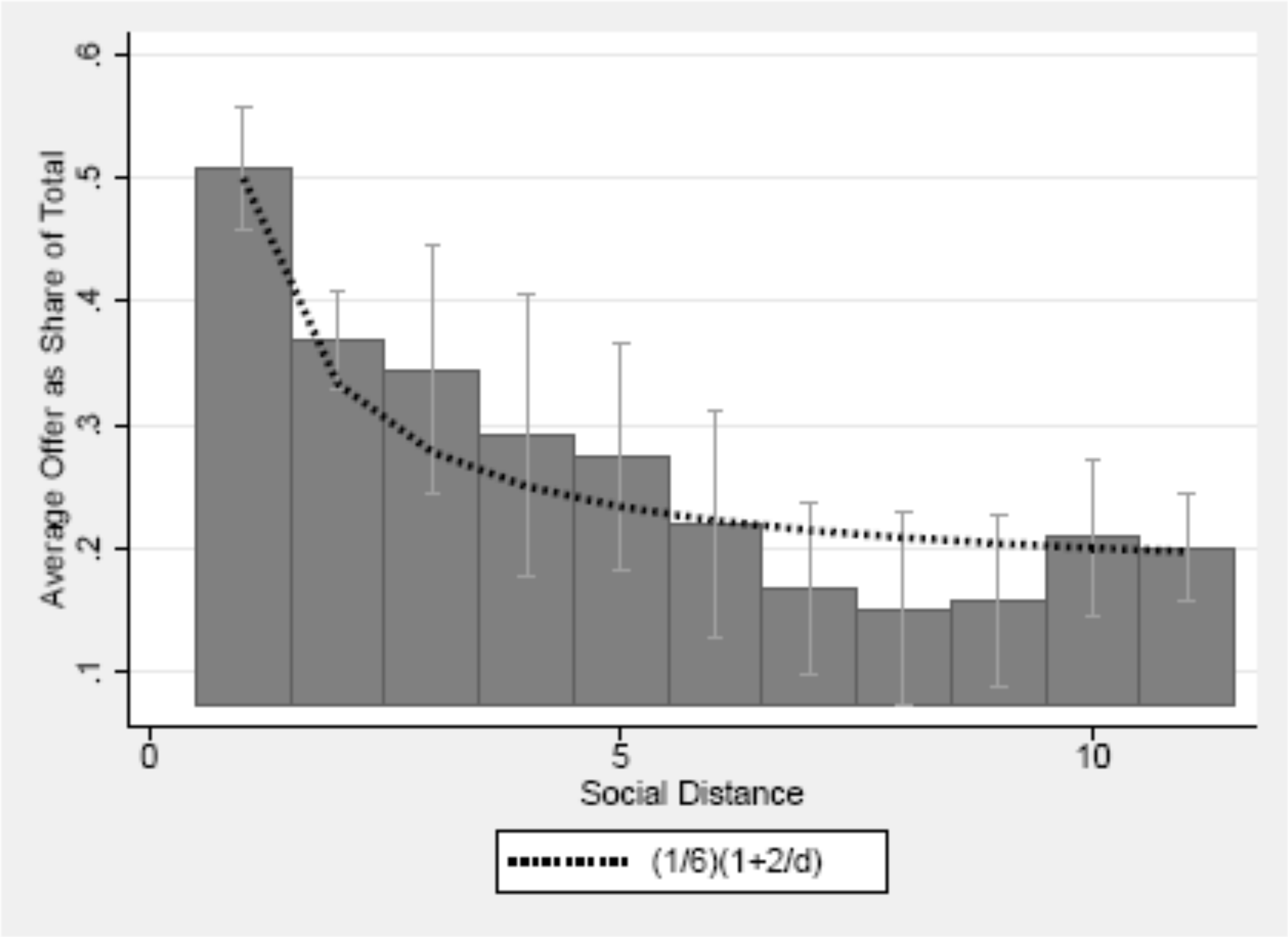
- **Strong Network Effects:** strangers receive 17%, first-degree friends receive an additional 36%, second-degree friends receive an additional 20%, etc.

- When network characteristics are included, fit improves significantly

- **Inverse Distance Law:**

$$\boldsymbol{y_{ij}} = \frac{1}{6}\left(1 + \frac{2}{\boldsymbol{d_{ij}}}\right)$$

Distance Pattern



Indirect Role of Network Characteristics

- Find evidence for homophily over race, confidence, popularity, and height

- Many triangles (a-la preferential attachment): **75% of links are to those who would have distance 2 without that link**

Attributes and Outcomes

- Having a majority attribute may entail more friends (homophily)
- Having more direct friends has an effect on expected outcomes
- Suggestive of the potential relationship between homophily and inequality

Main Insights

- Network structure has overwhelming power in explaining experimental results
- Personal characteristics (demographic and psychological) strongly related to network formation and indirectly affect outcomes

Other Work

- Altruism on Networks: Leider, Mobius, Quoc-Anh, and Rosenblat (2007), Branas-Garza, Duran and Espinosa (2005)

- Trust on Networks: Breza, Chandrasekhar, and Larreguy (2016)

