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Today, I basically worked on some mathematical tools mentioned in the reference paper, most of them are about optimizations like KKT, dual problems and so on.

Math language

- **subject to:**

  It is a way to specify constraints. To put it very simply, the problem "do 'X' subject to 'Y'" means that, you have to do "X" (whatever X is), but you have to do it such that "Y" is also satisfied in the process.

- **objective function:**

  The objective function in a mathematical optimization problem is the real-valued function whose value is to be either minimized or maximized over the set of feasible alternatives. In problem P above, the function f is the objective function.

- **Two different types of constraints:**
  - Inequality constraints :$g_i(x) = 0, 1 \leq i \leq p$
  - equality constraints: $h_j(x) \leq 0$,$1 \leq j \leq q$
- $R^n \to R^m$:

  A linear transformation T between two vector spaces R^n and R^m, written T:$R^n \to R^m$ just means that T is a function that takes as input n-dimensional vectors and gives you m-dimensional vectors. These properties are:

  $$1.T(v+w) = T(v) + T(w)$$
  $$2.T(av) = aT(v)$$

  for all $v, m \in R^n$ and a real number.

- **Lagrange dual problems:**

  For an optimization problems:

  minimize $f_0(x)$

  subject to $f_i(x) \leq 0 \quad i = 1,$

  $h_i(x) = 0 \ \ i = 1,$

  with variable $x \in R^n$,domain D, and optimal value p^*.

  The Lagrangian is a function L:

  $L(x, \lambda, \mu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} v_i h_i(x)$

  where λ_i is the Lagrange multiplier associated with $f_i(x) \leq 0$ and v_i is the Lagrange multiplier associated with $h_i(x) = 0$, **while they are all greater than 0**

  Then the Lagrange dual function can be defined as:

  $$g(\lambda, \mu) = \inf_{x \in D} L(x, \lambda, \mu)$$
  $$= \inf_{x \in D}(L(x, \lambda, \mu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} v_i h_i(x))$$

  The original primal problem is to find the minimum then we can obtain that by find the maximum of this dual problem to find the best lower bound.

- **Affine functions:**

An affine function is a function composed of a linear function + a constant and its graph is a straight line.

- **KKT conditions**:

  if we have an optimization problems:

$$
\begin{aligned}
&minf(x)\\
&s.t. g_i(x) \leq 0 (j=1,2,\ldots,m)\\
&h_k(x)=0(k=1,2,\ldots,l)
\end{aligned}
$$

If have an optimal value x^*, and we want to determine it is an optimal value for our optimization, then we can use KKT conditions to check

$$
\begin{cases}
\frac{\delta f}{\delta x_i}+\sum_{j=1}^{m}\mu_j\frac{\delta g_j}{\delta x_i}+\sum_{k=1}^{l}\lambda_k\frac{\delta h_k}{\delta_{x_i}}=0,(i=1,2,\ldots,n)\\
h_k(x)=,(k=1,2,\ldots,l)\\
\mu_j g_j(x)=0,(j=1,2,\ldots,m)\\
\mu_j \geq 0
\end{cases}
$$

Optimization

We are trying to maximize the target energy efficiency which belongs to SWIPT-Enabled D2D links.

$$
\begin{aligned}
P1: max \quad EE_i^D\\
s.t. \quad C1: 0 < P_i^D \leq P_{max}\\
C2: 0 \leq \lambda_i^e \leq 1\\
C3: T_i^D \geq T_{min}^D\\
C4: T_c^k \geq T_c^{min}\\
C5: P_i^R \geq P_{th}^1\\
C6: P_{th}^j \leq P_i^R \leq P_{th}^{j+1}, j \in 0,\ldots,L
\end{aligned}
$$

And for Constraints C1-C5, we have the Lagrange functions:

$$
L(P^i,\lambda_i^e,\alpha,\beta,\gamma,\delta,\in)=T_i^D-Q_i^D EC_i^D-\alpha(P_i^D-P_{max})-\beta(\lambda_i^e-1)+\gamma(T_i^D-T_{min}^D)+\\
\delta(T_i^C-T_C^{min})+\in(P_i^R-P_{th}^1)
$$

So for the original problem, we have the following optimization problems:

$$
\begin{aligned}
P1: max \quad EE_i^D\\
s.t. \quad C1: 0 < P_i^D \leq P_{max}\\
C2: 0 \leq \lambda_i^e \leq 1\\
C3: T_i^D \geq T_{min}^D\\
C4: T_c^k \geq T_c^{min}\\
C5: P_i^R \geq P_{th}^1\\
C6: P_{th}^j \leq P_i^R \leq P_{th}^{j+1}, j \in 0,\ldots,L
\end{aligned}
$$

So the original optimization problem which need to be found the maximum can be obtained to find its dual problem which needs to find the minimum:

$$
\min_{\{\alpha,\beta,\gamma,\delta,\in\geq 0\}} \qquad \max_{P_i^D,\lambda_i^e} \quad L(P_i^D,\lambda_i^e,\alpha,\beta,\gamma,\delta,\in), i \in Eha^D
$$

which subjects to constraint 6 C6.