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Today, I basically worked on some mathematical tools mentioned in the reference paper, most of them are about optimizations like KKT, dual problems and so on.

Math language

- **subject to:**

  It is a way to specify constraints. To put it very simply, the problem "do 'X' subject to 'Y'" means that, you have to do "X" (whatever X is), but you have to do it such that "Y" is also satisfied in the process.

- **objective function:**

  The objective function in a mathematical optimization problem is the real-valued function whose value is to be either minimized or maximized over the set of feasible alternatives. In problem P above, the function f is the objective function.

- **Two different types of constraints:**
  - Inequality constraints :$g_i(x) = 0$, $1 \leq i \leq p$
  - equality constraints: $h_j(x) \leq 0$,$1 \leq j \leq q$

- $R^n \to R^m$:

  A linear transformation T between two vector spaces R^n and R^m, written T:$R^n \to R^m$ just means that T is a function that takes as input n-dimensional vectors and gives you m-dimensional vectors. These properties are:

  $$1.T(v+w) = T(v) + T(w) \\ 2.T(av) = aT(v)$$

  for all $v, m \in R^n$ and a real number.

- **Affine functions:**

  An affine function is a function composed of a linear function + a constant and its graph is a straight line.

- **KKT conditions**:

  if we have an optimization problems:

  $$min f(x) \\ s.t.\, g_i(x) \leq 0 (j = 1, 2, \ldots, m) \\ h_k(x) = 0(k = 1, 2, \ldots, l)$$

If have an optimal value x^*, and we want to determine it is an optimal value for our optimization, then we can use KKT conditions to check

$$\begin{cases} \frac{\delta f}{\delta x_i} + \sum_{j=1}^{m} \mu_j \frac{\delta g_j}{\delta x_i} + \sum_{k=1}^{l} \lambda_k \frac{\delta h_k}{\delta_{x_i}} = 0, (i = 1, 2, \ldots, n) \\ h_k(x) =, (k = 1, 2, \ldots, l) \\ \mu_j g_j(x) = 0, (j = 1, 2, \ldots, m) \\ \mu_j \geq 0 \end{cases}$$

Optimization

We are trying to maximize the target energy efficiency which belongs to SWIPT-Enabled D2D links.

$$
\begin{gathered}
P1 : max \quad EE_i^D \\
s.t. \quad C1 : 0 < P_i^D \leq P_{max} \\
C2 : 0 \leq \lambda_i^e \leq 1 \\
C3 : T_i^D \geq T_{min}^D \\
C4 : T_c^k \geq T_c^{min} \\
C5 : P_i^R \geq P_{th}^1 \\
C6 : P_{th}^j \leq P_i^R \leq P_{th}^{j+1}, j \in 0, \ldots, L
\end{gathered}
$$

And for Constraints C1-C5, we have the Lagrange functions:

$$
\begin{gathered}
L(P^i, \lambda_i^e, \alpha, \beta, \gamma, \delta, \in) = T_i^D - Q_i^D EC_i^D - \alpha(P_i^D - P_{max}) - \beta(\lambda_i^e - 1) + \gamma(T_i^D - T_{min}^D) + \\
\delta(T_i^C - T_C^{min}) + \in (P_i^R - P_{th}^1)
\end{gathered}
$$

Suppose an objective function f(y) of the constrained optimization problem has a local minimum at the feasible point x. If f(x) and the various constraint functions are continuously differentiable near x, then there exists unit vector of Lagrange multipliers $\lambda_0, \ldots, \lambda_p$ and $\mu_1, \ldots \mu q$ such that:

$$
\lambda_0 \nabla f(x) + \sum_{i=1}^{p} \lambda_i \, g_i(x) + \sum_{j=1}^{q} \nabla h_j(x) = 0
$$