binomial-gamma-hurdle

March 20, 2019

0.1 Binomial-Gamma Hurdle Models accounting for zero-inflation and imperfect detection

0.1.1 Model description

We consider here two common problems in ecological data: zero-inflation and imperfect detection. Approach is presented using simulated data, that resembles real data such as e.g. species sightings per minute of effort.

Dealing with zero-inflation: Dependent variable, y is semi-continuous (i.e. a point mass in a single value and a continuous distribution elsewhere). The data generating process for this type of data can be modelled using a gamma distribution. The main problem is however that response variable has a high proportion of zeros (>90%), which is more than expected from a gamma distribution, hence it cannot be readily applied.

There are two common methods for dealing with zero-inflated data:

(1) Modelling a zero-inflation parameter that represents the probability a given 0 comes from the main distribution (say the negative binomial distribution) or is an excess 0;
(2) Modelling the zero and non-zero data with one model and then modelling the non-zero data with another. This is often called a hurdle model.

In (1), the response variable is modelled as a mixture of a Bernoulli distribution (a point mass at zero) and a Poisson distribution (or any other count distribution supported on non-negative integers). In (2), the basic idea is that a Bernoulli probability governs the binary outcome of whether a variable has a zero or positive realization. If the realization is positive, the hurdle is crossed, and the conditional distribution of the positives is governed by a truncated-at-zero model. Hurdle models model the zeros and non-zeros as two separate processes and can be useful in that they allow you to model the zeros and non-zeros with different predictors or different roles of the same predictors.

Zero-inflation models may be more elegant and informative if the same predictors are thought to contribute to the extra and real zeros. Hurdle models can be useful in that they allow you to model the zeros and non-zeros with different predictors or different roles of the same predictors. Maybe one process leads to the zero/non-zero data and another leads to the non-zero magnitude.

Here we shall focus on (2) and model the zeros separately from the non-zeros in a binomial-Gamma hurdle model.

Dealing with imprefect detection To isolate the effect of variables that might influence detectability and to partition out the annual signals, a mixed-effect modelling framework will be separately applied to each part of the hurdle model. A varying-intercept model will be fit with the explanatory variables that are expected to influence detectability (say x1). For the Bernoulli part of the hurdle model the response is a binary variable (presence/absence). For the Gamma part the response is the number of observations. The factor variable 'year' is included as a random effect. This approach is particularly useful when variation among years is of interest. Random effects are conditional modes calculated as the difference between the average predicted response for a given set of fixed-effect values (in this case x1 variable that may influence detectability) and the response (presence/absence or abundance) predicted for particular year. These conditional modes were then extracted for each part of the hurdle and were then included as the response variable in a series of general linear models that modelled the effect of x2 variable on y.

0.1.2 Load libraries

```
In [48]: library(lme4)
         library(effects)
         library(optimx)
         # require(devtools)
         # install_version("effects", version = "4.0-0")
```

```
In [2]: set.seed(4322)
```

0.1.3 Read in data

```
In [49]: dat <- read.csv(file = 'data.csv', row.names=1)
```

Variable y is a response variable, variables x1 and x2 are explanatory variables. Variable x1 is expected to influence detectibility of y, variable x2 is expected to relate to y.

```
In [50]: head(dat)
```

y	x1	x2	year
0.0000000	30	10.40875	1971
0.1184611	30	10.40875	1971
0.0000000	90	10.40875	1971
0.0000000	210	10.40875	1971
0.0000000	300	10.40875	1971
0.0000000	150	10.40875	1971

0.1.4 Scale data

```
In [51]: cols = c("x1", "x2")
         dat[, paste0(cols, "_", "sc")] <- scale(dat[ ,cols])
         summary(dat)
```

```
       y                    x1              x2                 year
 Min.   :0.000000   Min.   :  5   Min.   : 9.207    Min.   :1971
 1st Qu.:0.000000   1st Qu.: 30   1st Qu.:13.169    1st Qu.:1984
```

```
 Median :0.000000   Median : 60   Median :15.261   Median :1998
 Mean   :0.002485   Mean   :128   Mean   :14.621   Mean   :1996
 3rd Qu.:0.000000   3rd Qu.:180   3rd Qu.:16.288   3rd Qu.:2009
 Max.   :0.133199   Max.   :960   Max.   :18.168   Max.   :2017
                    NA's   :363   NA's   :61
     x1_sc              x2_sc
 Min.   :-0.8945   Min.   :-2.5651
 1st Qu.:-0.7126   1st Qu.:-0.6882
 Median :-0.4944   Median : 0.3032
 Mean   : 0.0000   Mean   : 0.0000
 3rd Qu.: 0.3786   3rd Qu.: 0.7894
 Max.   : 6.0531   Max.   : 1.6802
 NA's   :363       NA's   :61
```

