

## Question

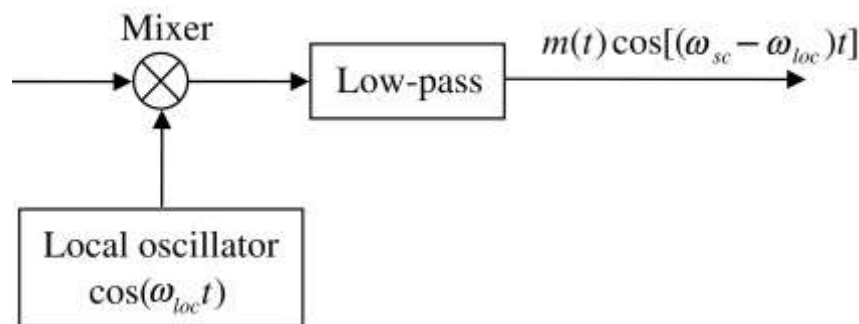
1. [why the mean value of the carrier and signal noise will be equal when in-phase and out-of-phase happens](#)
2. [why the final output signal in DSBSC is  \$\frac{1}{2} f\(t\)\$ , shouldn't it be  \$\frac{1}{2} S\_i\(t\)\$ ?](#)

## DSBSC(Double sideband suppress carrier)

- More efficient
- Higher cost
- More complex

### Coherent detection

It will be used to detect DSBSC.



## DSBLC(Double sideband large carrier)

- less efficient
- lower cost

## Noise in AM systems

Mean noise power in demodulator where a noise **voltage** is input into our demodulator

$$N_i = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{n_i^2}{R} dt$$

If the noise is in the form of a **current**, then the mean noise will be

$$N_i = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} n_i(t)^2 R dt$$

Usually we will assume that the noise power will flow into  $R = 1\Omega$ . But it is noticeable that in most cases  $R$  is matched to  $50\Omega$ .

## random noise

$$n_i(t) = n_c \cos(w_c t) - n_s(t) \sin(w_c t)$$

where the noise is defined in terms on **in-phase** and **out-of-phase**.

**in-phase:** If two waves coincide with peaks and troughs matching they are said to be in phase.

**out-of-phase :** A phrase used to characterize two or more signals whose phase relationship with each other is such that when **one is at its positive peak the other is at** (or near) its negative peak.

So the final noise will be:

$$N_i = \frac{1}{2T} \int_{-\frac{T}{2}}^{\frac{T}{2}} n_c(t)^2 + n_s(t)^2 [1 - \cos(w_c t)] - 2n_c(t)n_s(t) \sin(2w_c t) dt$$

And it basically used the trigonometry:

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

where the **R** is assumed to be **1Ω**.

the cos and sin terms average to zero to give,

$$N_i = \frac{1}{2T} \int_{-\frac{T}{2}}^{\frac{T}{2}} n_c^2(t) + n_s^2(t) dt = \frac{1}{2} n_c^2(t) + \frac{1}{2} n_s^2(t)$$

Which is the addition of two mean value of power.

If the noise is random, then the noise spikes will occur in-phase and out-of phase with equal frequency and so we can see

$$n_c^2(t) = n_s^2(t) = n_i^2(t)$$

## Q1

- ☒ **Why?** Cause when the noise is random, we assume that they are equal to each other although that I don't the exact reason.....

## Signal

$$S_i = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} s_i(t)^2 dt$$

## Amplitude demodulation

$$S_i(t) = f(t) \cos(w_c t)$$

Substitute it into the original equation:

$$S_i = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t) dt$$

since the double angles was 0 during the integral.

So as shown before, according to DSBSC demodulation:

$$S_i(t) = f(t) \cos(w_c t)$$

$$S_o(t) = S_i(t) \cos(w_c t) = f(t) \cos^2(w_c t)$$

Since the  $2w_c$  term will be filtered off. Then the obtained signal will be

$$S_o = \frac{1}{2} f(t)$$

## Q2

✓ Why? Typo.....

