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Still working on the paper ***Resource and Power Allocation in SWIPT-Enabled Device-to-Device Communications Based on a Nonlinear Energy Harvesting Model***. Record some notes for the piecewise linear EH model mentioned in this paper, and methods for transforming the linear programming model into nonlinear programming model.

Technical term

- **Piecewise linear functions**

  It is a function whose graph consists of straight line segments. it is a function of which the each piecewise is linear.

$$f(x) = \begin{cases} 2x & x <= 2 \\ -x+3 & x > 2 \end{cases}$$

  For the linear EH model mentioned in this paper:

$$EH_i^D = \begin{cases} 0 & P_i^R \in [P_{tn}^0, P_{th}^1] \\ k_j P^R + b_j, & P_i^R \in [P_{th}^j, P_{th}^{j+1}], j \in 1, \ldots, L-1 \\ P_{max}^{EH} & P_i^R \in [P_{th}^L, P_{th}^{L+1}] \end{cases}$$

  Where the EH_i^D is the power harvested by D2D receiver i , and P_i^R is the received power for EH at D2D receiver i when sharing the RB with CUE k, which can be expressed as:

$$P_i^R = \lambda_i^e (P_i^D + P_k^C h_{k,i} + N_0)$$

  Note that $P_{th} = \{P_{th} | 1 \le j \le L+1\}$ is the set of thresholds on P_i^R for $L+1$ linear segments. The k_j and b_j are the coefficients and the intercept of the linear function in the j_{th} segment. P_{th}^1 denotes the minimum received power requirement for activating the **RF EH** circuit, which is also the circuit sensitivity of the EH circuit, and te P_{max}^{EH} is the maximum power the **RF EH** circuit can harvest.

  So the above piecewise linear EH model shows the **different amount of energy that the system can harvest at different segment.**

- **Maximization of energy efficiency(EE)for SWIPT-enabled D2D links**

  The final equation of **Energy Efficiency(EE)** for D2D links can be expressed by

$$EE_i^D = \frac{T_i^D}{EC_i^D} = \frac{log_2(1 + \frac{P_i^D h_i^D}{(P_k^C h_{k,i} + N_0) + \frac{N_1}{1-\lambda_i^e}})}{P_i^D + 2P_{cir} - EH_i^D}$$

  As shown in the equation, if i want to find the maximum value of the Energy Efficiency, it is all about finding a optimal value for the transmission power at D2D link i (P_i^D), the harvest energy from the system(EH_i^D), the power splitting ratio(λ_i^e).

  And in this paper, the transmission power for **CUE link is constant for simulation。**

  Accordingly, In this paper the EE maximization problem was formulated as

$$
\begin{aligned}
\textbf{P1:} \quad & \max_{\{P_i^D,\lambda_e^i, i\in Eha^D\}} \quad \mathrm{EE}_i^D \\
\text{s.t.} \quad & \mathrm{C1}: 0 < P_i^D \leq P_{\max} \\
& \mathrm{C2}: 0 \leq \lambda_i^e \leq 1 \\
& \mathrm{C3}: T_i^D \geq \mathrm{T}_{\min}^{\mathrm{D}} \\
& \mathrm{C4}: T_k^C \geq \mathrm{T}_{\min}^{\mathrm{C}} \\
& \mathrm{C5}: P_i^R \geq P_{th}^1 \\
& \mathrm{C6}: P_{th}^j \leq P_i^R \leq P_{th}^{j+1}, \quad j \in 0, \ldots, L \quad (12)
\end{aligned}
$$

- **non-linear programming**

  In mathematics, **nonlinear programming** (**NLP**) is the process of solving an optimization problem where some of the constraints or the objective function are nonlinear

- **linear programming:**

  Linear programming is a simple technique where we **depict** complex relationships through linear functions and then find the optimum points. The important word in the previous sentence is depicted. The real relationships might be much more complex – but we can simplify them to linear relationships.

  As shown in the figure 1, it is a very classic example for people to use LP to save on fuel and time and find the shortest route.