0.1.5 Binomial model

We fit full model.

```
In [6]: summary(glm(ifelse(dat$y>0,1,0) ~
                    x1_sc +
                    x2_sc +
                    year,
                    data = dat,
                family = binomial(link = logit)))
```

```
Call:
glm(formula = ifelse(dat$y > 0, 1, 0) ~ x1_sc + x2_sc + year,
    family = binomial(link = logit), data = dat)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.3114  -0.2885  -0.2789  -0.2601   2.8235

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.152262  12.235934  -0.012   0.9901
x1_sc       -0.174758   0.102027  -1.713   0.0867 .
x2_sc       -0.052618   0.080658  -0.652   0.5142
year        -0.001567   0.006135  -0.255   0.7984
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1455.7  on 4609  degrees of freedom
Residual deviance: 1450.7  on 4606  degrees of freedom
```

```
  (365 observations deleted due to missingness)
AIC: 1458.7

Number of Fisher Scoring iterations: 6
```

We see that observer related variable (x1) is significant. We try to isolate its effect for each year. To do so we apply a mixed-effect modeling framework and fit a varying intercept model. This approach is useful when we are interested explicitly in variation among and by groups. Group level variables are specified using a special syntax: (1|year) to fit a linear model with a varying-intercept group effect using the variable year.

We include 'year' as random effect with noise variables.

```
In [52]: m.bin.full.re <- glmer(ifelse(dat$y>0,1,0) ~
                          x1_sc +
                          (1|year) ,
                        data = dat,
         #                control = glmerControl(optimizer ='optimx', optCtrl=list(method='nlminb'
                        family = binomial(link = logit))
```

```
In [53]: summary(m.bin.full.re)
```

```
Generalized linear mixed model fit by maximum likelihood (Laplace
  Approximation) [glmerMod]
 Family: binomial  ( logit )
Formula: ifelse(dat$y > 0, 1, 0) ~ x1_sc + (1 | year)
   Data: dat

     AIC      BIC   logLik deviance df.resid
  1457.4   1476.7   -725.7   1451.4     4609

Scaled residuals:
    Min      1Q  Median      3Q     Max
-0.2114 -0.2066 -0.2027 -0.1871  7.2335

Random effects:
 Groups Name        Variance Std.Dev.
 year   (Intercept) 0.002407 0.04907
Number of obs: 4612, groups:  year, 44

Fixed effects:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -3.27872    0.08461 -38.751   <2e-16 ***
x1_sc       -0.18641    0.09471  -1.968    0.049 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Correlation of Fixed Effects:
```

```
      (Intr)
x1_sc 0.147
```

Random effects are conditional modes - the difference between the average predicted response for a given set of fixed-effect values (observer related variables) and the response predicted for particular year.

In [54]:
```
ranef.bin.dat<-as.data.frame(ranef(m.bin.full.re))[c(3,4)]
colnames(ranef.bin.dat)[2] <- "reyear"
```

In [55]:
```
head(ranef.bin.dat)
```

grp	reyear
1971	0.0137390698
1972	0.0035901339
1973	-0.0042083611
1974	-0.0048717533
1975	0.0022349064
1976	0.0009954729

In [56]:
```
# merge with dat by column year
dat.bin.re<-merge(dat,ranef.bin.dat, by.x = "year", by.y = "grp")
```

In [57]:
```
head(dat.bin.re)
```

year	y	x1	x2	x1_sc	x2_sc	reyear
1971	0.0000000	30	10.40875	-0.7126393	-1.995932	0.01373907
1971	0.1184611	30	10.40875	-0.7126393	-1.995932	0.01373907
1971	0.0000000	90	10.40875	-0.2761402	-1.995932	0.01373907
1971	0.0000000	210	10.40875	0.5968579	-1.995932	0.01373907
1971	0.0000000	300	10.40875	1.2516065	-1.995932	0.01373907
1971	0.0000000	150	10.40875	0.1603588	-1.995932	0.01373907

Fit variable x2 against ranef.year.

