Glottal inversion with approximate vocal tract filter

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Contents

1 Introduction

A human vowel sound consists of a periodic sound signal created at the vocal folds (called the *glottal excitation signal*) and the vocal tract, through which the glottal signal is filtered. A synthetic vowel sound consists of a periodic signal to simulate the glottal excitation signal and a filter (i.e. a digital frequency filter) to simulate the vocal tract's effect on the glottal excitation signal.[9] With a given vocal tract filter the direct problem is *given a glottal excitation signal, create the vowel sound.* The inverse problem is *given a (recorded) vowel sound, find the glottal excitation signal.* In this study we will be concentrating on the inverse problem using both simulated vowel data and real recorded data.

The inversion from a vowel sound to the glottal signal is an important part of creating synthetic human voices and speech generators. To create a synthetic vowel both the glottal signal and the vocal tract filter are needed. However, the glottal signal cannot be directly measured, but it can be approximated with inversion of a recorded vowel. With this data models for simulating the glottal excitation signal can be created.

2 Materials and Methods

2.1 Glottal excitation signal

In this study the Rosenberg-Klatt model (RK-model) for the glottal excitation signal will be used for the generation of synthetic data and as a reference point for the obtained results. The RK-model is a simple model for the glottal signal, proposed in 1970 by Rosenberg.[8] The model is simple and easy to use, as it creates the signal only from two parameters, the sound frequency f and the so called Klatt-parameter Q.

The *airflow* for the glottal excitation signal created by the RK-model is defined as

$$g(t) = \begin{cases} at^2 + bt^3 & \text{if } 0 \leq t \leq QT \\ 0 & \text{if } QT < t \leq T, \end{cases} \tag{1}$$

where t is a time variable, $T = 1/f$ is the period of the pitch, $Q \in [0, 1]$ is the Klatt-parameter and a and b are variables defined in terms of $T_0 := QT$ as

$$a = \frac{27}{4T_0^2}, \quad b = -\frac{27}{4T_0^3}.$$

Here the parameter f defines the frequency of the generated signal and the Klatt-parameter Q defines the shape of the pulse.

The glottal excitation signal can be retrieved as the derivative g' of the airflow function. Here g' is the *pressure function*, and simulates the sound generated in the glottis. The pulse generated by the model can be seen in figure 1.

Another, more widely used, model for the glottal excitation signal worth mentioning is the Liljencrants-Fant model (LF-model).[3] It is regarded as more accurate than the RK-model, but it is also much more complex. It has also been shown, that the LF-model generates only marginally better approximations for the resulting vowel after the vocal tract filtering than the RK-model.[4] Due to this and the overall complexity of the LF-model we will be using the RK-model for the simulation of the glottal excitation signal in this study.

2.2 Vocal tract filter

In this study we will assume an approximate vocal tract filter to be known for the recorded vowel we want to invert. The digital filter, defined by a vector

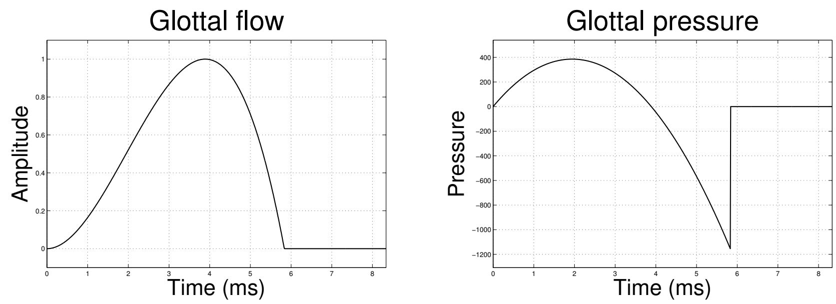


Figure 1: The airflow and pressure generated by the RK-model

$\boldsymbol{a} = (a_1, a_2, \ldots, a_{N_a})^T \in \mathbb{R}^{N_a}$, filters the data $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ as defined by the difference equation

$$\begin{cases} y_1 = x_1 \\ a_1 y_j = -\displaystyle\sum_{k=2}^{\min\{j-1, N_a\}} a_k y_{j-k}, \end{cases} \tag{2}$$

where $\boldsymbol{y} = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$ is the filtered data. We denote $\boldsymbol{y} = \varphi_{\boldsymbol{a}}(\boldsymbol{x})$.

Consider now the filter defined by $\boldsymbol{a} \in \mathbb{R}^{N_a}$ and the data $\boldsymbol{x} \in \mathbb{R}^n$ which we want to filter. Now the filter defined by (2) can be expressed by the the matrix $A \in \mathbb{R}^{n \times n}$, where

$$\begin{cases} A_{1,1} = a_1 & \\ A_{i,1} = -\displaystyle\sum_{k=1}^{\min\{i-1, N_a-1\}} a_{k+1} A_{i-k,1}, & 2 \leq i \leq n \\ A_{i+1,j+1} = A_{i,j}, & j \leq i \\ A_{i,j} = 0, & j > i. \end{cases} \tag{3}$$

Now $\varphi_{\boldsymbol{a}}(\boldsymbol{x}) = A\boldsymbol{x}$.

2.3 The matrix model

A vowel sound can be simulated by applying a digital filter $A \in \mathbb{R}^{n \times n}$, defined as in (3), to a sample of a glottal excitation signal $\boldsymbol{g} \in \mathbb{R}^n$ as

$$\boldsymbol{v} = A\boldsymbol{g}. \tag{4}$$

Here $\boldsymbol{v} \in \mathbb{R}^n$ is the simulated vowel.

In this study we will assume an approximation of the filter A to be known. Given the measurements $\boldsymbol{m} \in \mathbb{R}^n$ of a vowel corresponding approximately to the filter A, equation (4) can be expressed as

$$\boldsymbol{m} = A\boldsymbol{g} + \boldsymbol{\varepsilon}, \tag{5}$$

where $\boldsymbol{\varepsilon} \in \mathbb{R}^n$ denotes the measurement noise.

2.4 The inversion method

2.4.1 Tikhonov reguralization

The classical Tikhonov regularized solution for $\boldsymbol{m} = A\boldsymbol{g} + \boldsymbol{\varepsilon}$, defined in section 2.3, is usually denoted by the vector $T_\alpha(\boldsymbol{m}) \in \mathbb{R}^n$ that minimizes

$$\|AT_\alpha(\boldsymbol{m}) - \boldsymbol{m}\|^2 + \alpha \|T_\alpha(\boldsymbol{m})\|^2 \Leftrightarrow$$
$$T_\alpha(\boldsymbol{m}) = \underset{\boldsymbol{z} \in \mathbb{R}^n}{\operatorname{argmin}} \left\{ \|A\boldsymbol{z} - \boldsymbol{m}\|^2 + \alpha \|\boldsymbol{z}\|^2 \right\},$$

where $\alpha > 0$ is called a regularization parameter. The resulting $T_\alpha(\boldsymbol{m})$ can be understood as a compromise between two conditions, namely

I. $T_\alpha(\boldsymbol{m})$ should give a small residual $AT_\alpha(\boldsymbol{m}) - \boldsymbol{m}$.

II. $\|T_\alpha(\boldsymbol{m})\|_2$ should be small.

The α parameter is used in order to tune to balance between the two conditions above.

In generalized Tikhonov regularization some prior knowledge is assumed to be known. For example, in some cases g might be known to be smooth. This information can be incorporated into the regularization by choosing

$$T_\alpha(\boldsymbol{m}) = \underset{\boldsymbol{z} \in \mathbb{R}^n}{\operatorname{argmin}} \left\{ \|A\boldsymbol{z} - m\|^2 + \alpha \|L\boldsymbol{z}\|^2 \right\}, \tag{6}$$

where L is a discretized differential operator. As shown in [5], the regularized solution satisfies

$$\left(A^T A + \alpha L^T L\right) T_\alpha(\boldsymbol{m}) = A^T \boldsymbol{m}, \tag{7}$$

which can be used to calculate the solution numerically.

In our model proposed in section 2.1 we know the airflow of the excitation signal to be smooth in the interval $[0, QT]$ and to be zero in the interval $]QT, T]$. This can be incorporated in our model by customizing the discrete differential operator matrix, described in more detail in section 2.5.

