Tutorial 2

1.Find the radius of convergence of series

(i):$1+\frac{z}{1!}+\frac{z^2}{2!}+\frac{z^3}{3!}+\cdots=\sum_{n=0}^{n=\infty}\frac{z^n}{n!}$

$z_n=\frac{z^n}{n!}$

$z_{n+1}=\frac{z^{n+1}}{(n+1)!}$

$L=\lim_{n\to\infty}\frac{z_{n+1}}{z_n}=lim_{n\to\infty}\frac{z}{n}=0$

so $\rho=\infty$

(2):$1+2^2z+3^2z^2+4^2z^3=\sum_{n=1}^{n=\infty}n^2z^{n-1}$

$z_n=n^2z^{n-1}$

$z_{n+1}=(n+1)^2z^n$

$L=|\lim_{n\to-\infty}\frac{z_{n+1}}{z_n}|=\lim_{n\to-\infty}|z(\frac{n+1}{n})^2|=|z^2|<1$

so $\rho=1$

(3):$1-\frac{z}{2}+\frac{z^2}{3}-\frac{z^3}{4}+\cdots=\sum_{n=0}^{n=\infty}(-1)^n\frac{z^n}{n+1}$

$z_n=(-1)^n\frac{z^n}{n+1}$

$z_{n+1}=(-1)^{n+1}\frac{z^{n+1}}{n+2}$

$L=|\lim_{n\to-\infty}\frac{z_{n+1}}{z_n}|=\lim_{n\to\infty}|(-1)\frac{n+2}{n+1}z|=|z|<1$

$\rho=1$

(4):$1+1!z+2!z^2+3!z^3+\cdots=\sum_{n=1}^{n=\infty}n!(z^n)$

$z_n=n!(z^n)$

$z_{n+1}=(n+1)!(z^{n+1})$

$L=|\lim_{n\to-\infty}\frac{z_{n+1}}{z_n}|=\lim_{n\to\infty}|nz|=\infty$

$\rho=0$

2.

(1):

$z_n=\sum_{n=1}^{n=\infty}=\frac{(z+2)^{n-1}}{(n+1)^3 4^n}$

$z_{n+1}=\sum_{n=1}^{n=\infty}=\frac{(z+2)^n}{(n+2)^3 4^{n+1}}$

$L=\lim_{n\to\infty}|\frac{1}{4}(z+2)(\frac{n+1}{n+2})^3|=|\frac{1}{4}(z+2)|<1$

so $\rho = 4$ the region will be $|z+2| < 4$

(2):

$z_n = \sum_{n=1}^{n=\infty} \frac{(-1)^{n-1} z^{2n-1}}{(2n-1)!}$

$z_{n+1} = \sum_{n=1}^{n=\infty} \frac{(-1)^{n} z^{2n+3}}{(2n+1)!}$

$L = \lim_{n\to\infty} |\frac{z_{n+1}}{z_n}| = \lim_{n\to\infty} |\frac{z^4}{(2n+1)(2n)} = 0|$

So $\rho = \infty$ the region will be $|z| < \infty$

(5):

Geometric series

$$\frac{1}{1-z} = \{ {}^{\sum_{n=0}^{\infty} z^n, |z|<1}_{-\sum_{n=1}^{\infty} \frac{1}{z^n}, |z|>1}$$

Back to the question

(1)$\frac{1}{1+z} = \frac{1}{1+2i+z-2i} = \frac{1}{(1+2i)(1-\frac{2i-z}{1+2i})} = \frac{1}{1+2i} * \frac{1}{1-\frac{2i-z}{1+2i})}$

=$\frac{1}{1+2i} \sum_{n=0}^{n=\infty} (-1)^n (\frac{z-2i}{1+2i})^n$

(2):