TUTORIAL 2: complex series I.

1. Find the radius of convergence of the series

$$(i)\ 1+\frac{z}{1!}+\frac{z^2}{2!}+\frac{z^3}{3!}+\dots \qquad (ii)\ 1+2^2z+3^2z^2+4^2z^3+\dots$$

$$(iii)\ 1-\frac{1}{2}z+\frac{1}{3}z^2-\frac{1}{4}z^3+\dots \qquad (iv)\ 1+1!z+2!z^2+3!z^3+\dots$$

2. Find the region of convergence of the series

$$(i)\ \sum_{n=1}^{\infty}\frac{(z+2)^{n-1}}{(n+1)^3 4^n} \qquad (ii)\ \sum_{n=1}^{\infty}\frac{(-1)^{n-1}z^{2n-1}}{(2n-1)!}$$

3. Find the Taylor series expansion about $z=0$ of the functions

$$(i)\ \ \sin(z^2) \quad (ii)\ \frac{1}{(1+z)^2} \qquad (iii)\ \ \tanh z$$

4. Find the expansion of the following functions about the origin ($z=0$). For each case show, in the Argand diagram, the region of convergence and the poles of the function.

$$(i)\ \frac{1}{i+z} \quad (ii)\ \frac{z}{1+z^2} \quad (iii)\ \frac{1}{(1-z)(2-iz)}$$

5. Find the expansion of the following functions. Show the region of convergence on the Argand diagram.

$$(i)\ \frac{1}{1+z}\ \text{ about }\ z=2i \qquad (ii)\ \frac{1}{(1+z)(z-i)}\ \text{ about }\ z=-1+i$$

N.B. You can verify your solutions for 4 and 5 using Taylor Series.

Answers

1.
(i) $R=\infty$; (ii) $R=1$; (iii) $R=1$; (iv) $R=0$
2.
(i) $\rho=4$, $|z+2|<4$; (ii) $\rho=\infty$, $|z|<\infty$
3.
(i)

$$\sin z^2 = \frac{2z^2}{2!} - \frac{120z^6}{6!} + \cdots = \frac{z^2}{2} - \frac{z^6}{6} + \ldots$$

(ii)

$$\frac{1}{(1+z)^2} = 1 - 2z + \frac{6z^2}{2!} - \frac{24z^3}{3!} + \cdots = 1 - 2z + 3z^2 - 4z^3 + \ldots$$

(iii)

$$\tanh z = z - \frac{2z^3}{3!} + \cdots = z - \frac{z^3}{3} + \ldots$$

4. (i)

$$\frac{1}{j+z} = -j + z + jz^2 - z^3 + \ldots$$

(ii)

$$\frac{z}{1+z^2} = z - z^3 + z^5 - z^7 + \ldots$$

(iii)

$$\frac{1}{1+j-z^2} = \frac{1-j}{2} - \frac{j}{2}\, z^2 - \frac{1+j}{4}\, z^4 + \ldots$$

(iv)

$$\frac{1}{(1-z)(2-jz)} = \frac{1}{2} + \frac{2+j}{4}\, z + \frac{3+2j}{8}\, z^2 + \frac{6+3j}{16}\, z^3 + \ldots$$

5.
(i)

$$\frac{1}{1+z} = \frac{1}{1+2j}\left[1 - \frac{z-2j}{1+2j} + \left(\frac{z-2j}{1+2j}\right)^2 - \left(\frac{z-2j}{1+2j}\right)^3 + \ldots\right]$$

(ii)

$$\frac{1}{(1+z)(z-j)} = j - (1-j)(z+1-j) - (z+1-j)^2 + \ldots$$