So the average power of the output signal will be

$$S_{op} = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{1}{2} f(t)\right)^2 dt = \frac{1}{4} f(t)^2$$

**No matter what is at input both noise and signal will be demodulated accordingly:**

so the output noise will be  $n_o(t) = n_i(t) \cos(w_c t)$  which is the same as the input signal and output signal which is:

$$n_o(t) = \frac{1}{2} n_c(t) [1 + \cos(2w_c t)] - \frac{1}{2} n_s(t) \sin(2w_c t)$$

Again, after filtered off  $n_o(t) = \frac{1}{2} n_c(t)$ , so the corresponding average power will be:

$$N_o = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{1}{2} n_c(t)\right)^2 dt = \frac{1}{4} n_i^2(t)$$

## Final SNR Ratio

$$\frac{S_i}{N_i} = \frac{\frac{1}{2} f(t)^2}{n_i(t)^2}$$

$$\frac{S_o}{N_o} = \frac{\frac{1}{4} f(t)^2}{\frac{1}{4} n_i(t)^2}$$

Hence  $SNR_o = 2SNR_i$

## Find noise performance of DSBLC through envelope detector

$$S_i = (\alpha + f(t)) \cos(w_c t) = \text{Re}\{(\alpha + f(t))e^{jw_c t}\}$$

where  $\alpha$  is the carrier amplitude

So the input noise will be given as:

$$n_i(t) = n_c(t)\cos(w_ct) - n_s \sin(w_ct)$$

So the input to envelope detector would be the addition of signal and noise:

$$s_i(t) + n_i(t) = \text{Re}\{(\alpha + f(t) + n_c(t) + jn_s(t))e^{jw_ct}\}$$

So the modulus of the input waveform can be expressed as:

$$r(t) = \sqrt{(\alpha + f(t) + n_c(t))^2 + n_s(t)^2}$$

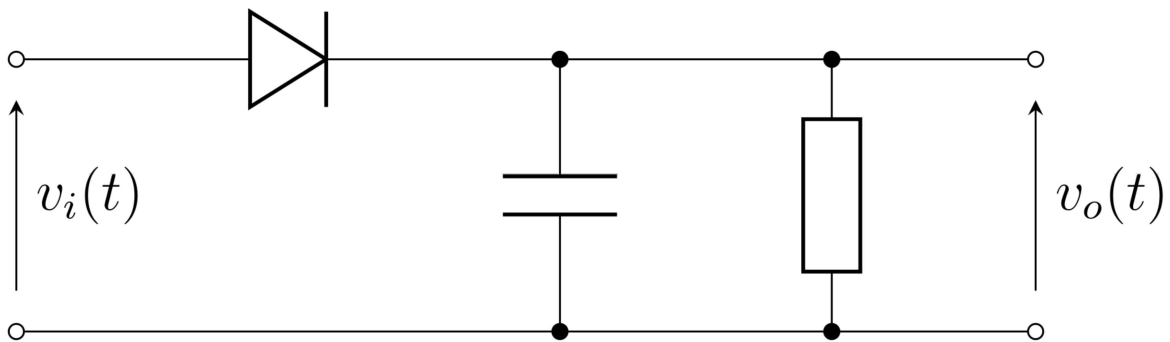
$$r(t) = (\alpha + f(t) + n_c(t))\sqrt{1 + \frac{n_s(t)^2}{\alpha + f(t) + n_c(t)^2}}$$

Since  $n_s(t) \ll \alpha$  and  $n_s \ll f_s(t)$ . So  $r(t) \approx (\alpha + f(t) + n_c(t))$ , which is addition of DC term, signal term and noise term.

And  $r(t)$  is the output of the envelope detector

✓ Why?

As for envelope detector:



So usually the envelope of the signal:

$$x(t) = (C + m(t))\cos(wt)$$

where  $C$  is the carrier amplitude and  $m(t)$  is the message signal. And the output is the envelope. That's the reason why the  $r(t)$  is the output of the envelope detector.

Since the DC term can be filtered out, so the remaining will be signal term and noise term.

which means that the output signal  $S_o = f(t)$ ,  $N_o = n_c(t)$ .

which means that their corresponding mean output signal and noise power can be given as:

$$S_o = f(\hat{t})^2$$

$$N_o = n_c(\hat{t})^2$$

Similarly, since  $S_i = a + f(t)^2 \cos(w_ct)^2$  where the  $2w_ct$  term will be filtered out. So the final input signal will be  $\frac{1}{2}(\alpha + f(\hat{t})^2)$

where the input noise is  $n_i(\hat{t})$  which is the similar as that in DSCSC.

so

$$\frac{S_i}{N_i} = 2 \frac{S_i}{N_i} - \frac{\alpha^2}{\hat{n}_i^2}$$

which means that  $SNR_O < 2SNR_i$

where in DSBSC  $SNR_O = 2SNR_i$ , that's the reason why DSBLC is less efficient than DSBSC.

## Signal power in FM systems

Limiter: To stop fluctuation, and set the certain value we want.