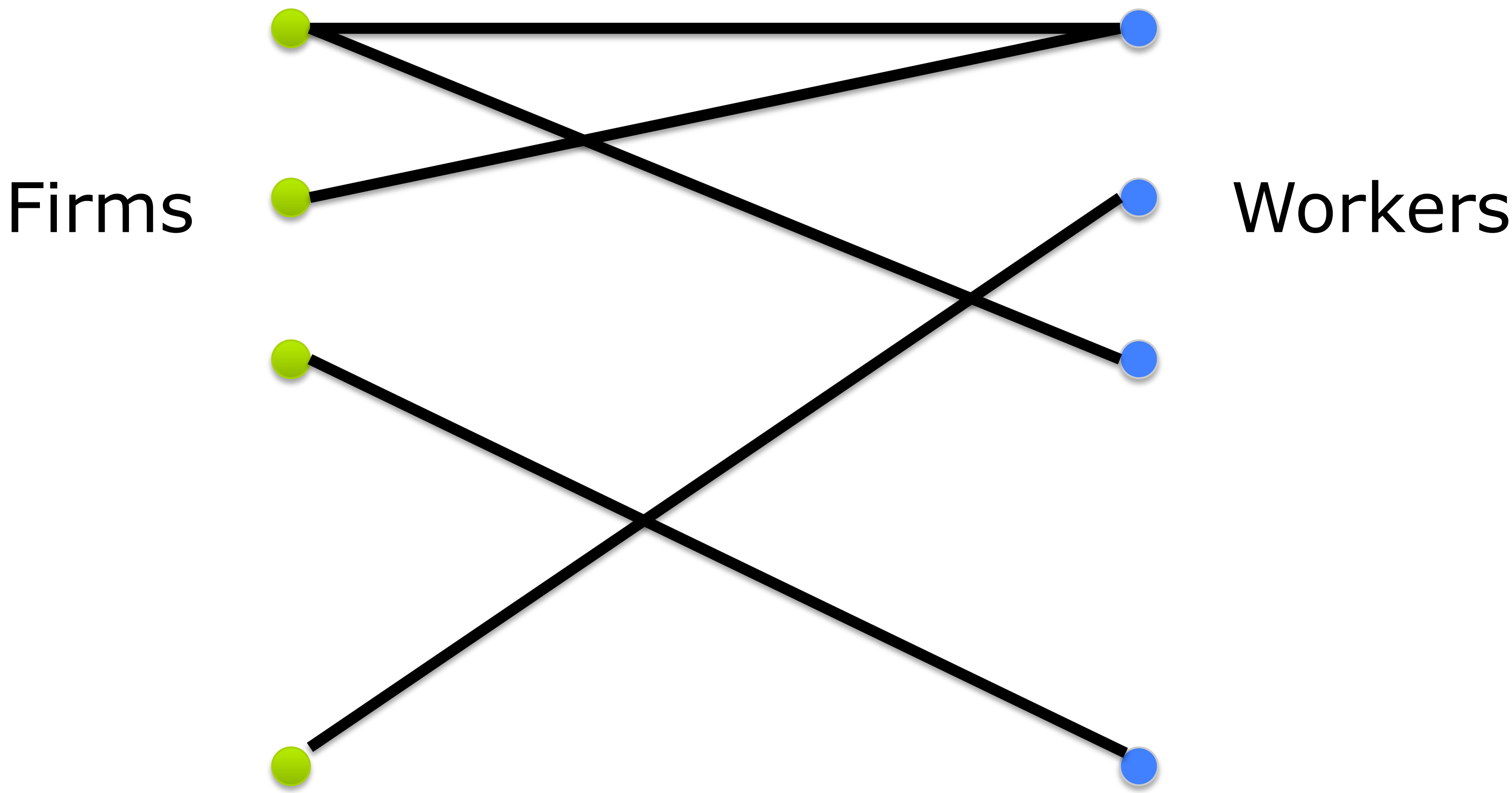
- Various basic elicitations: The Caltech Cohort Study, Snowberg and Yariv (2016)

Matching and Social Networks

- Can think of a one-to-one matching as a special (bi-partite) graph

Matching and Social Networks

- Can think of a one-to-one matching as a special (bi-partite) graph



Matching and Social Networks

- Can think of a one-to-one matching as a special (bi-partite) graph
- Literature on experimental matching markets more evolved, can potentially borrow techniques

Background Theory – Stability

- **Two sided market**: n Colors *C* and n Foods *F*
- **Metaphor for**: workers and firms, men and women, kids and schools, etc.

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  - Each color ranks the foods from 1 to n
  - Each fruit ranks the color from 1 to n

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  - Each fruit ranks the color from 1 to n
- **Stable Match: n pairs (color, food) with no overlap** (a mapping $\mu : C \cup F \to C \cup F, \mu(c) = f, \mu(f) = c$),

  such that there are no color and food who would rather have one another than their current partners

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  such that there are no color and food who would rather have one another than their current partners
- [can allow staying unmatched, imbalanced markets]

Experiments of Decentralized Markets

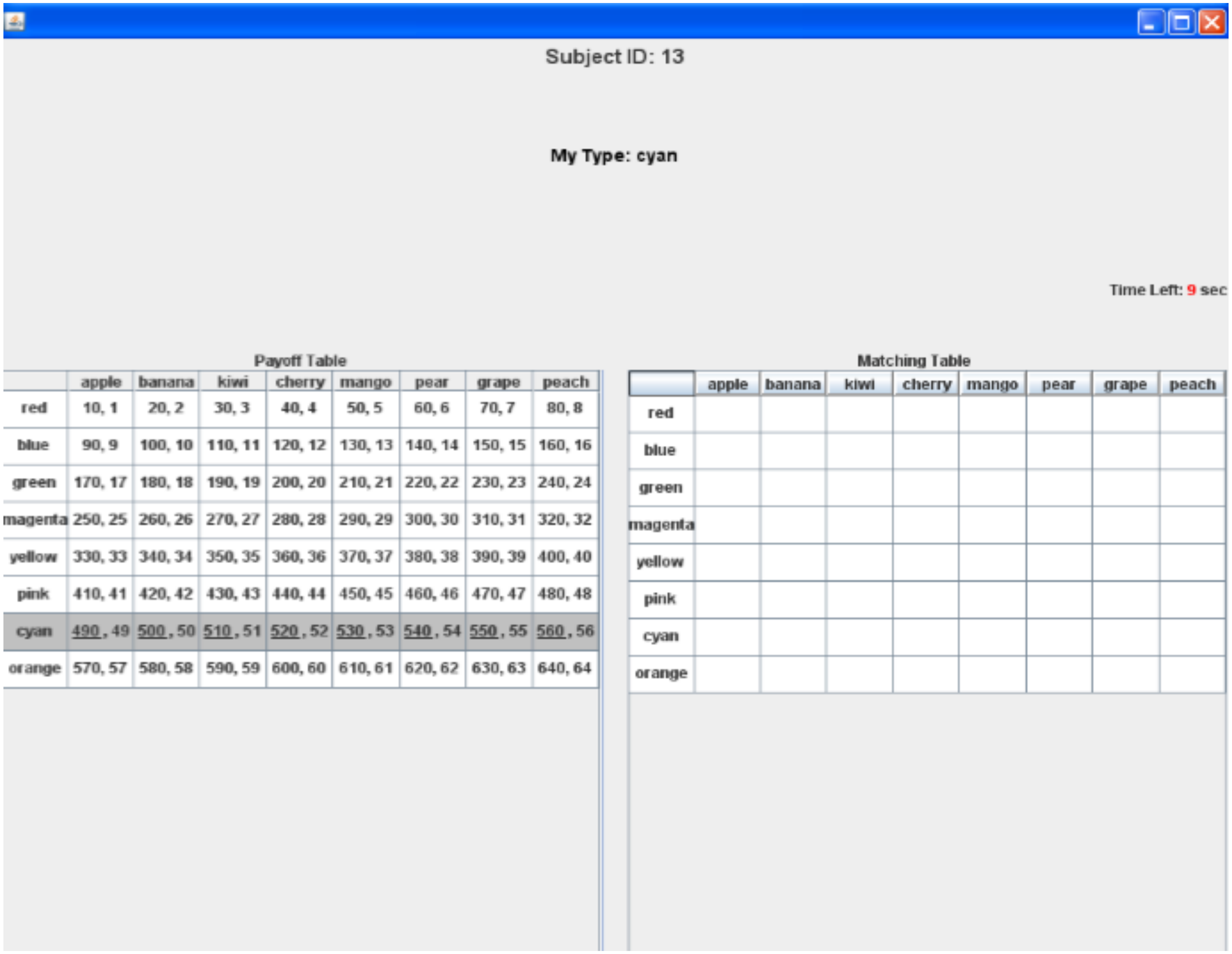
An Experimental Study of Decentralized Matching:
Echenique and Yariv (2013)