2.4.2 The conjugate gradient method

The conjugate gradient method is an iterative method for the quadratic optimization problem

$$\text{minimize } \frac{1}{2}\boldsymbol{x}^T Q\boldsymbol{x} - \boldsymbol{b}^T\boldsymbol{x}, \tag{8}$$

where $Q \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix. We will now briefly explain the algorithm and its use to our particular problem. For a more detailed explanation, see [5].

Let $\boldsymbol{b} \in \mathbb{R}^n$ fixed, $Q \in \mathbb{R}^{n\times n}$ a symmetric positive definite matrix, $\boldsymbol{x_0} \in \mathbb{R}^n$ the initial guess and define $\boldsymbol{d_0} = -\boldsymbol{g_0} = \boldsymbol{b} - Q\boldsymbol{x_0}$. Now for $k \geq 0$ let

$$\begin{aligned}
\alpha_k \quad &= -\frac{\boldsymbol{g_k^T d_k}}{\boldsymbol{d_k^T} Q\boldsymbol{d_k}}\\
\boldsymbol{x_{k+1}} &= \boldsymbol{x_k} + \alpha_k\boldsymbol{d_k}\\
\boldsymbol{g_{k+1}} &= Q\boldsymbol{x_{k+1}} - \boldsymbol{b}\\
\beta_k \quad &= \frac{\boldsymbol{g_{k+1}^T} Q\boldsymbol{d_k}}{\boldsymbol{d_k^T} Q\boldsymbol{d_k}}\\
\boldsymbol{d_{k+1}} &= -\boldsymbol{g_{k+1}} + \beta_k\boldsymbol{d_k}.
\end{aligned} \tag{9}$$

Now $\boldsymbol{x_k}$ converges toward the solution of (8). We now want to apply the conjugate gradient algorithm in the case of the optimization problem defined in (7) for the Tikhonov regularization.

Let $A \in \mathbb{R}^{n\times n}$, $L \in R^{n\times n}$ invertible and $\alpha > 0$. Now the square matrix $B := A^TA + \alpha L^TL$ is invertible. If we denote $\boldsymbol{f} := T_\alpha(\boldsymbol{m})$ in (7), the problem becomes to minimize the expression

$$\left\|B\boldsymbol{f} - A^T\boldsymbol{m}\right\|^2. \tag{10}$$

We see that

$$\begin{aligned}
\left\|B\boldsymbol{f} - A^T\boldsymbol{m}\right\|^2 &= \langle B\boldsymbol{f}, B\boldsymbol{f}\rangle - 2\left\langle B\boldsymbol{f}, A^T\boldsymbol{m}\right\rangle + \left\langle A^T\boldsymbol{m}, A^T\boldsymbol{m}\right\rangle\\
&= \boldsymbol{f}^TB^TB\boldsymbol{f} - 2\boldsymbol{m}^TAB\boldsymbol{f} + \left\|A^T\boldsymbol{m}\right\|^2.
\end{aligned} \tag{11}$$

Further, we notice that B^TB is a positive definite symmetric matrix, since

$$\boldsymbol{v}^T(B^TB)\boldsymbol{v} = (B\boldsymbol{v})^TB\boldsymbol{v} = \|B\boldsymbol{v}\|^2 > 0$$

for any $\boldsymbol{v} \in R^n$, $\boldsymbol{v} \neq 0$, due to the fact that B is invertible. Now we define

$$Q := 2B^T B \quad \text{and} \quad \boldsymbol{b}^T := 2\boldsymbol{m}^T AB.$$

As can be seen from (11), minimizing (10) is equivalent to minimizing the expression

$$\frac{1}{2}\boldsymbol{f}^T Q \boldsymbol{f} - \boldsymbol{b}^T \boldsymbol{f}, \tag{12}$$

and thus we can use the conjugate gradient method for the optimization.

2.4.3 Morozov's discrepancy principle

The problem of finding the optimal regularization parameter is, in general, considered to be unsolved. There are, however, methods that attempt to find an optimal choice of the regularization parameter, including the Morozov discrepancy principle, which is based on the noise level in the data.

Assume that we know the size of the noise in our model defined by (5) to be $\delta > 0$. Now $T_\alpha(\boldsymbol{m})$ is an acceptable reconstruction if

$$\|AT_\alpha(\boldsymbol{m}) - \boldsymbol{m}\| \leq \delta \quad \left(A \in R^{k\times n}\right). \tag{13}$$

The idea of the Morozov's discrepancy principle is to choose $\alpha > 0$ such that

$$\|AT_\alpha(\boldsymbol{m}) - \boldsymbol{m}\| = \delta \tag{14}$$

It can be proven (see [5]) that the α that satisfies the above expression is attained by solving

$$\sum_{j=1}^{\min\{k,n\}} \left(\frac{\alpha}{d_j^2 + \alpha}\right)^2 \left(m'_j\right)^2 + \sum_{j=\min\{k,n\}+1}^{k} \left(m'_j\right)^2 - \delta^2 = 0, \tag{15}$$

where d_j are the singular values of A and $\boldsymbol{m'} = U^T\boldsymbol{m}$ where U is an orthogonal matrix acquired from the singular value decomposition $A = UDV^T$.

As previously mentioned, we will use Morozov's discrepancy principle to choose the optimal regularization parameter.

2.5 The basis and materials

In this work we will mainly use synthetic data as our basis (for real data, see section 2.5.1). We first create a synthetic glottal excitation signal using the airflow function defined in (1). To this signal we apply a previously calculated vocal tract filter, as described in section 2.2 to create a simulated vowel sound. Finally we add some normally distributed noise to the data to simulate measurement noise. Different data is created by varying the sound frequency, the value of the Klatt-parameter Q and the noise-level.

The inversion of the vowel sound is done using another filter similar to the one used in creating the synthetic data. For example we might have created the data with a filter for a male vowel /a/, and use a filter for a female vowel /a/ for the inversion. This way we avoid inverse crime. The idea is that we can assume two different filters for the same vowel to be approximately the same. That is, given two different filters A_1 and A_2 for the same vowel and a glottal excitation signal $\boldsymbol{g}$, we assume that $A_1\boldsymbol{g} \approx A_2\boldsymbol{g}$. This is based on the fact that a vowel is defined by its two or three first *formant frequencies*, which can be assumed to be about the same for two different vowel sounds.[7]

We then attempt to solve the inverse problem by using the generalized Tikhonov regularization with a customized penalty matrix. We will assume the Klatt-parameter Q of the glottal excitation signal to be known (when dealing with synthetic data), and as explained in section 2.1 we know the pressure function to be zero in the interval $]QT, T]$. This will be incorporated in the model by assigning large values to the diagonal entries $l_i \in L$ for the values of i that correspond to the previously mentioned interval, namely $i \in \{j \in \mathbb{N} : Qn < j \leq n\}$ where n is the length of our data. We also know the pressure function to be smooth in the interval $[0, Q]$. This can be incorporated by adding differential operator properties to the penalty matrix.

We can thus assign the differential operator properties

$$\begin{cases} l_{i,i} & = 1 \\ l_{i,i+1} & = -1 \end{cases} \text{when } i \in \{j \in \mathbb{N} : 1 \leq j \leq Qn\},$$

and further, we assign the larger values described above as

$$l_{i,i} = C \geq 1, \quad \text{when } i \in \{j \in \mathbb{N} : Qn < j \leq n\}$$

resulting in the penalty matrix

$$L = \begin{pmatrix} 1 & -1 & 0 & 0 & & & \cdots & & 0 \\ 0 & 1 & -1 & 0 & & & \cdots & & 0 \\ \vdots & & \ddots & \ddots & & & & & \vdots \\ 0 & \cdots & 0 & 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & C & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & C & 0 & \cdots & 0 \\ \vdots & & & & & & \ddots & & \vdots \\ 0 & & & \cdots & & & 0 & C & 0 \\ 0 & & & \cdots & & & 0 & 0 & C \end{pmatrix} \qquad (16)$$

for a single period of the excitation signal. This procedure must of course be repeated as many times as we have periods in our measurement data.