The FM signal can be represented by:

$$s_i(t) = \alpha \cos \left( w_c t + c \int_0^t f(\tau) d\tau \right)$$

where  $\alpha$  is the signal amplitude and  $c$  is a constant.

And the instantaneous frequency can be written as the derivative of the phase:

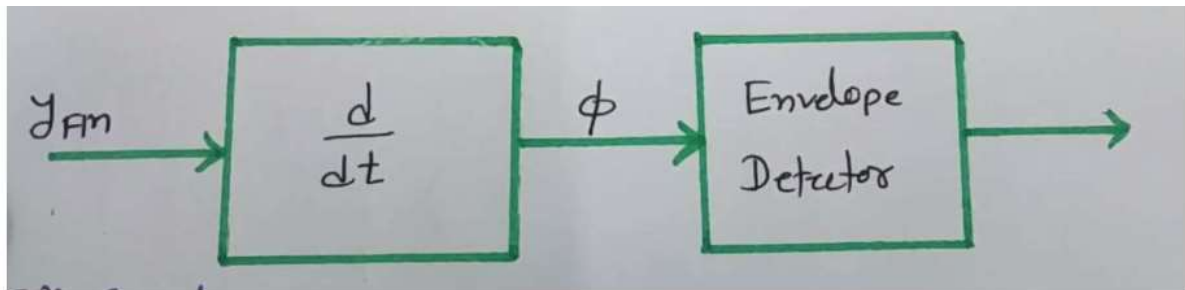
$$f_i = \frac{d}{dt} (w_c t + c \int_0^t f(\tau) d\tau) = w_c + c f(t)$$

Let's assume a FM receiver looks like this:



The signal after the demodulation is proportional to the difference between **instantaneous frequency and carrier frequency**.

Just a quick recall for FM demodulation



Let's say the  $y_{FM}$  is

$$y_{fm}(t) = Ec \cos \left( w_c t + c \int m(t) dt \right)$$

After the differentiator, the signal will be

$$-Ec(w_c + cm(t)) \sin(w_c t + c \int m(t) dt)$$

Then what we need to do just simply use the envelope detector to extract the amplitude

$$w_c + c(t)$$

Then the output will be proportional to  $w_c + c(t) - w_c$ , where  $w_c + c(t)$  is the **instantaneous frequency**

So that's the reason why the final output is  $s_o(t) = Kcf(t)$  where  $K$  depends on the receiver.

## Noise in FM

When considering the noise, we can just simply add them up to an unmodulated carrier frequency:

$$\begin{aligned} &= \alpha \cos(w_c t) + n_c(t) \cos(w_c t) - n_s(t) \sin(w_c t) \\ &= r(t) \cos(w_c t + \gamma(t)) \end{aligned}$$

In this phasor diagram, either  $n_c t$  or  $\alpha$  has been multiplied by  $w_c t$ , while  $-\sin \alpha = \cos(\alpha - 90^\circ)$ , so that's the reason why  $n_s(t)$  is clockwise  $90^\circ$  between the cos term.

And from the diagram, we can see that:

$$\begin{aligned} \alpha + n_c t &= r(t) \cos(\gamma(t)) \\ n_s(t) &= r(t) \sin(\gamma(t)) \end{aligned}$$

And substitute them into the original signal we can get the expression of  $r(t)$

According to FM, we need to know the instantaneous frequency:

$$\gamma(t) = \tan^{-1} \left( \frac{n_s(t)}{\alpha + n_s(t)} \right) \approx \tan^{-1} \left( \frac{n_s}{\alpha} \right) \approx \frac{n_s(t)}{\alpha} (\text{limit})$$

where we just assume that  $n_s(t) \ll \alpha$ ,  $n_s(t) \ll \alpha$  (**quite important!!!**)

And accordingly, the final output noise:

$$n_o(t) = \frac{K}{\alpha} \frac{d}{dt} n_s(t)$$

The output noise power could be calculated by using Parseval's theorem:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(w)|^2 dw$$

$$N_o = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{no}(w) dw$$

$$S_{no}(w) = \frac{K^2 w^2}{\alpha^2} \eta$$

where  $\eta$  is the white noise which has the same magnitude of output frequency

Hence the noise output from the demodulator will increase with the square of the frequency deviation.

## Risk

When increasing the frequency, the **output signal** will become **very weak**.

## Solution

**Pre-emphasis filter** and **De-emphasis filter** to set a certain frequency limit before the noise is introduced

## Aim of this Course

Find SNR<sub>i</sub> and SNR<sub>o</sub> which is SNR of input and SNR of output respectively