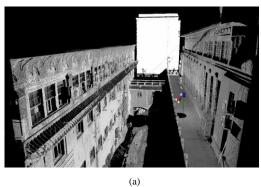
## **Supplementary Tests**

## Test 1

Test 1 is a demonstration of the presented method applied to registration of two point clouds acquired using a Zoller & Fröhlich IMAGER 5006i laser scanner in an outdoor environment (Fig. 1).



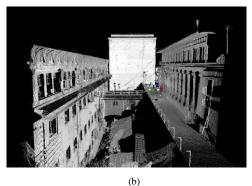
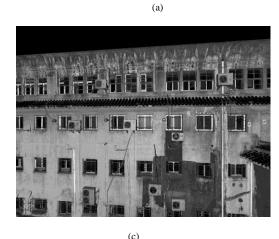


Fig. 1. Test point clouds: (a) Point cloud 1 with 29,289,429 points (b) Point cloud 2 with 29,274,388 points.

Four of the generated intensity images are shown in Fig. 2. Fig. 2(a) and Fig. 2(b) were generated from point cloud 1. Fig. 2(c) and Fig. 2(d) were generated from point cloud 2.







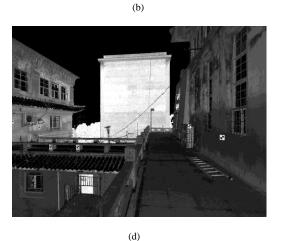


Fig. 2. Perspective intensity images generated: (a) and (b) Perspective intensity images generated from point cloud 1. (c) and (d) Perspective intensity images generated from point cloud 2.

The corner points extracted from Fig. 2(a)—Fig. 2(d) are marked by crosses shown in Fig. 3(a)—Fig. 3(d) respectively.

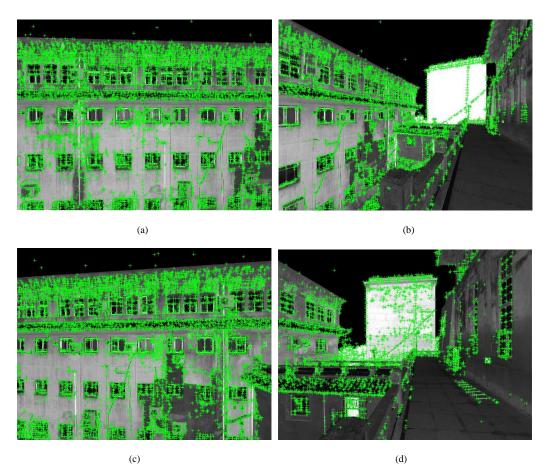


Fig. 3. Corner points extracted from the perspective intensity images in Fig. 2.

Six pairs of corner points are interactively selected from four intensity images as tie points. The 3D coordinates of the six pairs of tie points are listed in Table I and Table II.

 $\label{eq:table I} \mbox{3D coordinates of six tie points from point cloud 1 (UNITS: meters)}$ 

Tie Point  Coordinate	1	2	3	4	5	6
X	11.898	10.279	9.941	8.628	5.312	12.615
Y	-0.284	2.289	1.757	4.909	-13.310	-10.480
Z	-0.929	-0.935	-6.006	-4.393	-0.612	-3.937

Tie Point  Coordinate	1	2	3	4	5	6
X	-9.223	-11.040	-10.424	-12.872	5.291	-0.034
Y	3.910	1.462	1.344	-1.033	2.600	8.332
Z	-0.944	-0.966	-6.026	-4.427	-0.640	-3.912

The registration accuracy using all of the six pairs of tie points is given in Table III. The values in each column show the residual vector of one pair of tie points. Residual vectors are computed by subtracting coordinates of tie points in point cloud 2 from the transformed coordinates of their correspondences in point cloud 1. Three digits after the decimal point are kept for convenience. The largest residuals along each coordinate axis are bold-marked. It is shown that all of the absolute values of the three largest residuals are around 2cm. The last column of Table III is the root mean square error (RMSE) along each coordinate axis. The RMSE values show that the accuracy of registration using all of the six pairs of tie points is at the level of centimeters.