2.5.1 Real data

Apart from the synthetic data the method of inversion was also tested with real recorded data. As real data we used recordings of the vowel /a/ in different pitches by different male subjects. The recordings where cropped so that the signal would consist of ten periods and fundamental frequency of the signal was determined with pitch detection software. The frequency can also be seen in the frequency spectrum of the signal, if it doesn't overlap with a formant frequency. An example of this can be seen in figure 2.

The airflow of the reconstruction is also presented in the case of real data. The airflow can be acquired as the integral of the glottal pressure.

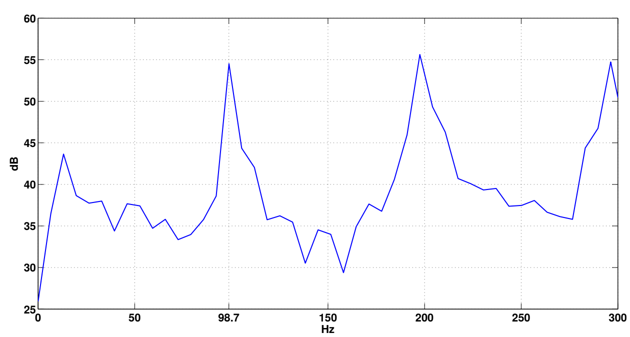


Figure 2: The frequency spectrum of a vowel /a/ attained by a fast Fourier transform. It can be seen that the fundamental frequency is $f \approx 98.7$ Hz.

2.5.2 Noise estimation

As previously mentioned, we will use Morozov's discrepancy principle for optimization of the regularization parameter α. We assume the noise to originate from a scaled standard multivariate normal distribution, i.e. if

$$\boldsymbol{\varepsilon'} = (Z_1, \dots, Z_n), \quad Z_i \sim N(0,1) \perp\!\!\!\perp \text{ for all } i \in \{1, \dots, n\} \tag{17}$$

then our assumed noise vector is a sample from $\boldsymbol{\varepsilon}$, defined as

$$\boldsymbol{\varepsilon} := c\boldsymbol{\varepsilon'} = (cZ_1, \dots, cZ_n), \quad c > 0. \tag{18}$$

Note that

$$cZ_i \sim N\left(0, c^2\right) \text{ for all } i \in \{1, \dots, n\}. \tag{19}$$

The scaling is done to make the error relative to the data, which again is preferable in order to generate consistent data. We want to estimate the magnitude of the noise. Let $X \sim \chi_n^2$ and $Y \sim \chi_n$ where χ_n^2 denotes the chi-squared distribution and χ_n the chi distribution with n degrees of freedom respectively. We now see ([1], [2]) that

$$\begin{aligned}
E\left(\|\boldsymbol{\varepsilon}\|\right) &= E\left(\|c\boldsymbol{\varepsilon'}\|\right) = E\left(c\,\|\boldsymbol{\varepsilon'}\|\right) = cE\left(\|\boldsymbol{\varepsilon'}\|\right) \\
&= cE\left(\sqrt{Z_1^2 + \ldots + Z_n^2}\right) = cE\left(\sqrt{X}\right) \\
&= cE\left(Y\right) = c\sqrt{2}\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)},
\end{aligned}$$

so the natural choice of δ is

$$\delta = c\sqrt{2}\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}. \tag{20}$$

This is numerically impossible to compute in a straightforward manner due to fast growth of the gamma function, so in practice we will use

$$\delta = c\sqrt{2}\exp\left[\log\Gamma\left(\frac{n+1}{2}\right) - \log\Gamma\left(\frac{n}{2}\right)\right] \tag{21}$$

where the $\log\Gamma$ function is, in our case, more numerically stable. Note that expressions (20) and (21) are equivalent.

In our case we know our approximated vocal tract filter to be at least to some extent erroneous. Therefore the measurement noise must be estimated somewhat differently to take the errors attained from the differences between the filters into account.

Let $\boldsymbol{m} \in \mathbb{R}^n$ be the given measurement data. As we know the sound frequency and the value of the Klatt-parameter we can generate a glottal excitation signal, and filter it with the approximated filter used in the inversion to create a similar vowel sound as our measurements. Let us denote it with $\boldsymbol{v} \in \mathbb{R}^n$. Now we can calculate an upper bound for the measurement noise as

$$\delta_{\max} = \|\boldsymbol{v} - \boldsymbol{m}\| . \tag{22}$$

However, we know our filter to be only an approximation of the filter used in generating the measurement data, not the exact same filter. Therefore it is reasonable to assume that the approximations done in the selection of the filter give rise to some differences in the vowel sound that cannot be regarded as measurement errors, but rather systematic errors attained by the approximations. Therefore we will use the estimation $\delta = \delta_{\max}/2$ for the noise level of the measurement data. This estimation of the measurement noise is not at all a trivial task, and should require more careful examination. This is however beyond the scope of this study.

When we have acquired an estimation for the measurement noise the solution of equation (15) is approximated with Newton's method [6].

2.6 Approximation of the Klatt-parameter

The inversion of the glottal sound is always done with a fixed guess of the Klatt-parameter Q when using synthetic data. However, the real value of Q can be approximated as long as the inversion is done with an approximation of Q that is no less than what the data is created with. This means that we can approximate the Klatt-parameter as long as we always *over-estimate* it a little bit.

The reason why we need to over-estimate the Klatt-parameter is the shape of the glottal excitation signal (see figure 1). The negative peak in the glottal pressure is located at precisely the point QT in time, where T is the length of the period. The peak is quite sharp, and thus if the parameter is at all under-estimated the peak will almost completely vanish due to the inversion method suppressing the signal toward zero for all points after the time QT (as explained in section 2.5).

When the inversion is done with a too large value of the parameter the result in the range $[0, QT]$ will give a good approximation for the signal, but the range $[QT, Q_{guess}T]$ will be very oscillatory although it should consist of zero values. The negative peak will, however, be clearly visible. We can thus select the value for the next guess for Q to be the point in the period where the least value is reached, plus a small constant to ensure that we still over-estimate the value. This procedure is then repeated.

As we always want to over-estimate the parameter we can start by selecting the initial guess $Q_1 = 1$. We then iterate for $i = 1, 2, \ldots, N$ the following. Solve the inverse problem as described in previous sections using Q_i as the Klatt-parameter. Call the result $\boldsymbol{g}^{(i)} \in \mathbb{R}^n$. Next, find the index for the least value in $\boldsymbol{g}^{(i)}$, that is

$$I_i = \underset{j \in \{1,\ldots,n\}}{\operatorname{argmin}} \boldsymbol{g}_j^{(i)}.$$

Then we get the new approximation for the Klatt-parameter simple by calculating $Q_{i+1} = I_i/n + c$, where $c > 0$ is a small constant.

3 Results

In this section we will present some of the results of our study. For obvious reasons we will not include sound data from our results in this paper. Note that in all the following graphs of the target glottal pressure and the reconstructed glottal pressure are represented by the colors green and blue, respectively. We will also denote the Klatt-parameter with Q.

The error in both the time domain and the frequency domain will be presented for the reconstructions. The relative error of the reconstruction in the time domain is calculated with the formula

$$\delta_{time} = \frac{\|\boldsymbol{g} - T_\alpha(\boldsymbol{m})\|_2}{\|\boldsymbol{g}\|_2} \cdot 100\% \tag{23}$$

and the relative error in the frequency domain with the formula

$$\delta_{freq} = \frac{\||\mathrm{FFT}\,(\boldsymbol{g})| - |\mathrm{FFT}\,(T_\alpha(\boldsymbol{m}))|\|_2}{\||\mathrm{FFT}\,(\boldsymbol{g})|\|_2} \cdot 100\%, \tag{24}$$

where $\boldsymbol{g} \in \mathbb{R}^k$ is the original glottal excitation signal, $T_\alpha(\boldsymbol{m})$ is the reconstruction calculated from the noisy data m with the regularization parameter α and FFT is the fast Fourier transform.