In [58]:
```
m.bin.full <- glm(reyear ~
                  x2_sc,
              data = dat.bin.re,
              family = gaussian)
```

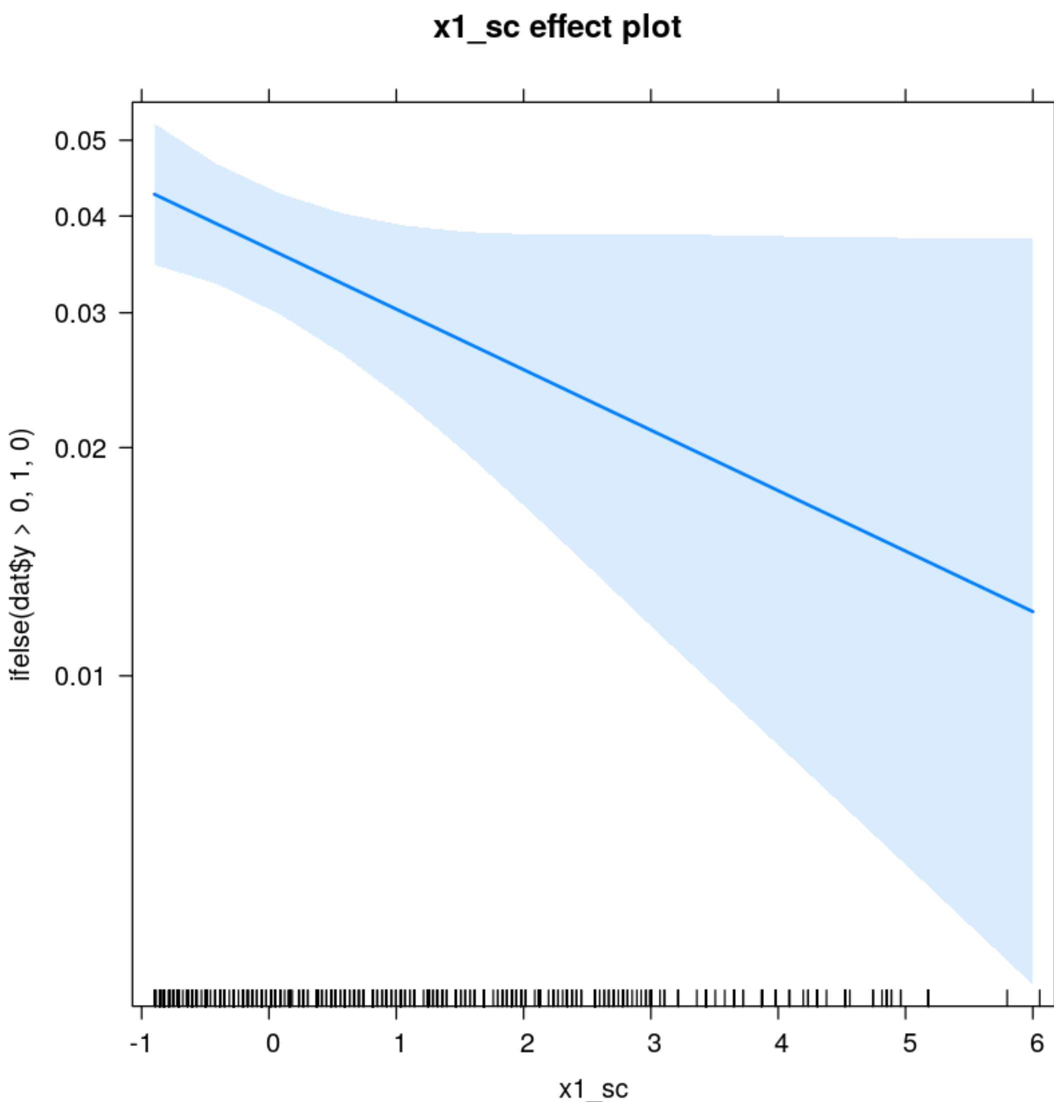
In [59]:
```
summary(m.bin.full)
```

```
Call:
glm(formula = reyear ~ x2_sc, family = gaussian, data = dat.bin.re)

Deviance Residuals:
      Min         1Q     Median         3Q        Max
-0.008938  -0.003452  -0.001559   0.003270   0.013473
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.589e-04  7.320e-05   4.903 9.73e-07 ***
x2_sc       -1.235e-04  7.321e-05  -1.687   0.0916 .
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

(Dispersion parameter for gaussian family taken to be 2.633023e-05)

    Null deviance: 0.12941  on 4913  degrees of freedom
Residual deviance: 0.12933  on 4912  degrees of freedom
  (61 observations deleted due to missingness)
AIC: -37868

Number of Fisher Scoring iterations: 2
```

The model predicted random effect for each year, assuming all other model variables (if there are any) remain constant.

```
In [60]: plot(allEffects(m.bin.full.re))
```