- Test experimentally whether decentralized markets converge to stable outcomes
- If so, to which ones and how
- The effects of market attributes:
  - Cardinal presentation of ordinal preferences
  - Number of stable matches
  - Market size

The Experiments

- CASSEL, 144 subjects, modified multi-stage
- Each treatment run for 10 rounds (+ 2 practice rounds)
- Randomly assigned to groups of 16 each round:
  - One side: 8 foods (banana, apple, kiwi, etc.)
  - One side: 8 colors (blue, red, green, etc.)
- Make offers freely to one another (acceptances non-binding), market ends with 30 seconds of inactivity
- Several markets: 7 ordinal, 20 cardinal - multiple stable matches, unique stable match (each round different)

Experimental Interface



Payoff Table

	apple	banana	kiwi	cherry	mango	pear	grape	peach
red	10, 1	20, 2	30, 3	40, 4	50, 5	60, 6	70, 7	80, 8
blue	90, 9	100, 10	110, 11	120, 12	130, 13	140, 14	150, 15	160, 16
green	170, 17	180, 18	190, 19	200, 20	210, 21	220, 22	230, 23	240, 24
magenta	250, 25	260, 26	270, 27	280, 28	290, 29	300, 30	310, 31	320, 32
yellow	330, 33	340, 34	350, 35	360, 36	370, 37	380, 38	390, 39	400, 40
pink	410, 41	420, 42	430, 43	440, 44	450, 45	460, 46	470, 47	480, 48
cyan	490, 49	500, 50	510, 51	520, 52	530, 53	540, 54	550, 55	560, 56
orange	570, 57	580, 58	590, 59	600, 60	610, 61	620, 62	630, 63	640, 64

Matching Table

	apple	banana	kiwi	cherry	mango	pear	grape	peach
red								
blue								
green								
magenta								
yellow								
pink								
cyan								
orange								

The Experiment

- In each round, you will randomly be assigned the role of a “color” or a “food” and a group
- There will be 8 colors and 8 foods in your group
- You will try to match with a member of the opposite group
- For example, if you are color blue you may match with “apple,” “banana,” etc.
- You derive different payoffs from different matches
- In each round of the experiment you will see all payoffs from each possible matching

Payoffs

Payoff Table

	apple	banana	kiwi	cherry	mango	pear	grape	peach
red	10, 1	20, 2	30, 3	40, 4	50, 5	60, 6	70, 7	80, 8
blue	90, 9	100, 10	110, 11	120, 12	130, 13	140, 14	150, 15	160, 16
green	170, 17	180, 18	190, 19	200, 20	210, 21	220, 22	230, 23	240, 24
magenta	250, 25	260, 26	270, 27	280, 28	290, 29	300, 30	310, 31	320, 32
yellow	330, 33	340, 34	350, 35	360, 36	370, 37	380, 38	390, 39	400, 40
pink	410, 41	420, 42	430, 43	440, 44	450, 45	460, 46	470, 47	480, 48
cyan	490 , 49	500 , 50	510 , 51	520 , 52	530 , 53	540 , 54	550 , 55	560 , 56
orange	570, 57	580, 58	590, 59	600, 60	610, 61	620, 62	630, 63	640, 64

Making Offers

You make an offer by clicking on the name you want to match with (right panel)

Matching Table

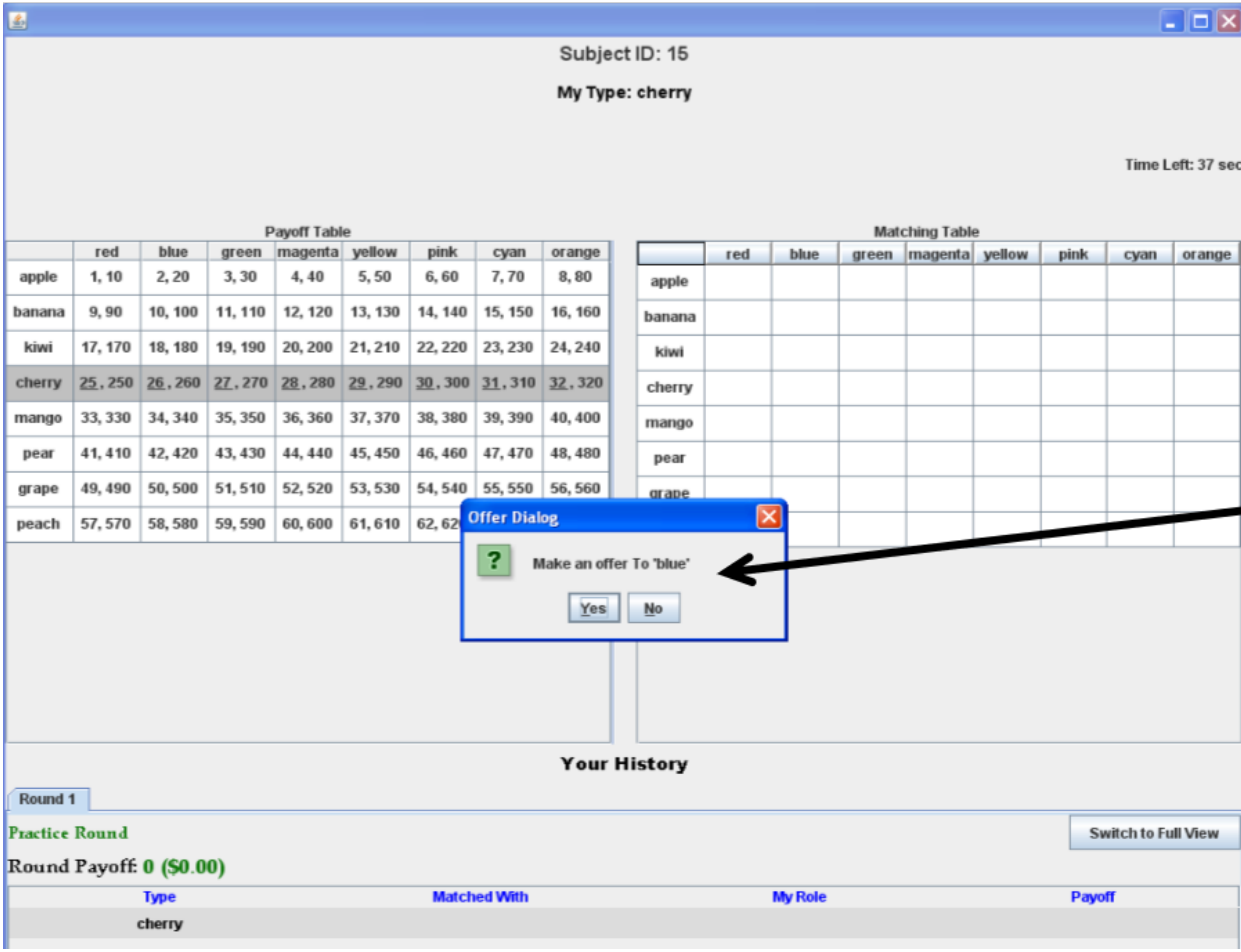
	apple	banana	kiwi	cherry	mango	pear	grape	peach
red								
blue								
green								
magenta								
yellow								
pink								
cyan								
orange								