3.1 Results of the glottal inversion of synthetic data

We will now present our results of the inversion of synthetic data. The inversion was done, in addition to the Tikhonov regularization strategy, also with the naïve inversion method in order to illustrate the ill-posedness of the inverse problem. A comparison of the methods is presented in figures 3 and 4.

In order to evaluate the accuracy and correctness of the α-value chosen according to Morozov's discrepancy principle the relative errors were calculated with different values of the parameters α. This was then compared with corresponding reconstruction with an α-value chosen according to Morozov's discrepancy principle. The results can be seen in figure 5. The comparison was done solely with the relative error in the time domain, as the errors in the time domain and the frequency domain are strongly correlated.

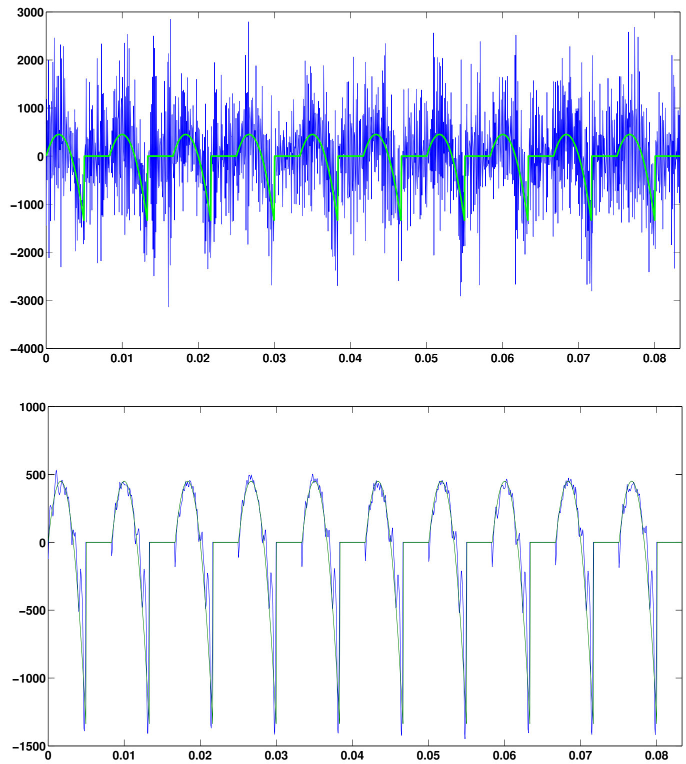


Figure 3: Comparison between the naïve inversion method and the Tikhonov regularization strategy with inverse crime.
- The data was created with the parameters $f = 90$ Hz and $Q = 0.7$.
- The regularization parameter used in the inversion: $\alpha \approx 63.7$.
- The relative errors of the reconstruction: $\delta_{time} \approx 43.9\%$ and $\delta_{freq} \approx 34.9\%$.

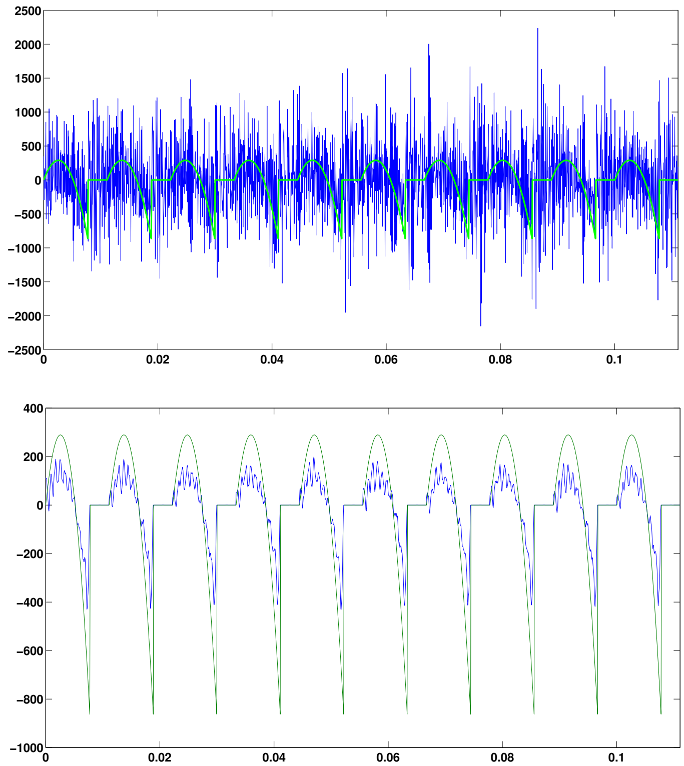


Figure 4: Comparison between the naïve inversion method and the Tikhonov regularization strategy without inverse crime.
- The data was created with the parameters $f = 90$ Hz and $Q = 0.7$.
- The regularization parameter used in the inversion: $\alpha \approx 63.2$.
- The relative errors of the reconstruction: $\delta_{time} \approx 63.1\%$ and $\delta_{freq} \approx 59.0\%$.

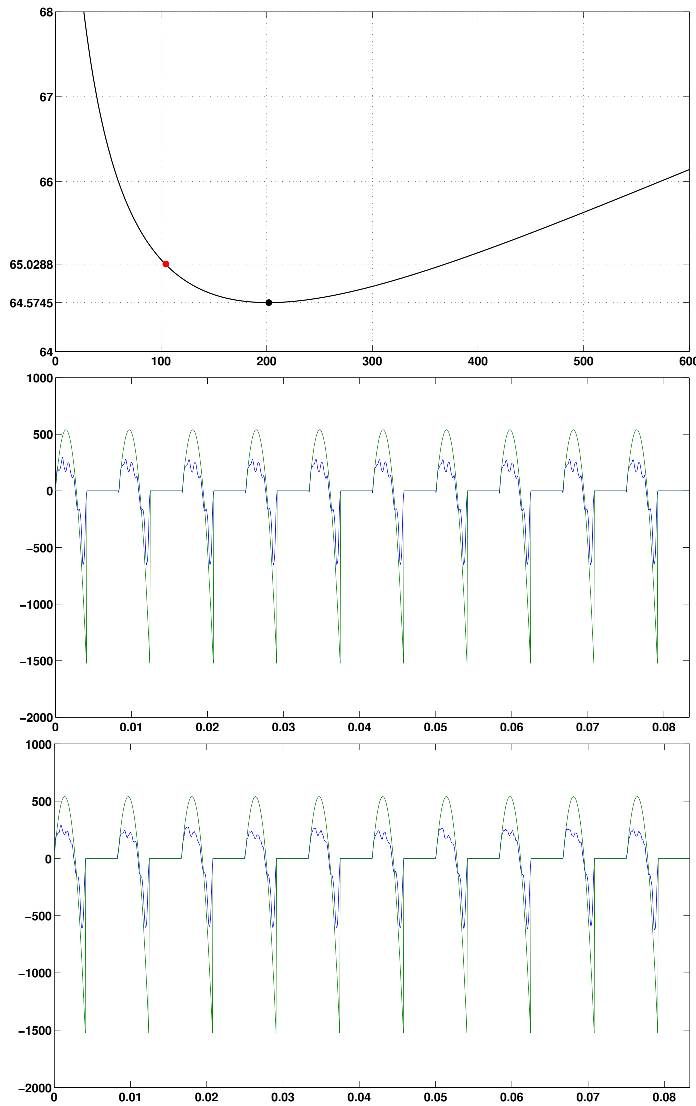


Figure 5: The uppermost picture plots the relative error values vs. α-values of the iterative calculation of the inversion. The red dot denotes the α-value acquired from Morozov's discrepancy principle and the black dot denotes the α-value corresponding to the least relative error in the reconstruction (from the iterative calculation). The middle picture graphs the reconstruction with the α-value acquired from Morozov's discrepancy principle and the bottom picture graphs the reconstruction α-value corresponding to the least relative error in the reconstruction.

Since the number of (a priori known) variables is quite high a large amount of data needed to be harvested in order to properly analyze the inversion method. The following figures demonstrate some of the data that was collected.