0.1.6 Gamma model

```
In [61]: # drop rows with nas
         no.na.dat <- na.omit(dat)
```

We fit Gamma model to the positive part of the model with log link using a similar set of predictors.

```
In [62]: summary(glm(y ~
                     year +
                     x1_sc +
                     x2_sc,
                     data = subset(no.na.dat, y>0),
                     family = Gamma(link = log)))
```

```
Call:
glm(formula = y ~ year + x1_sc + x2_sc, family = Gamma(link = log),
    data = subset(no.na.dat, y > 0))

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-2.29973  -0.58475   0.05655   0.41737   0.89432

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -10.032208   6.618597  -1.516    0.131
year          0.003676   0.003318   1.108    0.269
x1_sc        -0.044201   0.060329  -0.733    0.465
x2_sc         0.014744   0.047531   0.310    0.757

(Dispersion parameter for Gamma family taken to be 0.3448597)

    Null deviance: 104.02  on 169  degrees of freedom
Residual deviance: 103.22  on 166  degrees of freedom
AIC: -593.9

Number of Fisher Scoring iterations: 5
```

We separate observer related effect for each year.

```
In [64]: m.gamma.full.re <- glmer(y ~
                          x1_sc +
                          (1|year),
                         data = subset(no.na.dat, y>0),
                         control = glmerControl(optimizer ='optimx', optCtrl=list(method='nlminb')
                       family = Gamma(link = log))
```

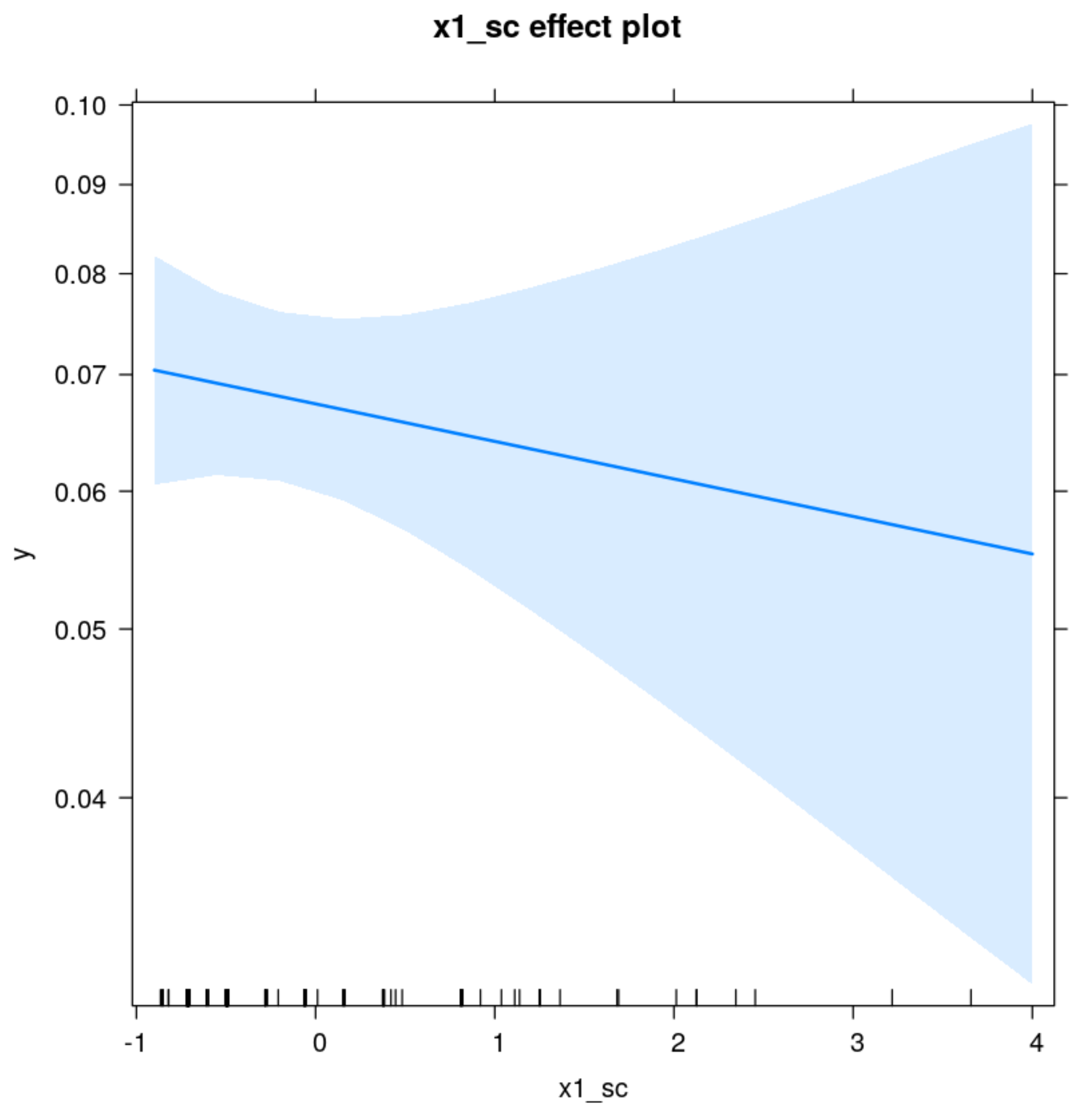
```
singular fit
```

```
In [65]: summary(m.gamma.full.re)
```

```
Generalized linear mixed model fit by maximum likelihood (Laplace
  Approximation) [glmerMod]
 Family: Gamma  ( log )
Formula: y ~ x1_sc + (1 | year)
   Data: subset(no.na.dat, y > 0)
Control: glmerControl(optimizer = "optimx", optCtrl = list(method = "nlminb"))

     AIC      BIC   logLik deviance df.resid
  -593.0   -580.4    300.5   -601.0      166
```

```
Scaled residuals:
    Min      1Q  Median      3Q     Max
-1.6776 -0.8645  0.1301  0.8736  1.7613

Random effects:
 Groups   Name        Variance Std.Dev.
 year     (Intercept) 0.0000   0.0000
 Residual             0.3362   0.5798
Number of obs: 170, groups:  year, 39

Fixed effects:
            Estimate Std. Error t value Pr(>|z|)
(Intercept) -2.69798    0.05806 -46.467   <2e-16 ***
x1_sc       -0.04962    0.06810  -0.729    0.466
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Correlation of Fixed Effects:
      (Intr)
x1_sc 0.175
convergence code: 0
singular fit
```

```
In [66]: plot(allEffects(m.gamma.full.re))
```




```
In [67]: # extract random effect
         ranef.gamma.dat<-as.data.frame(ranef(m.gamma.full.re))[c(3,4)]

In [68]: colnames(ranef.gamma.dat)[2] <- "reyear"

In [69]: dat.gamma.re <- merge(subset(no.na.dat, y>0),ranef.gamma.dat, by.x = "year", by.y = "g

In [70]: # fit the rest of the variables against ranef.year
         m.gamma.full <- glm(reyear ~
                     x2_sc,
                    data = dat.gamma.re,
                    family = gaussian)

In [71]: summary(m.gamma.full)
```

```
Call:
glm(formula = reyear ~ x2_sc, family = gaussian, data = dat.gamma.re)

Deviance Residuals:
   Min      1Q  Median      3Q     Max
     0       0       0       0       0

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)        0          0      NA       NA
x2_sc              0          0      NA       NA

(Dispersion parameter for gaussian family taken to be 0)

    Null deviance: 0  on 169  degrees of freedom
Residual deviance: 0  on 168  degrees of freedom
AIC: -Inf

Number of Fisher Scoring iterations: 1
```