Clickable buttons

Only one offer at a time

Can make an offer while matched with someone

Making Offers

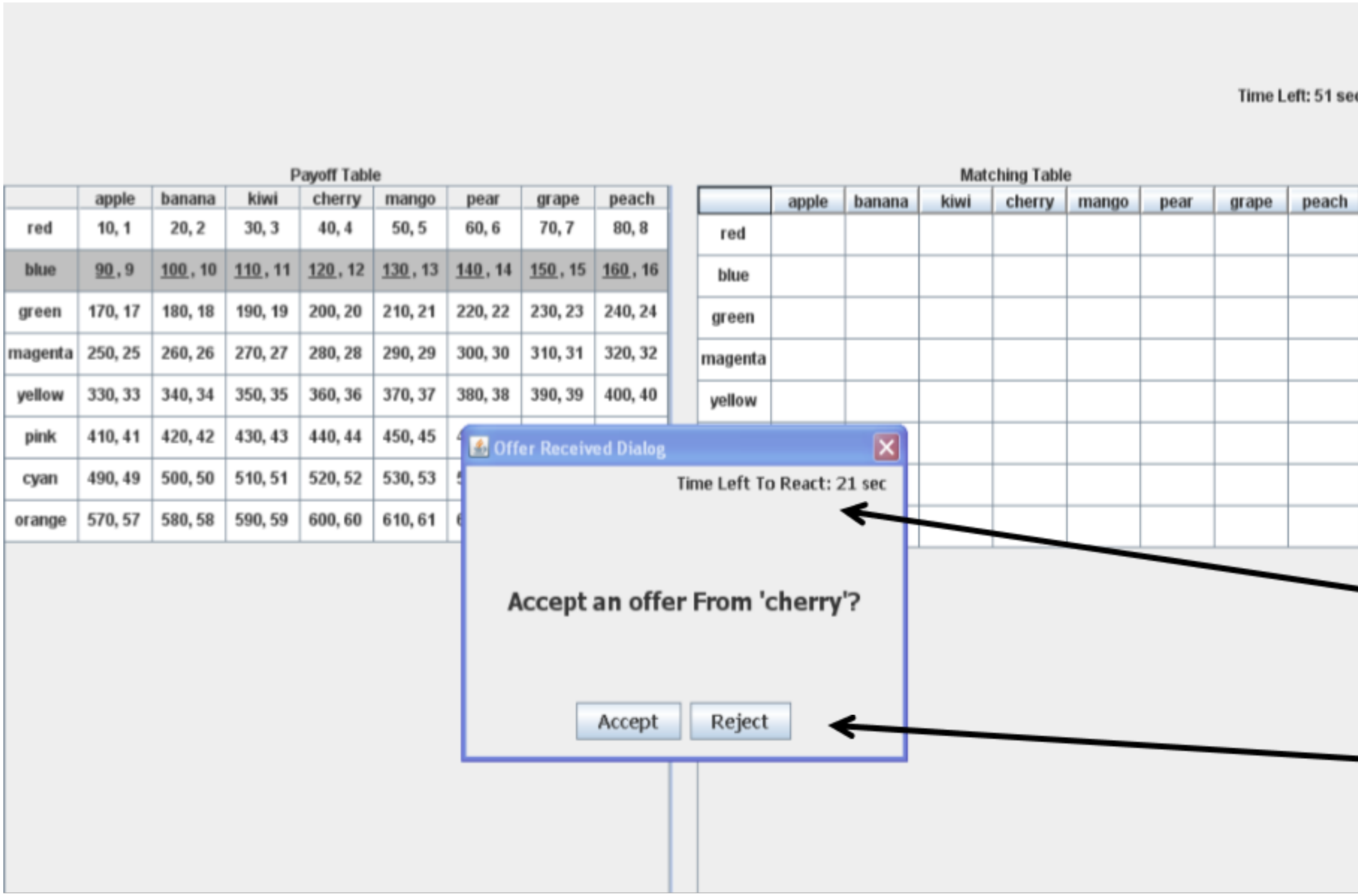


After choosing a target for an offer, you will have to confirm

Response to Offers

- If you receive an offer, a window opens
- You can accept or reject
- If you wait more than 10 seconds the offer is rejected for you