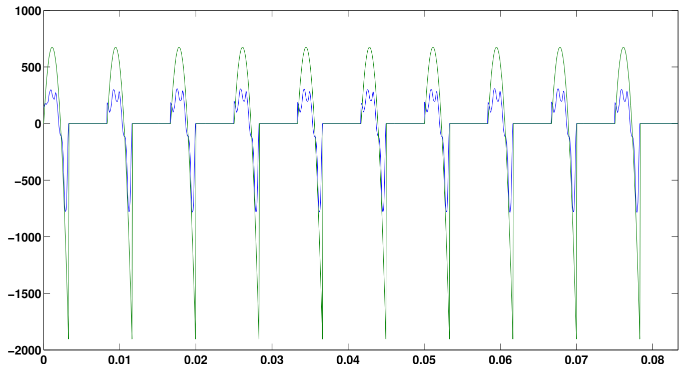


Figure 6:
- The data was created with the parameters $f = 120$ Hz and $Q = 0.4$.
- The regularization parameter used in the inversion: $\alpha \approx 123.9$.
- The relative errors of the reconstruction: $\delta_{time} \approx 69.1\%$ and $\delta_{freq} \approx 63.2\%$.

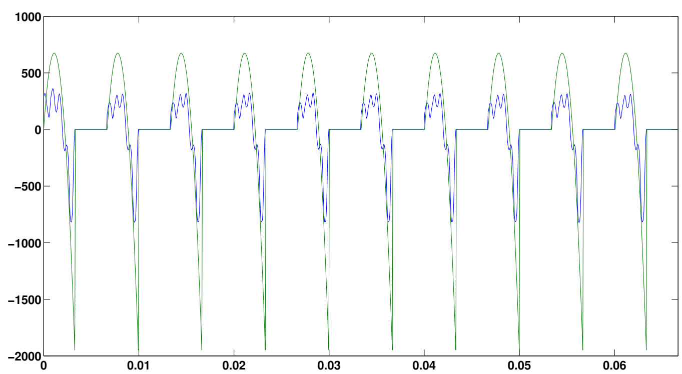


Figure 7:
- The data was created with the parameters $f = 150$ Hz and $Q = 0.5$.
- The regularization parameter used in the inversion: $\alpha \approx 86.0$.
- The relative errors of the reconstruction: $\delta_{time} \approx 70.1\%$ and $\delta_{freq} \approx 59.4\%$.

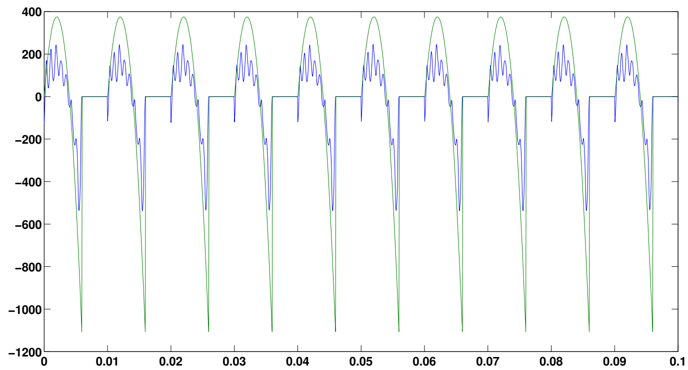


Figure 8:
- The data was created with the parameters $f = 100$ Hz and $Q = 0.6$.
- The regularization parameter used in the inversion: $\alpha \approx 66.6$.
- The relative errors of the reconstruction: $\delta_{time} \approx 64.1\%$ and $\delta_{freq} \approx 58.0\%$.

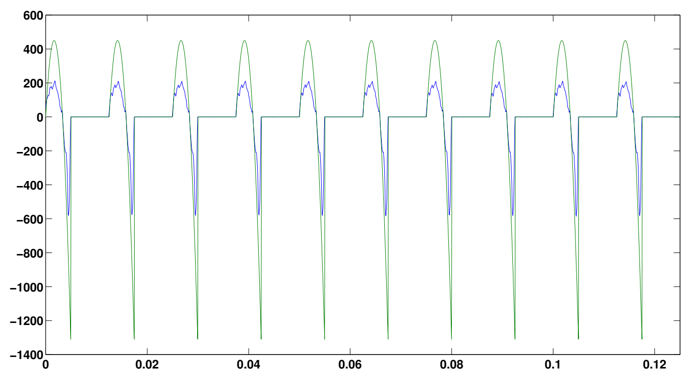


Figure 9:
- The data was created with the parameters $f = 80$ Hz and $Q = 0.3$.
- The regularization parameter used in the inversion: $\alpha \approx 84.5$.
- The relative errors of the reconstruction: $\delta_{time} \approx 64.6\%$ and $\delta_{freq} \approx 60.2\%$.

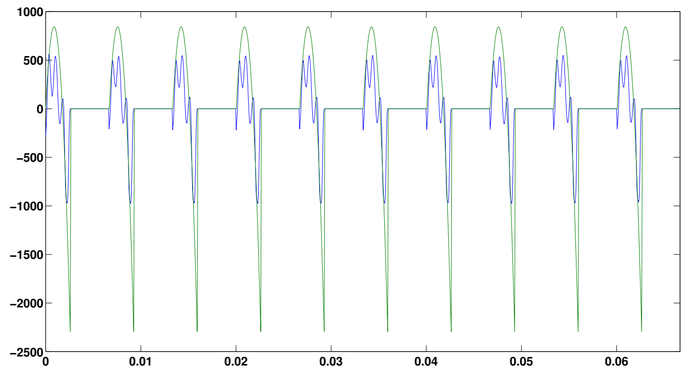


Figure 10:
- The data was created with the parameters $f = 150$ Hz and $Q = 0.4$.
- The regularization parameter used in the inversion: $\alpha \approx 48.5$.
- The relative errors of the reconstruction: $\delta_{time} \approx 74.0\%$ and $\delta_{freq} \approx 57.6\%$.

3.2 Results of the approximation method for approximating the Klatt-parameter

In this section we will present results obtained during the approximation of the Klatt-parameter (algorithm previously described in section 2.6). We will present the results as

- plots of four periods of both the target and reconstructed glottal excitation signal
- plots of only one period of the reconstructed glottal excitation signal.

The Q_{guess} parameter acquired from the algorithm is visualized as a black dot in the plots mentioned above (anchored to the x-axis). The approximation routine was applied on both synthetic and actual data.

In figures 11 to 16 the signal frequency is 120 Hz. The approximation algorithm acquired $Q_{guess} = 0.165$ at the end of the iterations, while the actual parameter $Q = 0.15$, relative error of the reconstruction $\approx 64.6\%$, $\alpha \approx 84.5$.

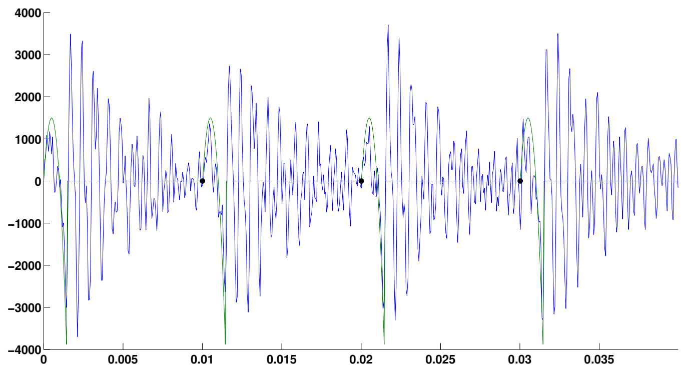


Figure 11: Iteration no. 1

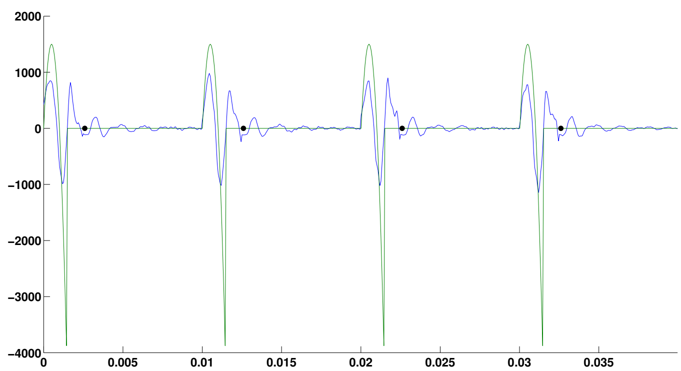


Figure 12: Iteration no. 2

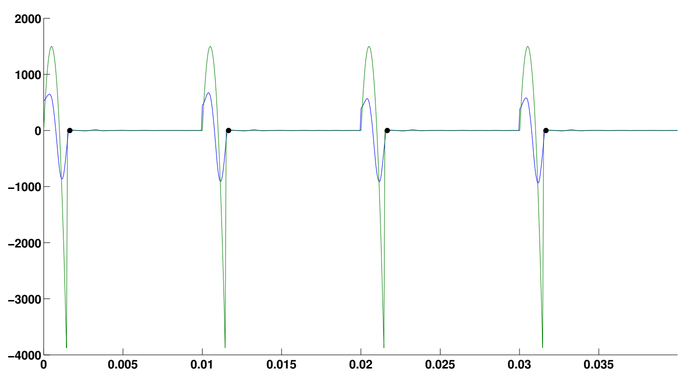


Figure 13: Iteration no. 5.