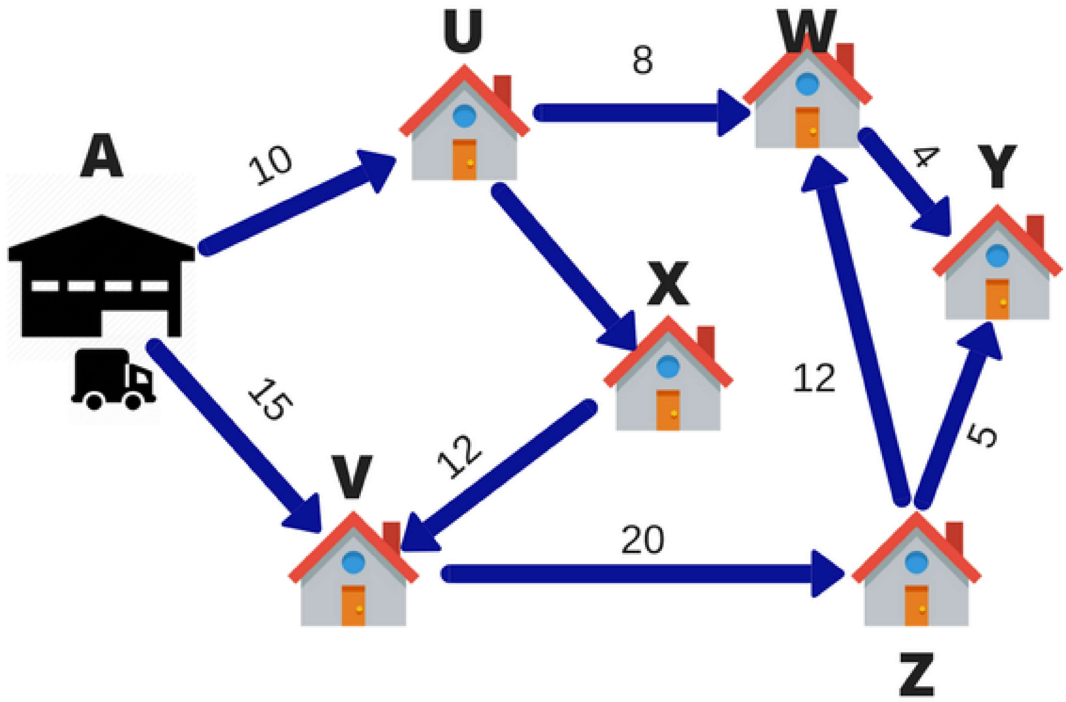

Figure 1: A simple example for linear programming

Notation definition

- $argmax_S f = \{x \in f(s) \quad for \quad all \quad s \in S\}$:$argmax$ is the set of points x for which $f(x)$ attains the function's largest value(if it exists). $Argmax$ may be the empty set, a singleton, or contain multiple elements.

Literature Review

1. ***On nonlinear fractional programming***:
   1. To figure out how did the authors transform a the nonconvex factional programming problem which is very difficult to solve into a nonfractional problem by employing nonlinear fractional programming.
      - Some mathematic concepts:
        - Compact space: In mathematics, specifically general topology, **compactness** is a property that generalizes the notion of a subset of Euclidean space being closed (containing all its limit points) and bounded (having all its points lie within some fixed distance of each other).
        - Euclidean space: Euclidean space is the fundamental space of classical geometry. Originally, it was the three-dimensional space of Euclidean geometry, but in modern mathematics there are Euclidean spaces of any nonnegative integer dimension,[1] including the three-dimensional space and the *Euclidean plane* (dimension two).
      - First of all, we need to figure out the relationship between nonlinear fractional nonlinear parametric Programming:

        Let's set two different real-valued functions $N(x)$ and $D(x)$, they are all continuous. In addition, we also assumed that:$D(x) > 0$ for all $x \in S$.

        To figure out the problems mentioned before, we need to think about two following questions:

        1. $max\{\frac{N(x)}{D(x)}|x \in S\}$
        2. $max\{N(x) - qD(x)|x \in S\}$ for $q \in E^1$ where E^1 is one dimensional Euclidean space

        According to Dickelbach, we have a couple of following Lemmas.

        - $Lemma\ 1$:$F(q) = max\{N(x) - qD(x)|x \in S\}$ is convex overE^1
        - $Lemma\ 2$:$F(q)$ is continuous for $q \in E^1$.
        - $Lemma\ 3$:$F(q) = max\{N(x) - qD(x)|x \in S\}$ is strictly monotonic decreasing.
        - $Lemma\ 4$:$F(q) = 0$ has an unique solution, q_0
        - $Lemma\ 5$: Let $x^+ \in S$ and $q^+ = \frac{N(x^+)}{D(x^+)})$ then $F(q^+) \geq 0$

        For any $q = q^*$, the maximum of $\{N(x) - q^*D(x)|x \in S\}$ is taken on, for instance at x^*; this may be indicated by writing $F(q^*, x^*)$. Now that we can get a theorem:

        $$q_0 = \frac{N(x_0)}{D(x_0)} = max\{\frac{N(x)}{D(x)}|x \in S\} \quad if \quad and \quad only \quad if,$$
        $$F(q_0) = F(q_0, x_0) = max\{N(x) - q_0D(x)|x \in S\} = 0$$

Now, back to the problem, for the original equation we have:

$$EE_i^D = \frac{T_i^D}{EC_i^D}$$

This is the target of the whole paper we want to find the maximum value of the Energy efficiency. Now we can use the method that we deduced above:

$$max\{T_i^D - Q_i^D EC_i^D\}, \quad i \in Eha^D$$

Where in this method T_i^D and EC_i^D is $N(x)$ and$D(x)$ respectively. And according to the nonlinear fractional programming method there is a Q_i^D which can let the $F(T_i^D, EC_i^D) = 0$.

Then remember that we have this equation and a couple constraints:

$$\begin{aligned} \textbf{P1}: \quad & \max_{\{P_i^D, \lambda_e^i, i \in Eha^D\}} \quad \mathrm{EE}_i^D \\ & \text{s.t.} \quad \mathrm{C1}: 0 < P_i^D \leq P_{\max} \\ & \qquad \mathrm{C2}: 0 \leq \lambda_i^e \leq 1 \\ & \qquad \mathrm{C3}: T_i^D \geq \mathrm{T}_{\min}^{\mathrm{D}} \\ & \qquad \mathrm{C4}: T_k^C \geq \mathrm{T}_{\min}^{\mathrm{C}} \\ & \qquad \mathrm{C5}: P_i^R \geq P_{th}^1 \\ & \qquad \mathrm{C6}: P_{th}^j \leq P_i^R \leq P_{th}^{j+1}, \quad j \in 0, \ldots, L \quad (12) \end{aligned}$$

So according to the usage of Lagrange multipliers, C1-C6 can all be thought of as different constraints. And if you want to find the Maximum value of EE_i^D, we can put them all into the Lagrange function:

$$EE_i^D = T_i^D - Q_i^D EC_i^D - \alpha(P_i^D - P_{max})$$
$$-\beta(\lambda_i^e - 1) + \gamma(T_i^D - T_{min}^D) + \delta(T_i^C - T_{min}^C) + \in (P_i^R - P_{th}^1)$$

Where all of these symbols are Lagrange multipliers for C1-C5 respectively.