Response to Offers - Interface

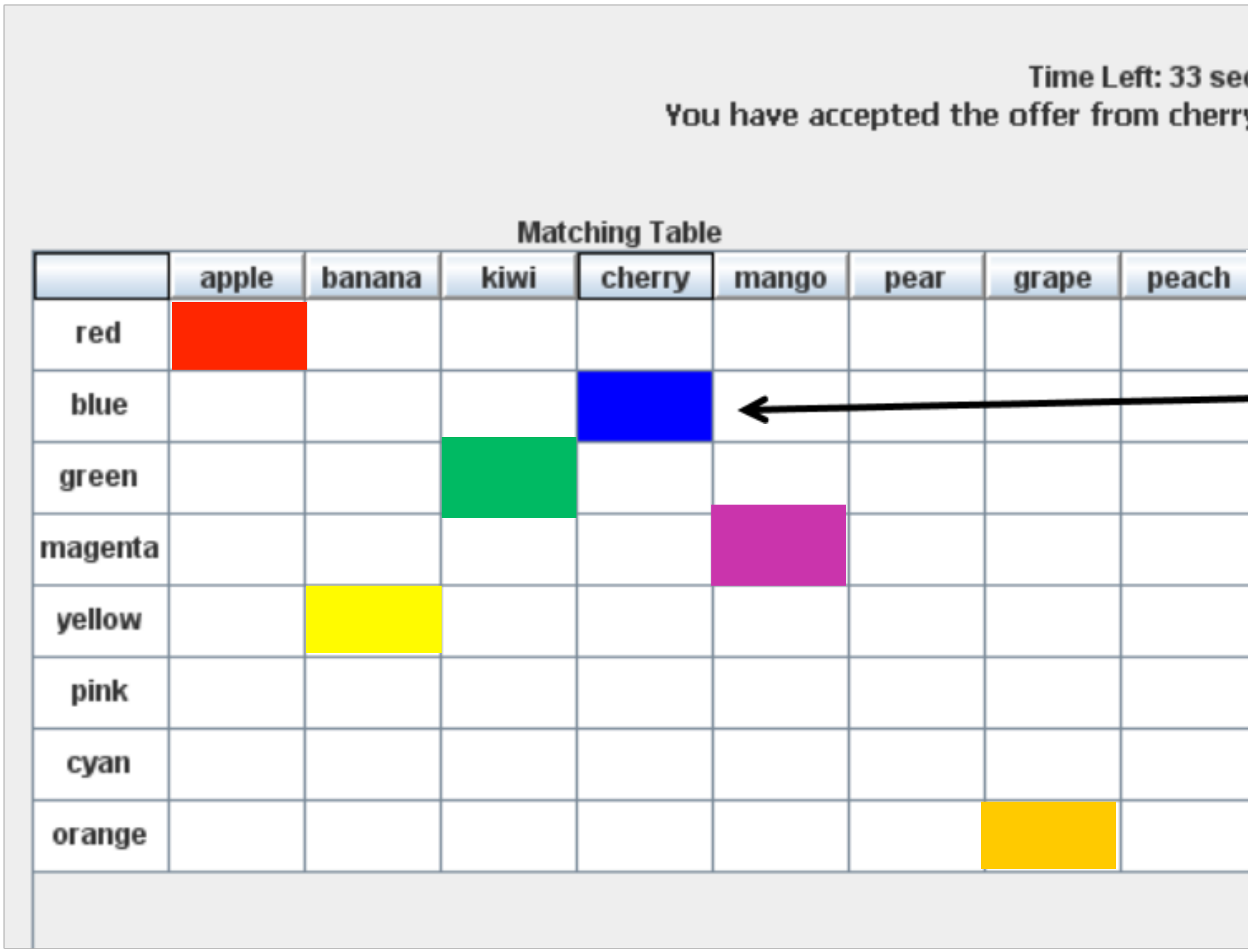


Offer Timer

Offer Decision

Current Matches

In each round, you see a table (right) with the current matches.



Response to offer

blue matched to cherry

Matches are Temporary

- Until the round ends, matches are temporary
- A match can be broken by accepting alternative offers, or by making another offer that gets accepted
- A round ends when there are no new offers during 30 seconds

Markets Used

- 8x8 markets
- **7 ordinal** markets, **20 cardinal** presentations (varying sums of payoffs to either side, marginal payoffs)
- In all markets, each agent had either **one, two, or three stable partners**

Markets Used – Example

	apple	banana	kiwi	cherry	mango	pear	grape	peach
red	300, 300	250, 250	200, 200	250, 150	600, 350	150, 100	100, 50	50, 50
blue	200, 250	300, 300	350, 400	150, 100	500, 250	250, 250	300, 450	200, 150
green	450, 550	400, 500	300, 300	400, 400	200, 150	0, 50	250, 150	400, 100
magenta	50, 100	350, 450	200, 250	300, 300	100, 100	400, 350	150, 100	100, 50
yellow	50, 150	250, 300	400, 500	100, 200	300, 300	50, 150	250, 400	350, 0
pink	150, 200	0, 100	50, 150	200, 250	400, 200	300, 300	150, 200	200, 200
cyan	100, 50	150, 200	50, 50	400, 350	250, 50	200, 200	300, 300	150, 50
orange	100, 150	350, 400	50, 100	150, 100	400, 200	100, 100	150, 0	300, 300

Markets Used – Example

	apple	banana	kiwi	cherry	mango	pear	grape	peach
red	300, 300	250, 250	200, 200	250, 150	600, 350	150, 100	100, 50	50, 50
blue	200, 250	300, 300	350, 400	150, 100	500, 250	250, 250	300, 450	200, 150
green	450, 550	400, 500	300, 300	400, 400	200, 150	0, 50	250, 150	400, 100
magenta	50, 100	350, 450	200, 250	300, 300	100, 100	400, 350	150, 100	100, 50
yellow	50, 150	250, 300	400, 500	100, 200	300, 300	50, 150	250, 400	350, 0
pink	150, 200	0, 100	50, 150	200, 250	400, 200	300, 300	150, 200	200, 200
cyan	100, 50	150, 200	50, 50	400, 350	250, 50	200, 200	300, 300	150, 50
orange	100, 150	350, 400	50, 100	150, 100	400, 200	100, 100	150, 0	300, 300

Results – Ultimate Matchings

- Almost all are matched:
  - **852 of 864** pairs are matched overall:
    - **Unique stable match:** 239 of 240 pairs are matched
    - **Two stable matches:** 351 of 352 are matched
    - **Three stable matches:** 262 of 272 are matched

Results – Ultimate Matchings

- Almost all are matched:
  - **852 of 864** pairs are matched overall:
    - **Unique stable match:** 239 of 240 pairs are matched
    - **Two stable matches:** 351 of 352 are matched
    - **Three stable matches:** 262 of 272 are matched

- Almost all **individual matches** are stable:
  - **Unique stable match: 95%** of matches are stable
  - **Two stable matches: 98%** of matches are stable
  - **Multiple stable match: 92%** of matches are stable