The accuracy of registration using RANSAC is shown in Table IV. The threshold  $T_{Sh}$  for inlier determination is set to 8mm and the number of samples is set to 100. Four of six pairs of tie points are selected by program as inliers. And the largest residuals along each coordinate axis are reduced dramatically from (-0.023, 0.019, -0.020) to (-0.013, 0.011, -0.006). And the RMSE values also show that the registration accuracy using RANSAC is at the level of millimeters. The estimated translation vector is  $(-5.145, -7.256, -0.030)^T$  and the estimated rotation matrix is

 $\begin{pmatrix} -0.368 & -0.930 & -0.003 \\ 0.930 & -0.368 & -0.002 \\ 0.001 & -0.003 & 1.000 \end{pmatrix}$ 

TABLE III

ACCURACY OF REGISTRATION USING ALL OF SIX PAIRS OF TIE POINTS

Tie Point  Residual	1	2	3	4	5	6	RMSE
X	-0.023	0.002	0.005	0.008	0.002	0.003	0.010
Y	0.001	-0.005	0.018	0.005	-0.008	0.019	0.012
Z	0.003	0.007	-0.003	-0.001	0.007	-0.020	0.009

TABLE IV

ACCURACY OF REGISTRATION USING RANSAC

Tie Point Residual	2	3	4	6	RMSE
X	-0.013	0.003	0.000	0.003	0.007
Y	0.000	0.011	0.004	0.007	0.007
Z	0.004	-0.004	-0.003	-0.006	0.005

## Test 2

Test 2 is a demonstration of the presented method applied to registration of two point clouds acquired using a Zoller & Fröhlich IMAGER 5006i laser scanner in another outdoor environment (Fig. 4).

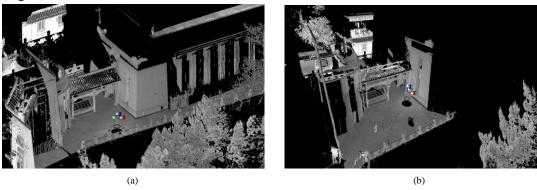
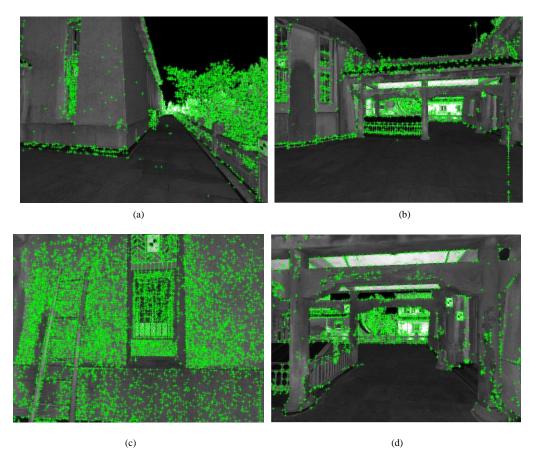


Fig. 4. Test point clouds: (a) Point cloud 3 with 23,871,941 points (b) Point cloud 4 with 28,139,727 points. Four of the generated intensity images are shown in Fig. 5. Fig. 5(a) and Fig. 5(b) were generated from point cloud 3. Fig. 5(c) and Fig. 5(d) were generated from point cloud 4.



Fig. 5. Perspective intensity images generated: (a) and (b) Perspective intensity images generated from point cloud 3. (c) and (d) Perspective intensity images generated from point cloud 4.

The corner points extracted from Fig. 5(a)—Fig. 5(d) are marked by crosses shown in Fig. 6(a)—Fig. 6(d) respectively.



 $Fig.\ 6.\quad Corner\ points\ extracted\ from\ the\ perspective\ intensity\ images\ in\ Fig.\ 5.$ 

Seven pairs of corner points are interactively selected from four intensity images as tie points. The 3D coordinates of the seven pairs of tie points are listed in Table V and Table VI.