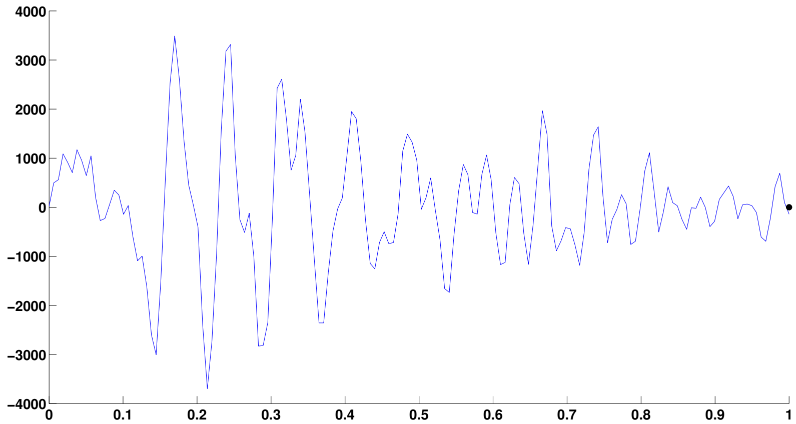


Figure 14: Iteration no. 1

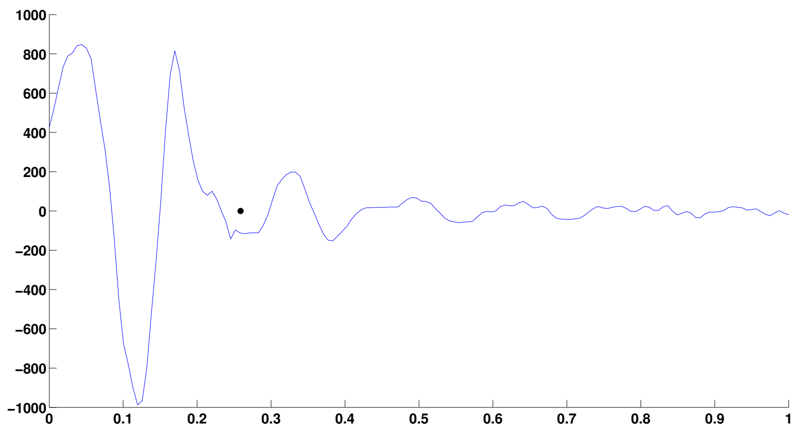


Figure 15: Iteration no. 2

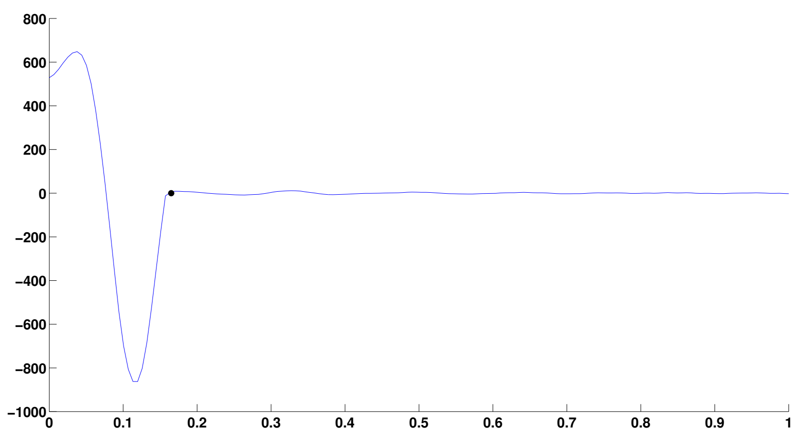


Figure 16: Iteration no. 5

In figures 17 to 22 the signal frequency is 123 Hz. The approximation algorithm acquired $Q_{guess} = 0.341$ at the end of the iterations, while the actual parameter $Q = 0.15$, relative error of the reconstruction $\approx 98.4\%$, $\alpha \approx 8.3$.

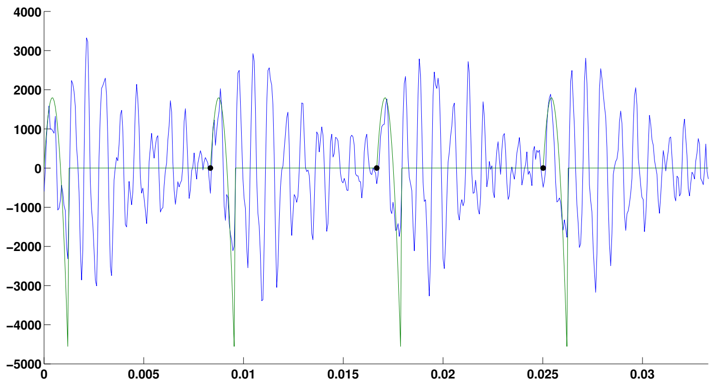


Figure 17: Iteration no.1

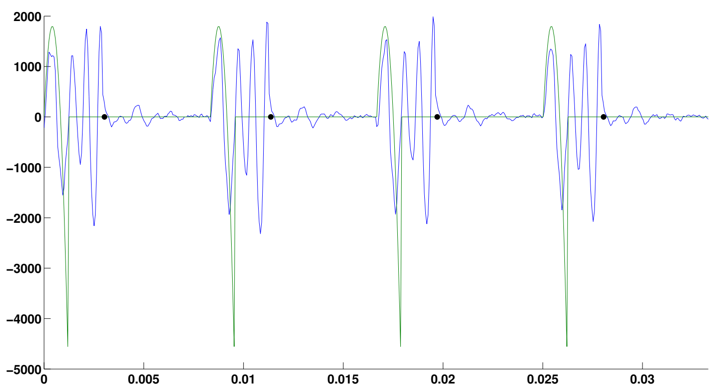


Figure 18: Iteration no.2

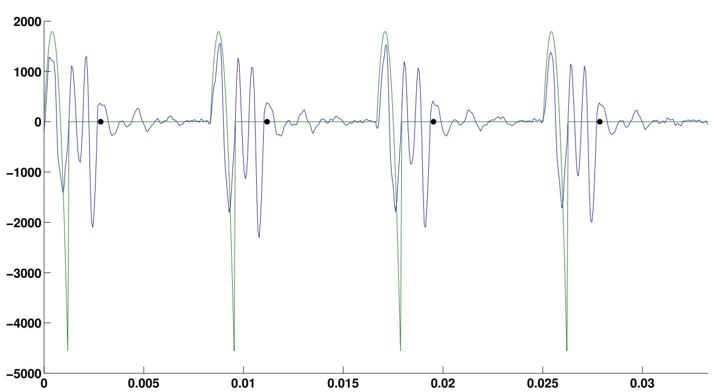


Figure 19: Iteration no.5.

