Inversion of the Laplace transform

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1 Introduction

Definition and some applications of the Laplace transform. Explanation of what will be done in this study. Mention the ill-posedness of the inversion of the Laplace transform. Include references to literature whenever necessary.

Let $f : [0, \infty) \to \mathbb{R}$. The Laplace transform F of f is defined by

$$F(s) = \int_0^\infty e^{-st} f(t) dt, \quad s \in \mathbb{C}, \tag{1}$$

provided that the integral converges. The direct problem is to determine F for a given function f according to (**??**). The inverse problem is: *given a Laplace transform F, find the corresponding function f.*

2 Materials and Methods

2.1 The matrix model

Assume we know the values of F at these real-valued points:

$$0 < s_1 < s_2 < \ldots < s_n < \infty.$$

Then we may approximate the integral in (**??**) for example with the trapezoidal rule as

$$\begin{aligned} \int_0^\infty e^{-st} f(t) dt \approx \frac{t_k}{k} \Bigg(& \frac{1}{2} e^{-st_1} f(t_1) + e^{-st_2} f(t_2) + e^{-st_3} f(t_3) + \ldots \\ & + e^{-st_{k-1}} f(t_{k-1}) + \frac{1}{2} e^{-st_k} f(t_k) \Bigg), \end{aligned} \tag{2}$$

where vector $t = [t_1 \ t_2 \ \ldots \ t_k]^T \in \mathbb{R}^k$, $0 \leq t_1 < t_2 < \ldots < t_k$, contains the points at which the unknown function f will be evaluated. By denoting $f_\ell = f(t_\ell),\ \ell = 1, \ldots, k$, and $m_j = F(s_j),\ j = 1, \ldots, n$, and using (**??**), we get a linear model of the form $m = Af + \epsilon$ with

$$A = \frac{t_k}{k} \begin{bmatrix} \frac{1}{2}e^{-s_1 t_1} & e^{-s_1 t_2} & e^{-s_1 t_3} & \ldots & e^{-s_1 t_{k-1}} & \frac{1}{2}e^{-s_1 t_k} \\ \frac{1}{2}e^{-s_2 t_1} & e^{-s_2 t_2} & e^{-s_2 t_3} & \ldots & e^{-s_2 t_{k-1}} & \frac{1}{2}e^{-s_2 t_k} \\ \vdots & & & & & \vdots \\ \frac{1}{2}e^{-s_n t_1} & e^{-s_n t_2} & e^{-s_n t_3} & \ldots & e^{-s_n t_{k-1}} & \frac{1}{2}e^{-s_n t_k} \end{bmatrix}. \tag{3}$$

2.2 The inversion method

3 Results

The Results section is for a detailed explanation of what happens when the methods of Section **??** *are applied to the materials. There should be no interpretation of what the results might mean, just a dry and factual report with numbers, charts and plots.*

(a) Compute numerically and plot the Laplace transform of

$$f(t) = \begin{cases} 1, & \text{for } 0 \leq t \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Construct matrix A given by (**??**) for a suitable choice of points t_ℓ and s_j. Compute the singular values of A. Do you detect ill-posedness?

(c) Use truncated SVD to compute the inverse Laplace transform of f.

4 Discussion

This section is for interpretations of the results presented in Section **??**. *Sometimes this section is called Conclusions, or Discussion and Conclusions.*