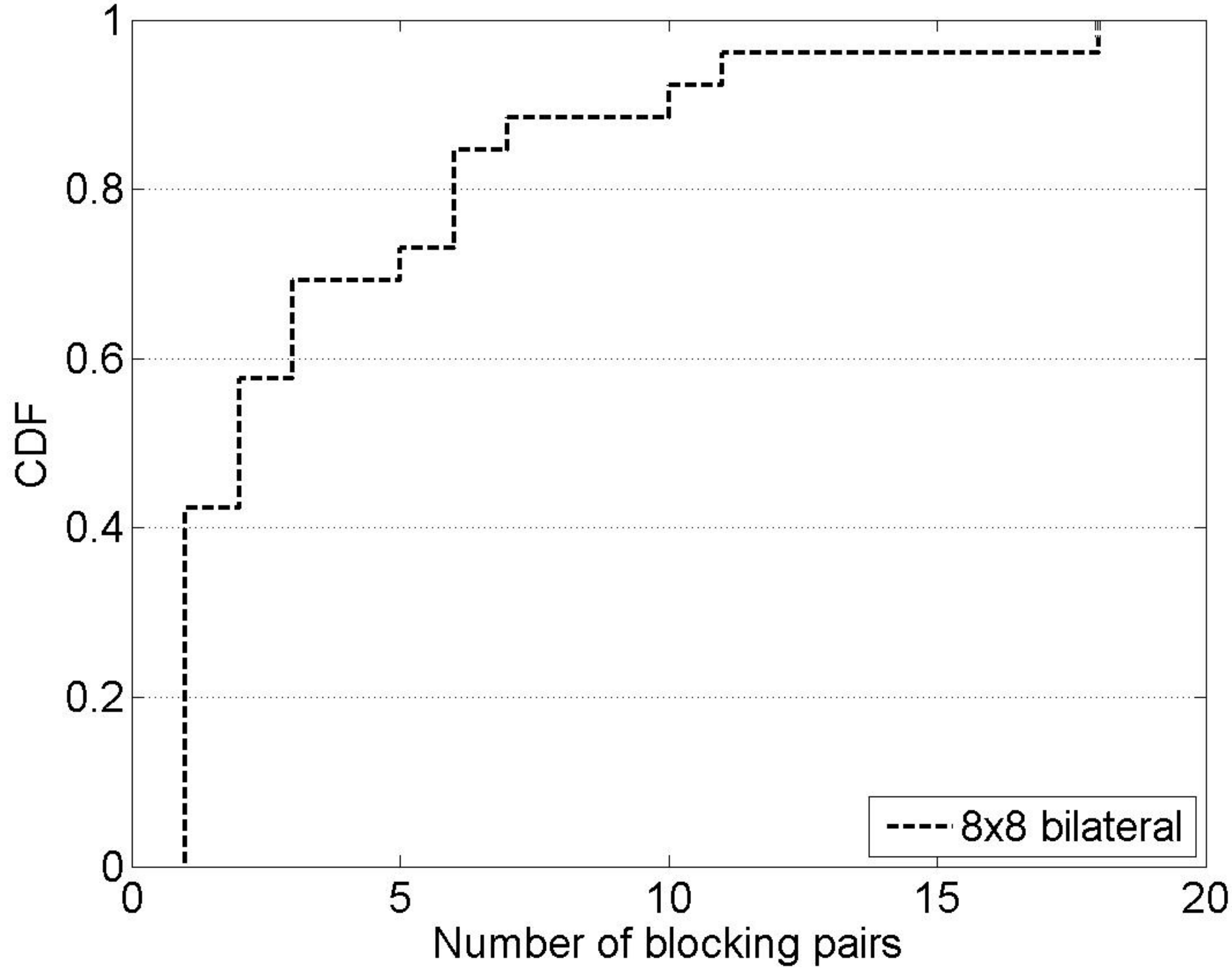
Results – Ultimate Matchings

- **76%** of market outcomes are stable:
  - **Unique stable match: 90%** stable
  - **Two stable matches: 89%** stable
  - **Three stable matches: 47%** stable

- More "complex" markets yield less fully stable outcomes, but they are "close" to stable (payoffs, blocking pairs)

Results – Ultimate Matchings

The Cumulative Distribution of Blocking Pairs for Unstable Markets:



Selection of Stable Matchings

Treatments with 3 stable partners for each subject:

Cardinal treatment	Type of match			
	Median	Optimal for color	Optimal for food	Total by cardinal treatment
8x8 multiple 70-70	80	11	8	99
8x8 multiple 70-20	32	8		40
8x8 multiple 20-70	20		15	35
8x8 multiple 20-20 cshift100	24	13		37
8x8 multiple 20-20	25	9	4	38
Total by type of match	181	41	27	249

Outcomes – Summary

- Frequent convergence to the stable match
- Median stable match has strong drawing power
- Cardinal presentation of ordinal markets matters

The End

ADDITIONAL SLIDES FOR EXPERIMENTAL NETWORK GAMES

Theoretical Predictions – Complete Information

Proposition 1. *Consider the scenario of strategic substitutes and complete information with network* g_O.

a) *There are three pure-strategy Nash equilibria*: (1,0,1,0,1), (1,0,0,1,0), and (0,1,0,0,1).

b) *The following strategy profiles, where agents use mixed strategies, are Nash equilibria:* $(m_A, 0.5, 1-\frac{0.5}{1-m_A}, 0, 1)$ *with* $m_A \in (0, 0.5]$, $(1, 0, 1-\frac{0.5}{1-m_E}, 0.5, m_E)$ *with* $m_E \in (0, 0.5]$, (0,0.5,0.5,0,1), (1,0,0.5,0.5,1), and (0,0.5,0,0.5,0).

c) *There are no other Nash equilibria.*

Theoretical Predictions – Complete Information

Proposition 2. *Consider the scenario of strategic substitutes and complete information with network g_G.*

a) *There are four pure-strategy Nash equilibria*: (1,0,1,0,1), (0,1,0,1,0), (1,0,0,1,0), and (0,1,0,0,1).

b) *The following strategy profiles, where agents use mixed strategies, are Nash equilibria*: $(m_A, 0.5, 1-\frac{0.5}{1-m_A}, 0, 1)$ *with* $m_A \in (0, 0.5)$, $(1, 0, 1-\frac{0.5}{1-m_E}, 0.5, m_E)$ *with* $m_E \in (0, 0.5]$, (0.5,0.5,0,0.5,0.5), (0.5,0.5,0,1,0), (0.5,0.5,0,0,1), (0,1,0,0.5,0.5), (1,0,0,0.5,0.5), (0,0.5,0.5,0,1), and (1,0,0.5,0.5,0).

c) *There are no other Nash equilibria*.

Theoretical Predictions – Complete Information

Proposition 3. *Consider the scenario of strategic substitutes and complete information with network g_P.*

a) *There are three pure-strategy Nash equilibria*: (1,0,1,0,1), (1,0,1,1,0), and (0,1,0,0,1).

b) *The following strategy profiles, where agents use mixed strategies, are Nash equilibria*: $(m_A, 0.5, 1-\frac{0.5}{1-m_A}, 0, 1)$ *with* $m_A \in (0, 0.5)$, $(1, 0, 1-\frac{0.5}{1-m_E}, 0.5, m_E)$ *with* $m_E \in (0, 0.5]$, (0,0.5,0,0.5,0), *and* (1,0,1,0.5,0.5).

c) *There are no other Nash equilibria.*

Theoretical Predictions – Complete Information

Proposition 4. *Consider the scenario of strategic complements and complete information with network* g_O.

a) *There are two pure-strategy Nash equilibria*: (0,0,0,0,0), and (0,1,1,1,0).

b) *The following strategy profiles, where agents use mixed strategies, are Nash equilibria*: (0,1,0.5,0.5,0), (0,0.5,1,0.5,0), (0,0.5,0.5,1.0), *and* (0,0.75,0.75,0.75,0).

c) *There are no other Nash equilibria*.

Theoretical Predictions – Complete Information

Proposition 5. *Consider the scenario of strategic complements and complete information. If the network is either* g_G *or* g_P*, there is a unique Nash equilibrium*: (0, 0, 0, 0, 0).

