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Lagrange multipliers caculation

The optimization problem can be formulated as:

$\min_{\{\alpha,\beta,\gamma,\delta,\in\geq 0\}} max\ L(P_i^D, \lambda_i^e, \alpha, \gamma, \beta, \delta, \in), i \in Eha^D$

$s.t.$ C6

where $\alpha, \gamma, \beta, \delta, \in$ are the corresponding Lagrange multipliers, and Constraint 6 C6 is $P_{th}^j \leq P_i^R \leq P_{th}^{j+1}, j \in 0, \ldots, L$

And the Lagrange function is:

$$L(P_D^i, \lambda_i^e, \alpha, \beta, \gamma, \delta, \in) = T_i^D - Q_i^D EC_i^D - \alpha(P_i^D - P_{max}) - \beta(\lambda_i^e - 1) + \gamma(T_i^D - T_{min}^D) + \delta(T_i^C - T_{min}^C) + \in (P_i^R - P_{th}^1)$$

First, we want to find the optimal value of power splitting ratio λ_i^e and the transmission power of D2D transmitter P_i^D.

According to the optimization method, we need to get the first derivative of Lagrange functions and let it equal to zero with respect to λ_i^e:

$$\frac{\partial L(P_i^D, \lambda_i^e, \alpha, \beta, \gamma, \delta, \in)}{\partial \lambda_i^e} = 0$$

For this equation, first we need to find which variable has λ_i^e: T_i^D, P_i^R in Lagrange functions:

- $T_i^D = \log_2(1 + \frac{P_i^D h_i^D}{(P_k^C h_{k,i} + N_0) + \frac{N_1}{(1-\lambda_i^e)}}), \gamma(T_i^D), T_i^D$
- $P_i^R = \lambda_i^e(P_i^D h_i^D + P_k^C h_{k,i} + N_0), \in P_i^R$
- $-\beta(\lambda_i^e)$
- $k_j P_i^R + b_j, P_i^R \in [P_{th}^j, P_{th}^{j+1}], j \in 1, \ldots, L$

Let those combination of irrelevant variables be a constant:

$P_i^D h_i^D = G$, $P_k^C h_{k,i} + N_0 = H$. Then the original $T_i^D = \log_2(1 + \frac{G}{H + \frac{N_1}{1-\lambda_i^e}})$

First find the first order derivative of T_i^D with respect to λ_i^e which can be substitude with x at the beginning:

$$T_i^D = \log_2(1 + \frac{G}{H + \frac{1}{1-x}})$$

So, after the equations were entered as an input in the MATLAB:

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syms  T_id p_id h_id p_ir Q_id EC_id EH_id p_kc h_ki N0 N1 kj bj pmax lambda T_ic T_dmin T_cmin P_th1 beta alpha gamma delta

%Partial derivative of Lagrange functions with respect to power splitting
%ratiro λ: lambda
A=1+(p_id*h_id)/((p_kc*h_ki+N0)+(N1/(1-lambda)));
T_id=log2(A);
p_ir=lambda*(p_id*h_id+p_kc*h_ki+N0);
EH_id=kj*p_ir+bj;
EC_id=p_id+2*p_ir-EH_id;
L=T_id-Q_id*EC_id-alpha*(p_id-pmax)-beta*(lambda-1)+gamma*(T_id-T_dmin)+delta*(T_ic-T_cmin)+in*(p_ir-P_th1);
d_lambda=diff(L,lambda);

%Partial derivative of Lagrange functions with respect transmission power
%P_id
d_pid=diff(L,p_id);
```