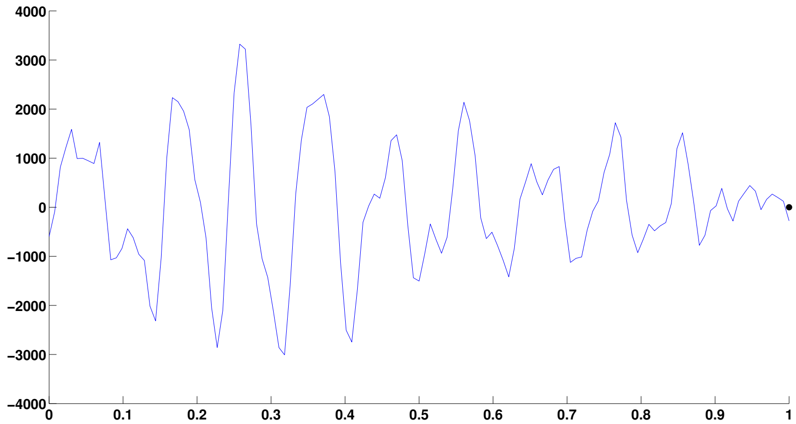


Figure 20: Iteration no.1

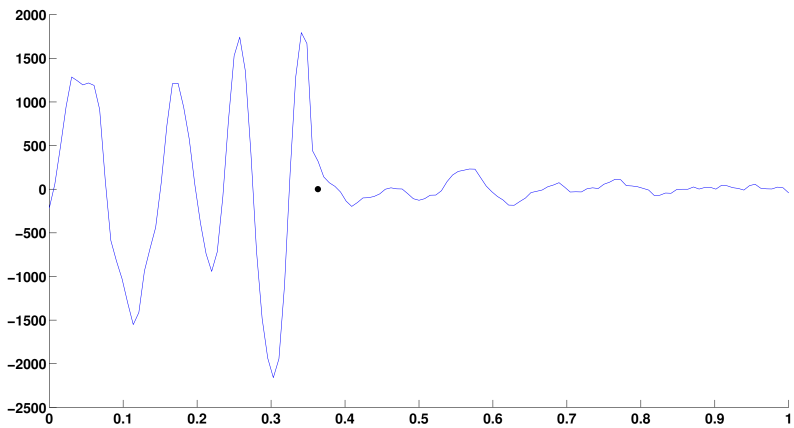


Figure 21: Iteration no.2

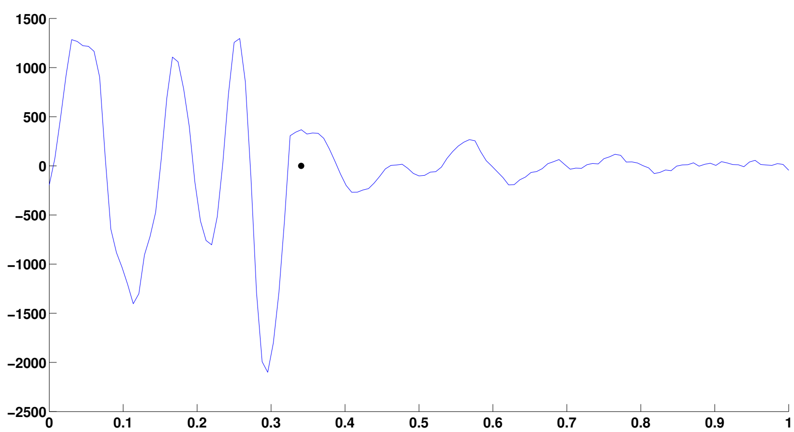


Figure 22: Iteration no.5

The Klatt-parameter approximation algorithm was run 100 times to see how well the algorithm works in different cases. In each case the relative error of the recovered Klatt-parameter was calculated and recorded. The fundamental frequency and the Klatt-parameter for the simulation of the data were chosen randomly from the ranges 100 Hz to 300 Hz and 0.3 to 0.8, respectively, for each case. In one of the cases the algorithm could not result in a good approximation of the Klatt-parameter (relative error about 70 %), but in all of the remaining 99 cases the relative error of the recovered Klatt-parameter was well below 10 %, with a mean of 2.9 % and a median of 2.7 %.

3.3 Results of the glottal inversion of real data

In the inversion of real data we used the Klatt-parameter approximation method described in section 2.6 in order to determine the length of the open phase of the vocal folds for the data. The results of the inversion of real data is presented in the graphs on the following pages. In the case of real data also the glottal flow resulting from the inversion is presented. Both the approximated target airflow and the reconstruction of the airflow are normalized, as the scaling of the flow only affects the amplitude of the resulting sound.

Note that the curves of target glottal pressure and flow in the figures of this section are only assumed target curves made according to values acquired from the Klatt-parameter approximation and the frequency spectrum analysis (see section 2.5.1). The target curves are not actual targets for the inversion but rather illustrations for the accuracy of the inversion.

The following inversion was made with an with approximation of 336 Hz for the fundamental frequency (see figure 23) of the signal.

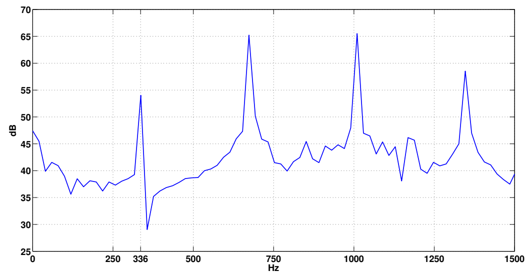
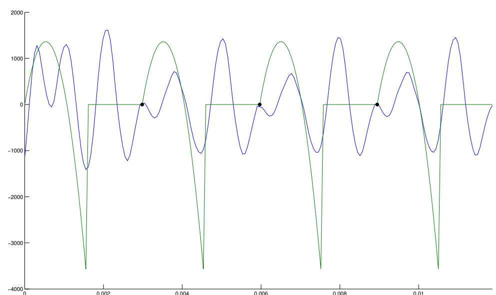
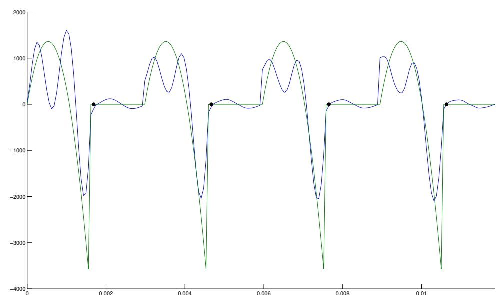


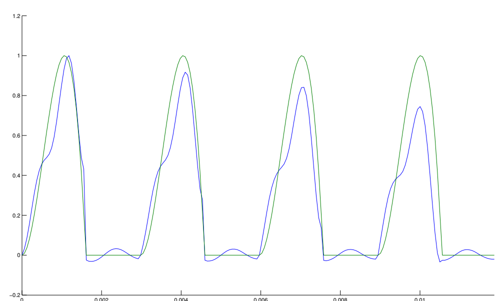
Figure 23: Cropped caption of the frequency spectrum of the signal.



(a) Iteration no. 1



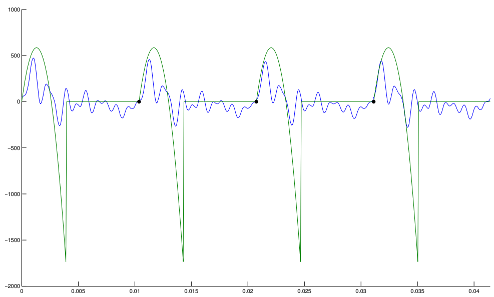
(b) Iteration no. 5



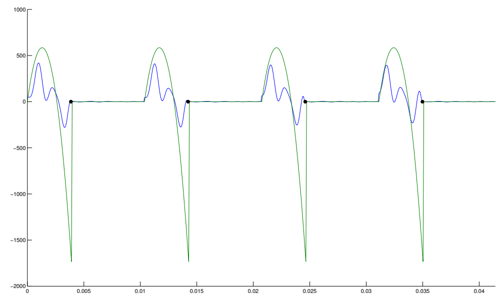
(c) Glottal airflow after iteration no. 5

Figure 24: The glottal pressure and airflow. The acquired parameters are $Q \approx 0.57$ and $\alpha \approx 82.2$. The approximated relative errors are $\delta_{time} \approx 74.3\%$ and $\delta_{flow} \approx 28.2\%$.