0.1.7 References

- https://www.ssc.wisc.edu/sscc/pubs/MM/MM_DiagInfer.html
- http://rstudio-pubs-static.s3.amazonaws.com/5691_192685385fc445c9b3fb1619960a20e2.html
- http://pj.freefaculty.org/guides/stat/Regression-GLM/Gamma/GammaGLM-01.pdf
- https://stats.stackexchange.com/questions/81457/what-is-the-difference-between-zero-inflated-and-hurdle-distributions-models
- http://seananderson.ca/2014/05/18/gamma-hurdle.html
- https://ms.mcmaster.ca/~bolker/R/misc/modelDiag.html
- https://stats.idre.ucla.edu/r/dae/logit-regression/
- https://stats.idre.ucla.edu/other/mult-pkg/faq/general/faq-how-do-i-interpret-odds-ratios-in-logistic-regression/
- http://environmentalcomputing.net/interpreting-coefficients-in-glms/
- https://www.sciencedirect.com/science/article/pii/S0167947308000169
- https://www.cambridge.org/core/services/aop-cambridge-core/content/view/S0021859611000608
- https://link.springer.com/article/10.1007/s10742-017-0169-9
- https://stats.stackexchange.com/questions/96972/how-to-interpret-parameters-in-glm-with-family-gamma/126225
- https://stats.stackexchange.com/questions/161216/backtransform-coefficients-of-a-gamma-log-glmm