Theoretical Predictions – Incomplete Information

Proposition 6. *Let $p \in (0,1)$. In the scenario of strategic substitutes and incomplete information there exists an equilibrium* $(1, s_2^*(p), 0)$, *where*

$$s_2^*(p) = \begin{cases} 0 & if\ p \leq \frac{1}{2} \\ 1 - \sqrt{\frac{2-3p}{1-p}} & if\ \frac{1}{2} < p < \frac{2}{3} \\ 1 & if\ p \geq \frac{2}{3} \end{cases}$$

Moreover, there are no other pure-strategy equilibria and, if $p \geq 0.2$, the equilibrium $(1, s_2^*(p), 0)$ *is unique.*

[[For p=0.2, additional mixed equilibria can be found]]

Theoretical Predictions – Incomplete Information

Proposition 7. *Let* $p' = (\sqrt{105} + 13)/32$, *and consider the scenario of strategic complements and incomplete information.*[12] *If* $p < 1/2$, *there is a unique equilibrium:* (0, 0, 0). *If* $p \geq 1/2$, *there are three equilibria:* (0, 0, 0), (0, 1, 1) and $(0, s_2^{;}(p), s_3^{;}(p))$, where

$$s_2^{;}(p) = \begin{cases} \dfrac{5-6p}{4(1-p)} & \textit{if } p < p' \\[2ex] \dfrac{3-30p+51p^2}{2(5p-1)^2} & \textit{otherwise} \end{cases} \quad \textit{and} \quad s_3^{;}(p) = \begin{cases} 1 & \textit{if } p < p' \\[2ex] \dfrac{3p+9p^2}{(5p-1)^2} & \textit{otherwise} \end{cases}$$

ADDITIONAL SLIDES FOR THE 1/D LAW OF GIVING

Homophily: Explaining Link Formation

- Examine theory of **homophily** from sociology: do subjects link to those with similar characteristics?
  - Calculate share of students in a grade of the same race: Expected
  - Calculate share of first degree friends of the same race: Actual
  - Comparing expected to actual: evidence of observed homophily

Homophily: Explaining Link Formation

- For example: 53% of first degree friends are of the same race while 41% of all other friends are of the same race

- Homophilic preferences by attributes:

Race	60%
Confidence	53%
Popularity	53%
Height	55%

Network Environment: Explaining Link Formation

- Links may depend not only on characteristics of the pair but also the nature of the network structure

- Barabasi and Albert (1999) and Jackson and Rogers (2006) suggest network formation dynamics that produces cliques (linking to friends of friends)

- Are links to individuals who are already socially close more likely?

Estimation Strategy

- We model the probability of a link as a function of the hypothetical distance between i and j without that link
- Include separate dummies for each hypothetical distance
- **In the data we see that 75% of links are to those who would have distance 2 without that link**
- Explain link using Recipient Attributes, Interactions, and controls for Hypothetical Distance

skip regression

Variable Link	Coefficient (Standard Error)	
	Model 1	**Model 2**
samerace	0.618***	0.475***
	(0.090)	(0.090)
sameheight	0.285***	0.275***
	(0.067)	(0.073)
samepopular	0.267***	0.268***
	(0.070)	(0.085)
sameconf	0.174**	0.148*
	(0.070)	(0.077)
sameboyfriend	0.836**	1.171**
	(0.316)	(0.271)
shy_recipient	0.032	0.026
	(0.033)	(0.037)
popular_recipient	0.276***	0.189***
	(0.015)	(0.019)
d 2		2.601***
		(0.135)
d 3		0.403**
		(0.179)
d 4		0.229
		(0.172)
reverse triad		1.731***
		(0.094)
Log Likelihood	-5239.3764	-3774.7329

Robust standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%

Results on Preferential Attachment

- Inclusion of network structure variables results in a dramatically improved fit: increase in likelihood of ~30%

- Tendency to link to those who are already "socially" close in the network decays quickly

Earnings and Characteristics

- Personal characteristics may affect earnings indirectly through network structure
- Evidence presented may be muted by randomness in the experimental design

Simulation of Earnings

- We simulate interaction over the entire network and generate a weighted sum

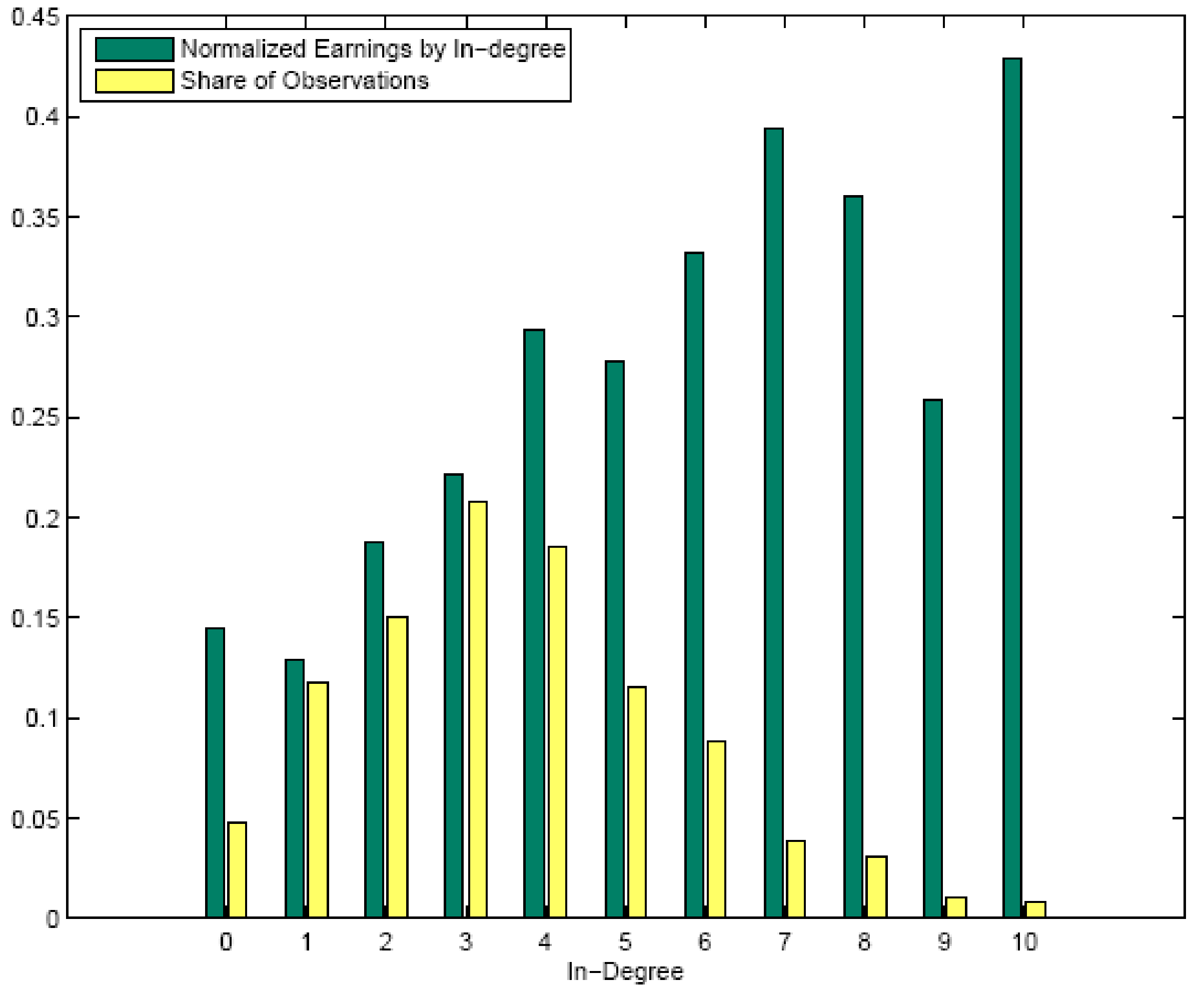
- Normalized earnings are the value an individual expected to receive from pairwise interaction with all other members of the network