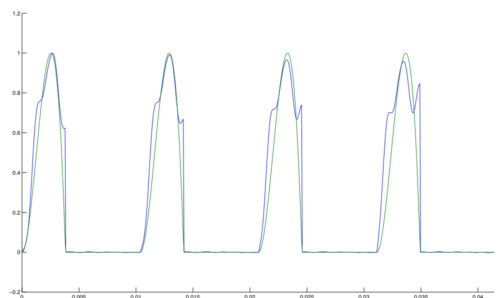
The following inversion was made with an with approximation of 98.7 Hz for the fundamental frequency.



(a) Iteration no. 1



(b) Iteration no. 5



(c) Glottal airflow after iteration no. 5

Figure 25: The glottal pressure and airflow. The acquired parameters are $Q \approx 0.37$ and $\alpha \approx 312$. The approximated relative errors are $\delta_{time} \approx 93.8\%$ and $\delta_{flow} \approx 28.6\%$.

4 Discussion

4.1 Ill-posedness

As it can be seen from figures 3 and 4 the inverse problem to determine the glottal impulse is clearly *ill-posed*. It can be seen that naïve inversion fails completely to achieve the shape of the glottal excitation signal, and thus a regularization method is required.

In this work the Tikhonov regularization strategy was chosen for solving the inverse problem. It can further be seen from figures 3 and 4 that when the same problem that failed with naïve inversion is solved with Tikhonov regularization the results are significantly better.

It must be noted that the situation displayed in figure 3 is done with inverse crime, which means that the same filter was used both in creating the data and in solving the inverse problem. This is the reason why the error of the reconstruction is so low; such low values could not be achieved with real data. The situation in figure 4 is a much better example of how reconstructions with real data could look like.

4.2 Tikhonov regularization

Apart from the naïve inversions displayed in figures 3 and 4 discussed above, all the reconstructions in figures 3 to 10 are done with the Tikhonov regularization strategy. For all reconstructions the relative error in both the time domain and the frequency domain was well below 75%, which can be regarded as a good result.

As seen from the figures 3 to 10, the shape of the reconstruction is in all the cases clearly the same as in the original data, although the reconstruction is somewhat flattened. This is due to the high values of the regularization parameter used to compensate for the noise. The problem here is that although the *sound* generated by the reconstruction is close to the original data the relative error of the reconstruction in the time domain can become quite high; as the reconstruction is flattened it only affects the *amplitude* of the generated sound as long as the *shape* is unaltered. Because of this it is important to also pay attention to the error in the frequency domain, as it describes more accurately the quality of the sound of the reconstruction. As it can be seen, the relative error in the frequency domain is in fact smaller than in the time domain for all of the reconstructions.

If the Klatt-parameter is approximated too low (a too large part of a period of the reconstruction is suppressed by the penalty matrix, see section 2.5) the negative peak of the excitation signal would also be suppressed, thus ruining the reconstruction. If the value of the Klatt-parameter is approximated too high (smaller part of a period suppressed) a part of the reconstruction that should have the value 0 will become oscillatory. As described in section 2.6 this property is used in the automatic approximation of the Klatt-parameter.

4.3 Morozov's discrepancy principle

The regularization parameters acquired with Morozov's discrepancy principle yielded good results. As can be seen from figure 5, the reconstruction with a regularization parameter acquired with Morozov's discrepancy principle gave almost as good a reconstruction as with the best possible value of the parameter (in the sense of the values in the iteration) when using the relative error of the reconstructions as a reference point. The difference in the audio data was indistinguishable to the human ear.

Considering that the selection of the regularization parameter with Morozov's discrepancy principle works completely without the need of human interaction, the method selection can be regarded as a success for this particular problem. With an optimization of the noise estimation process (see 2.5.2) potentially more accurate results can be acquired.

4.4 Verification of the approximation method for approximating the Klatt-parameter

In this section we will evaluate the efficiency of the approximation method for approximating the Klatt-parameter explained in section 2.6. The convergence of the method is, in general, quite fast; the characteristics of the glottal excitation signal almost surely guarantee that the negative peak will dominate the oscillatory behaviour of a reconstruction computed with a too large Klatt-parameter guess. One example of the reconstructions after different numbers of iterations of the algorithm can be seen in figures 11 to 16.

There are, however, some problems with the algorithm. One possible problem situation is when a negative peak, which is not the negative peak for the glottal excitation signal but rather a peak resulting from the random oscillation caused by the inversion, dominates the recovered signal. In this case the algorithm will have no chance to recover and a false result for the Klatt-parameter is reached. An example of this can be seen in figures 17 to 22.

The previously mentioned problem could be averted by improving the Klatt-parameter approximation algorithm. One idea could be to try to identify the "first negative peak" in the time domain reconstruction, which, due to the characteristics of the glottal excitation signal, always seems to be located near the (in practice just before) the point described by the Klatt-parameter. This is, however, not an easy problem, as can be understood from the description of the idea; there are many vaguely defined steps in the algorithm, such as defining which local minima should be included in "negative peaks".

Even though the approximation algorithm still could be improved it works very well in practice even as it is defined now; in most cases the algorithm reaches a Klatt-parameter with a relative error below 10 % with a mean and median below 3 % (see section 3.2).

4.5 Inversion on real data

The approximation algorithm of the Klatt-parameter plays a significant role in the inversion of real data; in the absence of the algorithm one would still be forced to make some assumptions on the length of the vocal folds' open phase or, in the worst case, approximate it manually (i.e. trial and error). This is, especially in the case of real data where there is no real target to compare to, a very tedious and time consuming job.

It is important to note that the target functions of the reconstructions (see figures 24 and 25) are only approximative as both the pitch and the length of the vocal folds' open phase of the original signal are unknown. But with mean and median of the error of the approximation algorithm below 3% (see section 3.2), we can conclude that the algorithm approximates the Klatt-parameter very accurately. As the pitch detection method can in our case be considered sufficient, we can use the approximative target glottal excitation signal for qualitative analysis of the actual reconstruction as long as we are aware of the approximative dimension in the system.

In the first case (signal fundamental frequency about 336 Hz) we get an relative error of 74.3% compared to the target signal. This can be considered a very good result as the error is within the error range of the inversions on synthetic data. The characteristics of a glottal excitation signal are also recognizable in the reconstruction; the negative peak is clearly distinctive from the positive part of the signal.

In the second case we get an relative error of 93.8% compared to the target signal, which also quite good. The shape of the reconstruction is however suppressed as the noise-estimation generates a quite high reguralization parameter.

In both the cases the normalized airflow approximates quite nicely the assumed target airflow. As the relative errors of the airflows are under 30% it can be concluded that in both cases the inversion was a success.

References

[1] Abramowitz, M & Stegun, I.A. (1965) *Handbook of mathematical functions with formulas, graphs and mathenatical tables* New York, NY: Dover, p. 940

[2] Evans, M., Hastings, N. & Peacock, B. (2000) *Statistical distributions.* New York: Wiley, p. 57

[3] Fant, G., Liljencrants, J., Lin, Q., (1985). *A four-parameter model of glottal flow.* STL-QPSR 26 (4), p. 1-13

[4] Fujisaki, H., Ljungqvist, M., 1986. Proposal and evaluation of models for the glottal source waveform. In: Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). Vol. 11. p. 1605–1608.

[5] Mueller, Jennifer L. & Siltanen Samuli, (2012). *Linear and Nonlinear Inverse Problems with Practical Applications.* SIAM, 1st edition.

[6] Press, W.H., Teukolsky, S.A., Vetterling, W.T. & Flannery, B.P. (2007) *Numerical Recipes: The Art of Scientific Computing.* New York: Cambridge University Press, 3rd edition, p. 362

[7] Rabiner, L. R., Schafer, R. W., (1987). *Digital processing of speech signals.* Englewood Cliffs: Prentice-Hall, p. 38-107.

[8] Rosenberg, A., (1971). *Effect of glottal pulse shape on the quality of natural vowels.* Journal of the Acoustical Society of America 49 (2B), p. 583–590.

[9] Touda, K., (2007) *Study on numerical method for voice generation problem.* PhD thesis. The University of Electro-Communications.