**Plots of noisy signal and original signal**

In Figure 1, the plot of the original signal versus time is plotted. The sampling frequency is 360 Hz, and the the time vector is established based on the sampling frequency, where the interval is equal to the sampling period and time span is 1 0s. Similarly, the plot of the noisy signal versus time is plotted in Figure 2.

图表

描述已自动生成

Figure 1: ECG Signal versus time

图表

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Figure 2: Noise versus time

**Heart Rate (Bpm)**

The heart rate is calculated as beats per minute (Bpm). In the ECG signal, the average heart rate can be calculated by dividing the the number of extrema by duration in one minute:

(1)

Where is the number of extrema and is the duration in one minute. The duration in one minute can be calculated by dividing the time span by 60 seconds. As mentioned before, the time span is 10 seconds then is . The extrema can be defined as the local maxima in ECG signal as shown in Figure 3. The final calculated Bpm is approximately 72.

图表

描述已自动生成

Figure 3: Number of extrema and calculated

**Original and Noisy signal in frequency domain**

Before designing the low pass filter, the original signal and Noise signal in frequency domain need to be shown so that we can find the frequency such that we can find the signal that we do not want. The plot of the ECG signal in frequency time domain is given by using fast fourier transform(fft) in Figure 4. As shown in this figure, the amplitude has been normalized, the frequency is established based on the sampling frequency and it is noted that the figure plotted by using fft is symmetric.

图表, 直方图

描述已自动生成

Figure 4: ECG signal in frequency domain

The reason why fft diagram is symmetric is because of the nature of fourier transform and it can be proved in a mathematical way: Define a signal in time domain . Then the fourier transform of this signal can be expressed as:

And the fourier transform at the negative frequency can be expressed as:

Let the term and the term .So, , since the cosine term is even and sine term is odd, which can be expressed as:

And the modulus of can be expressed as:

Then clearly when the frequency is negative, the modulus is still the same:

So (5) = (6), and this can also apply in fast fourier transform (fft). The plot of the Noise in frequency domain is given in Figure 5, where the mains hum is labeled at around 135 Hz, while the mains frequency in the US is around 60Hz.

图表, 直方图

描述已自动生成

Figure 5: Noise in frequency domain

**Cut off frequency**

To design a filter to filter the noise out, the cut off frequency needs to be determined. The cutoff frequency can be determined by comparing the Figure 5 with Figure 4 and it is noted that the noise is located at the frequency which is bigger than approximately 60 Hz. So, the cutoff frequency and it can be normalized as , where is the sampling frequency.

**Filter Design**

1. **IIR filter**

The infinite impulse response can be characterized as:

Then the corresponding difference equation can be expressed as:

Based on (8), the corresponding transfer function can be calculated as:

Where and are the corresponding coefficients. To design a IIR low pass filter, we need to first determine the transfer function of the low pass filter and the coefficients need to be determined. In this filter design, to achieve the transfer function and coefficients mentioned above, the Butterworth filter has been chosen since the linear phase response it has is more than that of Chebyshev Type I and Elliptic filters. The Butterworth polynomial can be expressed as:

According to bilinear transform, , where , and is the calculated cut-off frequency which can be calculated as:

(12)

And is the sampling period. is the selected cut off frequency based on Figure 5. It is noted that when the sampling frequency , . According to the bilinear transform, the final transfer function can be expressed as , where can be replaced by:.

1. **FIR filter**

The relationship between the input sequence and output sequence in a discrete-time FIR filter can be expressed as:

Where is the number of orders and is the coefficient, which can be expressed as: , then the Z-transform of it is (15).

Compare Figure 5 with Figure 4, the wanted frequency is around 60 Hz. So, in this FIR filter design, to allow for the error, the cut-off frequency is set as around 70 Hz and since the unwanted noise is located at around 118 Hz in the frequency domain as shown in Figure 5 an