- Weight represents the ratio of time spent on average with direct friends to the total time spent with all friends

Results: Popularity Pays

- Earnings and in-degree closely related
- Each time named as a friend, normalized earnings increase by 2.6%
- Share of pie 10% for someone with no friends
- Most popular person predicted to receive 36%

Simulated Earnings

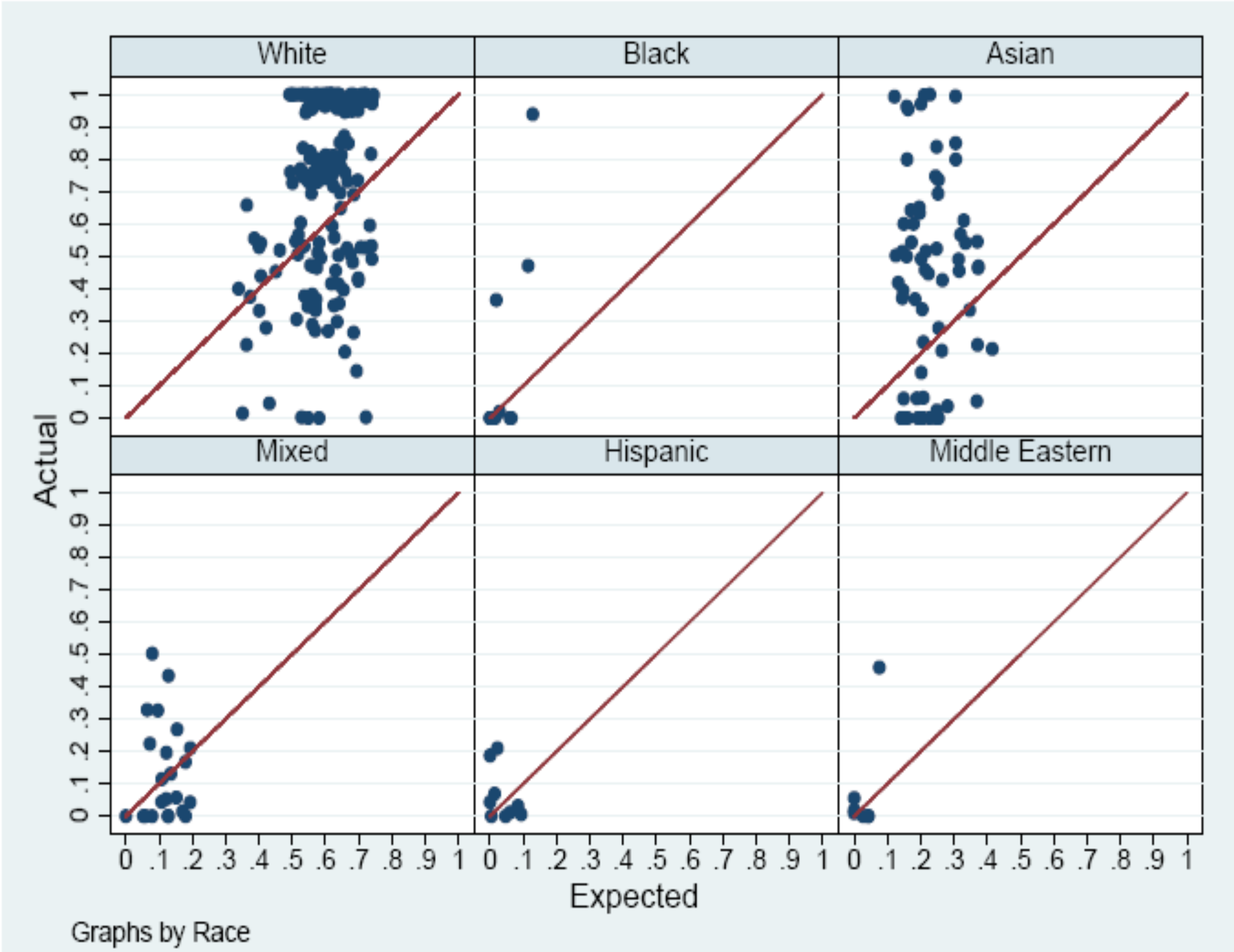


Explaining Offer Amounts by Network Structure

- Network structure measures not statistically significant and represent very weak effect

- Results large compared to other studies:
  - Social network of 10-12 year olds are concentrated at their school
  - Ingroup/outgroup effects may be much more pronounced among adolescents

Homophily: Data



Explain Link Formation

- Compare value of link to other hypothetical links the subject could have made

- Reduced form model based on a notion of stability: if there is a more valuable link, expect a shift in social resources meaning that the network would not be stable

Estimation Strategy: Explaining Link Formation

- Conditional Logit discrete choice model to explain choice of links
- Links defined based on a directed network, more consistent with use of a discrete choice model

Homophily – Regression Results

- Explain link using Recipient Attributes and Interactions
- **Results:** Shared characteristics are strong predictors of a network link
- Outstanding question: due to preferences over types of links or differences in the cost of maintaining different kinds of links over time?

Variable Link	Coefficient (Standard Error)
	Model 1
samerace	0.618***
	(0.090)
sameheight	0.285***
	(0.067)
samepopular	0.267***
	(0.070)
sameconf	0.174**
	(0.070)
sameboyfriend	0.836**
	(0.316)
shy_recipient	0.032
	(0.033)
popular_recipient	0.276***
	(0.015)
Log Likelihood	-5239.3764

Robust standard errors in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%