 $\label{eq:table V} \mbox{3D coordinates of seven tie points from point cloud 3 (UNITS: meters)}$ 

Tie Point  Coordinate	1	2	3	4	5	6	7
X	4.784	4.787	4.851	5.031	5.669	5.288	5.574
Y	2.676	2.650	2.958	7.150	9.698	6.962	6.950
Z	1.474	1.033	1.402	0.882	-0.487	-1.278	-1.165

TABLE VI
3D COORDINATES OF SEVEN TIE POINTS FROM POINT CLOUD 4 (UNITS: METERS)

Tie Point  Coordinate	1	2	3	4	5	6	7
X	1.303	1.521	1.077	-2.728	-4.827	-2.443	-2.320
Y	1.730	1.971	1.897	3.691	5.265	3.864	4.125
Z	1.494	0.535	1.433	0.906	-0.475	-1.268	-1.166

The registration accuracy using all of the seven pairs of tie points is given in Table VII. It is shown that all of the three largest residuals exceed 1cm. The RMSE values show that the plane accuracy of registration using all of the seven pairs of tie points is at the level of centimeters and the height accuracy of registration using all of the seven pairs of tie points is at the level of decimeters.

The accuracy of registration using RANSAC is shown in Table VIII. The threshold Tsh for inlier determination is set to 1cm and the number of samples is set to 100. Four of six pairs of tie points are selected by program as inliers. And the largest residuals along each coordinate axis are reduced dramatically from (-0.127, -0.152, 0.366) to (0.004, -0.006, 0.013). And the RMSE values also show that the registration accuracy using RANSAC is at the level of millimeters. The estimated translation vector is  $(1.940, -3.711, -0.016)^T$  and the estimated rotation matrix is

0.385 -0.923 -0.005 0.923 -0.385 -0.007 0.008 -0.002 1.000

 $\label{thm:constraint} \mbox{Table VII}$  Accuracy of registration using all of the seven pairs of tie points

Tie Point  Residual	1	2	3	4	5	6	7	RMSE
X	0.072	-0.127	0.059	0.0218	-0.036	-0.025	-0.031	0.064
Y	0.085	-0.152	0.085	0.006	-0.006	0.061	0.059	0.080
Z	-0.151	0.366	-0.155	-0.046	0.039	-0.030	-0.014	0.163

TABLE VIII

ACCURACY OF REGISTRATION USING RANSAC

Tie Point Residual	1	5	6	7	RMSE
X	0.003	0.004	0.003	0.000	0.003
Y	-0.004	-0.004	-0.004	-0.006	0.005
Z	-0.004	-0.007	-0.001	0.013	0.007

## Test 3

Test 3 is a demonstration of the presented method applied to registration of two point clouds acquired using a RIEGL VZ-1000 laser scanner in another indoor environment (Fig. 7).

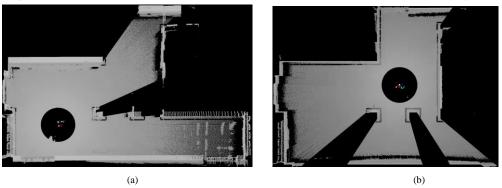


Fig. 7. Test point clouds: (a) Point cloud 5 with 15,161,360 points (b) Point cloud 6 with 15,011,150 points.

Four of the generated intensity images are shown in Fig. 8. Fig. 8(a) and Fig. 8(b) were generated from point cloud 5. Fig. 8(c) and Fig. 8(d) were generated from point cloud 6.

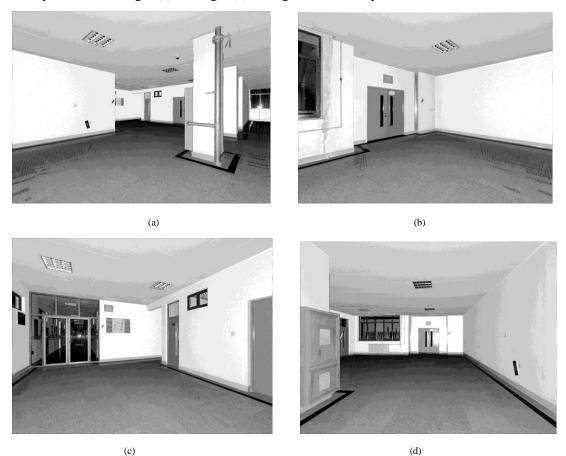


Fig. 8. Perspective intensity images generated: (a) and (b) Perspective intensity images generated from point cloud 5. (c) and (d) Perspective intensity images generated from point cloud 6.

