

Some basic equations for complex numbers

1. Multiplication of complex numbers

$$z = \cos\theta + i\sin\theta$$

$$w = \cos\alpha + i\sin\alpha$$

$$\begin{aligned}zw &= (\cos\alpha + i\sin\alpha)(\cos\theta + i\sin\theta) \\&= \cos\theta\cos\alpha + i\sin\alpha\cos\theta + i\sin\theta\cos\alpha - \sin\theta\sin\alpha \\&= \cos(\theta + \alpha) + i\sin(\alpha + \theta) \\arg(zw) &= arg(z) + arg(w) \\z_1z_2 &= r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]\end{aligned}$$

2. Division of complex numbers

$$\left|\frac{1}{z}\right| = \frac{1}{|z|}$$

$$\frac{1}{z} = \frac{x - iy}{x + iy}$$

$$arg\left(\frac{1}{z}\right) = -arg(z)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

3. De Moivre's theorem

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

4. nth root of a complex number

$$z^{\frac{1}{n}} = [r(\cos\theta + i\sin\theta)]^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right]$$