The corner points extracted from Fig. 8(a)—Fig. 8(d) are marked by crosses shown in Fig. 9(a)—Fig. 9(d) respectively.

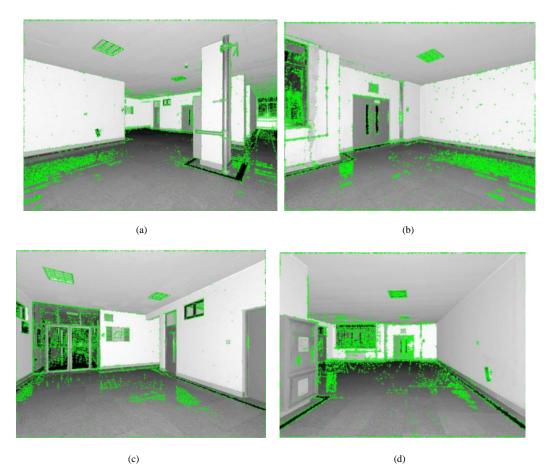


Fig. 9. Corner points extracted from the perspective intensity images in Fig. 5.

Six pairs of corner points are interactively selected from four intensity images as tie points. The 3D coordinates of the six pairs of tie points are listed in Table IX and Table X.

 $\label{table IX} \mbox{3D coordinates of seven tie points from point cloud 5 (UNITS: meters)}$ 

Tie Point  Coordinate	1	2	3	4	5	6
X	-7.853	-8.057	-2.881	-3.885	-5.257	-2.853
Y	10.066	9.190	10.011	-3.833	-3.182	-4.083
Z	0.888	1.522	0.812	1.350	0.363	0.818

 $\label{eq:Table X} Table \ X$  3D coordinates of seven tie points from point cloud 6 (UNITS: meters)

Tie Point  Coordinate	1	2	3	4	5	6
X	-5.556	-4.813	-3.682	8.839	7.737	9.457
Y	-3.957	-4.473	0.631	-5.368	-6.398	-4.491
Z	0.898	1.532	0.799	1.339	0.345	0.813

The registration accuracy using all of the six pairs of tie points is given in Table XI. It is shown that all of the three largest residuals exceed 1cm. The RMSE values show that the plane and height accuracy of registration using all of the seven pairs of tie points is at the level of millimeters.

The accuracy of registration using RANSAC is shown in Table XII. The threshold Tsh for inlier determination is set to 1cm and the number of samples is set to 100. Four of six pairs of tie points are selected by program as inliers. And the largest residuals along each coordinate axis decrease slightly. The RMSE values also show that the registration accuracy using RANSAC is at the level of millimeters, which is slightly better than that using all of the six pairs of tie points. The estimated translation vector is  $(6.696, -0.339, 0.008)^T$  and the estimated rotation matrix is

0.365 -0.931 -0.003 0.931 0.365 -0.005 0.006 -0.001 1.000

 $\label{eq:table XI} \mbox{Accuracy of registration using all of six pairs of tie points}$ 

Tie Point  Residual	1	2	3	4	5	6	RMSE		
X	0.010	0.009	0.007	0.003	-0.005	-0.006	0.007		
Y	-0.007	0.001	0.011	0.009	-0.001	0.000	0.007		
Z	-0.004	-0.004	0.004	-0.001	0.010	-0.010	0.006		

TABLE XII

ACCURACY OF REGISTRATION USING RANSAC

Tie Point Residual	6	3	5	4	RMSE
X	-0.004	0.004	0.000	0.003	0.003
Y	0.000	0.000	0.001	0.005	0.002
Z	0.000	0.000	-0.002	0.000	0.001